1 Notes

- A shortest may not exist when:
  - Negative weight cycle
  - The graph is not connected
- Two Properties of The Shortest Path
  - The optimal substructure of dynamic shortest path: A subpath of a shortest path itself is a shortest path
  - Triangle inequality
    - For all $u, v, x \in V$ we have $\delta(u, v) \leq \delta(u, x) + \delta(x, v)$
    - Shortest path from u to v is at most any particular path, e.g., the blue chain.
- Tractable Problems
  - Problems solvable in a polynomial time
- $P \subset NP$
- NPC: If any other problem (i.e., NP-Hard problems) p in NP is polynomial reducible to problem q

2 Psuedocode

2.1 Dijkstra(G,s)

```
d[s] = 0
for each vertex v in V−{s}
    d[v] = infinity
    S = emptySet
    Q = V

while Q is not empty
    u = Extract_Min(Q)
    S = S ∪ {u}
    for each vertex v adjacent to u
        if d[v] > d[u] + w(u,v)
            d[v] = d[u] + w(u,v)
```

2.2 Bellman-Ford

```
Given G=(V,E,w) and source vertex s
d[x] = distance estimate from s to x

Bellman-Ford(G,s)
d[s] = 0
for each vertex v in V-{s}
    d[v] = infinity

for i=1 to |V|-1
    for each edge (u,v) in E
        if d[v] > d[u] + w(u,v)
        d[v] = d[u] + w(u, v)

for each edge (u, v) in E
    if d[v] > d[u] + w(u, v)
        report that a negative-weight cycle exists
```

2.3 Floyd-Warshall

```
D^(0) = W
for k = 1 to n do
    for i = 1 to n do
        for j = 1 to n do
    d_ij(k) = min{d_ij^(k-1), d_ik^(k-1) + d_kj^(k-1)};

Return D^(n)
```

2.4 Ford Fulkerson

```
FORD-FULKERSON(G, s, t)
    for each edge (u,v) in E(G)
        do f[u, v] = 0
           f[v, u] = 0
    while there is a path p from s to t in the residual network Gf
        cf(p) = min{cf(u, v)-f[u, v]: (u, v) is in p}
            for each edge (u, v) on p
                if (u, v) in E
                    f[u, v] = f[u, v] + cf(p)
                else f[v, u] = f[v, u] - cf(p)
```

3 Complexities

Dijkstra's $\Theta(V) \times T_{\text{Extract-Min}} + \Theta(E) \times T_{\text{Decrease-Key}}$

Bellman-Ford $\mathcal{O}(VE)$

Floyd Warshall $\mathcal{O}(V^3)$

Ford-Fulkerson $\mathcal{O}(E|f^*|)$

Edmond Carp $\mathcal{O}(VE^2) = \mathcal{O}(V^5)$