Habib University
Course Code: EE366/CE366/CS380
Course Title: Introduction to Robotics
Instructor name: Dr. Basit Memon
Examination: Mid-term Exam Retake Exam Date: April 19, 2023
Total Marks: 100 Duration: 80 minutes

Instructions

1. You can refer to course slides, your notes, homework solutions, or any of the books included in the course syllabus. Where appropriate, you can cite the slides and don't have to redo what has already been done.
2. Disconnect your devices from the network.
3. You are welcome to utilize MATLAB in whichever way you find useful. You can submit the response to a question in the form of a MATLAB live script as well, but solution using numerical approach where question explicitly asks you to employ analytical methods will not be accepted.
4. The exam will be administered under HU student code of conduct (see Chapter 3 of `https://habibuniversity.sharepoint.com/sites/Student/code-of-conduct`). You may not talk, discuss, compare, copy from, or consult with any other human on this planet (except me) on questions or your responses during the exam.
5. Make sure that you show your work (intermediate steps). Most of the points for any question will be given based on the followed process.
6. Kindly use a black/blue pen to write your hand-written solutions, so that they are legible.
7. The questions or their associated points are not arranged by complexity or expected required time.

CLOs

- Use implicit and explicit representation of configuration and spatial velocities of a robot to mathematically describe its motion in 3D space.
- Analyze a serial manipulator's kinematic singularities and apply forward and inverse kinematics to transform between joint and end-effector positions and velocities.

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Questions

Problem 1
100 points

Consider the two robot setup of Figure 1 comprising of two identical planar 3R robot manipulators. In this setup, M is the primary robot and S is the secondary robot. In this problem,

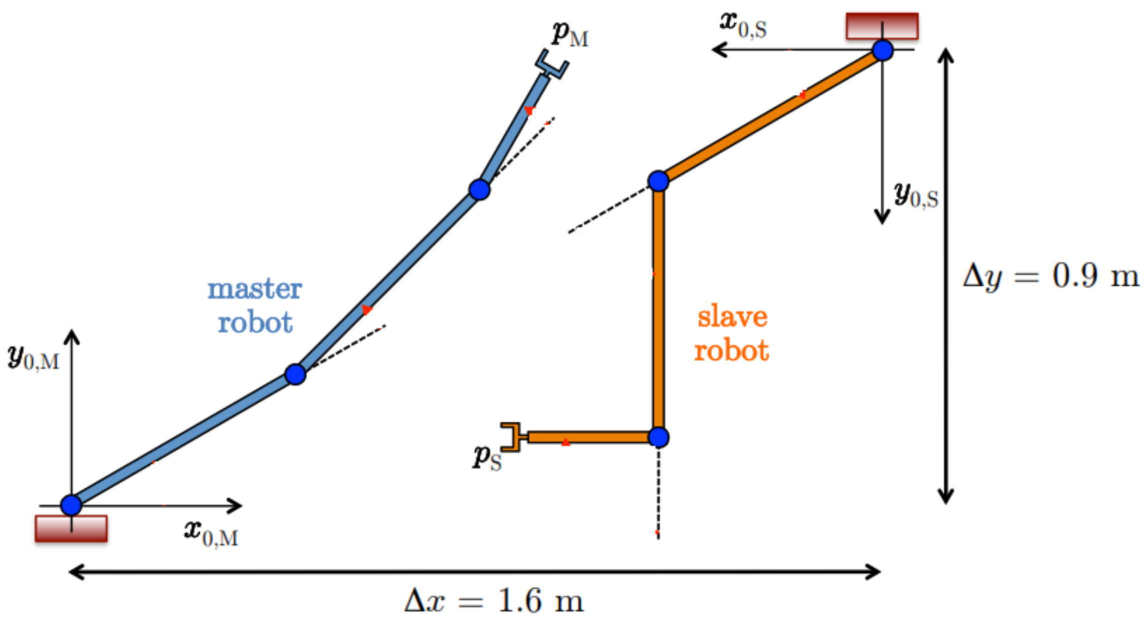

Figure 1: Relative positioning of two 3R primary and secondary robots

the two robots are to perform a coordinated task, which requires both their end-effectors to touch an object and move in exactly the same way. The following information is provided:

- The link lengths for both robots are $l_1 = l_2 = 0.5$ m and $l_3 = 0.25$ m;
- The base frames of the two robots are displaced by $(\Delta x, \Delta y, 0) = (1.6, 0.9, 0)$ m, and the frames are rotated by $180°$ relative to each other as shown in Figure 1. The z_0 is coming out of the page for both frames;
- The desired motion to be executed by two manipulators begins at time $t = t_0$ from the position $p_M(t_0)$, i.e. the initial position of the end-effector of the primary manipulator M, given by its initial configuration $(\theta_1, \theta_2, \theta_3) = (90°, -60°, 0)$;
- The desired motion will be along a straight line path, which is specified by the initial direction of the end-effector velocity of the primary manipulator, $v_M(t_0)$, resulting from the initial joint velocities $(\dot{\theta}_1(t_0), \dot{\theta}_2(t_0), \dot{\theta}_3(t_0)) = (-30°/s, 0, -90°/s)$. This is shown by the green line in Figure 2;

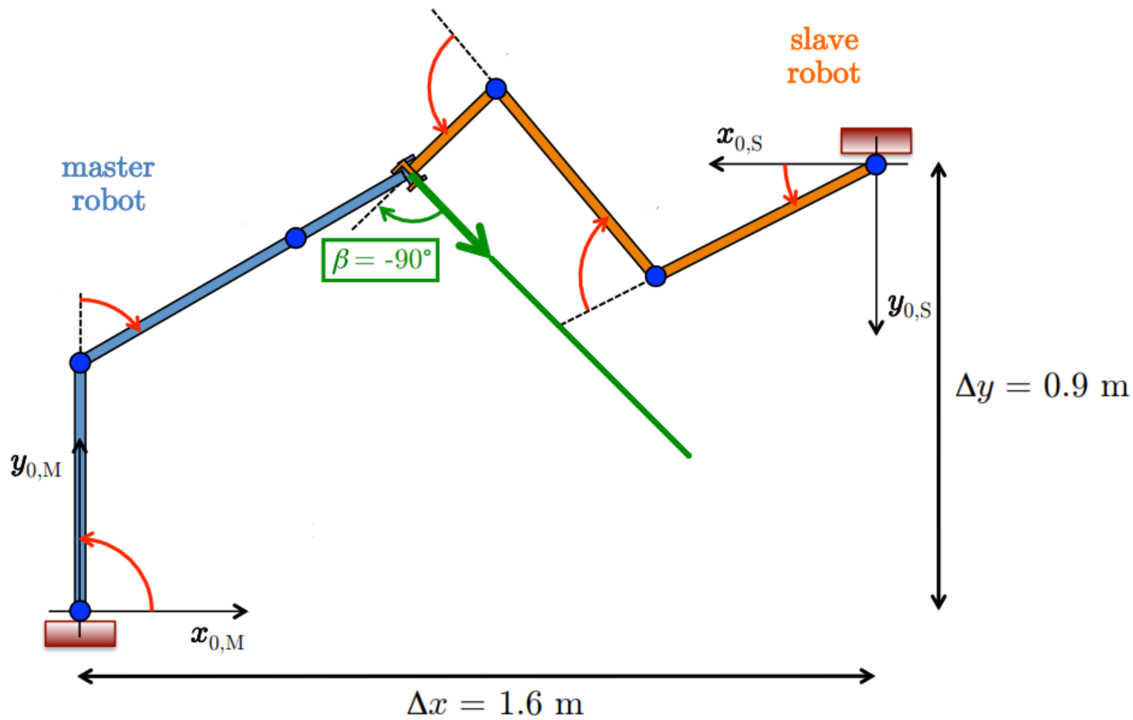

Figure 2: Desired Cartesian trajectory to be executed by two robots

- The end-effector of the secondary robot should always be oriented orthogonal to the linear path, as shown by the orientation of the orange end-effector relative to the green line in Figure 2.

Now,

(a) Determine an initial configuration, $q_S(t_0)$, of the secondary robot such that its end-effector position and orientation are initially matched with those required by the motion task;

(b) Determine the initial joint velocity vector, $\dot{q}_S(t_0)$, of the secondary robot to also match its initial Cartesian end-effector velocity with that of the primary robot, i.e. $v_S(t_0) = v_M(t_0)$.

Solution 1

We can determine p_M, the starting position of the end-effector, using forward kinematics. Let the joint angles for the primary manipulator be $\theta_{1,M}$, $\theta_{2,M}$, and $\theta_{3,M}$. Then, $({}^{M}x_p, {}^{M}y_p)$

can be found as:

$$
\begin{aligned}
{}^{M}x_p &= l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) + l_3 \cos(\theta_1+\theta_2+\theta_3) \\
{}^{M}y_p &= l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2) + l_3 \sin(\theta_1+\theta_2+\theta_3)
\end{aligned}
$$

MATLAB Symbolic Toolbox Help

- `help func` provides help for the MATLAB function `func`.
- `syms x f(x)` defines a symbolic variable `x` and a symbolic function `f(x)`.
- `deg2rad(arg)` and `rad2deg(arg)` can be used to convert between degrees and radians.
- `atan2(y,x)` and `atan2d(y,x)` are MATLAB equivalents of $\arctan 2$.
- `diff(f,x)` differentiates symbolic function `f(x)` with respect to `x`.
- `collect(expr,term)` collects all coefficients of `term` in the expression `expr`.
- `subs(expr,org,new)` substitutes `new` wherever `org` appears in the expression `expr`. It can also be used to substitute numbers for symbols.
- `det(A)` finds the determinant of matrix `A`, which can be a symbolic matrix as well.
- `simplify(expr)` simplifies a symbolic expression.