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# Chapter 6: Applications of Integration

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## 6.7 Physical Applications

### Mass of a One-Dimensional Object

Suppose a thin bar or wire is represented by a line segment on the interval $a \leq x \leq b$ with a density function $\rho$ (with units of mass per length). The mass of the object is

$$m = \int_a^b \rho(x)\,dx \tag{1}$$

### Work

The work done by a variable force $F$ in moving an object along a line from $x = a$ to $x = b$ in the direction of the force is

$$W = \int_a^b F(x)\,dx \tag{2}$$

### Solving Lifting Problems

1. Draw a $y$-axis in the vertical direction (parallel to gravity) and choose a convenient origin. Assume the interval $[a, b]$ corresponds to the vertical extent of the fluid.

2. For $a \leq y \leq b$, find the cross-sectional area $A(y)$ of the horizontal slices and the distance $D(y)$ the slices must be lifted.

3. The work required to lift the water is

$$W = \int_a^b \rho g A(y) D(y)\,dy \tag{3}$$

### Solving Force/Pressure Problems

1. Draw a $y$-axis on the face of the dam in the vertical direction and choose a convenient origin (often taken to be the base of the dam).

2. Find the width function $w(y)$ for each value of $y$ on the face of the dam.

3. If the base of the dam is at $y = 0$ and the top of the dam is at $y = a$, then the total force on the dam is

$$F = \int \rho g(a - y)w(y)\, dy \tag{4}$$