Control Systems Programming

Stanislav Tomashevich

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Sine model

$$u(t) = \sin t$$

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Get the first derivative

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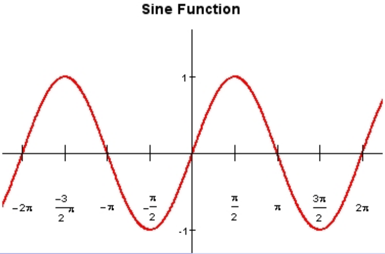

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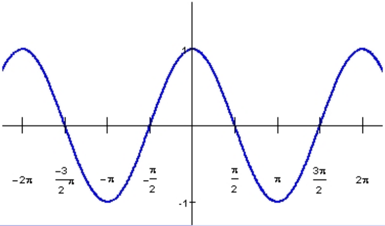
Cosine Function



Comparison of sine and cosine functions

$\sin$ $\cos$

Comparison of sine and cosine functions

sin	cos
$\frac{du}{dt^2} = -u(t)$	

Comparison of sine and cosine functions

sin	cos
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Comparison of sine and cosine functions

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How to distinguish a sine and cosine?

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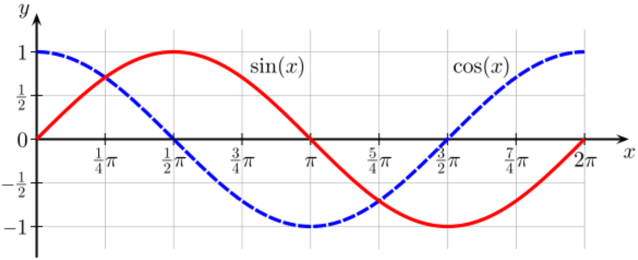

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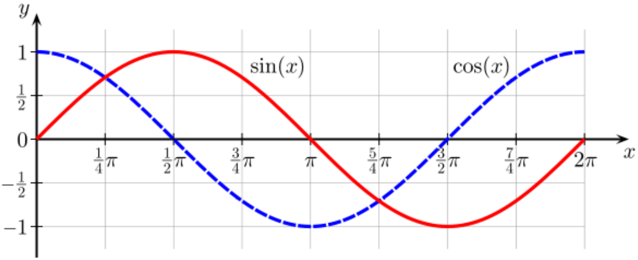

Oscillator can also have a different phase

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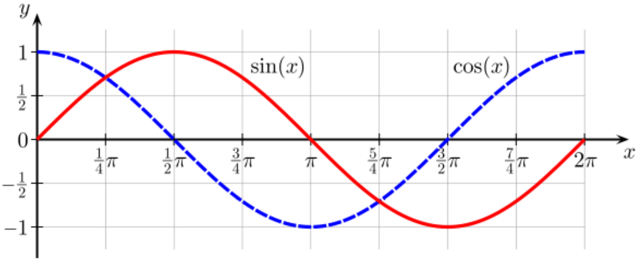

Oscillator can also have a different phase, frequency

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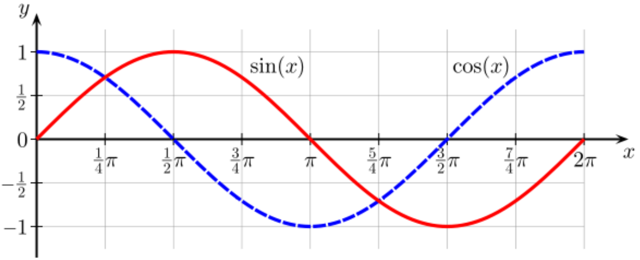

Oscillator can also have a different phase, frequency, amplitude

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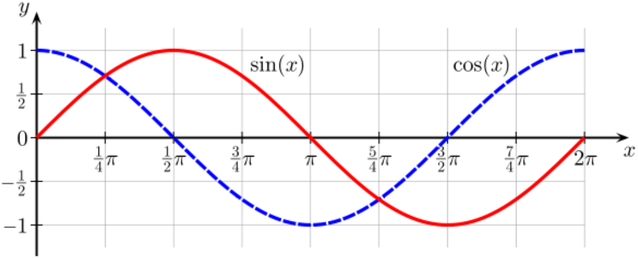

Oscillator can also have a different phase, frequency, amplitude, bias...

Generating sine trajectory

Lets generate $y(t) = \sin 2t$

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Solution of the exponent trajectories

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$$\ln|x| = -\frac{1}{\alpha}t + C$$

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$$x(t) = \pm e^{-\frac{1}{\alpha}t+C}$$

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$$y = u - \alpha\dot{y}$$

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Lets generate $y(t) = 10 - e^{-t}$

$$y = u - \alpha\dot{y}$$
$$\dot{y}(t) = e^{-t} \rightarrow y(t) = -e^{-t}$$

ADC

Analog

ADC

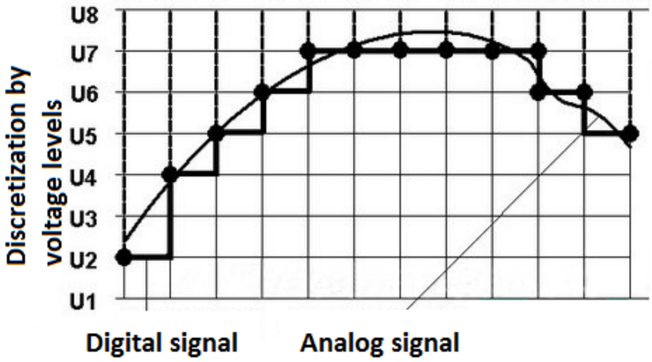
Analog Digital

ADC

Analog Digital Converter

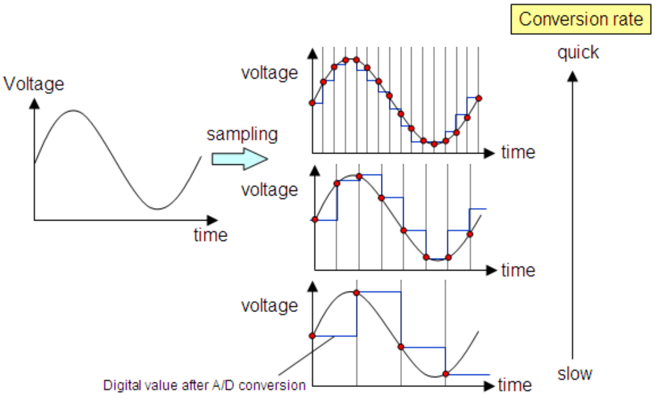
ADC

Analog Digital Converter



ADC

Analog Digital Converter



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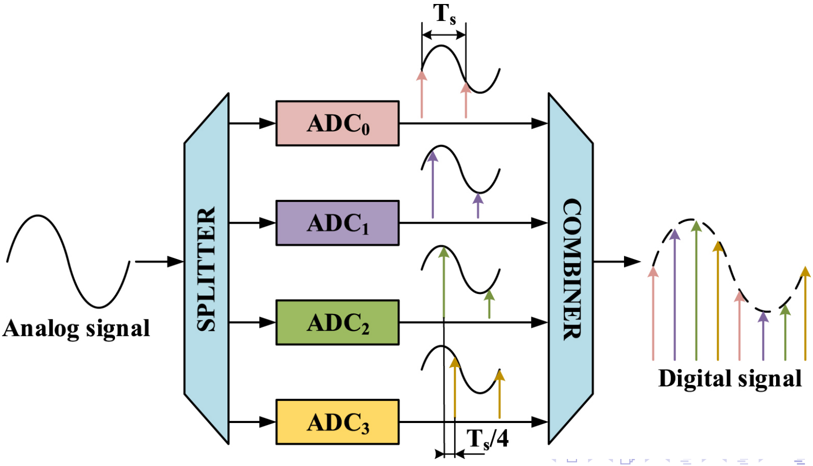


DAC

Digital

DAC

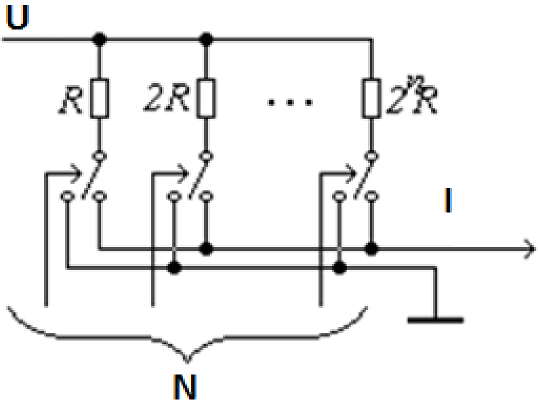
Digital Analog

DAC

Digital Analog Converter

DAC

Digital Analog Converter



DAC

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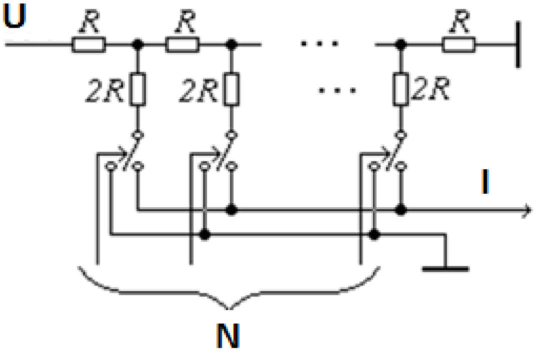

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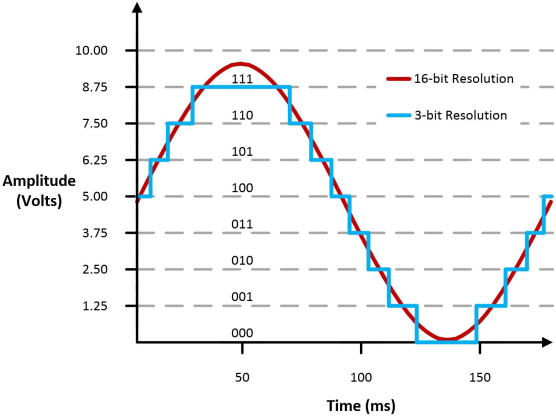
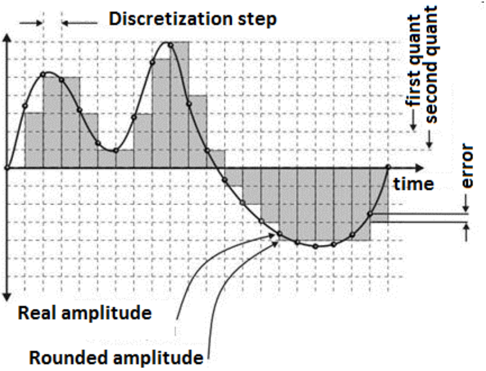


Discretization



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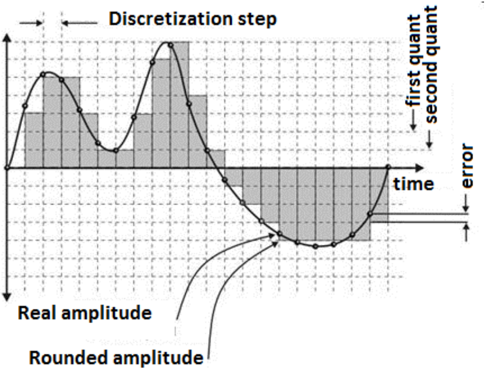

$\Delta t \to C =$ discrete

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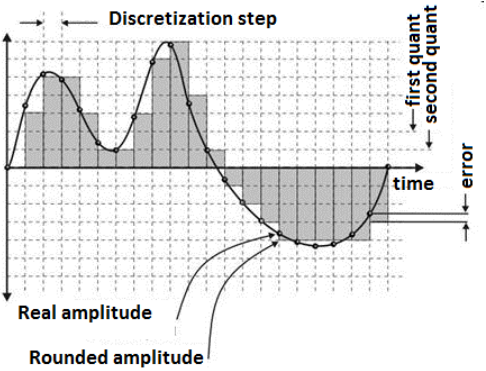

$\Delta t \to C =$ discrete

$\Delta t \to 0 \approx$ continuous

Integrator programming

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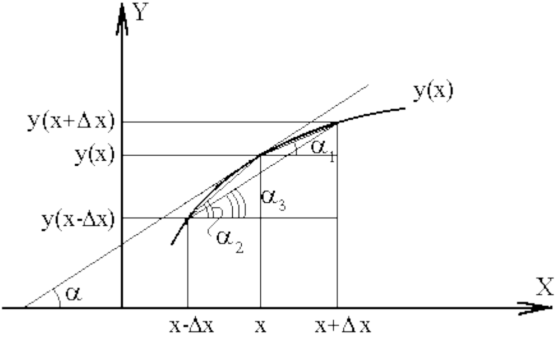
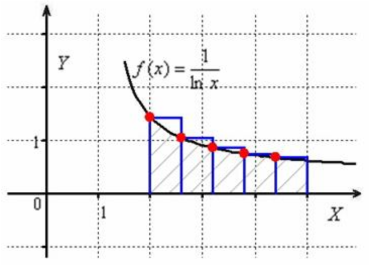


Rectangle method

Left



$$y[k] = y[k-1] + u[k-1]\Delta t \quad (1)$$

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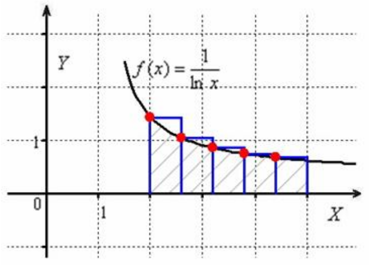

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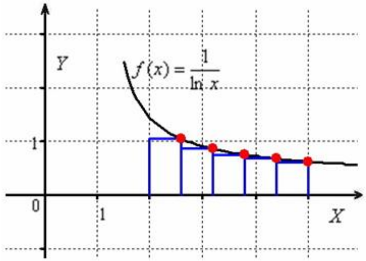
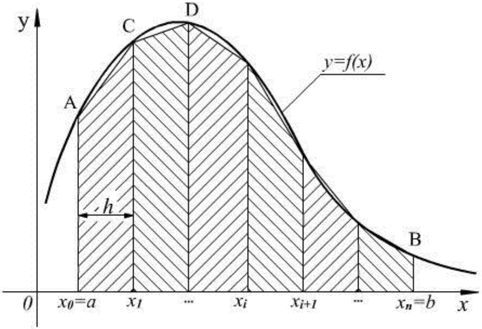


$$y[k] = y[k-1] + u[k-1]\Delta t \quad (1)$$

$$y[k] = y[k-1] + u[k]\Delta t \quad (2)$$

Trapezium method



$$y[k] = y[k-1] + (u[k] + u[k-1])\frac{\Delta t}{2} \quad (3)$$

C++ classes

```
class Student {
    std::string name;
    std::string last_name;
    int scores[5];
    float average_score;
};
```

It allows you to describe a typical object, its internal parameters and how to interact with other objects.

C++ objects

```
Student student;
```

An object can be defined as an instance of it's class. We can also view classes and objects in terms of data types and variable names, where the **class name** is the data type and the **object** is the variable name.

C++ objects

```
class Book {
  private:
    int year; //private member
    double price; //private member
  public:
    string title; //public member
    void printBook(); //public method
    void setPrice(double p); //public method
    void setYear(int y); //public method
};
```

C++ objects

```
void Book::setPrice(double p) {
  price = p;
}
void Book::setYear(int y) {
  year = y;
}
void Book::printBook() {
  cout << "Title: " << title << "\n";
  cout << "Price: " << price < "\n";
  cout << "Year: " << year << "\n";
}
```

C++ objects

```
Book book;
book.title = "hooked";
book.setYear(2013);
book.printBook();
book.price = 100;          // failed
book.year = 1988; // failed
```

Dynamic system model

```
class plant {

private:
    float state;    // save actual state
};
```

Dynamic system model

```
class plant {
public:
    plant(float init_state) {
        state = init_state;
        // set init state
    }
private:
    float state;
};
```

Dynamic system model

```
class Plant {
public:
    float update(float input) {
        state = state * a + input * b;
        return c * state;
        // output based on state,
        // input and parameters;
    }
private:
    float state;
    const float a = 0.1; b = 1; c = 1;
    // save parameters
};
```

Integrator class

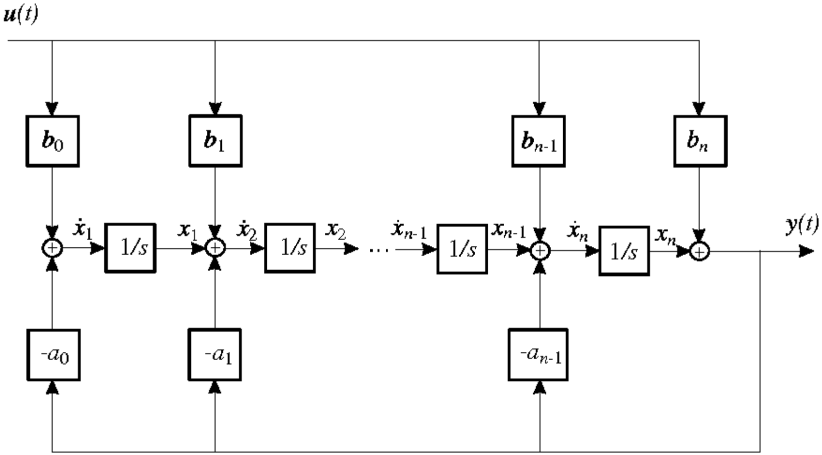
Using the trapezium method:

```
class Plant {
public:
    Plant(float init) { state = init; }
    float update(float input, float dt) {
        state = state +
            (prev_in + input) * dt / 2;
        prev_in = input;
        return state;
    }
private:
    float state = 0; float prev_in = 0;
};
```

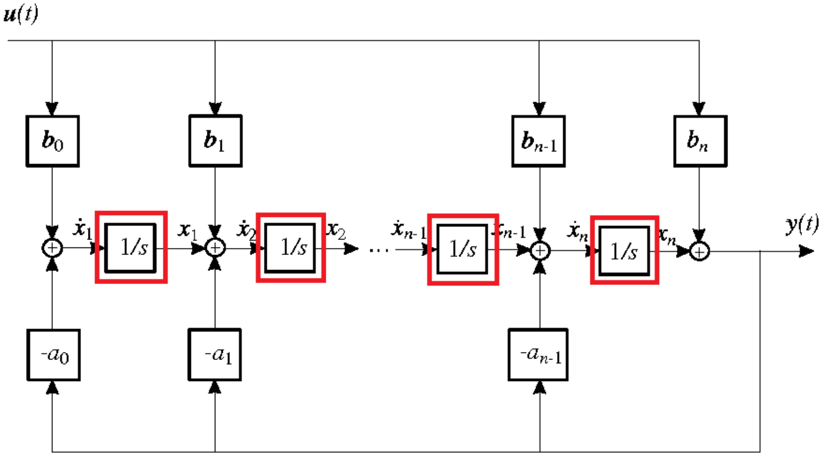
Integrator class

```
Plant plant(4.5);
for (int i = 0; i < 100; ++i) {
    plant.update(1);
}
plant.state = 10; // failed
cout << plant.state << "\n"; // failed
```

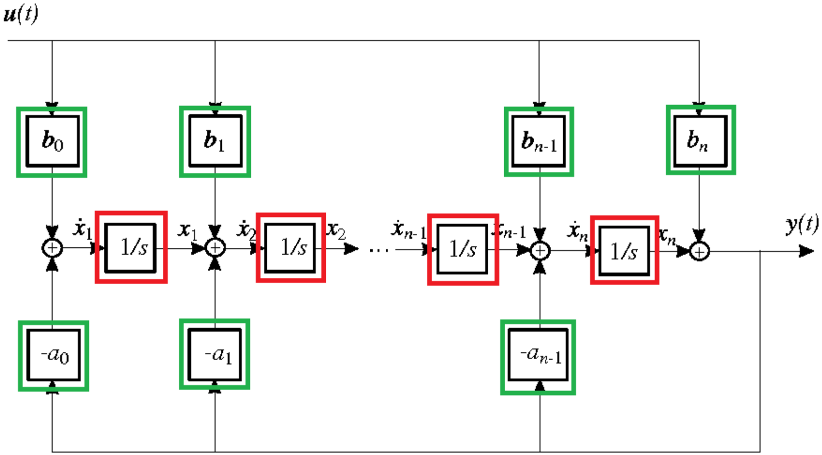
Whole system



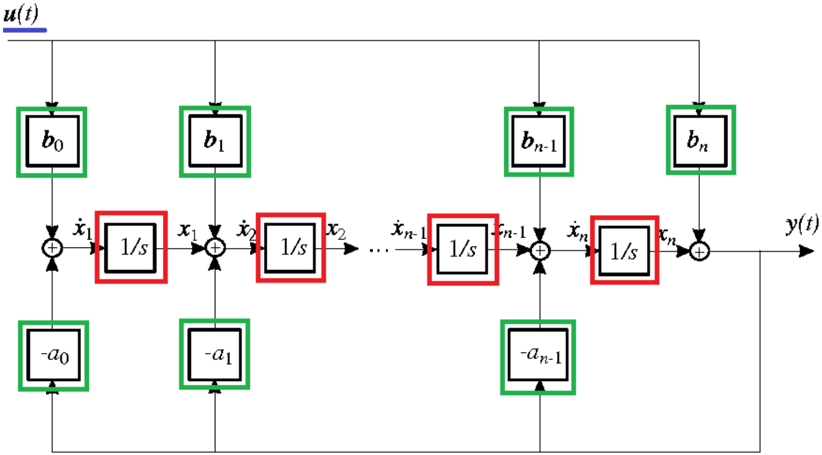
Whole system



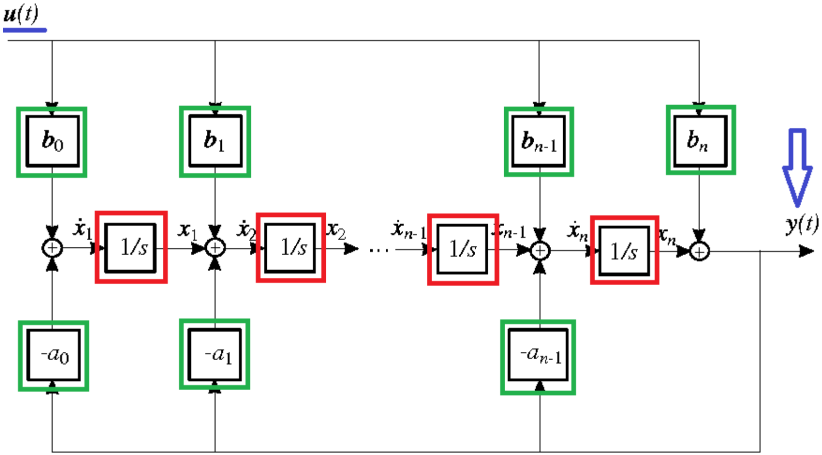
Whole system



Whole system



Whole system



Whole system

How to determine the order of subsequent actions to calculate y?