Observer based Sliding Mode Control Strategy for Vertical Take-Off and Landing (VTOL) Aircraft System

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***Abstract*—In this paper, a second order sliding mode (SOSM) controller with asymptotic convergence is proposed for stabilizing a vertical take-off and landing aircraft (VTOL) system affected by both matched and mismatched types of uncertainties. The proposed controller uses an adaptive gain tuning mechanism to design the sliding surface. Experimental results obtained by applying the adaptive SOSM to a laboratory model of VTOL system demonstrates the effectiveness of the proposed control scheme.**

***Index Terms*—Vertical take-off and landing aircraft (VTOL), second order sliding mode (SOSM), adaptive sliding surface.**

I. Introduction

The vertical take-off and landing (VTOL) aircraft is a highly complex nonlinear system whose aerodynamic parameters vary considerably during the flight. Real world examples of the VTOL aircraft system are aerospace vehicles like helicopters, rockets, balloons and harrier jets. All aerospace vehicles are difficult to model due to their changing aerodynamic parameters and environmental behavior during flight. Fig.1 shows the typical coordinate system for a VTOL aircraft in the vertical plane. VTOL aircraft system is a an uncertain system which is affected by both matched and mismatched types of uncertainties. For controlling such uncertain systems, the sliding mode controller (SMC) [2] has been applied widely because of its simplicity and inherent robustness. However, chattering in the control input is an undesired phenomenon in the conventional first order SMCs. Furthermore, the design prerequisite of advance knowledge about the upper bound of the uncertainty is a stringent condition in the case of conventional SMCs and is often difficult to meet in practice.
In this paper, a second order sliding mode (SOSM) controller is proposed to successfully mitigate chattering present in the control input of conventional first order sliding mode controllers. An adaptive tuning mechanism is employed in the proposed SOSM controller for estimating the upper bound of the system uncertainty. Prior knowledge about the upper bound of the system uncertainty, which is the design prerequisite of conventional first order sliding mode controllers, is eliminated by using this adaptive method. Moreover, the adaptive gain tuning mechanism also ensures that the gain is not overestimated with respect to the actual unknown value of the uncertainty.
This paper is organized as follows:
The design procedure of adaptive second order sliding mode controller is explained in Section II. The laboratory prototype of a single degree of freedom (DOF) VTOL system alongwith its mathematical model are described in Section III. Experimental results are presented in Section IV. Conclusions are drawn in Section V.

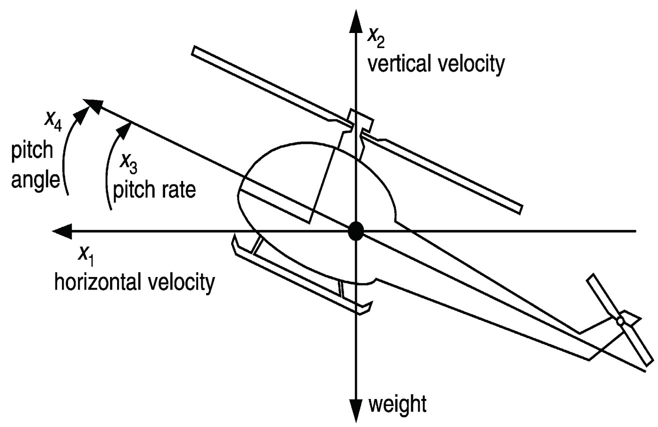

Fig. 1. A typical sketch of a VTOL aircraft in the vertical plane [1].

II. Design of Adaptive SOSM controller

Let us consider the following uncertain system

$$\dot{x}(t) = Ax(t) + Bu(t) + f(x,t) \tag{1}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input and the continuous function $f(x,t)$ represents matched and mismatched uncertainties together. Let us assume that the above system is in the regular form requiring no

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transformation. Thus the system can be written as,

$$\begin{aligned}\dot{x}_1(t) &= a_{11}x_1(t) + a_{12}x_2(t) + f_u(x,t) \\ \dot{x}_2(t) &= a_{21}x_1(t) + a_{22}x_2(t) + B_2u(t) + B_2f_m(x,t)\end{aligned} \quad (2)$$

where $x_1(t) \in R^{n-1}$, $x_2(t) \in R$, $f_u(x,t)$ is the mismatched perturbation and $f_m(x,t)$ is the matched one.
Let us consider the sliding surface given by

$$\begin{aligned} s &= ce \\ &= c_1e_1 + c_2e_2 \end{aligned} \quad (3)$$

where $e = x - x_{ref}, e_1 = x_1 - x_{1ref}, e_2 = x_2 - x_{2ref}, x_{ref} = [x_{1ref} \quad x_{2ref}]^T$, c_1, c_2 are matrices with proper dimension.

A. Stability during the sliding mode

During the sliding mode $s = 0$ and therefore (3) can be written as,

$$\begin{aligned} s &= cx = c_1e_1 + c_2e_2 = 0 \\ or, e_2 &= -c_2^{-1}c_1e_1 \end{aligned} \quad (4)$$

Using (4) in the state space model (2) yields,

$$\begin{aligned} \dot{x}_1 &= a_{11}e_1 - a_{12}c_2^{-1}c_1e_1 + f_u \\ &= (a_{11} - a_{12}c_2^{-1}c_1)e_1 + f_u \\ &= a_se_1 + f_u \end{aligned} \quad (5)$$

where $a_s = (a_{11} - a_{12}c_2^{-1}c_1)$. Furthermore, f_u is the uncertainty satisfying the condition $||f_u|| \leq \tilde{\lambda}||e_1||$ [3] where $\tilde{\lambda}$ is a bounded positive constant. It is to be noted that c_1 and c_2 are designed in such a way that the eigenvalues of $(a_{11} - a_{12}c_2^{-1}c_1)$ lie in the left half of the s-plane and there exits a positive definite matrix P [4] such that

$$a_s^TP + Pa_s = -R \quad (6)$$

where R is also a positive definite matrix. Let a Lyapunov function for the system be defined as $V_2 = e_1^TPe_1$. The time derivative of V_2 is obtained as,

$$\begin{aligned} \dot{V}_2 &= \dot{e}_1^TPe_1 + e_1^TP\dot{e}_1 \\ &= e_1^Ta_s^TPe_1 + f_u^TPe_1 + e_1^TPa_se_1 + e_1^TPf_u \\ &= e_1^T(a_s^TP + Pa_s)e_1 + f_u^TPe_1 + e_1^TPf_u \\ &= -e_1^TRe_1 + 2e_1^TPf_u \end{aligned} \quad (7)$$

It is known that [4], [5],

$$e_1^TRe_1 \geq \lambda_{min}(R)e_1^Te_1 = \lambda_{min}(R)||e_1||^2 \quad (8)$$

where λ_{min} is the minimum eigen value and so,

$$\dot{V}_2 \leq -\lambda_{min}(R)||e_1||^2 + 2e_1^TPf_u \quad (9)$$

If there exists a bounded positive constant $\tilde{\lambda}$ such that $\tilde{\lambda} < 0.5\lambda_{min}(R)/||P||$, then

$$2e_1^TPf_u \leq 2\tilde{\lambda}||P||||e_1||^2 < \lambda_{min}(R)||e_1||^2 \quad (10)$$

and

$$\dot{V}_2 \leq -\lambda_{min}(R)||e_1||^2 + 2e_1^TPf_u < 0 \quad (11)$$

Hence the stability of the sliding mode is proved.

B. Design of the control law

The time derivative of sliding surface

$$\begin{aligned} \dot{s} &= \frac{d}{dt}(ce) \\ &= c\dot{e} \\ &= c(Ax + Bu + f(x,t)) - c\dot{x}_{ref} \end{aligned} \quad (12)$$

$$\begin{aligned} \ddot{s} &= cA\dot{x} + cB\dot{u} + c\dot{f}(x,t) - c\ddot{x}_{ref} \\ &= cA^2x + cABu + cB\dot{u} + (cAf(x,t) \\ &\quad + c\dot{f}(x,t)) - c\ddot{x}_ref \end{aligned} \quad (13)$$

Assuming $s = y_1(x)$ and $\dot{s} = y_2(x)$, the system dynamics can be written as,

$$\begin{aligned} \dot{y}_1(x) &= y_2(x) \\ \dot{y}_2(x) &= \Phi[x,u] + \Psi[x]v \end{aligned} \quad (14)$$

where $v = \dot{u}$ and $\Phi[x,u]$ collects all the uncertain terms not involving $\dot{u}$. So a sliding mode controller for the above system can be designed to keep the system trajectories on the sliding manifold using the control input $\dot{u} = v$. Let the sliding function be considered as,

$$\sigma(x) = y_2(x) + \alpha y_1(x) \quad (15)$$

where α is a positive constant. Differentiating (15) yields,

$$\dot{\sigma}(x) = \dot{y}_2(x) + \alpha\dot{y}_1(x) \quad (16)$$

Using (12), (13) and (16) yields,

$$\begin{aligned} \dot{\sigma}(x) &= cA^2x + cABu + cB\dot{u} + (cAf(x,t) + c\dot{f}(x,t)) \\ &+ \alpha c(Ax + Bu + f(x,t)) - c\ddot{x}_{ref} - \alpha c\dot{x}_{ref} \\ &= cA^2x + \alpha cAx + (cAB + \alpha cB)u + cB\dot{u} \\ &+ c(Af(x,t) + \dot{f}(x,t) + \alpha Bf(x,t)) \\ &\quad -c\ddot{x}_{ref} - \alpha c\dot{x}_{ref} \end{aligned} \quad (17)$$

Using the constant plus proportional reaching law gives rise to,

$$\dot{\sigma}(x) = -k_1\sigma - k_2sign(\sigma) \quad (18)$$

Using (17) and (18), the control law is obtained as,

$$\begin{aligned} \dot{u} &= -(cB)^{-1}((cA^2 + \alpha cA)x + (cAB + \alpha cB)u \\ &+ k_1\sigma + k_2sign(\sigma) - c\ddot{x}_{ref} - \alpha c\dot{x}_{ref}) \end{aligned} \quad (19)$$

where $k_1 \geq 0$ and $k_2 > c(Af(x,t) + \dot{f}(x,t) + \alpha f(x,t)) = c\nabla F = Q$ to satisfy the reaching law condition $\sigma\dot{\sigma} < -\eta|\sigma|$.
Proof : A Lyapunov function is defined as $V = \frac{1}{2}\sigma^2$ and using the control law it is easy to find that,

$$\begin{aligned} \dot{V} &= \sigma\dot{\sigma} \\ &= \sigma[c\nabla F - k_1\sigma - k_2sign(\sigma)] \\ &= \sigma[Q - k_1\sigma - k_2sign(\sigma)] \\ &< Q|\sigma| - k_2|\sigma| < -\eta|\sigma| \end{aligned} \quad (20)$$

Clearly, (20) implies that if $\eta > 0, k_1 \geq 0$ and $k_2 > Q$, control law (19) forces the sliding manifold σ to zero in finite time.

C. Design of the adaptive tuning law

In practice, the uncertain term ∇F is often difficult to know. Hence an adaptive tuning law is designed to determine k_2. So (18) can be written as

$$\dot{\sigma}(x) = -k_1\sigma - \hat{T}sign(\sigma) \tag{21}$$

where $\hat{T}$ estimates the value of k_2.

Using (19) and (21), the control law is obtained as,

$$\begin{aligned}\dot{u} \quad = \quad & -(cB)^{-1}((cA^2+\alpha cA)x + (cAB+\alpha cB)u \\ & +k_1\sigma + \hat{T}sign(\sigma) - c\ddot{x}_{ref} - \alpha c\dot{x}_{ref})\end{aligned} \tag{22}$$

Defining the adaptation error as $\tilde{T} = \hat{T} - T$, the parameter $\hat{T}$ is estimated by using the adaptation law [6]–[8]

$$\dot{\hat{T}} = \frac{1}{\gamma}|\sigma| \tag{23}$$

where γ is a positive constant. A Lyapunov function is defined as $V = \frac{1}{2}\sigma^2 + \frac{1}{2}\gamma\tilde{T}^2$ and it is easy to find that,

$$\begin{aligned}\dot{V} \quad = \quad & \sigma\dot{\sigma} + \gamma\tilde{T}\dot{\hat{T}} \\ & = \sigma[c\nabla F - k_1\sigma - \hat{T}sign(\sigma)] + \gamma(\hat{T} - T)\dot{\hat{T}} \\ & = \sigma[Q - k_1\sigma - \hat{T}sign(\sigma)] + \gamma(\hat{T} - T)\dot{\hat{T}} \\ & < Q|\sigma| - T|\sigma| < -\eta|\sigma|\end{aligned} \tag{24}$$

Where $\eta > 0$, thus the above inequality holds if $\dot{\hat{T}} = \frac{1}{\gamma}|\sigma|$ and $T > Q$. This ensures the finite time convergence of σ and guarantees that the states converges asymptotically.

Practically, $|\sigma|$ cannot become exactly zero in finite time and thus the adaptive parameter $\dot{\hat{T}}$ may increase boundlessly. A simple way of overcoming this disadvantage is to modify the adaptive tuning law (23) by using the dead zone technique [2] as

$$\dot{\hat{T}} = \begin{cases} \frac{1}{\gamma}|\sigma|, & |\sigma| \geq \epsilon \\ 0, & |\sigma| < \epsilon \end{cases} \tag{25}$$

where ϵ is a small positive constant.

As is evident from (19), $\dot{u}$ is free from any discontinuous part and so integration of $\dot{u}$ yields a continuous control law u. Hence the undesired high frequency chattering of the control signal is eliminated.

Thus the above adaptive SOSM control method offers two main advantages. Firstly, the knowledge about the upper bound of the system uncertainties is not required. Secondly, the chattering in the control input is removed.

III. 1 DOF VTOL SYSTEM

The vertical take-off and landing (VTOL) system [9] with one degree of freedom (pitch motion) is considered here for practical demonstration of the proposed adaptive SOSM controller. Fig. 2 shows the laboratory set-up QNET VTOL which basically consists of a variable speed fan with a safety guard mounted on an arm. An adjustable counterweight is attached to the other end of the arm. This counterweight allows position of the weight to be changed which in turn affects the system dynamics. A rotary encoder shaft to measure the VTOL pitch position is attached to an arm assembly.

The nominal values of the VTOL parameters are given in Table I [9].

Fig. 2. QNET VTOL trainer on ELVIS II

TABLE I
PARAMETERS OF 1DOF VTOL SYSTEM

Variable name	Description	Values
L_m	Motor inductance	53.8mh
R_m	Motor resistance	3 Ω
k_t	Torque thrust constant	0.0108Nm/A
j	Moment of inertia	0.00347$kg-m^2$
b_v	Viscous damping	0.002Nms/rad
k	Stiffness constant	0.0373Nm/rad

Using the parameter values given above, the transfer function for the 1 DOF VTOL [9] is obtained as,

$$G(s) = \frac{\theta(s)}{V_m(s)} = \frac{57.78}{s^2 + 0.576s + 10.7} \tag{26}$$

where θ is the pitch angle and V_m is the motor voltage. Accordingly, the state space model for the above system in presence of matched uncertainty can be described as,

$$\begin{aligned}\begin{bmatrix}\dot{x}_1 \\ \dot{x}_2\end{bmatrix} &= \begin{bmatrix}0 & 1 \\ -10.7 & -0.576\end{bmatrix}\begin{bmatrix}x_1 \\ x_2\end{bmatrix} \\ &+ \begin{bmatrix}0 \\ 57.78\end{bmatrix}u + \begin{bmatrix}0 \\ 57.78\end{bmatrix}f_m(x,t) \\ y &= \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1 \\ x_2\end{bmatrix}\end{aligned} \tag{27}$$

where x_1 is the pitch angle and x_2 is the angular velocity. The uncertainty is chosen as,

$$f_m(x,t) = 0.25\sin(0.1x_1) \tag{28}$$

The control law (22) is applied to the above system where, $\alpha = 0.25$, $k_1 = 5$, $\gamma = 2$ and

$$c = \begin{bmatrix}5 & 1\end{bmatrix}. \tag{29}$$

The design prerequisite of the SOSM controller is the complete knowledge about the state vector which is practically difficult to get. Hence unavailable states of the VTOL are estimated by using the extended state observer (ESO) [10]. The ESO can estimate the uncertainties along with the states of the system. Unlike traditional (linear or nonlinear) observers, the ESO estimates the uncertainties, unmodeled dynamics and external disturbances as extended states of the original system [11].

A. Linear extended state observer (LESO) design

The idea of LESO is explained in the following single input single output (SISO) system,

$$\begin{aligned} x^n(t) &= f(x^{n-1}(t), x^{n-2}(t), ..., x(t), w(t), t) + bu(t) \\ y &= x(t) \end{aligned} \tag{30}$$

where x is the nth order state vector, y is the output, u is the input, b is a constant, $w(t)$ is the external disturbance, $f(\cdot)$ is an unknown function which can be viewed as the total uncertainties or disturbances, both internal and external, acting on the system. Now $h = \frac{df}{dt}$ is introduced such that if the function f is nonsmooth, h denotes the generalized derivative of $f(\cdot)$. Treating the uncertainty f as an extended state of the system 30, the equation 30 can be written in the state space form as,

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= x_3(t) \\ &. \\ &. \\ \dot{x}_{n-1}(t) &= x_n(t) \\ \dot{x}_n(t) &= x_{n+1}(t) + b_0 u \\ \dot{x}_{n+1}(t) &= h(\cdot) \end{aligned} \tag{31}$$

where $X = [x_1, x_2, ..., x_n, x_{n+1}]^T \in R^{n+1}$ represents the state of the system and b_0 is best estimate of b (30). Now, LESO for estimating both the states and the extended state for the uncertain system (30) can be obtained as follows [12]

$$\begin{aligned} \dot{z}_1 &= z_2 - \beta_1 e_1 \\ \dot{z}_2 &= z_3 - \beta_2 e_1 \\ &. \\ &. \\ \dot{z}_{n-1} &= z_n - \beta_{n-1} e_1 \\ \dot{z}_n &= z_{n+1} - \beta_n e_1 + b_0 u \\ \dot{z}_{n+1} &= -\beta_{n+1} e_1 \end{aligned} \tag{32}$$

where $Z = [z_1, z_2, ..., z_n, z_{n+1}]^T \in R^{n+1}$, $e_1 = z_1 - x_1$ and $\beta_i (i \in n+1)$ are the states of LESO, the observation error and observer gains, respectively. LESO (32) is designed to have the property, $z_i(t) \to x_i(t) (i \in n+1)$.
Writing the extended order system (31) in the state space form gives rise to,

$$\dot{X} = AX + Bu + Eh \tag{33}$$

where $X = [x_1, x_2, ..., x_n, x_{n+1}]^T$ is the state vector of the extended order system. Here A, B and E matrices are given by,

$$A = \begin{bmatrix} 0 & 1 & .... & 0 & 0 \\ 0 & 0 & 1 & .... & 0 \\ . & & & & \\ . & & & & \\ 0 & 0 & 0 & .... & 1 \\ 0 & 0 & 0 & .... & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ b_0 \\ 0 \end{bmatrix}, E = \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ 0 \\ 1 \end{bmatrix} \tag{34}$$

So (32) represents LESO for the system (33). The state space model of the LESO dynamics can be written as

$$\dot{Z} = AZ + Bu + L(y - CZ) \tag{35}$$

where $L = [\beta_1 \quad \beta_2 .... \beta_n \quad \beta_{n+1}]^T$ is the observer gain vector, y is output vector and $C = [1 \ 0 \ 0.....0]$ is the output matrix. The parameters are chosen in a special way as $s^{n+1} + \beta_1 s^n + ... + \beta_{n+1} = (s + \omega_0)^{n+1}$, where ω_0 denotes the bandwidth of the LESO (32) [13]. It is proved that if f is differentiable with respect to t and $h = \dot{f}$ is bounded, then the LESO (32) can estimate $f(t)$ with bounded error and also estimates the unknown states.

IV. Experimental Results

Experiments are carried out on the QNET VTOL Elvis II board using LABVIEW software for interfacing. Furthermore, Runge Kutta 4 algorithm with step size of 0.1 ms is used in a PC with 2.50 GHz Core I-7 processor having 4GB memory for simulation purpose. The LESO observer gain parameter is obtained as

$$L = \begin{bmatrix} 19.5 & 126.75 & 274.625 \end{bmatrix}^T \tag{36}$$

considering $\omega_0 = 6.5$. The initial state of the VTOL system

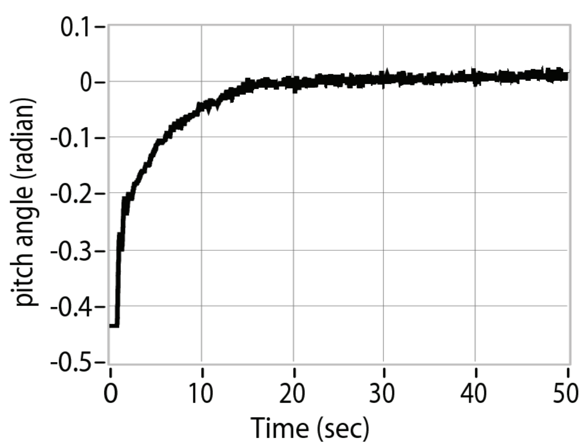

Fig. 3. Angular position (x_1) obtained by using the proposed adaptive SOSM controller

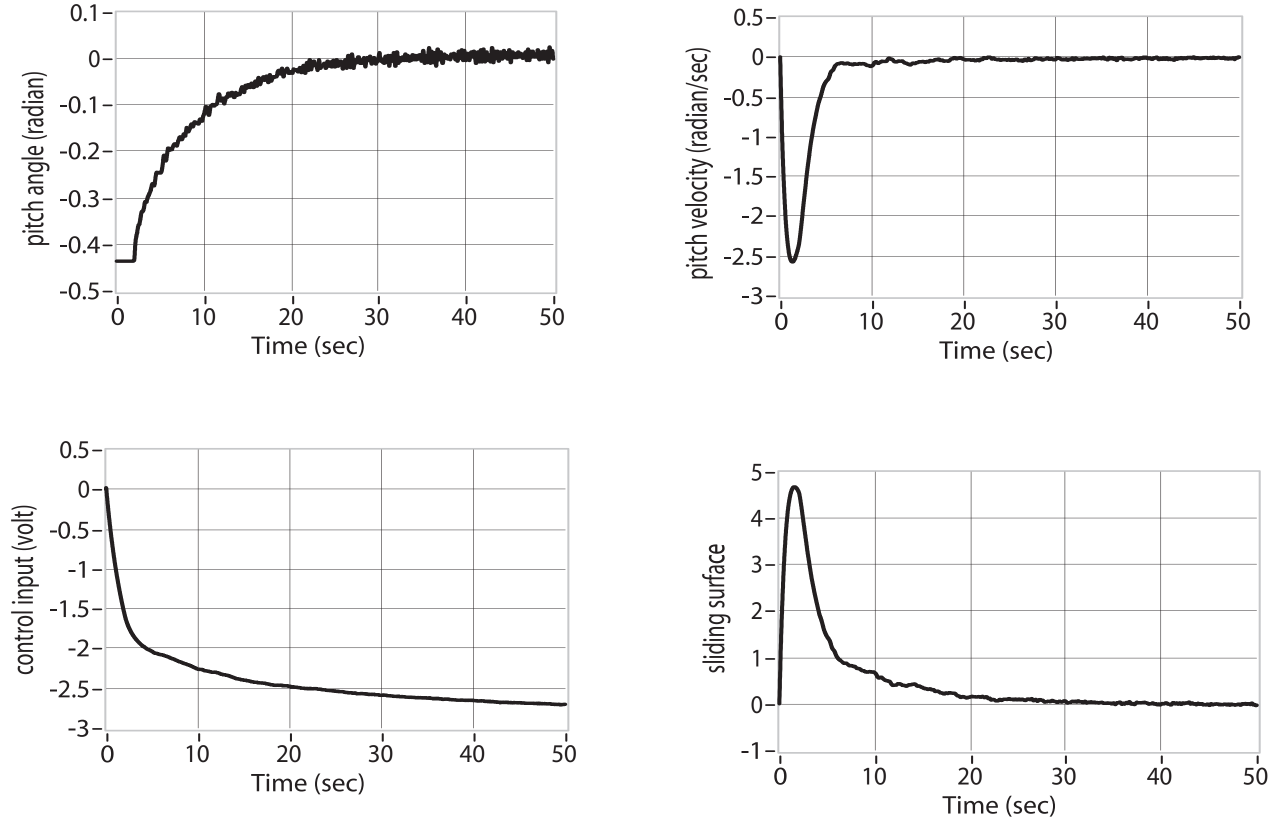

Fig. 8. Experimental results using the proposed method when parameters are changed by 25%

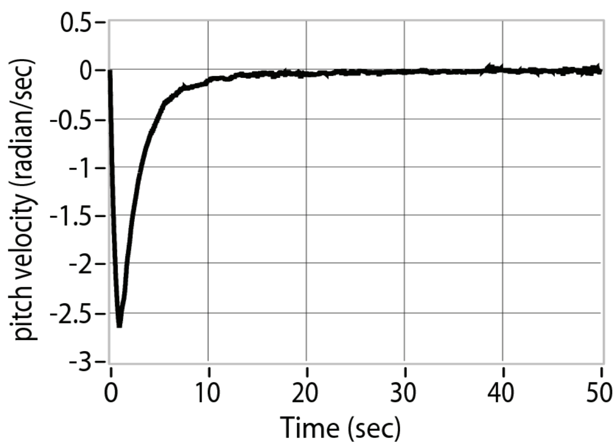

Fig. 4. Angular velocity (x_2) obtained by using the proposed adaptive SOSM controller

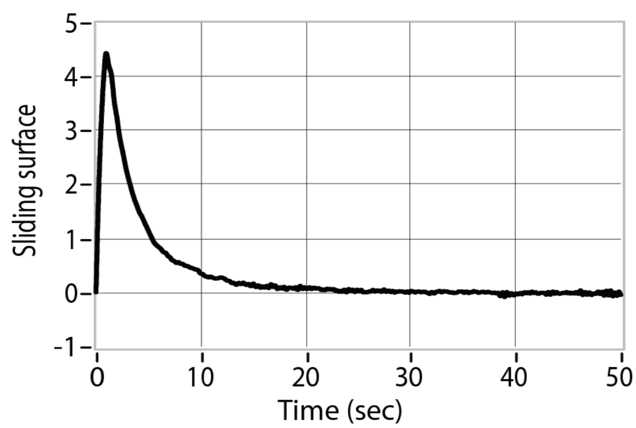

Fig. 5. Sliding surface s obtained by using the proposed adaptive SOSM controller

as well as the LESO observer is chosen as $[-0.45 \quad 0]^T$. The adaptive tuning law is designed as $\dot{\hat{T}} = 0.5|\sigma|$ with $T_0 = 0$ and the boundary layer ϵ is chosen as 0.05. The desired trajectory $x_d(t)$ to be tracked is chosen as $x_d(t) = 0$, i.e the VTOL system will have to position itself to the horizontal plane. The control signal is applied to the VTOL lab module through QNET's interfacing hardware board. In Fig. 3, the tracking performance is presented from where it is observed that the VTOL tracks the reference accurately. The angular velocity (x_2) is plotted in Fig. 4. The quick convergence of the sliding surface (s) is noted in Fig. 5. It is clearly observed from Fig

6 that the proposed control law is smooth and chattering free. The convergence of the adaptive gain is confirmed in Fig. 7.

In order to check the robustness of the proposed adaptive

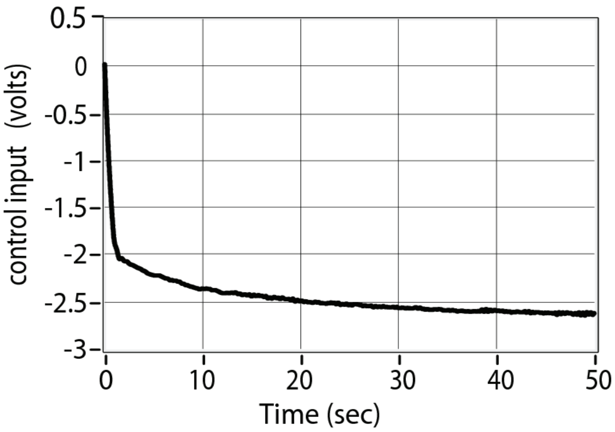

Fig. 6. Control input u obtained by using the proposed adaptive SOSM controller

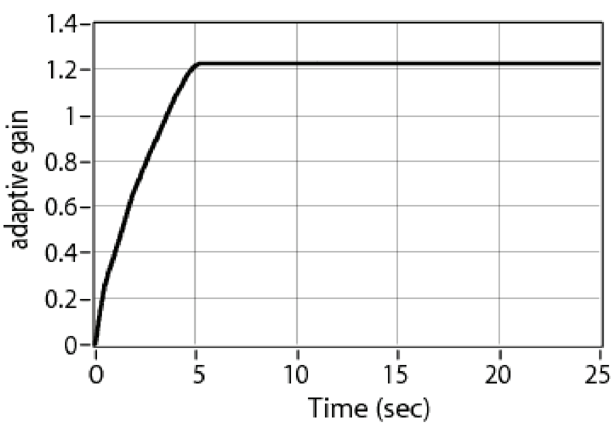

Fig. 7. Adaptive gain ($\hat{T}$) obtained by using the proposed adaptive SOSM controller

SOSM controller, the plant parameter matrices are perturbed by 25% and the new plant parameters are denoted by $\hat{A} = 1.25A$ and $\hat{B} = 1.25B$. The experiment is repeated and results are shown in Fig. 8. It is observed from Fig. 8 that the proposed adaptive SOSM controller integrated with the LESO observer still works reliably demonstrating the robustness of the proposed controller - observer pair.

V. Conclusion

In this paper a second order sliding mode (SOSM) controller with asymptotic convergence is designed by using adaptive gain tuning mechanism. The adaptive SOSM controller does not require advance knowledge about the upper bound of the uncertainty which is the design prerequisite of conventional first order sliding mode controllers. Furthermore, the proposed adaptive SOSM controller eliminates the undesired chattering in the control input. The proposed controller is applied for stabilizing a single degree of freedom vertical take-off and landing (VTOL) aircraft system affected by mismatched uncertainty. A linear extended state observer (LESO) is used to estimate the pitch velocity which is an unavailable state in the 1 DOF VTOL. Experimental results demonstrate that the proposed controller is able to bring the 1 DOF VTOL system to the equilibrium state quickly and the input signal is smooth and chattering free.

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