

# Nigeria Fiscal-Monetary-Structural Simulator:

## Complete Technical Documentation

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### **Abstract**

This document provides the complete mathematical formulation of the Nigeria Fiscal-Monetary-Structural Simulator, a dynamic stochastic general equilibrium (DSGE)-inspired model with Nigeria-specific structural characteristics. The model integrates fiscal policy, monetary policy, structural reforms, distributional impacts, and external sector dynamics in a unified framework for policy analysis and simulation.

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# 1 Model Architecture Overview

The simulator implements a **dynamic stochastic general equilibrium (DSGE)**-inspired framework with Nigeria-specific structural features, running annual simulations over a 5-year horizon. The model integrates four policy channels:

- **Fiscal Policy:** Revenue dynamics, spending composition, debt sustainability
- **Monetary Policy:** Central Bank reaction function, inflation targeting, exchange rates
- **Structural Reforms:** Institutional improvements, formalization, poverty reduction
- **Distributional Impacts:** Fiscal incidence, inequality dynamics, pro-poor targeting

# 2 Core Model Equations

## 2.1 National Income Identity (Enhanced)

The core growth equation incorporates multiple policy channels:

$$GDP_t = GDP_{t-1} \times \left(1 + \frac{g_t}{100}\right) \quad (1)$$

$$g_t = g^B + \Delta^M + \Delta^f + \Delta^x + \Delta^s + \Delta^D - \Omega \quad (2)$$

**Where:**

- $g^B$ : Baseline growth assumption (exogenous)
- $\Delta^M$ : Monetary policy impact
- $\Delta^f$ : Fiscal policy impact
- $\Delta^x$ : Net trade effect
- $\Delta^s$ : Structural reform boost
- $\Delta^D$ : Distributional equality effect
- $\Omega$ : Crowding-out effect

### 3 Fiscal Policy Block

#### 3.1 Revenue Composition

$$R_t^{Oil} = P_t^{Oil} \times \kappa^{Oil} \quad (3)$$

$$R_t^{Non-Oil} = R_{t-1}^{Non-Oil} \times \left(1 + \frac{\tau}{100}\right) \times \left(1 + \frac{g^B}{100}\right) \quad (4)$$

$$R_t^{Total} = R_t^{Oil} + R_t^{Non-Oil} \quad (5)$$

**Where:**

- $P_t^{Oil} = P_0^{Oil} \times \left(1 + \frac{\varepsilon^{Oil}}{100}\right)$ : Oil price with shock
- $\kappa^{Oil} = 0.33$ : Oil revenue sensitivity (\$0.33B per \$1 oil price)
- $\tau$ : Tax policy change (%)

#### 3.2 Government Spending

$$G_t = G_{t-1} \times \left(1 + \frac{\gamma}{100}\right) \times \eta \quad (6)$$

**Where:**

- $\gamma$ : Spending change (%)
- $\eta$ : Spending efficiency (1.0 base, 1.1 with reforms)

### 3.3 Debt Dynamics

$$D_t = D_{t-1} + (G_t + S_t^{Debt} - R_t^{Total}) \quad (7)$$

$$S_t^{Debt} = D_{t-1} \times \left( \frac{r_t^{Sov}}{100} \right) \quad (8)$$

$$r_t^{Sov} = r_t^{Policy} + \rho_t^{Risk} \quad (9)$$

### 3.4 Risk Premium Mechanism

$$\rho_t^{Risk} = \rho^{Debt} + \rho^{Reserve} + \rho^{Reform} \quad (10)$$

$$\rho^{Debt} = \max \left( 0, \left( \frac{Debt}{GDP} - 50 \right) \times 0.15 \right) \quad (11)$$

$$\rho^{Reserve} = \begin{cases} 3.0 & \text{if ReserveCover} < 3 \text{ months} \\ 1.5 & \text{if ReserveCover} < 6 \text{ months} \\ 0.0 & \text{otherwise} \end{cases} \quad (12)$$

$$\rho^{Reform} = \begin{cases} -2.0 & \text{if structural reforms} \\ 0.0 & \text{otherwise} \end{cases} \quad (13)$$

## 4 Monetary Policy Block

### 4.1 Central Bank Reaction Function

$$\Delta r_t^{Policy} = f(\pi_{t-1} - \pi^*, \text{economic condition, monetary stance}) \quad (14)$$

#### 4.1.1 Hawkish Reaction

$$\begin{aligned} \text{If } (\pi - \pi^*) > 5 : \quad \Delta r &= \min(4.0, (\pi - \pi^*) \times 0.5) - \phi^{Reform} \\ \text{If } (\pi - \pi^*) > 2 : \quad \Delta r &= (\pi - \pi^*) \times 0.3 - \phi^{Reform} \end{aligned}$$

#### 4.1.2 Dovish Reaction

$$\begin{aligned} \text{If } (\pi - \pi^*) > 10 : \quad \Delta r &= (\pi - \pi^*) \times 0.2 - \phi^{Reform} \\ \text{If } (\pi - \pi^*) > 5 : \quad \Delta r &= (\pi - \pi^*) \times 0.1 - \phi^{Reform} \end{aligned}$$

#### 4.1.3 Automatic Reaction

$$\begin{aligned}
\text{If } (\pi - \pi^*) > 8 : \quad \Delta r = (\pi - \pi^*) \times 0.4 - \phi^{Reform} \\
\text{If } (\pi - \pi^*) > 4 : \quad \Delta r = (\pi - \pi^*) \times 0.25 - \phi^{Reform} \\
\text{If recession and } (\pi - \pi^*) < 2 : \quad \Delta r = -1.0 - \phi^{Reform}
\end{aligned}$$

Where:

- $\pi^* = 9.0\%$ : CBN inflation target
- $\pi_t = \pi_{t-1} - (\Delta r_t^{Policy} \times \text{transmission effect})$
- $\phi^{Reform} = 0.2$  if structural reforms, else 0

## 4.2 Monetary Impact on Growth

$$\Delta^M = -\Delta r_t^{Policy} \times (0.2 + \phi^{Reform}) \quad (15)$$

## 5 Exchange Rate and External Sector

### 5.1 Exchange Rate Determination

$$\Delta E_t = \frac{\theta^A + \theta^{Oil} + \theta^R + \theta^B + \theta^{Reform}}{10} \quad (16)$$

Components:

- $\theta^A = -\frac{CA_{t-1}}{GDP_0} \times 50$ : Current account pressure
- $\theta^{Oil} = \frac{R_t^{Oil} - R_0^{Oil}}{R_0^{Oil}} \times (-100)$ : Oil revenue effect
- $\theta^R = (r_t^{Policy} - 6.0) \times (-2)$ : Interest differential
- $\theta^B$ : Reserve adequacy effect
- $\theta^{Reform} = -20$  if reforms, else 0: Reform confidence effect

$$E_t = E_{t-1} \times \left(1 + \frac{\Delta E_t}{100}\right) \quad (17)$$

### 5.2 Reserve Dynamics

$$FXR_t = FXR_{t-1} - (\text{intervention intensity} \times FXR_{t-1} \times 0.1) \quad (18)$$

$$\text{intervention intensity} = \min \left( 1.0, \frac{|\Delta E_t|}{10} \right) \quad (19)$$

### 5.3 Trade Balance Effects

$$M_t = GDP_t \times 0.14 \times \left(1 + \frac{g_t}{100} \times 0.8\right) \quad (20)$$

$$X_t = GDP_t \times 0.11 \times \left(1 + \frac{\varepsilon_t^{exp}}{100}\right) \quad (21)$$

$$\varepsilon_t^{exp} = \max(0, \Delta E_t) \times 0.6 \quad (22)$$

## 6 Structural Reforms Block

### 6.1 Reform Growth Boost

$$\Delta^s = 0.2 \times \mu_t \times (1 + \phi^P + \phi^I) \quad (23)$$

Where:

- $\mu_t = \min(2.0, 1 + t \times 0.3)$ : Time multiplier
- $\phi^P = \max(0, \frac{\text{PovertyRate} - 20}{100})$ : Poverty reduction potential
- $\phi^I = \max(0, \frac{\text{InformalShare} - 30}{100})$ : Formalization potential

### 6.2 Structural Transformation

$$\text{PovertyRate}_t = \max(20, \text{PovertyRate}_{t-1} - \Delta^s \times 3) \quad (24)$$

$$\text{InformalShare}_t = \max(30, \text{InformalShare}_{t-1} - \Delta^s \times 2) \quad (25)$$

## 7 Fiscal Incidence and Distribution

### 7.1 Inequality Dynamics

$$\Delta \text{Gini}_t = f(\tau, \gamma, \text{progressivity flags}) \quad (26)$$

#### 7.1.1 Tax Progressivity Effects

If progressive tax and  $\tau > 0$  :  $\Delta \text{Gini} = \Delta \text{Gini} - 0.5 \times \left(\frac{\tau}{10}\right)$

If not progressive and  $\tau > 0$  :  $\Delta \text{Gini} = \Delta \text{Gini} + 0.3 \times \left(\frac{\tau}{10}\right)$

### 7.1.2 Spending Progressivity Effects

$$\begin{aligned} \text{If pro-poor spending and } \gamma > 0 : \quad \Delta\text{Gini} &= \Delta\text{Gini} - 0.8 \times \left( \frac{\gamma}{10} \right) \\ \text{If not pro-poor and } \gamma > 0 : \quad \Delta\text{Gini} &= \Delta\text{Gini} - 0.2 \times \left( \frac{\gamma}{10} \right) \end{aligned}$$

$$\text{Gini}_t = \max(20, \min(60, \text{Gini}_{t-1} + \Delta\text{Gini} \times 2)) \quad (27)$$

## 7.2 Consumption Share Redistribution

$$\begin{aligned} \text{If } \Delta\text{Gini} < 0 : \quad \text{PoorShare}_t &= \min(30, \text{PoorShare}_{t-1} + |\Delta\text{Gini}| \times 0.5) \\ &\quad \text{RichShare}_t = \max(40, \text{RichShare}_{t-1} - |\Delta\text{Gini}| \times 0.5) \\ \text{If } \Delta\text{Gini} > 0 : \quad \text{PoorShare}_t &= \max(10, \text{PoorShare}_{t-1} - \Delta\text{Gini} \times 0.3) \\ &\quad \text{RichShare}_t = \min(70, \text{RichShare}_{t-1} + \Delta\text{Gini} \times 0.3) \end{aligned}$$

## 7.3 Equality-Growth Link

$$\Delta^D = \Delta\text{Gini} \times 0.1 \quad (28)$$

# 8 Fiscal Impulse and Stance

## 8.1 Fiscal Impulse Calculation

$$FI_t = \frac{\gamma - \tau \times 0.7}{\text{divisor}} + \text{cyclical component} \quad (29)$$

$$\text{divisor} = \begin{cases} 5 & \text{in recession} \\ 8 & \text{in boom} \\ 6 & \text{otherwise} \end{cases} \quad (30)$$

$$\text{cyclical component} = -\text{OutputGap}_{t-1} \times 0.2 \quad (31)$$

## 8.2 Output Gap Dynamics

$$\text{OutputGap}_t = (g_t - g^B) \times 0.8 \quad (32)$$

## 8.3 Structural Balance

$$\text{StructuralBalance}_t = \text{StructuralBalance}_{t-1} - FI_t \quad (33)$$

## 9 Multiplier System

### 9.1 Time-Varying Multipliers

Table 1: State-Dependent Fiscal Multipliers

Economic Condition	Spending Multiplier	Tax Multiplier
Recession	0.8	0.5
Normal	0.6	0.3
Boom	0.4	0.2
High Debt	0.3	0.1
High Inflation	0.2	0.1

### 9.2 Fiscal Impact Calculation

$$\Delta^f = \frac{\gamma \times (G_t - S_t^{Debt}) \times \text{Multiplier}^S + \tau \times R_t^{Non-Oil} \times \text{Multiplier}^T}{GDP_{t-1}} \quad (34)$$

## 10 Crowding-Out Effect

$$\Omega = \max \left( 0, \frac{S_t^{Debt} - \left( S_t^{Debt} \times \left( \frac{D_{t-1}}{GDP_{t-1}} \right) \right)}{GDP_{t-1}} \right) \times 100 \quad (35)$$

## 11 Labor Market and Social Indicators

### 11.1 Okun's Law Implementation

$$\Delta \text{Unemployment}_t = -0.5 \times (g_t - g^B) \quad (36)$$

$$\text{Unemployment}_t = \max (2.0, \text{Unemployment}_{t-1} + \Delta \text{Unemployment}_t) \quad (37)$$

### 11.2 Social Progress Index

$$SPI_t = (100 - \text{PovertyRate}_t) + \frac{(100 - \text{InformalShare}_t)}{2} \quad (38)$$

Table 2: Initial Parameter Values (2024)

Parameter	Value and Description
$GDP_0$	\$450B - Initial GDP
Debt/ $GDP_0$	31% - Initial debt ratio
Oil Revenue Share	55% - Oil dependency
Inflation <sub>0</sub>	28% - Current inflation
Policy Rate <sub>0</sub>	18.75% - CBN MPC rate
Gini <sub>0</sub>	35.1 - Income inequality
Poverty Rate <sub>0</sub>	40% - Population below poverty line
Informal Share <sub>0</sub>	55% - Informal economy size

Table 3: Model Elasticity Parameters

Parameter	Value and Description
Oil Revenue Sensitivity	0.33 - \$0.33B per \$1 oil price
Import Elasticity	0.8 - Responsiveness to income
Export Elasticity	0.6 - Responsiveness to exchange rate
Automatic Stabilizers	0.2 - Cyclical sensitivity
Okun's Coefficient	0.5 - Unemployment-growth relationship

## 12 Model Calibration Parameters

### 12.1 Nigeria-Specific Initial Conditions

### 12.2 Elasticity Parameters

## 13 Economic Condition Assessment

$$\text{assess\_condition}(GDP, \text{Debt}/GDP, \text{Inflation}) = \begin{cases} \text{"high\_debt"} & \text{if } \text{Debt}/GDP > 60 \\ \text{"high\_inflation"} & \text{if } \text{Inflation} > 25 \\ \text{"recession"} & \text{if } GDP < GDP_0 \times 0.98 \\ \text{"boom"} & \text{if } GDP > GDP_0 \times 1.05 \\ \text{"normal"} & \text{otherwise} \end{cases} \quad (39)$$

Table 4: Policy Thresholds and Zones

Indicator	Safe Zone	Caution Zone	Danger Zone
Debt/GDP	30%	30-50%	50%
Debt Service/Revenue	15%	15-30%	30%
Reserve Cover	6 months	3-6 months	3 months
Oil Dependency	30%	30-60%	60%
Gini Coefficient	30	30-40	40

## 14 Non-linearities and Thresholds

### 14.1 Critical Thresholds

## 15 Feedback Loops and System Dynamics

### 15.1 Primary Feedback Mechanisms

#### 15.1.1 Debt-Growth Vicious Cycle

Higher Debt → Higher Risk Premium → Higher Interest Rates  
 → More Debt Service → Less Productive Spending  
 → Lower Growth → Higher Debt/GDP Ratio

#### 15.1.2 Reform-Confidence Virtuous Cycle

Structural Reforms → Improved Institutions → Higher Confidence  
 → Lower Risk Premium → More Investment  
 → Higher Growth → More Reform Capacity

#### 15.1.3 Oil Revenue Volatility Cycle

Oil Price Shock → Revenue Change → Fiscal Space Change  
 → Spending Consistency → Economic Stability  
 → Investor Confidence

#### 15.1.4 Inequality-Growth Nexus

Progressive Policies → Reduced Inequality → Higher Consumption  
→ Stronger Domestic Demand → More Inclusive Growth  
→ Poverty Reduction

## 16 Policy Transmission Channels

Table 5: Policy Transmission Mechanisms

Policy Instrument	Primary Channel	Secondary Effects
Fiscal Expansion	Direct Demand Stimulus	Multiplier Effects, Crowding-out
Tax Reform	Incentive Effects	Behavioral Responses, Revenue Mobilization
Monetary Tightening	Interest Rate Channel	Exchange Rate Impact, Expectations
Structural Reforms	Efficiency Gains	Institutional Quality, Confidence

## 17 Model Limitations and Boundary Conditions

### 17.1 Simplifications

- Constant elasticities within simulation periods
- Immediate rather than delayed policy impacts
- No explicit monetary policy reaction to fiscal stance
- Simplified expectations formation
- No regional or sectoral disaggregation
- No explicit financial sector modeling
- No demographic dynamics

### 17.2 Boundary Conditions

- Minimum policy rate: 12.0%

- Maximum annual exchange rate change:  $\pm 20\%$
- Minimum foreign reserves: \$5B
- Minimum poverty rate: 20%
- Minimum informal sector share: 30%
- Maximum debt/GDP ratio: 100%
- Maximum inflation: 50%

## 18 Model Enhancements Potential

### 18.1 Short-term Extensions

- Sectoral disaggregation (oil vs non-oil)
- Regional fiscal federalism
- Explicit expectations formation
- Delayed policy impacts

### 18.2 Medium-term Extensions

- Demographic dynamics
- Climate change impacts
- Digital economy effects
- Human capital accumulation

### 18.3 Long-term Extensions

- Political economy constraints
- Institutional quality metrics
- Global spillover effects
- Technological change modeling

## **19 Conclusion**

The Nigeria Fiscal-Monetary-Structural Simulator represents a sophisticated integration of macroeconomic theory with Nigeria-specific structural characteristics. The model provides a comprehensive framework for policy analysis, incorporating fiscal sustainability, monetary policy effectiveness, structural transformation, and distributional impacts in a unified dynamic system.

The mathematical formulation presented in this document ensures transparency and reproducibility, making the simulator suitable for academic research, policy analysis, and educational purposes. The model's modular design allows for continuous enhancement and adaptation to evolving economic conditions and policy priorities.