Habib University
Course Code: EE366/CE366/CS380
Course Title: Introduction to Robotics
Instructor name: Dr. Basit Memon
Examination: Mid-term Exam Exam Date: March 25, 2024
Total Marks: 100 Duration: 65 minutes

Instructions

1. You can refer to course slides, your notes, homework solutions, or any of the books included in the course syllabus. Where appropriate, you can cite the slides to avoid redoing what has already been done.

2. Disconnect your devices from the network.

3. You are welcome to utilize MATLAB in whichever way you find useful. You can submit the response to a question in the form of a MATLAB live script as well, but solution using numerical approach when question explicitly asks you to employ analytical methods will not be accepted.

4. The exam will be administered under HU student code of conduct (see Chapter 3 of `https://habibuniversity.sharepoint.com/sites/Student/code-of-conduct`). You may not talk, discuss, compare, copy from, or consult with any other human on this planet (except me) on questions or your responses during the exam.

5. Make sure that you show your work (intermediate steps). Most of the points for any question will be given based on the followed process.

6. Kindly use a black/blue pen to write your hand-written solutions, so that they are legible.

7. You're encouraged to make any justified assumptions and state them in your responses.

CLOs

- Use implicit and explicit representation of configuration and spatial velocities of a robot to mathematically describe its motion in 3D space.

- Analyze a serial manipulator's kinematic singularities and apply forward and inverse kinematics to transform between joint and end-effector positions and velocities.

Questions

Problem 1
100 points

Figure 1 depicts an RPR planar robot, with the first and third joints revolute and the second joint prismatic. Answer the following questions about this robot:

30 points

(a) Assign frames from the base frame, $\{0\}$, to the end-effector frame, $\{3\}$, according to the DH convention, consistent with the already specified frames and the indicated positive direction $(+)$ of rotation and translation of joints in the figure. Then, find the corresponding DH parameters. Make sure to clearly indicate the origin and the directions of $\hat{z}$ and $\hat{x}$ for each frame. Mark each non-zero length in the DH parameters on the figure and indicate which of the DH length parameters are negative.

30 points

(b) Solve the inverse kinematics problem analytically for this manipulator provided (p_x, p_y, ϕ), the position of the origin of the end-effector frame and the angle between $\hat{x}_3$ and $\hat{x}_0$. Clearly state the number of possible inverse kinematics solutions for the non-singular case and determine all of them. You're allowed to define the joint variables for this part differently from the previous part.

40 points

(c) Now, assume that another robot B is also installed on the table along with the robot of Figure 1, hereafter called A. Find a configuration of robot A such that its end-effector is located at the right position and orientation to pick up an object placed on the table by robot B, according to the following transformations.

Let the lengths of the first and third links of robot A be 1. Let the base frame of robot B be $\{w\}$ and

$$^{w}T_0 = \begin{bmatrix} 0.5 & -0.866 & 0 & 1 \\ 0.866 & 0.5 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

After the object has been placed by robot B, its pose is given by

$$^{w}T_o = \begin{bmatrix} 0 & 1 & 0 & 1 \\ -1 & 0 & 0 & 0.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

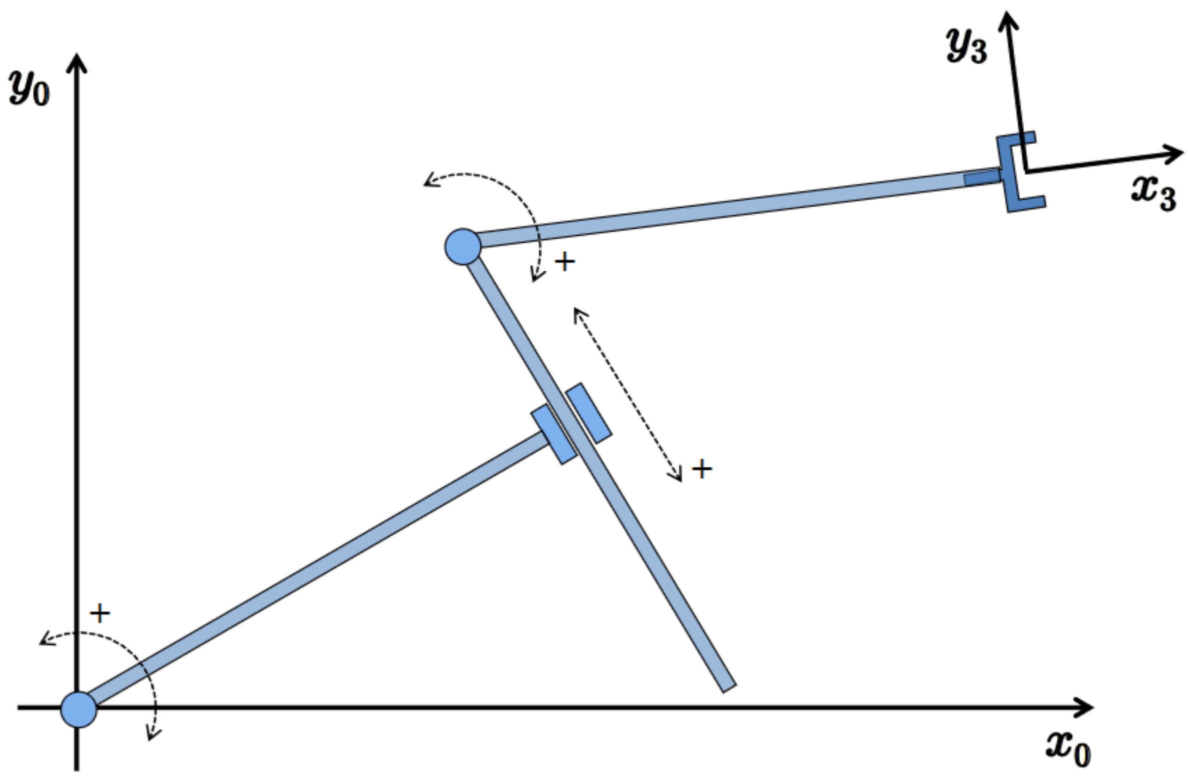

Figure 1: An RPR Spatial Robot

Solution 1

(a) The DH frames are assigned in Figure 2, making sure that $\hat{z}$ are chosen to align conventional counterclockwise rotation with the indicated positive direction of rotation. The

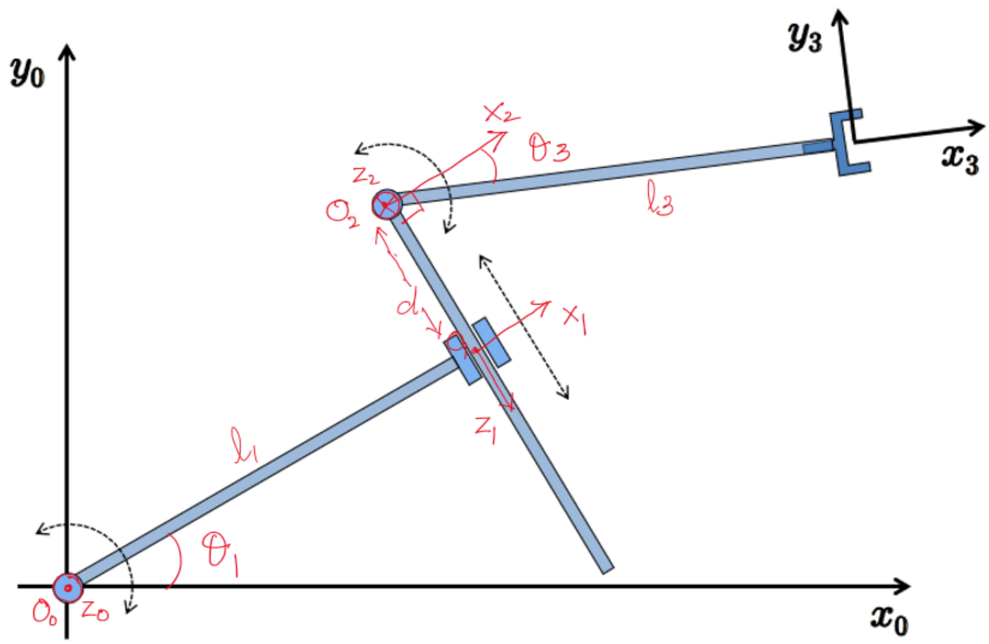

Figure 2: DH frame assignment for provided robot

corresponding DH parameters are:

Link	a_i	α_i	d_i	θ_i
1	l_1	$90°$	0	θ_1
2	0	$90°$	$-d$	0
3	l_3	$180°$	0	θ_3

Note that $d \geq 0$ is a length. Correspondingly, the forward kinematics mapping is:

$$
{}^0T_3 = \begin{bmatrix} \cos(\theta_1-\theta_3) & -\sin(\theta_1-\theta_3) & 0 & l_1\cos\theta_1 - d\sin\theta_1 + l_3\cos(\theta_1-\theta_3) \\ \sin(\theta_1-\theta_3) & \cos(\theta_1-\theta_3) & 0 & d\cos\theta_1 + l_1\sin\theta_1 + l_3\sin(\theta_1-\theta_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.
$$

(b) Let's use algebraic approach to find the inverse kinematics solution. The definitions of θ used in this part are same as the ones defined in the previous part. An example of how this makes an impact is that θ_3 as shown in the figure is a positive angle. Correspondingly, as seen from the homogeneous transformation,

$$\begin{aligned}
\phi &= \theta_1 - \theta_3 && (1)\\
p_x &= l_1 \cos\theta_1 - d\sin\theta_1 + l_3\cos(\theta_1-\theta_3)\\
p_y &= d\cos\theta_1 + l_1\sin\theta_1 + l_3\sin(\theta_1-\theta_3)\\
p_x - l_3\cos\phi &= l_1\cos\theta_1 - d\sin\theta_1 && (2)\\
p_y - l_3\sin\phi &= d\cos\theta_1 + l_1\sin\theta_1 && (3)\\
\Rightarrow l_1^2 + d^2 &= (p_x - l_3\cos\phi)^2 + (p_y - l_3\sin\phi)^2\\
\Rightarrow d &= \sqrt{(p_x - l_3\cos\phi)^2 + (p_y - l_3\sin\phi)^2 - l_1^2} && (4)
\end{aligned}$$

From (2) and (3),

$$\begin{aligned}
\cos\theta_1 &= \frac{l_1(p_x - l_3\cos\phi) + d(p_y - l_3\sin\phi)}{l_1^2 + d^2}\\
\sin\theta_1 &= \frac{-d(p_x - l_3\cos\phi) + l_1(p_y - l_3\sin\phi)}{l_1^2 + d^2}\\
\Rightarrow \theta_1 &= \arctan 2(\sin\theta_1, \cos\theta_1) && (5)\\
\theta_3 &= \theta_1 - \phi && (6)
\end{aligned}$$

Expressions (4), (5), and (6) provide an IK solution to the problem. There is only one solution.

Geometric approach. Let us now use geometric approach to obtain the IK solution. For the geometric approach, we'll define θ_1 and θ_3 as usual, whih means that definition of θ_3 here is negative of θ_3 used in the algebraic approach. From the definition of ϕ, we can see that

$$\phi = \theta_1 + \theta_3.$$

Let the coordinates of O_2 be (x', y'). Then,

$$\begin{aligned}
x' &= p_x - l_3\cos\phi,\\
y' &= p_y - l_3\sin\phi.
\end{aligned}$$

Consider $\triangle O_0O_1O_2$. It is a right-angled triangle. Thus,

$$\begin{aligned}
d^2 &= (x'^2 + y'^2) - l_1^2\\
\Rightarrow d &= \sqrt{(x'^2 + y'^2) - l_1^2}.
\end{aligned}$$

From the same triangle,

$$\begin{aligned}\cos\alpha &= \frac{l_1}{\sqrt{x'^2+y'^2}}\\ \Rightarrow \theta_1 &= \arctan 2(y', x') - \alpha.\end{aligned}$$

Note that since the lengths will always be positive, the two argument version of arctan is not required. Finally,

$$\theta_3 = \phi - \theta_1.$$

The sign difference due to the complementary definition of θ_3 is evident in this expression compared to the previous. We can also establish equivalence between the previous θ_1 expression (5) and this one. Starting with the previous one,

$$\begin{aligned}\theta_1 &= \arctan\left(\frac{-dx' + l_1 y'}{l_1 x' + dy'}\right)\\ &= \arctan\left(\frac{-d + l_1\frac{y'}{x'}}{l_1 + d\frac{y'}{x'}}\right)\\ &= \arctan\left(\frac{-\frac{d}{l_1} + \frac{y'}{x'}}{1 - \frac{d}{l_1}\frac{y'}{x'}}\right)\\ &= \arctan\left(\frac{y'}{x'}\right) + \arctan\left(-\frac{d}{l_1}\right)\\ &= \arctan\left(\frac{y'}{x'}\right) - \arctan\left(\frac{d}{l_1}\right)\\ &= \arctan 2(y', x') - \alpha.\end{aligned}$$

(c) To find the configuration for robot A, we need to use our obtained IK solution in the previous part. Correspondingly, we require (p_x, p_y, ϕ). The pose of the object with respect to the base frame $\{0\}$ of robot A is:

$$\begin{aligned}{}^0T_o &= {}^wT_0^{-1}\,{}^wT_o\\ &= \begin{bmatrix} -0.866 & 0.5 & 0 & -2.1651\\ -0.5 & -0.866 & 0 & -1.2501\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}.\end{aligned}$$

Correspondingly, $(p_x, p_y) = (-2.1651, -1.2501)$. The desired orientation is a rotation about the $\hat{z}_0$ axis by an angle ϕ. Comparing with canonical rotation matrix $R_z(\phi)$, we can see that $(\sin\phi, \cos\phi) = (-0.5, -0.866)$, or correspondingly $\phi = -150°$.

The configuration to achieve this end-effector position and orientation is:

$$d = \sqrt{x'^2 + y'^2 - l_1^2} = 1.118$$

$$\theta_1 = \arctan 2(y', x') - \arccos\left(\frac{l_1}{\sqrt{x'^2 + y'^2}}\right) = -198.2^\circ = 161.8^\circ$$

$$\theta_3 = \phi - \theta_1 = 48.2^\circ$$

MATLAB Symbolic Toolbox Help

- `help func` provides help for the MATLAB function `func`.
- `syms x f(x)` defines a symbolic variable `x` and a symbolic function `f(x)`.
- `deg2rad(arg)` and `rad2deg(arg)` can be used to convert between degrees and radians.
- `atan2(y,x)` and `atan2d(y,x)` are MATLAB equivalents of $\arctan 2$.
- `diff(f,x)` differentiates symbolic function `f(x)` with respect to `x`.
- `collect(expr,term)` collects all coefficients of `term` in the expression `expr`.
- `subs(expr,org,new)` substitutes `new` wherever `org` appears in the expression `expr`. It can also be used to substitute numbers for symbols.
- `det(A)` finds the determinant of matrix `A`, which can be a symbolic matrix as well.
- `simplify(expr)` simplifies a symbolic expression.