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# Evaluation of the Influence of Cross-connections Accounting in the Simplified Mathematical Model of the Quadrotor Motion in Three-dimensional Space

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**Abstract**

Nowadays unmanned aerial vehicles are in active development and are used in many areas of human activity. One of the main problems in their application is the description of their motion with a given level of accuracy. The use of nonlinear systems of high-order differential equations is associated with the complexity, and often the inability to obtain an analytical solution of the problem of transferring the drone to a given state. Many authors work in the direction of simplifying the mathematical model of the quadrotor motion, discarding the influence of cross-connections caused by the presence of gyroscopic momentums during the rotation of the quadrotor. In this paper we evaluate the accuracy of the description of such a model in comparison with a system of nonlinear differential equations, taking into account the presence of cross-connections and perturbations in the motion of the quadrotor. Conclusions about the need to take into account these factors in the modeling process due to the significant values of the resulting mismatches while using a simplified model are made.



*Keywords:* mathematical model; quadrotor; UAV; control; cross-connection; mismatch.

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## 1. Introduction

Areas of development of science, technology and technology related to unmanned aerial vehicles (UAV) are promising. Models describing the motion of multirotor UAV are particularly relevant in connection with the active development and application of this type of devices.

Most of the authors attempt to describe the motion of UAV using a system of linearized equations, because the analytical synthesis of the control system is difficult for nonlinear equations.

Description of the mathematical model of UAV motion using Newton-Euler equations with cross-connections is presented in [1-4]. With the help of generalized coordinates and the Lagrange method, a mathematical model of UAV motion is formed in [5]. Linearized and simplified models of UAV motion are the basis of synthesis of regulators and filters: LQR-regulators [6], LQG-regulators [7], Kalman filter [8], L1-optimization [9]; control with sliding mode [1,10]. In [11] the synthesis of regulators by feedback linearization is considered. In [12] the problem of synthesis of quadrotor control is solved by one of the symbolic regression methods (variational analytical programming method [13]).

The purpose of this study was to develop models that take into account the effect of cross-connections and the evaluation of this impact on the accuracy of the model. This work may be useful for specialists in various fields describing the motion of the quadrotor, as well as the synthesis of its control system.

## 2. The mathematical model of quadrotor motion in three-dimensional space with respect to perturbations and cross connections

Fig. 1 represents the relative positions of the normal ground coordinate system $O_0x_gy_gz_g$ and the associated coordinate system *Oxyz*, the positive direction of reference of yaw angle $\psi$, pitch angle $\theta$ and roll angle $\gamma$, thrust force $P_i$ and torque $M_i$ generated by the propellers $i = \overline{1,4}$, the direction of rotation and angular velocities $\omega_i$ of the propellers.

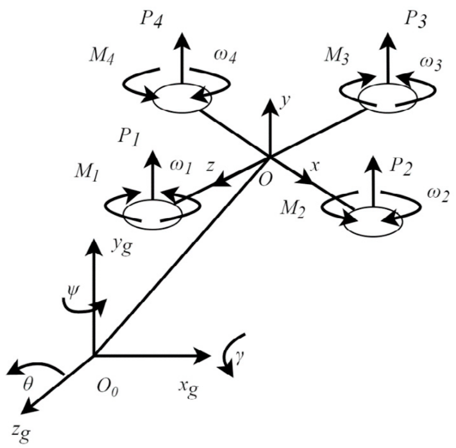


Fig. 1. Coordinate systems of quadrotor.

The transition from the associated coordinate system to the normal ground coordinate system can be performed using the transition matrix [4]:

$$\mathbf{R} = \begin{bmatrix} \cos\theta\cos\psi & -\cos\gamma\cos\psi\sin\theta + \sin\gamma\sin\psi & \sin\gamma\cos\psi\sin\theta + \cos\gamma\sin\psi \\ \sin\theta & \cos\gamma\cos\theta & -\sin\gamma\cos\theta \\ -\cos\theta\sin\psi & \cos\gamma\sin\psi\sin\theta + \sin\gamma\cos\psi & -\sin\gamma\sin\psi\sin\theta + \cos\gamma\cos\psi \end{bmatrix}. \tag{1}$$

The following assumptions are made for the modeling: the quadrotor is symmetrical; the center of mass is located at the origin of the associated coordinate system; the thrust coefficients of the propellers are equal; the torque coefficients created by the propellers are equal. Cross-connections are represented by gyroscopic momentums that occur during the rotation of the quadrotor.

The system of nonlinear differential equations in the form of Cauchy, describing the motion of the quadrotor in three-dimensional space, taking into account perturbations and cross-connections, has the following form (the development of the system of equations is given in [4]):

$$\begin{cases} \dot{x}_1 = x_2, \quad \dot{x}_2 = -\dfrac{f_x - \left(\sin x_{13} \sin x_{14} - \cos x_{13} \cos x_{14} \sin x_{15}\right) P}{m}, \\ \dot{x}_3 = x_4, \quad \dot{x}_4 = -\dfrac{\left(f_y + gm - \cos x_{13} \cos x_{15} P\right)}{m}, \\ \dot{x}_5 = x_6, \quad \dot{x}_6 = -\dfrac{f_z - \left(\cos x_{14} \sin x_{13} + \cos x_{13} \sin x_{14} \sin x_{15}\right) P}{m}, \\ \dot{x}_7 = x_8, \quad \dot{x}_8 = \dfrac{M_{R_x}}{I_x} + \dfrac{I_y - I_z}{I_x} x_{10} x_{12}, \\ \dot{x}_9 = x_{10}, \quad \dot{x}_{10} = \dfrac{M_{R_y}}{I_y} + \dfrac{-I_x + I_z}{I_Y} x_8 x_{12}, \\ \dot{x}_{11} = x_{12}, \quad \dot{x}_{12} = \dfrac{M_{R_Z}}{I_z} + \dfrac{I_x - I_y}{I_z} x_8 x_{10}, \\ \dot{x}_{13} = x_8 \cos x_{15} - x_{10} \sin x_{15}, \\ \dot{x}_{14} = x_{10} \cos x_{15} + x_8 \dfrac{\sin x_{15}}{\cos x_{13}}, \\ \dot{x}_{15} = x_{12} + x_{10} \cos x_{15} \operatorname{tg} x_{13} + x_8 \sin x_{15} \operatorname{tg} x_{13}, \end{cases} \tag{2}$$

where $x_1 = x,\ x_3 = y,\ x_5 = z$ are coordinates of the center of mass of the quadrotor;

$x_2 = \dot{x},\ x_4 = \dot{y},\ x_6 = \dot{z}$ are projections of the linear velocity vector of the center mass of quadrotor;

$x_7 = w_x,\ x_9 = w_y,\ x_{11} = w_z$ are projections of the angular velocity vector of the quadrotor in the associated coordinate system;

$x_8 = \dot{w}_x,\ x_{10} = \dot{w}_y,\ x_{12} = \dot{w}_z$ are projections of the derivative of the angular velocity vector of the quadrotor in the associated coordinate system;

$x_{13} = \gamma,\ x_{14} = \psi,\ x_{15} = \vartheta$ are roll angle, yaw angle and pitch angle of the quadrotor accordingly;

$f_x, f_y, f_z$ are projections of the force of air resistance in normal ground coordinate system;

$P$ is summary thrust of the quadrotor;

$m$ is the mass of the quadrotor;

$g$ is the acceleration of gravity;

$M_{R_x}, M_{R_y}, M_{R_z}$ are projections of the resultant momentum;

$I_x, I_y, I_z$ are axial moments of inertia of the quadrotor.

## 3. Simplified mathematical model of quadrotor motion in the tree-dimensional space without accounting of perturbations and cross-connections

A simplified mathematical model of quadrotor motion in the tree-dimensional space without accounting of perturbations and cross-connections can be written as follows:

$$\begin{cases} \ddot{x} = \dfrac{(\sin\gamma\sin\psi - \cos\gamma\cos\psi\sin\theta)P}{m}, & \ddot{\gamma} = \dfrac{M_{qx}}{I_x}, \\ \ddot{y} = \dfrac{-gm + \cos\gamma\cos\theta P}{m}, & \ddot{\psi} = \dfrac{M_{qy}}{I_y}, \\ \ddot{z} = \dfrac{(\cos\psi\sin\gamma + \cos\gamma\sin\psi\sin\theta)P}{m}, & \ddot{\theta} = \dfrac{M_{qz}}{I_z}. \end{cases} \tag{3}$$

The thrust force $P$ and control momentums for the system of equations (3) can be expressed as follows:

$$\begin{cases} P = m\dfrac{\ddot{y}+g}{\cos\gamma\cos\theta}, \\ M_{qx} = \ddot{\gamma}I_x, \\ M_{qy} = \ddot{\psi}I_y, \\ M_{qz} = \ddot{\theta}I_z. \end{cases} \tag{4}$$

The squares of the angular velocities of rotation of the propellers necessarily to create the control moments and the thrust force can be expressed as follows:

$$\begin{cases} \omega_1^2 = \dfrac{P}{4k} - \dfrac{M_{qx}}{2kl} - \dfrac{M_{qy}}{4b}, \\ \omega_2^2 = \dfrac{P}{4k} + \dfrac{M_{qz}}{2kl} + \dfrac{M_{qy}}{4b}, \\ \omega_3^2 = \dfrac{P}{4k} + \dfrac{M_{qx}}{2kl} - \dfrac{M_{qy}}{4b}, \\ \omega_4^2 = \dfrac{P}{4k} - \dfrac{M_{qz}}{2kl} - + \dfrac{M_{qy}}{4b}. \end{cases} \tag{5}$$

Thus, the problem of the quadrotor control can be reduced to the problem of determining the required thrust force and control momentums.

## 4. The simulation results of quadrotor motion

The simulation is performed for the systems of equations (2) and (3) for the subsequent analysis and comparison of quadrotor motion with and without cross-connections.

The initial data for modeling is presented in the Table 1. This data is similar to the original data in [14]. Aerodynamic resistance in the absence of disturbances is dissipative and was not taken into account in the simulation. Control acceleration through the channels of the roll $u_\gamma$, pitch $u_\psi$ and yaw $u_\theta$ are restricted $|u_\gamma| \le [a_\gamma]$, $|u_\psi| \le [a_\psi]$, $|u_\theta| \le [a_\theta]$ respectively. The desired acceleration created by the thrust force is also limited: $|P_d| \le [P_d]$.

Table 1. Initial data for the modeling.

| Parameter | Value | Dimension |
|---|---|---|
| $g$ | 9.81 | $m/s^2$ |
| $m$ | 0.468 | $kg$ |
| $l$ | 0.225 | $m$ |
| $k$ | $2.98 \cdot 10^{-6}$ | $rad$ |
| $b$ | $1.140 \cdot 10^{-7}$ | $rad$ |
| $I_M$ | $3.357 \cdot 10^{-5}$ | $kg \cdot m^2$ |
| $I_x$ | $4.856 \cdot 10^{-3}$ | $kg \cdot m^2$ |
| $I_y$ | $8.801 \cdot 10^{-3}$ | $kg \cdot m^2$ |
| $I_z$ | $4.856 \cdot 10^{-3}$ | $kg \cdot m^2$ |
| $\omega_i$ | $[300,\ 900]$ | $rad/s$ |
| $[a_\gamma]$, $[a_\psi]$, $[a_\theta]$ | 1 | $rad/s$ |
| $[u_y]$ | 19.62 | $m/s^2$ |

The desired values of pitch $\theta_d(t)$ and roll angles $\gamma_d(t)$ were changed in time as follows:

$$\theta_d(t) = \begin{cases} \Delta\theta, t < 3; \\ 0, t \geq 3, \end{cases} \quad \gamma_d(t) = \begin{cases} \Delta\gamma, t < 3; \\ 0, t \geq 3. \end{cases} \tag{6}$$

The quadrotor orientation control system is built using two PID controllers for the roll and pitch channels. For both channels were used the following coefficients: $k_P = 4,\ k_I = 0{,}1,\ k_D = 4.$ The yaw channel were not controlled.

The desired control momentums on the roll and pitch channels are determined as follows:

$$\begin{aligned} & r_\gamma(t) = k_p\varepsilon_\gamma(t) + k_d\dot{\varepsilon}_\gamma, && r_\theta(t) = k_p\varepsilon_\theta(t) + k_d\dot{\varepsilon}_\theta, \\ & \varepsilon_\gamma(t) = \gamma_d(t) - \gamma(t), && \varepsilon_\theta(t) = \theta_d(t) - \theta(t), \\ & u_\gamma = I_x r_\gamma(t), && u_\theta = I_z r_\theta(t), \\ & |u_\gamma| \leq [a_\gamma], && |u_\theta| \leq [a_\theta], \end{aligned} \tag{7}$$

at the same time $M_{qx} = u_\theta,\ M_{qz} = u_\gamma,\ M_{qy} = u_\psi.$

The motion control system of the center of mass of the quadrotor was built using a PD controller for height channel with the coefficients $k_P = 6,\ k_D = 4.5.$ The desired thrust force is calculated as follows:

$$P_d = \frac{m(u_y + g)}{\cos\gamma\cos\theta}, \quad u_{pd} = k_P\varepsilon_y + k_D\dot{\varepsilon}_y, \quad \varepsilon_y = y_d - y, \quad |P_d| \leq [P_d]. \tag{8}$$

According to the obtained desired control accelerations and thrust force, the necessary angular speeds of rotation of the propellers are calculated by (5).

Simulation of the motion of the quadrotor was performed in the software package MATLAB. As a method of numerical integration, the Euler method with a fixed step of 0,001 s was used.

Fig. 2 presents the results of a simulation of the motion of the quadrotor with (left column – a) and without (right column – b) considering cross-connections and perturbations.

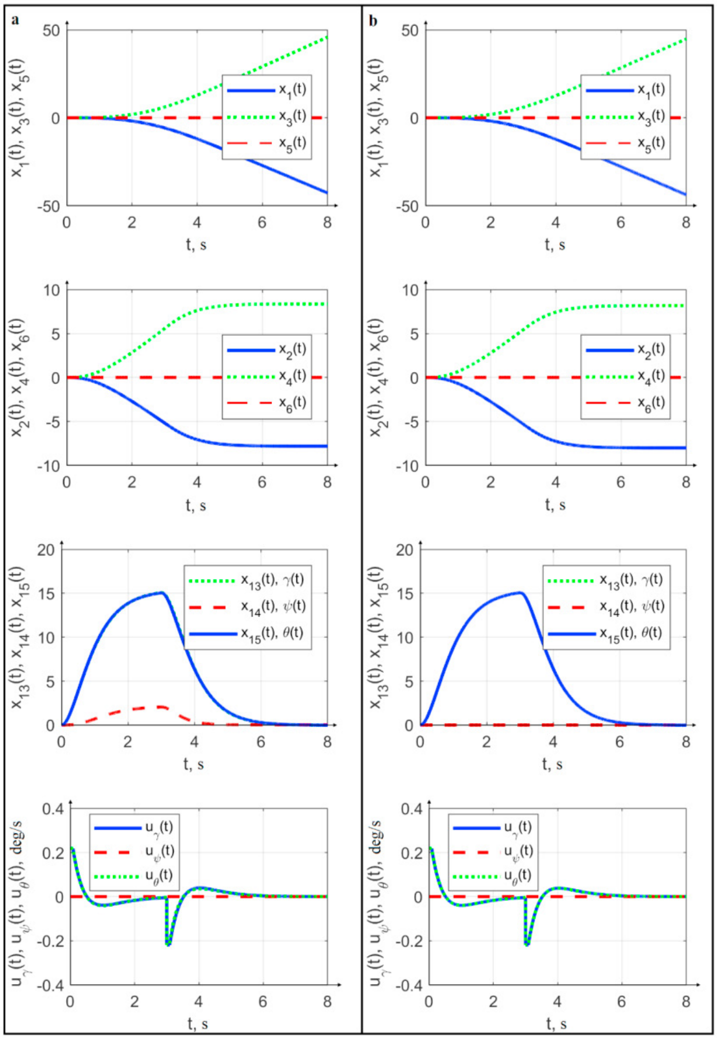


Fig. 2. Results of modeling the motion of quadrotor with (a) and without (b) accounting of the perturbations and cross-connections.

## 5. Evaluation of the effect of cross-connections on the motion of the quadrotor

Evaluation of the effect of cross-connection on the motion of the quadrotor is calculated by two criteria:
$\Delta\psi$ – the value of the yaw angle at the end of the simulation for the model described by the system of equations (2);
$\Delta\varepsilon$ – mismatch, the distance between the position of the quadrotor in the horizontal plane at the end of the simulation for the models described by the systems of equations (2) and (3), respectively.

The desired pitch angles $\Delta\theta$ change from 0 up to 25 degrees in increments of 1 degree.

Fig. 3 demonstrates the dependence of the criterion value $\Delta\psi$ on $\Delta\theta$, the dependence of the criterion value $\Delta\varepsilon$ on $\Delta\theta$ and the dependence of the criterion value $\Delta\varepsilon$ on the flight distance $\rho$ at the end of the simulation are presented. The dependencies were interpolated on the intervals between the points.

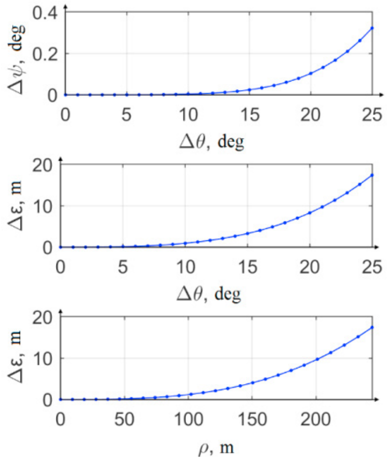


Fig. 3. Evaluation of the effect of cross-connections accounting in mathematical model on the accuracy of motion of the quadrotor.

From the analysis of Fig. 3 it follows that with an increase in the desired pitch angle $\Delta\theta$ , a non-linear growth of the yaw angle $\Delta\psi$ and a non-linear growth of the distance mismatch $\Delta\varepsilon$ occurs. At the same time, when the desired pitch angle $\Delta\theta$ is more than 10 degrees, distance mismatch $\Delta\varepsilon$ exceeds 1 m at a distance of 102.1 m from the point (0;0) (at 11 degrees – 1.257 m).

## 6. Conclusion

Two mathematical models of quadrotor motion in three-dimensional space as a control object with and without taking into account disturbances and cross-connections are developed. The linearized mathematical model of quadrotor motion in the vertical plane is obtained. The modeling of quadrotor motion in three-dimensional space with and without cross-connections accounting is performed and the impact of these connections is evaluated.

Based on the simulation results, a recommendation can be made to use the obtained system taking into account cross-connections, since this system increases the accuracy of the description of the quadrotor motion. Without taking into account the influence of cross-connections, the solution of the problem of accurate transfer of the quadrotor to a given state is significantly complicated due to the nonlinear growth of the mismatch.

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