# Statistics validations

August 3, 2025

# 1 Statistic checks of the collected data

```
[]: import numpy as np
from scipy import stats
from matplotlib import pyplot as plt

import pandas as pd
from scipy.signal import medfilt

%matplotlib inline
alpha = 0.05
```

#### 1.0.1 Verifying if the data is normally distribuated and has similar variance

Loading the test dataset of 3 Aspect Ratios. Each dataset contain 10 experiments. We will verify the normallity and the variance on the 10 values x3 datasets.

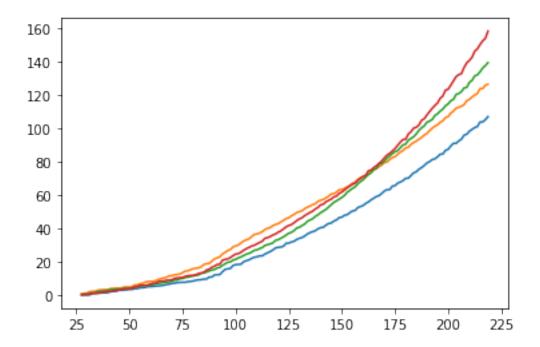
```
[]: def import_CSV(fileslist):
         expNbr = len(fileslist)
         expData = []
         for i in range(expNbr):
             expData.append(pd.read_csv(fileslist[i]))
         expDataSize = list(len(df['force'].values) for df in expData)
         dataCrop = min(expDataSize)
         #Clean data
         return_table = dict()
         medKernel = 5
         for i,data in enumerate(expData):
             data['force'] = medfilt(data['force'], kernel_size=medKernel) # Apply_
      \hookrightarrow median filter
             data['force'] = data['force'][0:-medKernel]
         dataCrop -= medKernel
         #Extract indicators
```

```
return_table["depth"] = expData[0]['steps'].values[:dataCrop]
for i in range(expNbr):
    return_table["exp_" + str(i)] = expData[i]['force'].values[:dataCrop]
    # print(np.max(expData[i]['steps'].iloc[-1]))
return pd.DataFrame(return_table)
```

```
[]: # Aspect ratio 1
     group_1_files =["./expData/dataTest/AR1/FR2_0.csv",
                     "./expData/dataTest/AR1/FR2_1.csv",
                     "./expData/dataTest/AR1/FR2 2.csv",
                     "./expData/dataTest/AR1/FR2_3.csv",
                     "./expData/dataTest/AR1/FR2 4.csv",
                     "./expData/dataTest/AR1/FR2_5.csv",
                     "./expData/dataTest/AR1/FR2_6.csv",
                     "./expData/dataTest/AR1/FR2_7.csv",
                     "./expData/dataTest/AR1/FR2_8.csv",
                     "./expData/dataTest/AR1/FR2_9.csv"]
     # Aspect ratio 2
     group_2_files =["./expData/dataTest/AR25/FR5_0.csv",
                     "./expData/dataTest/AR25/FR5 1.csv",
                     "./expData/dataTest/AR25/FR5_2.csv",
                     "./expData/dataTest/AR25/FR5 3.csv",
                     "./expData/dataTest/AR25/FR5_4.csv",
                     "./expData/dataTest/AR25/FR5 5.csv",
                     "./expData/dataTest/AR25/FR5_6.csv",
                     "./expData/dataTest/AR25/FR5_7.csv",
                     "./expData/dataTest/AR25/FR5_8.csv",
                     "./expData/dataTest/AR25/FR5_9.csv"]
     # Aspect ratio 3
     group_3_files =["./expData/dataTest/AR4/FR8_0.csv",
                     "./expData/dataTest/AR4/FR8_1.csv",
                     "./expData/dataTest/AR4/FR8_2.csv",
                     "./expData/dataTest/AR4/FR8_3.csv",
                     "./expData/dataTest/AR4/FR8_4.csv",
                     "./expData/dataTest/AR4/FR8_5.csv",
                     "./expData/dataTest/AR4/FR8 6.csv",
                     "./expData/dataTest/AR4/FR8_7.csv",
                     "./expData/dataTest/AR4/FR8_8.csv",
                     "./expData/dataTest/AR4/FR8_9.csv"]
     group_1 = import_CSV(group_1_files)
     group_2 = import_CSV(group_2_files)
     group_3 = import_CSV(group_3_files)
```

```
plt.plot(group_2["depth"],group_2["exp_1"])
plt.plot(group_2["depth"],group_2["exp_2"])
plt.plot(group_2["depth"],group_2["exp_3"])
plt.plot(group_2["depth"],group_2["exp_4"])
```

## []: [<matplotlib.lines.Line2D at 0x29ed9a88a30>]

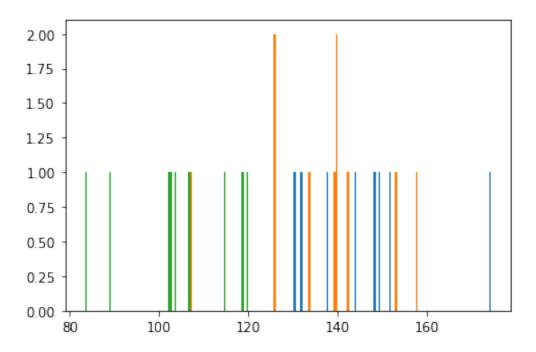


Extract the last depth values

```
[]: group_1_sub, group_2_sub, group_3_sub = [], [], []
for i in range(10):
        group_1_sub.append(group_1["exp_"+str(i)].iloc[-1])
        group_2_sub.append(group_2["exp_"+str(i)].iloc[-1])
        group_3_sub.append(group_3["exp_"+str(i)].iloc[-1])

[]: print(group_1_sub)
    plt.hist(group_1_sub,bins=100);
    plt.hist(group_2_sub,bins=100);
    plt.hist(group_3_sub,bins=100);
```

[125.81, 125.9, 174.4, 137.9, 152.03, 130.57, 143.83, 148.58, 149.23, 132.09]



```
for group_x in [group_1_sub,group_2_sub, group_3_sub]:
    #Check the normality
    shapiro_test = stats.shapiro(group_x)
    if(shapiro_test.pvalue > alpha):
        print(" Normal distribution accepted" + f" ({shapiro_test.pvalue})")
    else:
        print(" Normal distribution rejected")

#Check the variance
levene = stats.levene(group_1_sub, group_2_sub,group_3_sub, center='mean')
if(levene.pvalue > alpha):
    print(" Homegeinity of variance accepted" + f" ({levene.pvalue})")
else:
    print(" Homegeinity of variance rejected")
```

Normal distribution accepted (0.2532499204145696) Normal distribution accepted (0.6629100671090086) Normal distribution accepted (0.4633562895813254) Homegeinity of variance accepted (0.7233546183413981)

## 1.0.2 Appliying T-Test on the data

Once the conditions are verifyed, we can process a t-test on the vertical and horizontal force data. Comparing the data to AR1, which is the reference.

#### 1.0.3 Figure 1: Vertical penetration AR1 AR2.5 AR4 AR1asym

```
[]: group AR1 = [7.0503125, 9.3308125, 7.895068750000001]
     group_AR1_asym = [10.5254625, 10.33055, 9.971512500000003]
     group AR25 = [4.971775000000001, 4.30201875, 4.914106250000001]
     group_AR4 = [3.1286, 2.767200000000008, 2.5889250000000006]
     print("AR1 vs AR1 asym")
     result = stats.ttest_ind(group_AR1, group_AR1_asym, alternative='two-sided')
     if result.pvalue > alpha:
         print("HO: The two independent samples have identical average (expected) ⊔
      ⇔values."+ f" ({result.pvalue})")
     else:
         print("H1: The means of the distributions underlying the samples are⊔

unequal."+ f" ({result.pvalue})")
     print("AR1 vs AR25")
     result = stats.ttest_ind(group_AR1, group_AR25, alternative='two-sided')
     if result.pvalue > alpha:
         print("HO: The two independent samples have identical average (expected) ⊔
      →values."+ f" ({result.pvalue})")
     else:
         print("H1: The means of the distributions underlying the samples are⊔
      →unequal."+ f" ({result.pvalue})")
     print("AR1 vs AR4")
     result = stats.ttest_ind(group_AR1, group_AR4, alternative='two-sided')
     if result.pvalue > alpha:
         print("HO: The two independent samples have identical average (expected) ⊔

yalues."+ f" ({result.pvalue})")
     else:
         print("H1: The means of the distributions underlying the samples are⊔
      →unequal."+ f" ({result.pvalue})")
    AR1 vs AR1 asym
    H1: The means of the distributions underlying the samples are unequal.
    (0.03330004605113471)
    AR1 vs AR25
    H1: The means of the distributions underlying the samples are unequal.
    (0.008592803303155155)
    AR1 vs AR4
    H1: The means of the distributions underlying the samples are unequal.
    (0.0015366260089722042)
```

#### 1.0.4 Figure 2: Horizontal penetration

```
[]: depths = [120, 220, 320]
     group_AR1 = [[4.478293750000001, 3.85711875, 4.165350000000001, 3.7627, 2.
      0638625000000004, 2.5025312499999997
                  [5.79563125, 3.2915750000000004, 3.23479375, 3.6237749999999997],
                   [14.488743750000001, 5.205331250000001, 13.42079375, 7.6122, 8.
      →576387500000001, 7.5819624999999995]]
     group AR25 = [[3.7700125, 3.4090937500000003, 3.9712187500000002, 2.94235, 2.
      →234912499999996, 2.28149375, 2.7633875000000003],
                  [5.092081249999999, 2.9481375, 2.3913375, 3.016124999999999],
                 [13.544818750000001, 5.876668749999999, 4.24003125, 4.
      →8320187500000005, 5.6397125]]
     group AR4 = [[3.721174999999997, 4.508993749999999, 4.787049999999999, 2.
      △8526875, 2.0131437500000002, 2.841612499999999, 2.1451625],
                 [6.5362937500000005, 4.2832375, 4.519093750000001, 4.
      -265725000000001, 6.7435, 3.084275000000003, 2.918325000000003, 3.
      45413937500000006, 5.81105, 3.0568437499999996, 3.130125, 3.4471562500000004],
                 [11.30195, 2.8212375000000005, 4.3357375000000005, 3.
      →7234812500000003]
     for i,d in enumerate(depths):
        print("AR1 vs AR25 "+ str(d))
        result = stats.ttest_ind(group_AR1[i], group_AR25[i],__
      →alternative='two-sided',equal_var=False)
         if result.pvalue > alpha:
            print("\tHO: The two independent samples have identical average_
      →(expected) values.")
        else:
            print("\tH1: The means of the distributions underlying the samples are⊔

unequal.")

        print("\t",result.pvalue)
        print("AR1 vs AR4 "+ str(d))
        result = stats.ttest_ind(group_AR1[i], group_AR4[i],__
      →alternative='two-sided',equal_var=False)
         if result.pvalue > alpha:
            print("\tHO: The two independent samples have identical average_
      ⇔(expected) values.")
            print("\tH1: The means of the distributions underlying the samples are⊔

unequal.")

        print("\t",result.pvalue)
        print("----")
```

```
print("Comparing same tips at different depths")
print("AR1 1 vs 2")
result = stats.ttest_ind(group_AR1[0], group_AR1[1],__
 ⇔alternative='two-sided',equal_var=False)
if result.pvalue > alpha:
   print("\tHO: The two independent samples have identical average (expected),
⇔values.")
else:
   print("\tH1: The means of the distributions underlying the samples are⊔

unequal.")

print("\t",result.pvalue)
print("AR1 2 vs 3")
result = stats.ttest_ind(group_AR1[1], group_AR1[2],_
 ⇔alternative='two-sided',equal_var=False)
if result.pvalue > alpha:
   print("\tHO: The two independent samples have identical average (expected)
⇔values.")
else:
   print("\tH1: The means of the distributions underlying the samples are⊔

unequal.")

print("\t",result.pvalue)
print("----")
print("AR25 1 vs 2")
result = stats.ttest_ind(group_AR25[0], group_AR25[1],__
 ⇔alternative='two-sided',equal_var=False)
if result.pvalue > alpha:
   print("\tHO: The two independent samples have identical average (expected)
 ⇔values.")
else:
   print("\tH1: The means of the distributions underlying the samples are⊔
ounequal.")
print("\t",result.pvalue)
print("AR25 2 vs 3")
result = stats.ttest_ind(group_AR25[1], group_AR25[2],__
 ⇔alternative='two-sided',equal_var=False)
if result.pvalue > alpha:
   print("\tHO: The two independent samples have identical average (expected) ⊔
⇔values.")
else:
   print("\tH1: The means of the distributions underlying the samples are⊔

unequal.")

print("\t",result.pvalue)
print("----")
print("AR4 1 vs 2")
```

```
result = stats.ttest_ind(group_AR4[0], group_AR4[1],_
 ⇔alternative='two-sided',equal_var=False)
if result.pvalue > alpha:
    print("\tHO: The two independent samples have identical average (expected) ⊔
 ⇔values.")
else:
    print("\tH1: The means of the distributions underlying the samples are⊔

unequal.")

print("\t",result.pvalue)
print("AR4 2 vs 3")
result = stats.ttest_ind(group_AR4[1], group_AR4[2],__
 ⇔alternative='two-sided',equal_var=False)
if result.pvalue > alpha:
    print("\tHO: The two independent samples have identical average (expected) ⊔
 ⇔values.")
else:
    print("\tH1: The means of the distributions underlying the samples are⊔

unequal.")

print("\t",result.pvalue)
print("----")
AR1 vs AR25 120
       HO: The two independent samples have identical average (expected)
values.
         0.3985513059435344
AR1 vs AR4 120
       HO: The two independent samples have identical average (expected)
values.
        0.7275363084218764
AR1 vs AR25 220
       HO: The two independent samples have identical average (expected)
values.
         0.4904385586893891
AR1 vs AR4 220
       HO: The two independent samples have identical average (expected)
values.
        0.7028498930405361
AR1 vs AR25 320
       HO: The two independent samples have identical average (expected)
values.
        0.27317721245430543
AR1 vs AR4 320
        HO: The two independent samples have identical average (expected)
values.
         0.15728336091132192
```

```
Comparing same tips at different depths
    AR1 1 vs 2
            HO: The two independent samples have identical average (expected)
    values.
             0.5069928997827517
    AR1 2 vs 3
            H1: The means of the distributions underlying the samples are unequal.
             0.012651297225629864
    AR25 1 vs 2
            HO: The two independent samples have identical average (expected)
    values.
             0.6575911936308831
    AR25 2 vs 3
            HO: The two independent samples have identical average (expected)
    values.
             0.1137917055638882
    AR4 1 vs 2
            HO: The two independent samples have identical average (expected)
    values.
             0.09900805583446837
    AR4 2 vs 3
            HO: The two independent samples have identical average (expected)
    values.
             0.5650679374616405
    _____
    Compare asymetric orientation
[]: group_AR1 = [4.478293750000001, 3.85711875, 4.165350000000001, 3.7627, 2.
     →0638625000000004, 2.5025312499999997]
     group_AR1sym_UP = [4.56710625, 4.630962500000001, 4.4870937500000005, 2.
     95395625, 3.7320312499999995, 2.712149999999994, 2.56056875, 2.
     →8769687499999996, 2.8425437500000004, 1.945124999999998]
     group_AR1sym_DOWN = [2.740825, 1.840125, 1.76305625]
     print("AR1 vs DOWN")
     result = stats.ttest_ind(group_AR1, group_AR1sym_DOWN,_
      →alternative='two-sided',equal_var=False)
     if result.pvalue > alpha:
         print("\tHO: The two independent samples have identical average (expected) ⊔
      →values. " + str(result.pvalue))
         print("\tH1: The means of the distributions underlying the samples are⊔
      →unequal. " + str(result.pvalue))
```

```
print("AR1 vs UP")
result = stats.ttest_ind(group_AR1, group_AR1sym_UP,_
 ⇔alternative='two-sided',equal_var=False)
if result.pvalue > alpha:
   print("\tHO: The two indeppendent samples have identical average (expected) ∪
 ⇔values. " + str(result.pvalue))
   print("\tH1: The means of the distributions underlying the samples are⊔
 →unequal. " + str(result.pvalue))
print("UP vs DOWN")
result = stats.ttest_ind(group_AR1sym_UP,group_AR1sym_DOWN,__
 ⇔alternative='two-sided',equal_var=False)
if result.pvalue > alpha:
   print("\tHO: The two independent samples have identical average (expected)

¬values. " + str(result.pvalue))
else:
   print("\tH1: The means of the distributions underlying the samples are⊔
 →unequal. " + str(result.pvalue))
```

AR1 vs DOWN

 ${\rm H1:}$  The means of the distributions underlying the samples are unequal. 0.03237051806118191

AR1 vs UP

 $\,$  H0: The two indeppendent samples have identical average (expected) values. 0.7820231117277838

UP vs DOWN

 $\mbox{H1:}$  The means of the distributions underlying the samples are unequal. 0.03024950164915398

# 2 Active Strategies

#### 2.0.1 Air blowing system

```
print("\tH1: The means of the distributions underlying the samples are \cup ounequal. " + str(result.pvalue))
```

AR1 vs AIR

 $\,$  HO: The two independent samples have identical average (expected) values. 0.6578573459461617

## 2.0.2 Vacuuming system

AR1 vs VACUUM

 $\,$  H1: The means of the distributions underlying the samples are unequal. 0.014782227276510938