Physics C Unit 7 FRQs

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March 2, 2025

1 FRQ 1

1.1 Part A

1.1.1 Part i

For a physical pendulum, the period of oscillation is given by the formula

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{mgd}{I}}} = 2\pi\sqrt{\frac{I}{mgd}}$$

where I is the moment of inertia of the pendulum, m is the mass of the pendulum, g is the acceleration due to gravity, and d is the distance from the pivot point to the center of mass of the pendulum.

Thus, we first need to derive a formula for I and d in terms of the given variables (M, λ, and H).

For I,

$$I = \int_0^H dmr^2 = \int_0^H \lambda y^2 dy = c\int_0^H y^3 dy = c\frac{H^4}{4}$$

For d, we need to find the center of mass of the pendulum.

$$y_{com} = \frac{\int ydm}{M} = \frac{\int_0^H y\lambda dy}{M} = \frac{\int_0^H cy^2 dy}{M} = \frac{cH^3}{3M}$$

Substituting these into the formula for T,

$$T = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{c\frac{H^4}{4}}{Mg\frac{cH^3}{3M}}} = 2\pi\sqrt{\frac{3H}{4g}}$$

1.1.2 Part ii

The key thing to note is that D, the distance between the COM and the pivot point is constant between the two objects. So,

So, the only thing that matters is the moment of inertia of the two objects. We know that because the mass remains constant but the total distance increases, the moment of inertia of the second object must be greater.

$$d_1 = d_2, I_1 < I_2 \implies T_1 < T_2$$

1.2 Part B

First, note that $I_{2,com}$ is basically the same as $I_{1,com}$ except 4 times as great because distances are squared in the moment of inertia defenition. Finding $I_{1,com}$:

$$I_{1,com} = I_{1,pivot} - MD^2$$

Finding $I_{2,com}$:

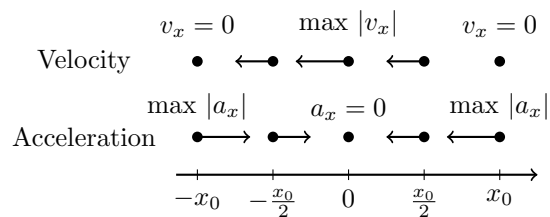$$I_{2,com} = 4I_{1,com} = 4(I_{1,pivot} - MD^2) = 4I_{1,pivot} - 4MD^2$$

Finding $I_{2,pivot}$:

$$I_{2,pivot} = I_{2,com} + MD^2 = 4I_{1,pivot} - 3MD^2$$

2 FRQ 2

2.1 Part A

The key thing to note here is that they're looking for $\frac{T}{2} \leq t \leq T$, so that can throw you off.

$$
\begin{array}{c}
v_x = 0 \qquad \max |v_x| \qquad v_x = 0 \\
\text{Velocity} \qquad \bullet \quad \longleftarrow\!\bullet \quad \longleftarrow\!\bullet \quad \longleftarrow\!\bullet \qquad \bullet
\end{array}
$$

$$
\begin{array}{c}
\max |a_x| \qquad a_x = 0 \qquad \max |a_x| \\
\text{Acceleration} \quad \bullet\!\longrightarrow \quad \bullet\!\longrightarrow \quad \bullet \quad \longleftarrow\!\bullet \quad \longleftarrow\!\bullet
\end{array}
$$

$$
-x_0 \qquad -\tfrac{x_0}{2} \qquad 0 \qquad \tfrac{x_0}{2} \qquad x_0
$$

2.2 Part B

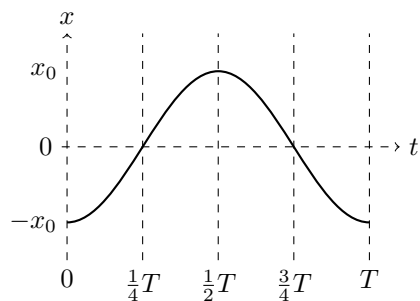Max potential energy = max kinetic energy, so just solve those equations.
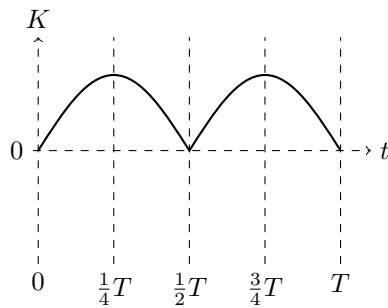
$$U_{max} = K_{max}$$

$$mgh = \frac{1}{2}mv_{max}^2 \implies v_{max} = \sqrt{2gh}$$

$$v_{max} = \sqrt{2gh} = \sqrt{2gBx_0^2} = x_0\sqrt{2gB}$$

2.3 Part C

Now we're back to the interval $0 \leq t \leq T$.

K

0

t

| 0 | $\frac{1}{4}T$ | $\frac{1}{2}T$ | $\frac{3}{4}T$ | T |

2.4 Part D

In the formula for v_{max} derived in Part B, x_0 is only to the first power, so the ratio would simply be $2:1$. This is checked pretty easily because twice the x means four times the height, which means four times the potential energy. This means max kinetic energy is four times as much, and because v is squared in kinetic energy, the resultant factor is simply 2.

3 FRQ 3

3.1 Part A

For each spring with a known spring constant, attach the rock to the spring, and compress the spring any distance. Then, time how long it takes the spring compress and extend 50 times. Record this time and the spring constant. Do this 10 times for each spring, and average the time result.

3.2 Part B

Take the time it takes for 50 compression/extensions and divide by 50 to get the period. Then, plot k on the x-axis and plot $(\frac{2pi}{T})^2$ on the y-axis. The slope of the best line of fit will be approximately equal to $\frac{1}{m}$.

3.3 Part C

3.3.1 Part i

A relationship between L_s and t_{10} can be obtained by using the equation for period of a pendulum. Note for a linear relationship, L_s must be square rooted.
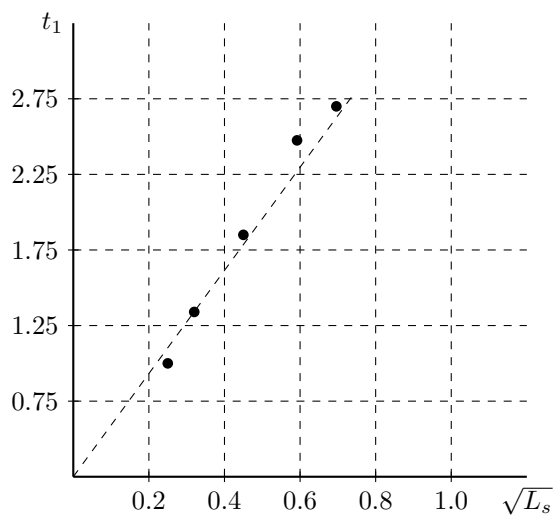
$$T = 2\pi\sqrt{\frac{L}{g}}$$

Horizontal Axis: $\sqrt{L_s}$
Vertical Axis: t_1

Time for 1 oscillation (s)	$\sqrt{L_s}$ $(m^{\frac{1}{2}})$
2.72	0.693
2.41	0.592
1.83	0.447
1.33	0.316
1.01	0.224

3.3.2 Part ii

3.4 Part D

Approximate the best line of fit with a slope of:

$$\frac{2.25 - 1.25}{0.6 - 0.3} = 3.333$$

$$3.333 = \frac{T}{\sqrt{L}} = \frac{2\pi\sqrt{\frac{L}{g}}}{\sqrt{L}} = \frac{2\pi}{\sqrt{g}} \implies (\frac{2\pi}{3.333})^2 = g$$

$$g \approx 3.554\frac{m}{s^2}$$

7

4 FRQ 4

4.1 Part A

The spring constant of f_{123} is less than f_2 because it's in a series, so there's less spring force, meaning less acceleration and less frequency.

4.2 Part B

We need to use the spring series formula and just sub in until we have the effective spring constant in terms of k_2, then plug into the frequency equation.

$$\frac{1}{k_{123}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} = \frac{10}{3k_2} \implies k_{123} = \frac{3k_2}{10}$$

$$f_{123} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{3k_2}{10m}}$$

4.3 Part C

It would be still be greater than the oscillation period for the springs in a series because in the equation there is a factor of $\frac{3}{10}$, and subbing in $2m$ for m in the general equation would only mean a factor of $\frac{1}{2}$, which is greater than $\frac{3}{10}$.