$\mathbf{P} = \{x_1, x_2, ..., x_n\}$
$var(l_i) = var(\bar{l_i}) = x_i$
$s(l_i) = s_i$
$s(\bar{l_i}) = 1 - s_i$
weight $w_i \in \mathbf{R_0^+}$

1 Number of Horn formulae

$a)$ maximize $\sum_{\forall C} z_C$ — *HORN_FORMULA*

$b)$ maximize $\sum_{\forall C} w_C \cdot z_C$ — *RESPECT_DECOMPOSITION_HORN_FORMULA*

$c)$ maximize $\sum_{\forall C} |C| \cdot z_C$ — *LENGTH_WEIGHTED_HORN_FORMULA*

$d)$ maximize $\sum_{\forall C} |C|^2 \cdot z_C$ — *SQUARED_LENGTH_WEIGHTED_HORN_FORMULA*

$e)$ maximize $\sum_{\forall C} \frac{1}{|C|} \cdot z_C$ — *INVERSE_LENGTH_WEIGHTED_HORN_FORMULA*

$f)$ maximize $\sum_{\forall C} \frac{1}{|C|^2} \cdot z_C$ — *SQUARED_INVERSE_LENGTH_WEIGHTED_HORN_FORM*

subject to

$$\sum_{\forall l \in C} s(\bar{l}) \leq |C| - z_C \cdot (|C| - 1), \quad \forall C$$

$$z_C \in \{0, 1\}, \quad \forall C$$

$$s_i \in \{0, 1\}, \quad \forall x_i \in \mathbf{P}$$

2 Number of edges

$a)$ minimize $\sum_{\forall x_i, x_j \in \mathbf{P}:\ i<j} e_{i,j}$ — *NUMBER_OF_EDGES*

$b)$ minimize $\sum_{\forall x_i, x_j \in \mathbf{P}:\ i<j} w_{i,j} \cdot e_{i,j}$ — *RESPECT_DECOMPOSITION_NUMBER_OF_EDGES*

subject to

$$s(\bar{l_i}) + s(\bar{l_j}) \leq 1 + e_{i,j}, \quad \forall C:\ \forall l_i, l_j \in C:\ i < j$$

$$e_{i,j} \in \{0, 1\}, \quad \forall x_i, x_j \in \mathbf{P}:\ i < j$$

$$s_i \in \{0, 1\}, \quad \forall x_i \in \mathbf{P}$$

3 Number of vertices

$$a) \text{ minimize } \sum_{\forall x_i \in \mathbf{P}} v_i \qquad \textit{NUMBER\_OF\_VERTICES}$$

$$b) \text{ minimize } \sum_{\forall x_i \in \mathbf{P}} w_i \cdot v_i \qquad \textit{RESPECT\_DECOMPOSITION\_NUMBER\_OF\_VERTICES}$$

$$\begin{aligned}
\text{subject to} \quad & \sum_{\forall l \in C} s(\bar{l}) \leq |C| - z_c \cdot (|C| - 1), && \forall C \\
& s(\bar{l}_i) \leq z_C + v_i, && \forall C : \forall l_i \in C \\
& v_i \in \{0,1\}, && \forall x_i \in \mathbf{P} \\
& s_i \in \{0,1\}, && \forall x_i \in \mathbf{P} \\
& z_C \in \{0,1\}, && \forall C
\end{aligned}$$

4 Vertex cover

$$a) \text{ minimize } \sum_{\forall x_i \in \mathbf{P}} c_i \qquad \textit{VERTEX\_COVER}$$

$$b) \text{ minimize } \sum_{\forall x_i \in \mathbf{P}} w_i \cdot c_i \qquad \textit{RESPECT\_DECOMPOSITION\_VERTEX\_COVER}$$

$$\begin{aligned}
\text{subject to} \quad & s(\bar{l}_i) + s(\bar{l}_j) \leq 1 + c_i + c_j, && \forall C : \ \forall l_i, l_j \in C : \ i < j \\
& c_i \in \{0,1\}, && \forall x_i \in \mathbf{P} \\
& s_i \in \{0,1\}, && \forall x_i \in \mathbf{P}
\end{aligned}$$