Introduction to Information Retrieval

Scoring, Term Weighting and the Vector Space Model

This lecture

- Ranked retrieval
- Scoring documents
- Term frequency
- Collection statistics
- Weighting schemes
- Vector space scoring

Ranked retrieval

- Thus far, our queries have all been Boolean.
    - Documents either match or don’t.
- Good for expert users with precise understanding of their needs and the collection.
    - Also good for applications: Applications can easily consume 1000s of results.

Ranked retrieval

- Not good for the majority of users.
    - Most users incapable of writing Boolean queries (or they are, but they think it’s too much work).
    - Most users don’t want to wade through 1000s of results.
        - This is particularly true of web search.

Problem with Boolean search: feast or famine

- Boolean queries often result in either too few (=0) or too many (1000s) results.
  - Query 1: “standard user dlink 650” → 200,000 hits
  - Query 2: “standard user dlink 650 no card found”: 0 hits
- It takes a lot of skill to come up with a query that produces a manageable number of hits.
  - AND gives too few; OR gives too many

Ranked retrieval models

- Rather than a set of documents satisfying a query expression, in **ranked retrieval,** the system returns an ordering over the (top) documents in the collection for a query
- **Free text queries**: Rather than a query language of operators and expressions, the user's query is just one or more words in a human language
- In principle, there are **two separate choices** here, but in practice, ranked retrieval has normally been associated with free text queries and vice versa

Feast or famine: not a problem in ranked retrieval

- When a system produces a ranked result set, large result sets are not an issue
    - Indeed, the size of the result set is not an issue
    - We just show the top k (≈ 10) results
    - We don’t overwhelm the user
    - **Premise: the ranking algorithm works**

Scoring as the basis of ranked retrieval

- We wish to return in order the documents most likely to be useful to the searcher
- How can we rank-order the documents in the collection with respect to a query?
- Assign a score – say in [0, 1] – to each document
- This score measures how well document and query “match”.

Query-document matching scores

- We need a way of assigning a score to a query/document pair
- Let's start with a one-term query
- If the query term does not occur in the document: score should be 0
- The **more frequent** the query term in the document, the **higher the score** (should be)
- We will look at a number of alternatives for this.

Take 1: Jaccard coefficient

- A commonly used measure of overlap of two sets A and B

$$\text{jaccard}(A,B) = |A \cap B| \,/\, |A \cup B|$$

$$\text{jaccard}(A,A) = 1$$

$$\text{jaccard}(A,B) = 0 \text{ if } A \cap B = 0$$

A and B don't have to be the same size.

- Always assigns a number between 0 and 1.

Jaccard coefficient: Scoring example

- What is the query-document match score that the Jaccard coefficient computes for each of the two documents below?

Query: ides of march

Document 1: caesar died in march –1/6

Document 2: the long march – 1/5

Issues with Jaccard for scoring

- It doesn’t consider term frequency (how many times a term occurs in a document)
- Rare terms in a collection are more informative than frequent terms. Jaccard doesn’t consider this information
- We need a more sophisticated way of normalizing for length

Recall (Lecture 1): Binary term-document incidence matrix

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	1	1	0	0	0	1
Brutus	1	1	0	1	0	0
Caesar	1	1	0	1	1	1
Calpurnia	0	1	0	0	0	0
Cleopatra	1	0	0	0	0	0
mercy	1	0	1	1	1	1
worser	1	0	1	1	1	0

Each document is represented by a binary vector $\in \{0,1\}|V|$

Term-document count matrices

- Consider the number of occurrences of a term in a document:
  - Each document is a **count vector** in ℕv: a column below

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	157	73	0	0	0	0
Brutus	4	157	0	1	0	0
Caesar	232	227	0	2	1	1
Calpurnia	0	10	0	0	0	0
Cleopatra	57	0	0	0	0	0
mercy	2	0	3	5	5	1
worser	2	0	1	1	1	0

Bag of words model

- Vector representation doesn't consider the ordering of words in a document
- *John is quicker than Mary and Mary is quicker than John have the same vectors*
- This is called the bag of words model.
- In a sense, this is a step back: The positional index was able to distinguish these two documents.
- We will look at "recovering" positional information later in this course.
- For now: bag of words model

Term frequency tf

- The term frequency $tf_{t,d}$ of term t in document d is defined as the number of times that t occurs in d.
- We want to use tf when computing query-document match scores. But how?

Term frequency tf

- Raw term frequency is not what we want:
  - A document with 10 occurrences of the term is more relevant than a document with 1 occurrence of the term.
  - But not 10 times more relevant.

- Relevance does not increase proportionally with term frequency.

NB: frequency = count in IR

Log-frequency weighting

- The log frequency weight of term t in d is

$$w_{t,d} = \begin{cases} 1 + \log_{10} \mathrm{tf}_{t,d}, & \text{if } \mathrm{tf}_{t,d} > 0 \\ 0, & \text{otherwise} \end{cases}$$

- 0 → 0, 1 → 1, 2 → 1.3, 10 → 2, 1000 → 4, etc.
- Score for a document-query pair: sum over terms t in both q and d:
- score

$$= \sum_{t \in q \cap d} (1 + \log \mathrm{tf}_{t,d})$$

- The score is 0 if none of the query terms is present in the document.

Document frequency

- Rare terms are more informative than frequent terms
    - Recall stop words
- Consider a term in the query that is rare in the collection (e.g., arachnocentric)
- A document containing this term is very likely to be relevant to the query arachnocentric
- We want a high weight for rare terms like arachnocentric.

Document frequency, continued

- Frequent terms are less informative than rare terms
- Consider a query term that is frequent in the collection (e.g., high, increase, line)
- A document containing such a term is more likely to be relevant than a document that doesn't

Document frequency, continued

- But it’s not a sure indicator of relevance.
- → For frequent terms, we want high positive weights for words like high, increase, and line
- But lower weights than for rare terms.
- We will use document frequency (df) to capture this.

idf weight

- df_t is the document frequency of t: the number of documents that contain t
  - df_t is an inverse measure of the informativeness of t
  - df_t <= N
- We define the idf (inverse document frequency) of t by

$$\mathrm{idf}_t = \log_{10}\left(N/\mathrm{df}_t\right)$$

- We use log (N/df_t) instead of N/df_t to "dampen" the effect of idf.

Will turn out the base of the log is immaterial.

idf example, suppose N = 1 million

term	df_t	idf_t
calpurnia	1	Log10(1,000,000) = 6
animal	100	4
sunday	1,000	3
fly	10,000	2
under	100,000	1
the	1,000,000	0

$$\mathrm{idf}_t = \log_{10}\left(N/\mathrm{df}_t\right)$$

There is one idf value for each term t in a collection.

Effect of idf on ranking

- Does idf have an effect on ranking for one-term queries, like
  - iPhone
- idf has no effect on ranking one term queries
  - idf affects the ranking of documents for queries with at least two terms
  - For the query **capricious person**, idf weighting makes occurrences of **capricious** count for much more in the final document ranking than occurrences of **person**.

Collection vs. Document frequency

- The collection frequency of t is the number of occurrences of t in the collection, counting multiple occurrences.
- Example:

Word	Collection frequency	Document frequency
insurance	10440	3997
try	10422	8760

- Which word is a better search term (and should get a higher weight)?

tf-idf weighting

- The tf-idf weight of a term is the product of its tf weight and its idf weight.

$$w_{t,d} = \log(1 + \mathrm{tf}_{t,d}) \times \log_{10}(N / \mathrm{df}_t)$$

- Best known weighting scheme in information retrieval
- Note: the “-” in tf-idf is a hyphen, not a minus sign!
- Alternative names: tf.idf, tf x idf

tf-idf weighting

- Increases with the number of occurrences within a document
- Increases with the rarity of the term in the collection

Score for a document given a query

$$\mathrm{Score}(q,d) = \sum_{t \in q \cap d} \mathrm{tf.idf}_{t,d}$$

- There are many variants
  - How “tf” is computed (with/without logs)
  - Whether the terms in the query are also weighted
  - …

Term-document count matrices

- Consider the number of occurrences of a term in a document:
    - Each document is a **count vector** in ℕv: a column below

Next slide we will turn the column into tf-idf weight column

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	157	73	0	0	0	0
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worser	2	0	1	1	1	0

Binary → count → weight matrix

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	1.049					
Brutus	0.21					
Caesar	0.187					
Calpurnia	0					
Cleopatra	1.372					
mercy	0.037					
worser	0.084					

$\log_{10}(1+157)*\log_{10}(6/2)$

$\log_{10}(0+1)*\log_{10}(6/1)$

Same way we can calculate for other terms also

Each document is now represented by a real-valued vector of tf-idf weights ∈ R|V|

Documents as vectors

- So we have a $|V|$-dimensional vector space
- Terms are axes of the space
- Documents are points or vectors in this space
- Very high-dimensional: tens of millions of dimensions when you apply this to a web search engine
- These are very sparse vectors - most entries are zero.

Queries as vectors

- Key idea 1: Do the same for queries: represent them as vectors in the space
- Key idea 2: Rank documents according to their proximity to the query in this space
- proximity = similarity of vectors
- proximity ≈ inverse of distance

Queries as vectors

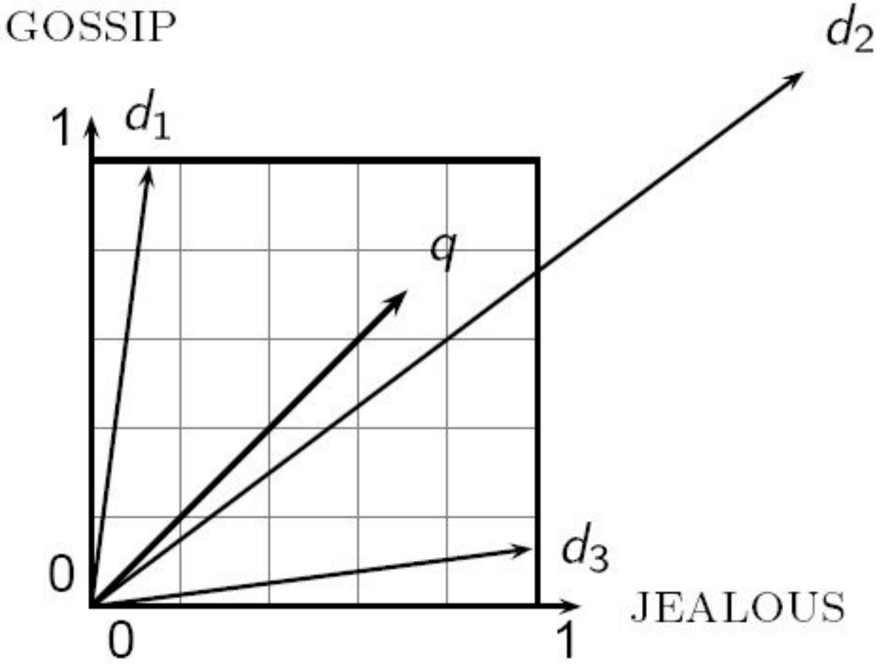
- Recall: We do this because we want to get away from the you’re-either-in-or-out Boolean model.
- Instead: rank more relevant documents higher than less relevant documents

Formalizing vector space proximity

- First cut: distance between two points
    - (= distance between the end points of the two vectors)
- Euclidean distance?
- Euclidean distance is a bad idea . . .
- . . . because Euclidean distance is large for vectors of different lengths.

Why distance is a bad idea

The Euclidean distance between **q and d2** is large even though the distribution of terms in the query q and the distribution of terms in the document d2 are very similar.



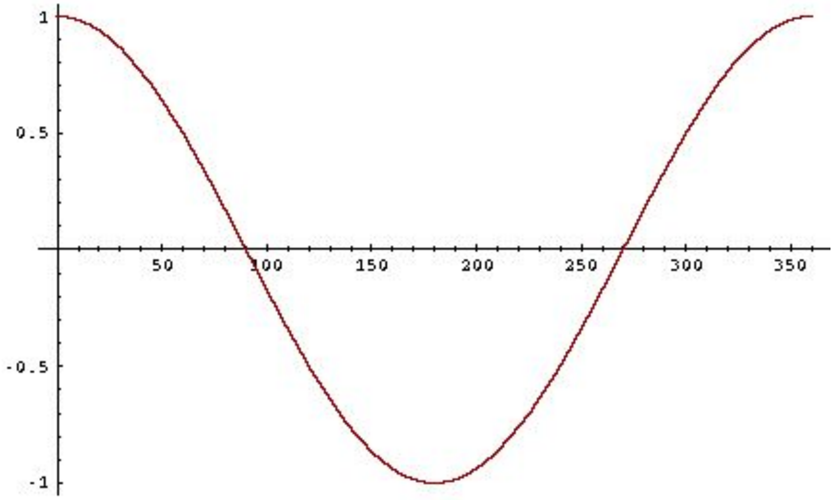
Use angle instead of distance

- Thought experiment: take a document d and append it to itself. Call this document d′.
- “Semantically” d and d′ have the same content
- The Euclidean distance between the two documents can be quite large
- The angle between the two documents is 0, corresponding to maximal similarity.
- **Key idea**: Rank documents according to angle with query.

From angles to cosines

- The following two notions are equivalent.
    - Rank documents in decreasing order of the angle between query and document
    - Rank documents in increasing order of cosine(query,document)
- Cosine is a monotonically decreasing function for the interval [0o, 180o]

From angles to cosines



- But how – and why – should we be computing cosines?

Length normalization

- A vector can be (length-) normalized by dividing each of its components by its length – for this we use the L2 norm:

$$\left\| \vec{x} \right\|_2 = \sqrt{\sum_i x_i^2}$$

- Dividing a vector by its L2 norm makes it a unit (length) vector (on surface of unit hypersphere)

Length normalization

- Effect on the two documents d and d′ (d appended to itself) from earlier slide: they have identical vectors after length-normalization.
    - Long and short documents now have comparable weights

cosine(query,document) q.d = |q||d|cos(\theta)

Dot product

Unit vectors

$$\cos(\vec{q},\vec{d}) = \frac{\vec{q} \bullet \vec{d}}{|\vec{q}||\vec{d}|} = \frac{\vec{q}}{|\vec{q}|} \bullet \frac{\vec{d}}{|\vec{d}|} = \frac{\sum_{i=1}^{|V|} q_i d_i}{\sqrt{\sum_{i=1}^{|V|} q_i^2}\sqrt{\sum_{i=1}^{|V|} d_i^2}}$$

- qi is the tf-idf weight of term i in the query
- di is the tf-idf weight of term i in the document

- cos(q,d) is the cosine similarity of q and d … or,
- equivalently, the cosine of the angle between q and d.

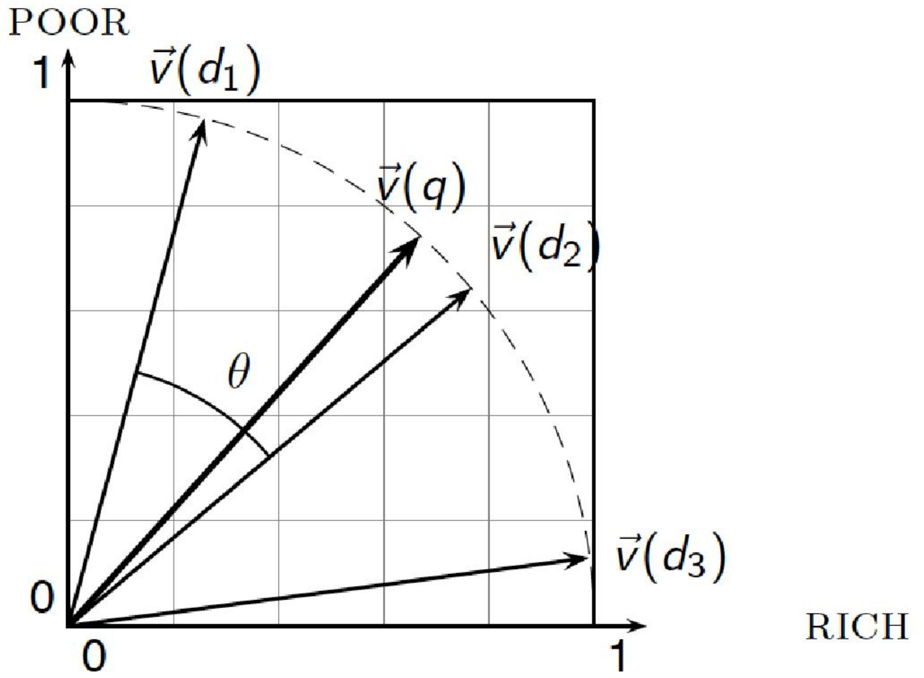
Cosine for length-normalized vectors

- For length-normalized vectors, cosine similarity is simply the dot product (or scalar product):

$$\cos(\vec{q}, \vec{d}) = \vec{q} \cdot \vec{d} = \sum_i q_i \cdot d_i$$

for q, d length-normalized.

Cosine similarity illustrated



Cosine similarity amongst 3 documents

How similar are the novels
SaS: Sense and Sensibility
PaP: Pride and Prejudice, and
WH: Wuthering Heights?

term	SaS	PaP	WH
affection	115	58	20
jealous	10	7	11
gossip	2	0	6
wuthering	0	0	38

Note: To simplify this example, we don't do idf weighting.

3 documents example contd.

Log frequency weighting

term	SaS	PaP	WH
affection	3.06	2.76	2.30
jealous	2.00	1.85	2.04
gossip	1.30	0	1.78
wuthering	0	0	2.58

After length normalization

term	SaS	PaP	WH
affection	0.789	0.832	0.524
jealous	0.515	0.555	0.465
gossip	0.335	0	0.405
wuthering	0	0	0.588

term	SaS	PaP	WH
affection	115	58	20
jealous	10	7	11
gossip	2	0	6
wuthering	0	0	38

$\text{Log}_{10}(115) + 1 = 3.06$

3.06/ sqrt((**3.06^2 + 2.00^2 + 1.30^2**))

0.789^2 + 0.515^2 + 0.335^2 = 1

3 documents example contd.

Log frequency weighting

term	SaS	PaP	WH
affection	3.06	2.76	2.30
jealous	2.00	1.85	2.04
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After length normalization

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wuthering	0	0	0.588

cos(SaS,PaP) ≈

$0.789 \times 0.832 + 0.515 \times 0.555 + 0.335 \times 0.0 + 0.0 \times 0.0$

≈ 0.94

cos(SaS,WH) ≈ 0.79

cos(PaP,WH) ≈ 0.69

Why do we have cos(SaS,PaP) > cos(SaS,WH)?

Computing cosine scores

CosineScore(q)

float $Scores[N] = 0$
float $Length[N]$
for each query term t
do calculate $w_{t,q}$ and fetch postings list for t
    for each pair(d, $\mathrm{tf}_{t,d}$) in postings list
    do $Scores[d] += w_{t,d} \times w_{t,q}$
Read the array $Length$
for each d
do $Scores[d] = Scores[d]/Length[d]$
return Top K components of $Scores[]$

tf-idf weighting has many variants log(n-df_t/df_t) df_t > N/2

Term frequency		Document frequency		Normalization	
n (natural)	$\mathrm{tf}_{t,d}$	n (no)	1	n (none)	1
l (logarithm)	$1 + \log(\mathrm{tf}_{t,d})$	t (idf)	$\log \frac{N}{\mathrm{df}_t}$	c (cosine)	$\frac{1}{\sqrt{w_1^2+w_2^2+\ldots+w_M^2}}$
a (augmented)	$0.5 + \frac{0.5\times\mathrm{tf}_{t,d}}{\max_t(\mathrm{tf}_{t,d})}$	p (prob idf)	$\max\{0, \log \frac{N-\mathrm{df}_t}{\mathrm{df}_t}\}$	u (pivoted unique)	$1/u$
b (boolean)	$\begin{cases} 1 & \text{if } \mathrm{tf}_{t,d} > 0 \\ 0 & \text{otherwise} \end{cases}$			b (byte size)	$1/CharLength^{\alpha}$, $\alpha < 1$
L (log ave)	$\frac{1+\log(\mathrm{tf}_{t,d})}{1+\log(\mathrm{ave}_{t\in d}(\mathrm{tf}_{t,d}))}$				

Columns headed 'n' are acronyms for weight schemes.

Why is the base of the log in idf immaterial?

Weighting may differ in queries vs documents

- Many search engines allow for different weightings for queries vs. documents
- SMART Notation: denotes the combination in use in an engine, with the notation **ddd.qqq**, using the acronyms from the previous table
- A very standard weighting scheme is: **lnc.ltc**

Weighting may differ in queries vs documents

- Document: logarithmic tf (l as first character), no idf and cosine normalization
- Query: logarithmic tf (l in leftmost column), idf (t in second column), no normalization …

tf-idf example: lnc.ltc 2.3 = $\log_{10}(N/5000)$

Document: car insurance auto insurance

Query: best car insurance

Term	Query						Document				Prod
	tf-raw	tf-wt	df	idf	wt	n'lize	tf-raw	tf-wt	wt	n'lize	
auto	0	0	5000	2.3	0	0	1	1	1	0.52	0
best	1	1	50000	1.3	1.3	0.34	0	0	0	0	0
car	1	1	10000	2.0	2.0	0.52	1	1	1	0.52	0.27
insurance	1	1	1000	3.0	3.0	0.78	2	1.3	1.3	0.68	0.53

Exercise: what is *N*, the number of docs?

Doc length $= \sqrt{1^2 + 0^2 + 1^2 + 1.3^2} \approx 1.92$

Score = 0+0+0.27+0.53 = 0.8

- $IDF = \log10(N/DF)$
- $2.3 = \log(N/5000)$
- $2.3 = \log(N) – \log(5000) = \log (N) – \log(10000) + \log(2) = \log (N) - 4 + .3$
- $6 = \log(N)$
- $N = 1000000$

Summary – vector space ranking

- Represent the query as a weighted tf-idf vector
- Represent each document as a weighted tf-idf vector
- Compute the cosine similarity score for the query vector and each document vector
- Rank documents with respect to the query by score
- Return the top K (e.g., K = 10) to the user