Solutions to exercises sheet 2

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1 Exercise 1

b) As $(-1)^2 + 2^2 = 5$, we get from the definition of R that every point $(x, y) \in \mathbb{R}^2$ s.t (such that) $(x, y)R(-1, 2)$ satisfies the equation $x^2 + y^2 = 5$.

   We recognize here the cartesian equation of a circle (Kreise) of radius $\sqrt{5}$ and center $(0, 0)$, which is then the equivalence class of $(-1, 2)$.

c) As stated in question above, the equivalence class of a point $(a, b) \in \mathbb{R}^2$ is the circle of radius $\sqrt{a^2 + b^2}$ and center $(0, 0)$.

   A representation system has then to take exactly one point in every of those circles. A smart and simple choice could be to fix one of the coordinates to 0, let say the second one. The representation system is then the ray $\mathbb{R}_{\geqslant 0} \times \{0\} \subset \mathbb{R}^2$, the represent of the equivalence class of $(a, b) \in \mathbb{R}^2$ being $(\sqrt{a^2 + b^2}, 0)$.

   See exercise 4 for a more comprehensive description of what happens

2 Exercise 2

You have to work with the equivalence relation $\sim$ given by $x \sim y :\iff f(x) = f(y)$.

3 Exercise 3

Let denote $\forall x, y \in \mathbb{R}^2 : [x, y] := \{tx + (1 - t)y | t \in \mathbb{R}\}$, i.e "the line between x and y". We should remark that when $x = y$, $[x, y]$ is nothing but the point $\{x\} = \{y\}$.

Let $\Phi : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear fonction, and $a, b \in \mathbb{R}^2$ s.t $a \neq b$.
$\forall t \in [0, 1]$, $\Phi(ta + (1-t)b) = \Phi(ta) + \Phi((1-t)b) = t\Phi(a) + (1-t)\Phi(b)$ by linearity of Φ, which means:

$$\Phi([a, b]) = [\Phi(a), \Phi(b)]$$

As $a \neq b$ by hypothesis, $[a, b]$ is here a true line (i.e not reduced to a point).
For $\Phi([a, b]) = [\Phi(a), \Phi(b)]$:

- if $\Phi(a) \neq \Phi(b)$, then $[\Phi(a), \Phi(b)]$ is a true line also.
- if $\Phi(a) = \Phi(b)$, then $[\Phi(a), \Phi(b)]$ is a reduced to the point $\{\Phi(a)\} = \{\Phi(b)\}$.

4 Exercise 4

a) Let be $x = (x_1, x_2) \in \mathbb{R}^2$. Then r is given by $r = \sqrt{x_1^2 + x_2^2}$.

   We then define $\tilde{x} = (\tilde{x}_1, \tilde{x}_2) := x/r$ and remark that $\tilde{x}_1^2 + \tilde{x}_2^2 = 1$ which means that $\exists \theta \in [0, 2\pi)$ s.t $(\tilde{x}_1, \tilde{x}_2) = (\cos(\theta), \sin(\theta))$.

   Ich weiss nicht ob Ihr habt schon das beweisst. Wenn nicht, können wir das zusammen beweissen