Statistical Learning Theory (SLT) offers a mathematical framework to address the problem of binary classification in Machine Learning. In the context of binary classification, the goal is to construct a function (f: X → Y) that minimizes the risk (R(f)), which is the expected loss of the function (f) over all points (X ∈ X). However, the underlying probability distribution (P) is unknown at the time of training, making it impossible to compute the Bayes error or the risk of a function (f) without knowledge of (P).

SLT steps in to analyze this situation and provide solutions and guarantees on the quality of these solutions. It allows for the formulation of the binary classification problem by considering a set of training points ((X1, Y1), ..., (Xn, Yn)) drawn independently from an unknown probability distribution (P) and a loss function l(X,Y,f(X)). The goal is to construct a function (f: X → Y) that approximates the risk of the Bayes classifier as closely as possible.

SLT addresses the challenge of generalization by evaluating the training error or empirical risk of a classifier on the training points, even though the true underlying risk cannot be computed. This analysis helps in assessing the performance of the classifier and its ability to generalize to unseen data. By building on a mathematical framework, SLT aims to answer fundamental questions about learning tasks, assumptions required for successful machine learning, key properties of learning algorithms, and performance guarantees on the results of certain learning algorithms.

In supervised learning, SLT focuses on the formal setup of an input space (X) and an output space (Y), particularly in the case of binary classification where the label space is identified with the set {-1, +1}. The learning process involves finding a function (f: X → Y) that minimizes the risk, which is the expected loss of the function over all points generated according to the underlying distribution (P). The risk quantifies how many elements in the instance space (X) are misclassified by the function (f).

Overall, SLT provides a theoretical foundation for machine learning, especially in the context of supervised learning and binary classification, by addressing key questions, defining formal setups, and analyzing the performance and generalization capabilities of learning algorithms.