

# Homework #1

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**Theorem 1.** *There does not exist a level set with more than one bottom level.*

*Proof.* Suppose that there exists two, different levels  $P$  and  $Q$  in the level set  $M$  (where  $M$  may contain two —  $P$  and  $Q$  — or more levels). Per our definition,  $P$  and  $Q$  are bottom levels of the level set  $M$  if there exists a level  $x$  in  $M$  where  $x$  is not above  $P$  and  $Q$ . This implies  $P$  cannot be above  $Q$  and  $Q$  cannot be above  $P$ , for a level  $x$  must not be above  $P$  and  $Q$ . However, this violates Axiom 3 (which states that  $P$  is above  $Q$  or  $Q$  is above  $P$ ). This has lead us to a contradiction.  $\therefore$  There does not exists a level set with more than one bottom level.  $\square$