Physics C Unit 7 FRQs

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1 FRQ 1

1.1 Part A

1.1.1 Part i

For a physical pendulum, the period of oscillation is given by the formula

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{mgd}{I}}} = 2\pi\sqrt{\frac{I}{mgd}}$$

where I is the moment of inertia of the pendulum, m is the mass of the pendulum, g is the acceleration due to gravity, and d is the distance from the pivot point to the center of mass of the pendulum.

Thus, we first need to derive a formula for I and d in terms of the given variables (M, λ, and H).

For I,

$$I = \int_0^H dmr^2 = \int_0^H \lambda y^2 dy = c\int_0^H y^3 dy = c\frac{H^4}{4}$$

For d, we need to find the center of mass of the pendulum.

$$y_{com} = \frac{\int ydm}{M} = \frac{\int_0^H y\lambda dy}{M} = \frac{\int_0^H cy^2 dy}{M} = \frac{cH^3}{3M}$$

Substituting these into the formula for T,

$$T = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{c\frac{H^4}{4}}{Mg\frac{cH^3}{3M}}} = 2\pi\sqrt{\frac{3H}{4g}}$$

1.1.2 Part ii

The key thing to note is that D, the distance between the COM and the pivot point is constant between the two objects. So,

So, the only thing that matters is the moment of inertia of the two objects. We know that because the mass remains constant but the total distance increases, the moment of inertia of the second object must be greater.

$$d_1 = d_2, I_1 < I_2 \implies T_1 < T_2$$

1.2 Part B

First, note that $I_{2,com}$ is basically the same as $I_{1,com}$ except 4 times as great because distances are squared in the moment of inertia defenition. Finding $I_{1,com}$:

$$I_{1,com} = I_{1,pivot} - MD^2$$

Finding $I_{2,com}$:

$$I_{2,com} = 4I_{1,com} = 4(I_{1,pivot} - MD^2) = 4I_{1,pivot} - 4MD^2$$

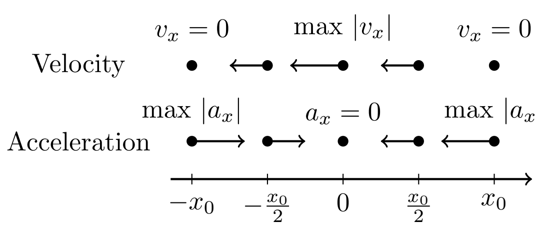
Finding $I_{2,pivot}$:

$$I_{2,pivot} = I_{2,com} + MD^2 = 4I_{1,pivot} - 3MD^2$$

2 FRQ 2

2.1 Part A

The key thing to note here is that they're looking for $\frac{T}{2} \leq t \leq T$, so that can throw you off.



2.2 Part B

Max potential energy = max kinetic energy, so just solve those equations.

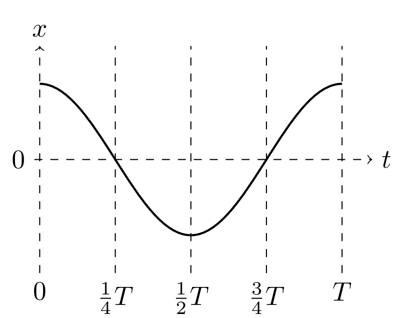
$$U_{max} = K_{max}$$

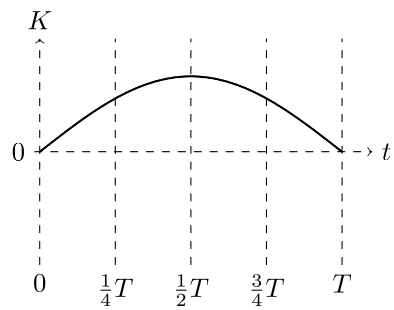
$$mgh = \frac{1}{2}mv_{max}^2 \implies v_{max} = \sqrt{2gh}$$

$$v_{max} = \sqrt{2gh} = \sqrt{2gBx_0^2} = x_0\sqrt{2gB}$$

2.3 Part C

Now we're back to the interval $0 \leq t \leq T$.





2.4 Part D

In the formula for v_{max} derived in Part B, x_0 is only to the first power, so the ratio would simply be $2:1$. This is checked pretty easily because twice the x means four times the height, which means four times the potential energy. This means max kinetic energy is four times as much, and because v is squared in kinetic energy, the resultant factor is simply 2.