

Different Types of Bootstraps

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Below are three bootstraps discussed in the paper Resampling Multilevel Models by Rien van der Leeden et al..

Overall model

$$y_i = X_i\beta + Z_ib + \epsilon_i$$

$$\epsilon_i \sim (iid)N(0, \sigma_{\epsilon_i}^2)$$

$$b_i \sim (iid)N(0, D)$$

$$Cov(\epsilon_i, b) = 0$$

These are two equations used in the following outlines:

$$(1) \hat{\theta}_B = \hat{\theta} - Bias_B = 2\hat{\theta} - \theta_{(.)}^*$$

$$(2) \hat{se}_B = \sqrt{\hat{Var}(\theta^*)}$$

Parametric Bootstrap

1. Draw J vectors of level-2 residuals from a multivariate normal distribution with mean zero and covariance matrix \hat{D} .
2. Draw J vectors ϵ_j^* of sizes n_j containing level-1 residuals from a normal distribution with means zero and covariance matrices $\hat{\sigma}^2 I_{n_j}$.
3. Generate the bootstrap sample y_j^* from $y_j^* = X_j\hat{\beta} + Z_j\delta_j^* + \epsilon_j^*$.
4. Compute estimates for all parameters of the two-level model.
5. Repeat steps 1-4 B times and compute bias-corrected estimates and bootstrap standard errors using formulas 1 and 2.

Residual Bootstrap

1. Draw a sample δ_j^* of size J with replacement from the set of estimated level-2 residuals.
2. Draw J samples ϵ_{ij}^* of size n_j with replacement from the elements $\hat{\epsilon}_{ij}$.
3. Generate the bootstrap samples y_j^* from $y_j^* = X_j\hat{\beta} + Z_j\delta_j^* + \epsilon_j^*$.
4. Compute estimates for all parameters of the two-level model.
5. Repeat steps 1-4 B times and compute bias-corrected estimates and bootstrap standard errors using formulas 1 and 2.

Cases Bootstrap

1. Draw a sample of size J with replacement from the level-2 units(unit numbers)
2. For each k , draw a sample of entire cases, with replacement, from the original level-2 unit. This sample has the same size as the original unit from which the cases are drawn. Then, for each k , we have a set of data
3. Compute estimates for all parameters of the two-level model
4. Repeat steps 1-3 B times and compute bias-corrected estimates and bootstrap standard errors using formulas 1 and 2.