

$$\begin{aligned}
\mathbf{P} &= \{x_1, x_2, \dots, x_n\} \\
var(l_i) &= var(\bar{l}_i) = x_i \\
s(l_i) &= s_i \\
s(\bar{l}_i) &= 1 - s_i \\
\text{weight } w_i &\in \mathbf{R}_0^+
\end{aligned}$$

1 Number of Horn clauses

$$\begin{aligned}
a) \text{ maximize } & \sum_{\forall C} z_C && \text{HORN_FORMULA} \\
b) \text{ maximize } & \sum_{\forall C} w_C \cdot z_C && \text{RESPECT_DECOMPOSITION_HORN_FORMULA} \\
c) \text{ maximize } & \sum_{\forall C} |C| \cdot z_C && \text{LENGTH_WEIGHTED_HORN_FORMULA} \\
d) \text{ maximize } & \sum_{\forall C} |C|^2 \cdot z_C && \text{SQUARED_LENGTH_WEIGHTED_HORN_FORMULA} \\
e) \text{ maximize } & \sum_{\forall C} \frac{1}{|C|} \cdot z_C && \text{INVERSE_LENGTH_WEIGHTED_HORN_FORMULA} \\
f) \text{ maximize } & \sum_{\forall C} \frac{1}{|C|^2} \cdot z_C && \text{SQUARED_INVERSE_LENGTH_WEIGHTED_HORN_FORMULA} \\
\\
\text{subject to } & \sum_{\forall l \in C} s(\bar{l}) \leq |C| - z_C \cdot (|C| - 1), \quad \forall C \\
& z_C \in \{0, 1\}, && \forall C \\
& s_i \in \{0, 1\}, && \forall x_i \in \mathbf{P}
\end{aligned}$$

2 Number of edges

$$\begin{aligned}
a) \text{ minimize } & \sum_{\forall x_i, x_j \in \mathbf{P}: i < j} e_{i,j} && \text{NUMBER_OF_EDGES} \\
b) \text{ minimize } & \sum_{\forall x_i, x_j \in \mathbf{P}: i < j} w_{i,j} \cdot e_{i,j} && \text{RESPECT_DECOMPOSITION_NUMBER_OF_EDGES} \\
\\
\text{subject to } & s(\bar{l}_i) + s(\bar{l}_j) \leq 1 + e_{i,j}, \quad \forall C : \forall l_i, l_j \in C : i < j \\
& e_{i,j} \in \{0, 1\}, && \forall x_i, x_j \in \mathbf{P} : i < j \\
& s_i \in \{0, 1\}, && \forall x_i \in \mathbf{P}
\end{aligned}$$

3 Number of vertices

$$a) \text{ minimize } \sum_{\forall x_i \in \mathbf{P}} v_i \quad \text{NUMBER.OF.VERTICES}$$

$$b) \text{ minimize } \sum_{\forall x_i \in \mathbf{P}} w_i \cdot v_i \quad \text{RESPECT_DECOMPOSITION_NUMBER.OF.VERTICES}$$

$$\text{subject to } \sum_{\forall l \in C} s(\bar{l}) \leq |C| - z_c \cdot (|C| - 1), \quad \forall C$$

$$s(\bar{l}_i) \leq z_C + v_i, \quad \forall C : \forall l_i \in C$$

$$v_i \in \{0, 1\}, \quad \forall x_i \in \mathbf{P}$$

$$s_i \in \{0, 1\}, \quad \forall x_i \in \mathbf{P}$$

$$z_C \in \{0, 1\}, \quad \forall C$$

4 Vertex cover

$$a) \text{ minimize } \sum_{\forall x_i \in \mathbf{P}} c_i \quad \text{VERTEX.COVER}$$

$$b) \text{ minimize } \sum_{\forall x_i \in \mathbf{P}} w_i \cdot c_i \quad \text{RESPECT_DECOMPOSITION_VERTEX.COVER}$$

$$\text{subject to } s(\bar{l}_i) + s(\bar{l}_j) \leq 1 + c_i + c_j, \quad \forall C : \forall l_i, l_j \in C : i < j$$

$$c_i \in \{0, 1\}, \quad \forall x_i \in \mathbf{P}$$

$$s_i \in \{0, 1\}, \quad \forall x_i \in \mathbf{P}$$