Math 231 Lecture 3 9/5/18

- All homeworks from last week due this Friday 8:30AM.
- New homework HW3 and Pre-lecture 3 due Monday 8:30AM.
- Tutoring room open 5-8pm M,Tu,W,Th in 347 AH.
- My office hours: M,W 1-2 (024 Illini Hall).

Last time: Substitution and Integration by Parts – no recipe or guarantee
Today: integrals for which recipes can be developed and guarantees made.

7.2 Trig Integrals

Recall trig identities (see Appendix D):

$$\boxed{\sin^2 x + \cos^2 x = 1}$$

$$\boxed{\sin(x+y) = \sin x \cos y + \cos x \sin y}$$
$$\boxed{\cos(x+y) = \cos x \cos y - \sin x \sin y}$$

$$\boxed{\int \sin^m x \cos^n x \, \mathrm{d}x}$$

Case 1: at least one power odd

$$\boxed{\sin^2 x + \cos^2 x = 1}$$

Ex 1: $\int \sin^3 x \cos^6 \mathrm{d}x$

Ex 2: $\int_0^{\pi/2} \dfrac{\cos^5 x}{\sqrt{\sin x}} \, \mathrm{d}x =$

(A) 24/15
(B) 32/15
(C) 64/45
(D) 82/9

Case 2: both powers even

$$\boxed{\sin^2 x = \tfrac{1}{2}(1 - \cos(2x))}$$
$$\boxed{\cos^2 x = \tfrac{1}{2}(1 + \cos(2x))}$$

Ex 3: $\int \sin^4 x \cos^2 \mathrm{d}x$

$$\boxed{\int \sec^m x \tan^n x \, dx}$$

Recall: $(\sec x)' = \sec x \tan x, \quad (\tan x)' = \sec^2 x$

Case 1: even power of $\sec x$

$$\boxed{\sec^2 x = 1 + \tan^2 x}$$

Ex 4: $\int \sec^4 x \tan^n x \, dx$

(A) $\int u^2(u^n - 1) \, du$
(B) $\int u^4(u^2 - 1)^n \, du$
(C) $\int (1 + u^2)u^n \, du$
(D) $\int (1 + u^2)^2 u^n \, du$

Case 2: odd power of $\tan x$

$$\boxed{\tan^2 x = \sec^2 x - 1}$$

Ex 5: $\int \sec x \tan^3 x \, dx$

Case 3: odd power of $\sec x$ **and even power of** $\tan x$

$$\boxed{\int \tan x \, dx = \ln|\sec x| + C}$$

$$\boxed{\int \sec x \, dx = \ln|\sec x + \tan x| + C}$$

7.3 Trig Substitution

Goal: evaluate integrals containing $\sqrt{a^2 - x^2}$, $\sqrt{a^2 + x^2}$, $\sqrt{x^2 - a^2}$
Method:

Ex 6: $\int \sqrt{1 - x^2}\, \mathrm{d}x$