# Task 1

$A = \{(1,2),(3,3),(4,0)\}$
$R(A) = \frac{1}{2}\mathbb{E}\left[\underset{a\in A}{sup}\sum_{i=1}^{2}\sigma_i a_i\right] =$
$= \frac{1}{2}(\frac{1}{4}max(1+2,3+3,4+0)+\frac{1}{4}max(1-2,3-3,4-0)+$
$+\frac{1}{4}max(-1+2,-3+3,-4+0)\frac{1}{4}max(-1-2,-3-3,-4-0)) = 1$

# Task 2

$S = \{(0,-1),(1,1),(1/3,1))\},\ x\in[0,1], y\in\{\pm1\}$
$H = \{h_1,h_2\},\ h_1(x) = 2x-1,\ h_2(x) = x$
$l(h,x) = max(0, 1-y\cdot h(x))$
$y(x) = \begin{cases} 1 & x > 1/4, \\ -1 & x \le 1/4 \end{cases}$
$\text{Rep}(\text{l} \circ H, S) = \underset{h\in H}{sup}(L_D(h) - L_S(h))$
1)

$$h_1(x) = 2x-1,\ h_1(x_1) = -1,\ h_1(x_2) = 1,\ h_1(x_3) = -1/3$$
$$L_D(h_1) = \mathbb{E}\left[max(0,1-y\cdot(2x-1))\right] =$$
$$=\int_0^1 max(0,1-y\cdot(2x-1))dx =$$
$$=\int_0^{1/4} max(0,1+(2x-1))dx + \int_{1/4}^1 max(0,1-(2x-1))dx =$$
$$=\int_0^{1/4} 2xdx + \int_{1/4}^1 2-2xdx = \frac{5}{8}$$
$$L_s(h_1) = \frac{1}{m}\sum_{i=1}^{m} l(h,x_i) = \frac{1}{3}(max(0,0)+max(0,0)+max(0,\frac{4}{3})) = \frac{4}{3}$$
$$l_D(h_1) - L_S(h_1) = \frac{13}{72}$$

2)

$$h_2(x) = x,\ h_2(x_1) = 0,\ h_2(x_2) = 1,\ h_2(x_3) = 1/3$$
$$L_D(h_2) = \frac{9}{16}$$
$$L_S(h_2) = \frac{5}{9}$$
$$l_D(h_2) - L_S(h_2) = \frac{1}{144}$$

$Rep(l\circ H, S) = \frac{13}{72}$

# Task 3

$R(\,ca + a_0 : a\in A\,) = \frac{1}{m}\mathbb{E}\left[\underset{a\in A}{sup}\sum_{i=1}^{m}\sigma_i(ca_i + a_{0i})\right] =$
$= \frac{1}{m}\mathbb{E}\left[\underset{a\in A}{sup}\sum_{i=1}^{m}\sigma_i(ca_i)\right] + \frac{1}{m}\mathbb{E}\left[\underset{a\in A}{sup}\sum_{i=1}^{m}\sigma_i a_{0i}\right] = \frac{1}{m}\mathbb{E}\left[\underset{a\in A}{sup}\sum_{i=1}^{m}\sigma_i(ca_i)\right] \le |c|\,\frac{1}{m}\mathbb{E}\left[\underset{a\in A}{sup}\sum_{i=1}^{m}\sigma_i a_i\right] =$
$= |c|\,R(A)$

$$R(A_1+A_2) = \frac{1}{m}\mathbb{E}\left[\underset{a_1\in A_1, a_2\in A_2}{sup}\sum_{i=1}^{m}\sigma_i(a_{1i}+a_{2i})\right] = \frac{1}{m}\mathbb{E}\left[\underset{a_1\in A_1}{sup}\sum_{i=1}^{m}\sigma_i a_{1i} + \underset{a_2\in A_2}{sup}\sum_{i=1}^{m}\sigma_i a_{2i}\right] =$$
$$= \frac{1}{m}\mathbb{E}\left[\underset{a_1\in A_1}{sup}\sum_{i=1}^{m}\sigma_i a_{1i}\right] + \frac{1}{m}\mathbb{E}\left[\underset{a_2\in A_2}{sup}\sum_{i=1}^{m}\sigma_i a_{2i}\right] = R(A_1)+R(A_2)$$

# Task 4

Воспользуемся неравенством Маркова:

$$1 - P\left[L_D(ERM_H(S)) - L_D(h^*) \le \epsilon\right] = P\left[L_D(ERM_H(S)) - L_D(h^*) \ge \epsilon\right] \le \delta,$$
$$\mathrm{P}[L_D(ERM_H(S)) - L_D(h^*) \ge \epsilon] \le \frac{\mathbb{E}[L_D(ERM_H(S)) - L_D(h^*)]}{\varepsilon} \le$$
$$\le \frac{2}{\varepsilon}\underset{S\backsim D^m}{\mathbb{E}}R(l\circ H\circ S),$$
$$\delta = \frac{2}{\varepsilon}\underset{S\backsim D^m}{\mathbb{E}}R(l\circ H\circ S) \Rightarrow \epsilon = \frac{2}{\delta}\underset{S\backsim D^m}{\mathbb{E}}R(l\circ H\circ S)$$