One Factor Experiments

$$H_0 = \mu_1 = \ldots = \mu_1$$

$$H_1 = \text{two or more of the } \mu_i \text{ are different}$$

$$SSTr = \sum_{i=1}^{I} J_{i.}(\bar{X}_{i.} - \bar{X}_{..})^2$$
$$= \sum_{i=1}^{I} J_i \bar{X}_{i.}^2 - N\bar{X}_{..}^2$$

$$SSE = \sum_{i=1}^{I} \sum_{j=1}^{J_i} (X_{ij} - \bar{X}_i)^2$$
$$= \sum_{i=1}^{I} \sum_{j=1}^{J_i} X_{ij}^2 - \sum_{i=1}^{I} J_i \bar{X}_i^2$$

$$MSTr = \frac{SSTr}{I-1} \qquad MSE = \frac{SSE}{N-I}$$

$$F = \frac{MSTr}{MSE}$$

$$SST = SSTr + SSE$$

$$SST = \sum_{i=1}^{I} \sum_{j=1}^{J_i} (X_{ij} - \bar{X}_{..})^2$$
$$= \sum_{i=1}^{I} \sum_{j=1}^{J_i} X_{ij}^2 - N\bar{X}^2$$

$p - \text{value} < \alpha \implies$ reject H_0

$p - \text{value} \geq \alpha \implies$ fail to reject H_0

Two Factor Experiments

$$X_{ijk} = \mu + \alpha_i + \beta_j + \gamma_{ij} + \epsilon_{ijk}$$
$$\alpha_i = \bar{\mu}_{i.} - \mu$$
$$\beta_j = \bar{\mu}_{.j} - \mu$$
$$\gamma_{ij} = \mu_{ij} - \bar{\mu}_i - \bar{\mu}_j + \mu$$
$$\epsilon_{ijk} = X_{ijk} - \mu_{ij}$$

$H_{0,AB} : \gamma_{11} = \gamma_{12} = \cdots = \gamma_{IJ} = 0$

$H_{H,AB}$: at least one of the γ_{ij} is nonzero

$H_{0,A} : \alpha_1 = \alpha_2 = \cdots = \alpha_I = 0$

$H_{1,A}$: at least one of the α_i is nonzero

Sum of Squares for Two-Way ANOVA

- Treatment sum of squares: SSTr (d.f. = I*J – 1)
- Row sum of squares: $SSA = JK\sum_{i=1}^{I} \hat{\alpha}_i^2$ (d.f. = I – 1)
- Column sum of squares: $SSB = IK\sum_{j=1}^{J} \hat{\beta}_j^2$ (d.f. = J – 1)
- Interaction sum of squares: $SSAB = K\sum_{i=1}^{I}\sum_{j=1}^{J} \hat{\gamma}_{ij}^2$ (d.f. = (I – 1)(J – 1))
- Error sum of squares: SSE (d.f. = I*J(K – 1))
- Total sum of squares: SST (d.f.= I*J*K – 1)

$$F_{AB}^* = \frac{MSAB}{MSE} \qquad F_A^* = \frac{MSA}{MSE} \qquad \frac{MSB}{MSE}$$

$$MSA = \frac{SSA}{I-1} \qquad MSB = \frac{SSB}{J-1} \qquad MSAB + \frac{SSAB}{(I-1)(J-1)}$$

Bernoulli Distribution

$$\mu_x = p$$
$$\sigma_x^2 = p(1-p)$$

Binomial Distribution

$$p(x) = P(X = x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x} & x = 0, 1, \ldots, n \\ 0 & \text{otherwise} \end{cases}$$

where $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

$$\mu_x = np$$
$$\sigma_x^2 = np(1-p)$$

$$\bar{p} = \frac{\text{number of successes}}{\text{number of trials}} = \frac{X}{n}$$

$$\sigma_{\overline{p}} = \sqrt{Var(\overline{p})} \approx \sqrt{\frac{\overline{p}(1-\overline{p})}{n}}$$

Poisson Distribution

$$p(x) = P(X = x) = \begin{cases} \frac{e^{-\lambda}\lambda^x}{x!} & x = 0, 1, \ldots, n \\ 0 & \text{otherwise} \end{cases}$$

$$\mu_x = \lambda \qquad \sigma_x^2 = \lambda$$

The Normal Distribution

If a continuous random variable x with mean μ and variance σ^2 has the following probability density function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$Z = \frac{X - \mu}{\sigma}$$

The Exponential Distribution

The notation is as follows $X \tilde{} Exp(\lambda)$

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x = 0, 1, \ldots, n \\ 0 & \text{otherwise} \end{cases}$$

$$F(x) = \begin{cases} 0 & x \leq 0 \\ 1 - e^{-\lambda x} & x > 0 \end{cases}$$

$$P(T > t + s | T > s) = P(T > t)$$

$$s_i^2 = \sigma_i^2 = \frac{1}{J_i - 1} \sum_{j=1}^{J_i} (X_{ij} - \bar{X_{i.}})^2$$