Automatic Transaction Systems
Information Asymmetry and Price Discovery

Project Report

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1 Introduction

Information asymmetry, where a market participant possesses private knowledge about an asset's fundamental value, plays a central role in the process of price formation. Kyle (1985) provides a foundational framework that formalizes how private information is incorporated into market prices through strategic trading. His model features a risk-neutral insider who chooses trading quantities to maximize profits, while market makers set prices based on observed order flow (both informed and noise trades).

In the single-period model, Kyle shows that prices are set as a linear function of total order flow. Because the market maker cannot distinguish informed trading from noise, prices reflect the conditional expectation of the asset's value given observed flow. This leads to partial information revelation in equilibrium: the price adjusts in the correct direction on average but does not fully reflect the insider's private signal. The slope of the pricing rule, denoted by λ, captures the sensitivity of price to order flow and is endogenously determined by the relative variances of the asset's fundamental value and the noise in order flow. It reflects both the market's depth and how much information about the true value is expected to be conveyed through trades.

Kyle extends this model to a multi-period setting, modeling a sequential auction in which the insider trades over several discrete auctions. In this dynamic environment, the insider balances the desire to profit from private information with the need to conceal it to avoid moving prices too aggressively. Prices adjust gradually as the market incorporates information over time, and market depth, measured as the inverse of λ, evolves endogenously across trading periods.

This report builds on Kyle's single- and multi-period models to study how private information is incorporated into prices over time. We analyze a simplified discrete-time extension of the model and focus on the evolution of two key equilibrium parameters: the insider's trading aggressiveness β_n and the price impact coefficient λ_n. Our main objective is to understand how the dynamics of these variables capture the process of information revelation in sequential trading. Specifically, we investigate how price discovery is influenced by the number of trading rounds N, the intensity of noise trading σ_u^2, and the presence of multiple informed traders M, as introduced in the extension by Holden and Subrahmanyam (1992).

This extension captures imperfect competition, where multiple informed traders strategically interact, each anticipating the impact of their trades on market prices and on the actions of others. The resulting equilibrium leads to faster information revelation. Another extension is considered by Admati and Pfleiderer (1988), who develop a model of strategic interaction between informed and uninformed traders. By allowing some uninformed traders to choose when to trade, they show that the resulting Nash equilibrium leads to concentrated bouts of trading, similar to the surges in order flow observed at the opening and closing of many continuous markets. Unfortunately, we were not able to fully implement this model within our project, but we include it here for its conceptual relevance. Answering these questions requires both analytical derivation and numerical analysis. Analytically, we derive expressions for λ_n and β_n using backward induction.

Numerically, we implement the models in Python, replicating the single-period and multi-period Kyle equilibria, extending the framework to include multiple informed traders, and simulating outcomes under long-lived private information. We examine how λ_n and β_n evolve with the number of trading auctions N, noise trader volatility σ_u, true price volatility σ_v, and the number of informed traders M.

The main research questions guiding this study are:

- How is private information incorporated into prices over time?
- How does the informed trader's strategy interact with market liquidity?
- How does the price impact parameter λ_n evolve across periods?
- In what ways does information incorporation in the multi-period case differ from the static, single-period model?
- How does competition among informed traders affect the speed of information revelation and market depth?

The remainder of this report is organized as follows. Section 2 introduces the single-period Kyle model and derives the equilibrium conditions, followed by the Holden and Subrahmanyam extension. Section 4 extends these models to a multi-period setting, presenting both analytical results and numerical simulations under varying conditions, followed by multi-period extension. Section 6 concludes with a summary of findings.

2 Single-Period Kyle Model

2.1 Model Assumptions and Equilibrium

In the single-period Kyle model, trading occurs in a single round during which both an informed trader and a noise trader submit orders to a risk-neutral market maker. The model relies on the following assumptions:

- The asset has an unknown fundamental value $v \sim \mathcal{N}(p_0, \Sigma_0)$, known only to the informed trader.
- Noise traders submit random orders $u \sim \mathcal{N}(0, \sigma_u^2)$, independent of v.
- The informed trader submits an order $x = \beta(v - p_0)$, based on his knowledge of v.
- The total order flow observed by the market maker is $y = x + u$.
- The market maker cannot distinguish between informed and uninformed orders and sets the price p based on the observed y such that his expected profit is zero:

$$p = E[v \mid y].$$

The market maker is assumed to follow a linear pricing rule:

$$p(y) = \mu + \lambda y,$$

where λ is the price impact parameter. The informed trader chooses x to maximize expected profits:

$$\max_x \; E\left[(v - p(y))x \,|\, v\right].$$

Substituting $y = x + u$ and applying the pricing rule, we get:

$$E[(v - \mu - \lambda(x + u))x] = (v - p_0 - \lambda x)x,$$

since $E[u] = 0$. Maximizing with respect to x gives:

$$x = \frac{1}{2\lambda}(v - p_0).$$

This implies that,

$$x = \beta(v - p_0), \quad where \beta = \frac{1}{2\lambda}.$$

Again, the market maker observes total order flow $y = x + u = \beta(v - p_0) + u$, and sets the price based on the conditional expectation:

$$p = E[v \mid y].$$

Since v and y are jointly normally distributed, the conditional expectation is linear, and:

$$\lambda = \frac{Cov(v, y)}{Var(y)} = \frac{\beta\Sigma_0}{\beta^2\Sigma_0 + \sigma_u^2}.$$

Substituting $\beta = 1/(2\lambda)$ into this expression:

$$\lambda = \frac{\sqrt{\Sigma_0}}{2\sigma_u}, \quad \beta = \frac{\sigma_u}{\sqrt{\Sigma_0}}.$$

In equilibrium, prices incorporate only partial information about the asset's true value:

$$p = p_0 + \lambda y = p_0 + \lambda(x + u).$$

Due to the presence of noise trading, only a fraction of the insider's information is reflected in the price. Specifically:

$$p = \frac{1}{2}(v + p_0) + \lambda u,$$

so the price lies halfway between the prior mean p_0 and the true value v, plus a noise term. This incomplete revelation is a core feature of the Kyle model and illustrates how noise trading facilitates profitable informed trading by obscuring the insider's information.

2.2 Simulation of Bid–Ask Dynamics

In our implementation of the single-period Kyle model, we simulate how the market maker's bid and ask quotes evolve over discrete time steps as informed and noise traders submit orders. The core Python routine is `get_kyle1_quotes()`, which proceeds as follows:

1. **Initial quotes.** We start from a "guess price" p_0 and a fixed half-spread Δ:

$$bid_0 = p_0 + \Delta, \qquad ask_0 = p_0 - \Delta.$$

2. **Equilibrium parameters.** The price impact λ and insider aggressiveness β are set according to the analytic single-period solution:

$$\lambda = \frac{\sigma_v}{2\,\sigma_u}, \quad \beta = \frac{1}{2\lambda},$$

where σ_v (`true_asset_stdev`) and σ_u (`uninf_stdev`) are the standard deviations of the fundamental value and noise order flow, respectively.

3. **Iterative updates.** For each time step $t = 1, \ldots, T$:

   (a) Compute the mid-price:
   $$p_{t-1}^{\text{mid}} = \frac{bid_{t-1} + ask_{t-1}}{2}.$$

   (b) Draw a noise order $u_t \sim \mathcal{N}(0, \sigma_u^2)$ and compute the insider's order
   $$x_t = \beta(v - p_{t-1}^{\text{mid}}),$$
   where v is the (fixed) true asset value known to the insider.

   (c) Form the total order flow
   $$y_t = u_t + M\,x_t,$$
   with M the number of insiders (in this case $M = 1$).

   (d) Adjust the quotes by the price impact:
   $$\Delta p_t = \lambda\,y_t, \quad bid_t = bid_{t-1} + \Delta p_t, \quad ask_t = ask_{t-1} + \Delta p_t.$$

4. **Output.** After T iterations, the function returns the trajectories $\{bid_t, ask_t\}_{t=0}^{T}$ as well as the equilibrium parameters λ and β.

Thus, each new order flow y_t is incorporated linearly into both bid and ask quotes with weight λ; the resulting mid-price
$$p_t^{\text{mid}} = \frac{bid_t + ask_t}{2} = p_0 + \lambda \sum_{s=1}^{t} y_s$$
traces the gradual incorporation of private information into price. This simulation faithfully reproduces the discrete-time analogue of Kyle's continuous-time equilibrium and provides the basis for our multi-period extensions.

2.3 Obtained results

Our simulation of the single-period Kyle (1985) model provides a clear illustration of price discovery in an informationally asymmetric market. For this specific run, the initial parameters were set, and a true asset value was randomly drawn from the prior distribution.

Simulation Parameters and Key Outcomes:

- **Initial Guess Price:** 110
- **Market Maker Spread:** 2
- **Number of Iterations:** 50
- **Calculated Lambda (λ):** 0.25
- **Calculated Beta (β):** 2.0
- **Simulated True Asset Value:** 101.24

The derived values for λ and β reflect the market's illiquidity and the informed trader's aggressiveness, respectively, based on the σ_v (`true_asset_stdev`) and σ_u (`uninf_stdev`) parameters defined earlier in the code.

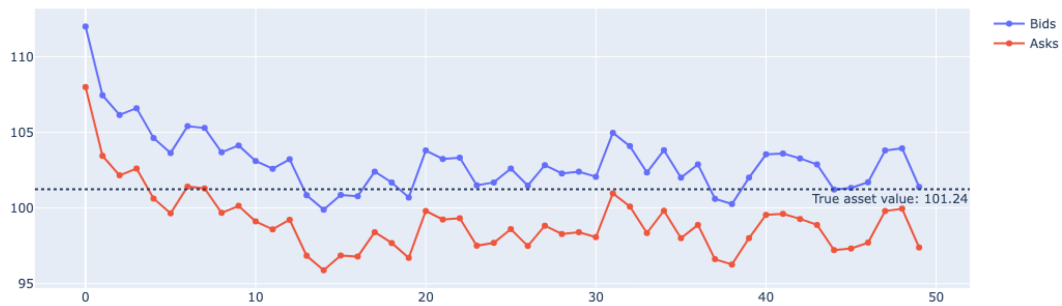

Figure 1: The plot shows the bid and ask prices over 50 iterations, converging towards the true asset value of 101.24. The calculated $\lambda = 0.25$ and $\beta = 2.0$ govern the price adjustment mechanism.

This figure shows the dynamic process of price adjustment over 50 iterations. As observed, the bid and ask prices gradually converge towards the true asset value of 101.24 as information is revealed through the order flow. The spread between the bids and asks represents the market maker's compensation for providing liquidity and bearing the risk of trading against an informed party.

2.4 Analysis of Market Liquidity (Lambda Surface)

To further investigate the properties of market liquidity, specifically Kyle's lambda (λ), we generated a surface plot illustrating how λ varies with different values of the asset's true value standard deviation (σ_v) and the uninformed trader order standard deviation (σ_u).

Recall that λ is calculated as $\lambda = \frac{\sigma_v}{2\sigma_u}$. This formula suggests an inverse relationship with uninformed trading noise and a direct relationship with the uncertainty of the true asset value.

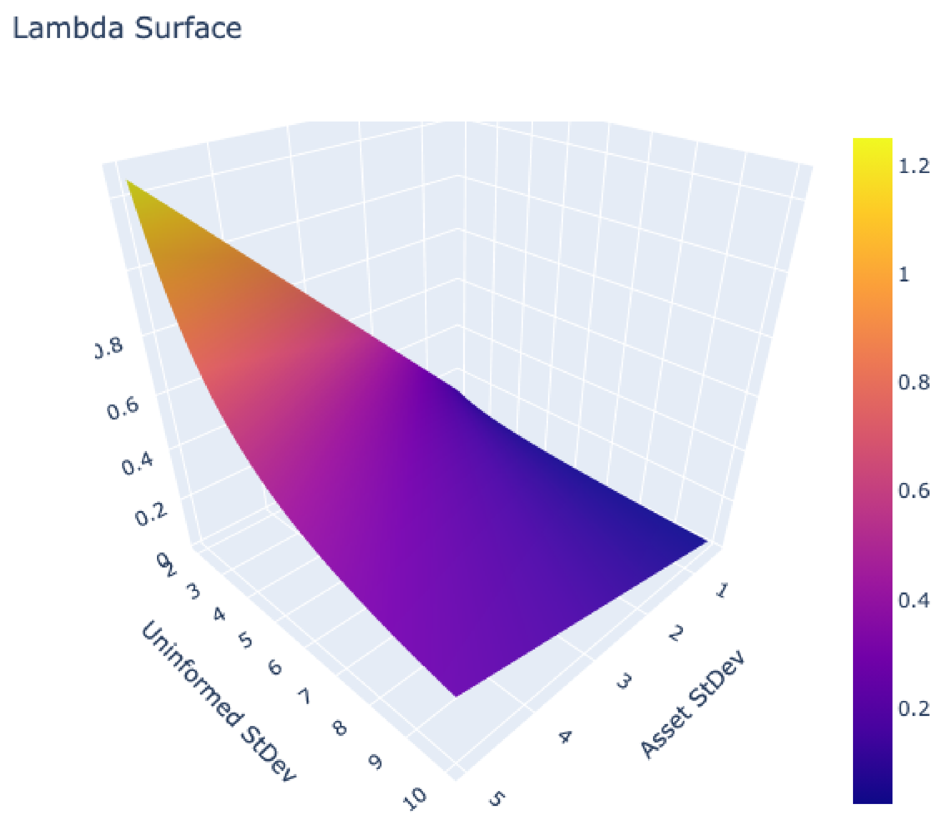

Figure 2: Lambda Surface: Variation of Market Illiquidity (λ) with Asset Standard Deviation (σ_v) and Uninformed Trader Order Standard Deviation (σ_u).

As depicted in Figure 2,

- Lambda increases as the Asset Standard Deviation (σ_v) increases. This indicates that when there is greater uncertainty about the true value of the asset, the market becomes less liquid, leading to a higher price impact for trades.
- Lambda decreases as the Uninformed Standard Deviation (σ_u) increases. A higher volume of noise trading provides more "camouflage" for informed traders, making it harder for market makers to infer private information from order flow, thus reducing the price impact and increasing liquidity.

This visualization confirms the theoretical underpinnings of Kyle's lambda as a measure of market illiquidity and its sensitivity to both fundamental uncertainty and the presence of noise trading.

2.5 Exploring Non-Linear Price Impact Adjustments

The foundational Kyle model assumes a linear relationship between order flow and price adjustment. However, in real markets, market makers' responses to order flow might be more complex and non-linear. To investigate this, we implemented several alternative price adjustment functions and simulated their impact on bid and ask price dynamics. These adjustments modify how market makers update their quotes based on the total observed order flow, moving beyond the simple $\lambda \times total\_order\_flow$ rule.

Defined Adjustment Functions

The following functions were implemented to simulate different non-linear price impact behaviors:

- **Kyle (Linear):** This represents the standard linear adjustment. In this case, no specific adjustment function is passed to the simulation, which leads to the price adjustment being calculated as $price\_adjustment = lmbd \cdot total\_order\_flow$.

- **Square Root (`sqrt_adjustment`):** The price adjustment is proportional to the square root of the absolute order flow: $lmbd \cdot sign(total\_order\_flow) \cdot \sqrt{1 + |total\_order\_flow|}$. This implies that larger order flows have a proportionally diminishing marginal price impact compared to linear.

- **Squared (`squared_adjustment`):** The price adjustment is proportional to the square of the absolute order flow, capped at 10: $lmbd \cdot sign(total\_order\_flow) \cdot \min((abs(total\_order\_flow))^2, 10)$. This suggests that larger order flows lead to a proportionally increasing marginal price impact.

- **Logarithmic (`log_adjustment`):** The adjustment uses a logarithmic transformation: $lmbd \cdot sign(total\_order\_flow) \cdot \ln(1 + |total\_order\_flow|)$. This implies that initial order flow has a significant impact, but the marginal impact diminishes for very large order flows.

- **Exponential (`exp_adjustment`):** The adjustment is based on an exponential function, also capped at 10: $lmbd \cdot sign(total\_order\_flow) \cdot \min(e^{|total\_order\_flow|} - 1, 10.0)$. This suggests that larger order flows result in exponentially increasing price impacts.

- **Power-Law (`power_law_adj`):** A generalized power-law adjustment with specific parameters ($\gamma = 0.6, \alpha = 0.5$): $\alpha \cdot lmbd \cdot sign(total\_order\_flow) \cdot (|total\_order\_flow|)^{\gamma}$. This allows for flexible non-linear relationships.

Each of these functions was then incorporated into the `get_kyle1_quotes` simulation by passing them as the `adjustment` parameter.

The figure below illustrates the evolution of bid and ask prices under each of these adjustment mechanisms.

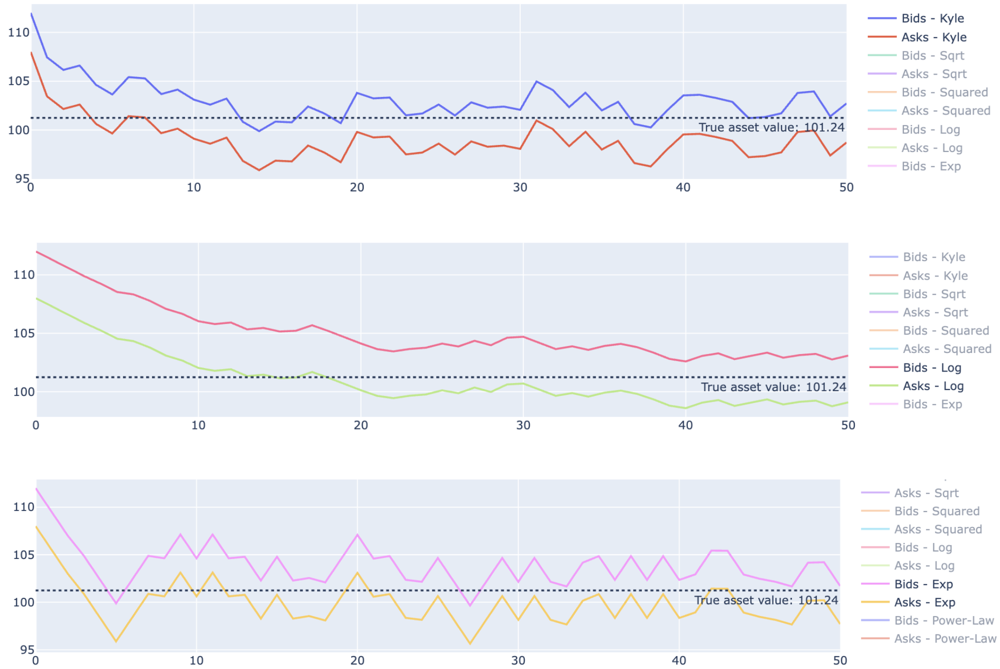

Figure 3: Concave adjustments (e.g., log, sqrt) vs Convex adjustments (e.g., squared, exp)

The comparison reveals significant differences in the speed and smoothness of price convergence:

- **Linear (Kyle):** This serves as the benchmark, showing a relatively steady, albeit sometimes volatile, convergence towards the true asset value.
- **Non-Linear Adjustments:**
  - Functions like 'Squared' and 'Exponential' tend to exhibit more rapid or aggressive price movements, especially for larger order flows. This can potentially lead to faster initial convergence but also higher volatility if order flow is erratic. These functions include built-in caps to prevent explosive price adjustments from very large order flows.
  - Conversely, 'Square Root' and 'Logarithmic' adjustments might dampen the impact of very large orders compared to the linear model. This can lead to smoother price paths but potentially slower overall information revelation.
  - The 'Power-Law' adjustment offers flexibility, allowing for tunable responses to order flow. This can mimic various market conditions depending on the chosen parameters (γ and α).

3 Holden and Subrahmanyam (1992) — Single-Period Equilibrium with Multiple Informed Traders

3.1 Model Definition

Holden and Subrahmanyam (1992) extended Kyle's single-period model by allowing for $M > 1$ informed traders who compete strategically in the market. Although the overall structure remains largely intact, the presence of multiple insiders changes how private information is incorporated into prices. A competitive, risk-neutral market maker observes only total order flow $y = \sum x_i + u$ and sets the price $p = E[v|y]$. Traders follow linear strategies $x_i = \beta(v - p)$, and the market maker uses a pricing rule $p = \mu + \lambda y$. A unique linear equilibrium exists.

3.2 Model Structure and Solution

Each informed trader maximizes their expected profit, assuming the others follow the same linear strategy. The maximization function becomes:

$$\max_{x_i} E[(v-p)x_i|v] = (v - \mu - \lambda(x_i + \sum_{j\neq i} x_j + u))x_i$$

Assuming symmetric strategies $x_j = \beta(v - \mu)$, this simplifies to:

$$\max_{x_i}(v - \mu - \lambda(x_i + (M-1)\beta(v-\mu)))x_i$$

Solving the FOC gives:

$$x_i = \frac{1 - \lambda\beta(M-1)}{2\lambda}(v - \mu)$$

From symmetry, $x_i = \beta(v - \mu)$, so:

$$\beta = \frac{1}{\lambda(M+1)}$$

The market maker observes total order flow $y = \sum_{i=1}^{M} x_i + u = M\beta(v - p_0) + u$, and sets the price using conditional expectation:

$$p = E[v|y] = p_0 + \frac{Cov(v,y)}{Var(y)}(y - E[y])$$

Since $y = M\beta(v - p_0) + u$, we compute:

$$E[y] = 0, \quad Cov(v,y) = M\beta\Sigma_0, \quad Var(y) = M^2\beta^2\Sigma_0 + \sigma_u^2$$

Thus,

$$\lambda = \frac{M\beta\Sigma_0}{M^2\beta^2\Sigma_0 + \sigma_u^2}$$

Substituting $\beta = \frac{1}{\lambda(M+1)}$, we get the closed-form equilibrium solution:

$$\lambda = \sqrt{\frac{M\Sigma_0}{\sigma_u^2(M+1)}}, \quad \beta = \frac{1}{\lambda(M+1)}$$

3.3 Model Simulation

This section explores how the model behaves as more informed traders are introduced. The focus is on how λ and β respond as M increases, and how the market maker adjusts prices toward the true asset value. The goal is to see if adding more informed traders speeds up price discovery.

Figure 4: Single-period model allowing for M informed traders.

Figure shows that both the single-period Kyle and the Holden-Subrahmanyam (M=3) models drive prices toward the true asset value, but the HS model adjusts faster. With multiple informed traders, prices show slightly higher volatility due to more aggressive competition. Quotes also begin closer to fair value, reflecting greater pricing efficiency. Overall, more informed traders make the market react faster to information. Additionally, as more informed traders are introduced, their incremental impact on price discovery diminishes.

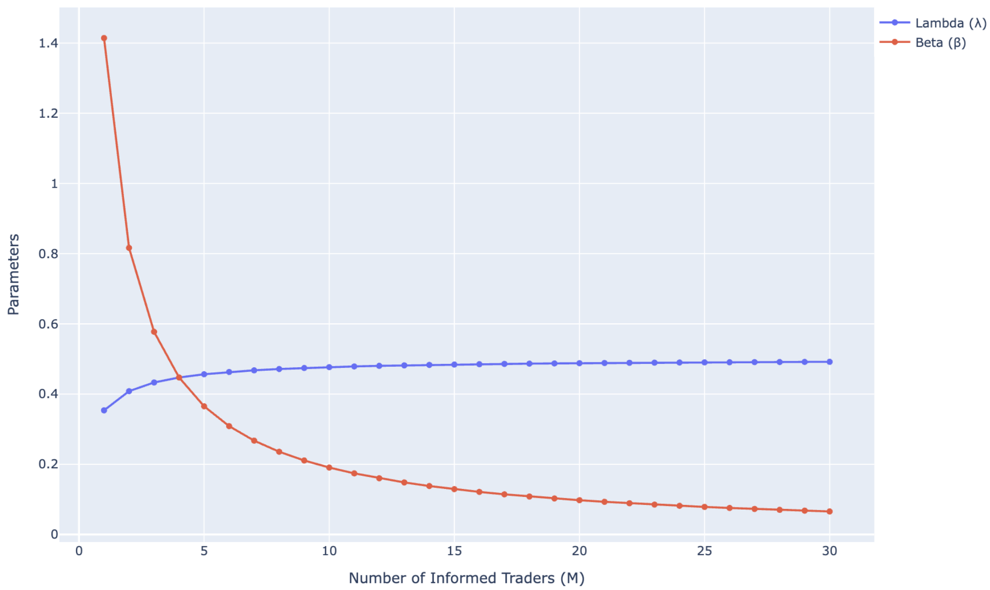

Figure 5: Parameter sensitivity to M informed traders.

Figure shows that as the number of informed traders (M) increases, λ rises but more slowly, meaning prices become less sensitive to each additional trader. β drops quickly, indicating that each trader becomes less aggressive.

4 Multi-Period Kyle Model

While the single-period Kyle model provides fundamental insights, real-world markets often involve continuous trading over multiple periods. The multi-period Kyle model extends this framework to a dynamic setting, allowing for a more nuanced understanding of how information is gradually revealed over time.

4.1 Model Structure

In this multi-period setup, we consider a sequence of N auctions happening at discrete times t_k, where $0 = t_0 < t_1 < \ldots < t_N = 1$. For implementation purposes, we assume equally spaced intervals, so $\Delta t_k = t_k - t_{k-1} = \frac{1}{N}$. Let $u(t)$ denote a Brownian motion process with instantaneous variance σ_u^2, and define $u_n = u(t_n)$ and $\Delta u_n = u_n - u_{n-1}$. We assume that the quantity traded by noise traders at the n-th auction is Δu_n. The Brownian motion assumption implies that Δu_n is normally distributed with zero mean and variance $\sigma_u^2 \Delta t_n$, and that the quantity traded at one auction is independent of the quantity traded at other auctions.

The liquidation value of the asset, v, is assumed to be drawn from a normal distribution, $v \sim \mathcal{N}(p_0, \Sigma_0)$. Importantly, v is a fixed value drawn once at the beginning of the trading period. It is known only to the informed trader and remains constant throughout the trading horizon.

From the market maker's perspective, it is a random variable. The realization of v is independent of the entire process $u(t)$.

The market features the same three types of agents as the single-period model:

- **Informed Trader:** A monopolist informed trader who knows the true value of v.
- **Noise Traders:** Their quantity traded is modeled as a Brownian motion process, $u_n = u(t_n)$. The change in noise trade, $\Delta u_n = u_n - u_{n-1}$, is normally distributed with zero mean and variance $\sigma_u^2 \Delta t_n$. Critically, quantities traded at one auction are independent of those at other auctions.
- **Market Maker:** Observes total order flow and updates beliefs to set prices.

Let x_n denote the aggregate position of the insider after the n-th auction, such that $\Delta x_n = x_n - x_{n-1}$ is the quantity traded at the n-th auction. Correspondingly, p_n is the market clearing price at the n-th auction. When deciding what quantity to trade, the insider uses the liquidation value v and past prices:

$$x_n = X_n(p_1, \ldots, p_{n-1}, v) \tag{1}$$

Then, market makers set a market-clearing price using current and past order flows:

$$p_n = P_n(x_1 + u_1, \ldots, x_n + u_n) \tag{2}$$

The profit of the insider on the positions acquired at auctions $n, \ldots, N$ is defined as:

$$\pi_n = \sum_{k=n}^{N} (v - p_k) x_k \tag{3}$$

A sequential auction equilibrium is defined such that profit maximization is achieved, meaning we find trading rules $X = (X_1, \ldots, X_N)$ that maximize the expected profit:

$$E\{\pi_n(X, P) \mid p_1, \ldots, p_{n-1}, v\} \tag{4}$$

Additionally, the market efficiency condition must hold:

$$p_n = E\{v \mid x_1 + u_1, \ldots, x_n + u_n\} \tag{5}$$

Kyle (1985) considers a recursive linear equilibrium, where X_k and P_k are linear, and:

$$\Delta x_n = \beta_n (v - p_{n-1}) \Delta t_n \tag{6}$$

$$\Delta p_n = \lambda_n (\Delta x_n + \Delta u_n) \tag{7}$$

In this equilibrium, price increments are normally and independently distributed. The distribution of the pricing process is characterized by a sequence of conditional variances:

$$\Sigma_n = Var\{v \mid x_1 + u_1, \ldots, x_n + u_n\} \tag{8}$$

4.2 Model Solution

The equilibrium defined above exists and is unique. Given Σ_0, the values of λ_n, β_n, and Σ_n are uniquely determined by the following recursive system, subject to $\alpha_N = 0$ and the second-order condition $\lambda_n(1 - \alpha_n \lambda_n) > 0$:

$$\beta_n = \frac{1 - 2\alpha_n \lambda_n}{2\lambda_n \Delta t_n (1 - \alpha_n \lambda_n)} \tag{9}$$

$$\lambda_n = \frac{\beta_n \Sigma_n}{\sigma_u^2} \tag{10}$$

$$\alpha_{n-1} = \frac{1}{4\lambda_n (1 - \alpha_n \lambda_n)} \tag{11}$$

$$\Sigma_{n-1} = \frac{\Sigma_n}{1 - \beta_n \lambda_n \Delta t_n} \tag{12}$$

The variable α_n represents the sensitivity of market makers' pricing rule to new information, and is computed backward starting from $\alpha_N = 0$.

This system is typically solved using backward induction. First, a guess for Σ_N is provided. Then, for each auction in the order $N, N-1, \ldots, 1$, Kyle showed that λ_n is the middle root of the cubic equation:

$$\left(1 - \frac{\lambda_n^2 \sigma_u^2 \Delta t_n}{\Sigma_n}\right)(1 - \alpha_n \lambda_n) = \frac{1}{2} \tag{13}$$

At each iteration n, α_n and Σ_n are given. We solve (13) for λ_n, and then compute β_n, α_{n-1}, and Σ_{n-1}. At the end, the computed Σ_0 is compared to the known true value. If they are sufficiently close, the system is solved. Otherwise, the initial guess for Σ_N must be adjusted. Methods such as bisection or Newton-Raphson are used to find the correct value that minimizes the discrepancy.

4.3 Model Analysis and Simulation Results

This section provides an analysis of the multi-period Kyle model's behavior under various conditions, primarily focusing on the evolution of key parameters and market characteristics across auction periods.

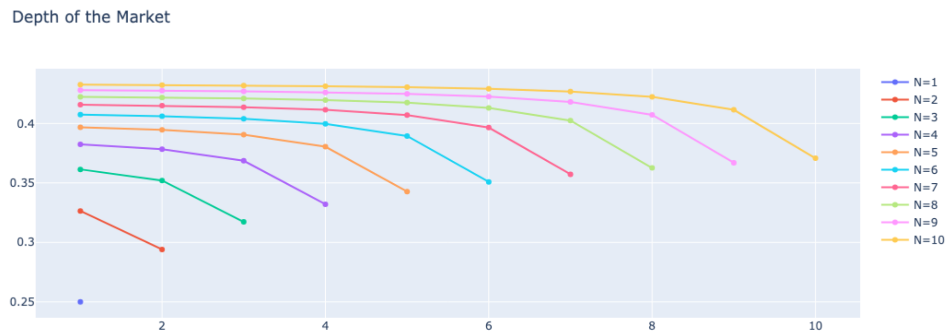

Figure 6: λ_n depending on the number of auctions N.

Figure 6 illustrates the behavior of λ_n as a function of the auction number N. It shows that the terminal value λ_N increases with N. This implies that in a multi-period setting with a monopolistic informed trader, market depth decreases as more auctions are held.

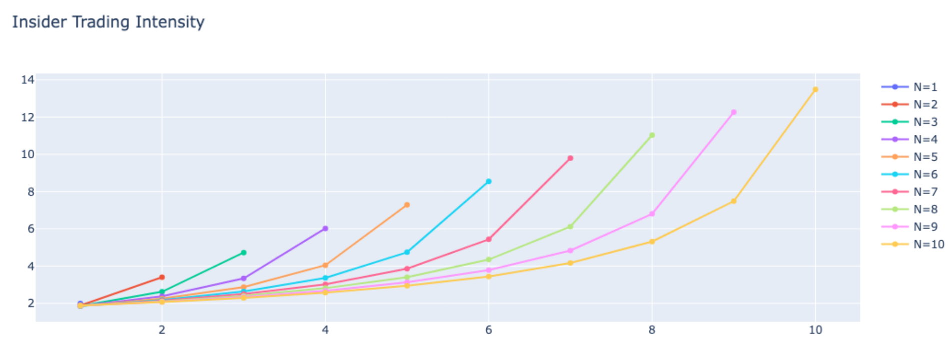

Figure 7: β_n depending on the number of auctions N.

As shown in Figure 7, the insider's trading intensity, β_n, increases with the number of auctions N, though this increase is gradual.

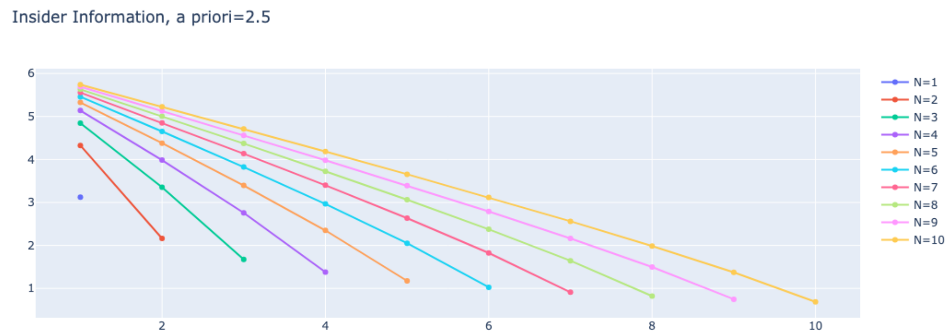

Figure 8: Σ_n depending on the number of auctions N.

Figure 8 displays Σ_n, which represents the variance of prices or the amount of information not yet incorporated. It demonstrates that Σ_n decreases slowly, indicating that information is gradually revealed over time, but not at a rapid pace.

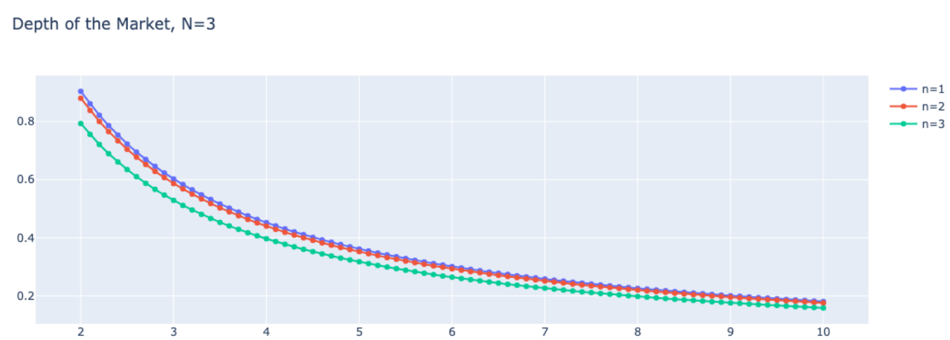

Figure 9: λ_n vs σ_u.

Figure 9 illustrates the relationship between λ_n and σ_u. It indicates that increasing the amount of noise trading generally leads to an increase in market depth, meaning a more liquid market.

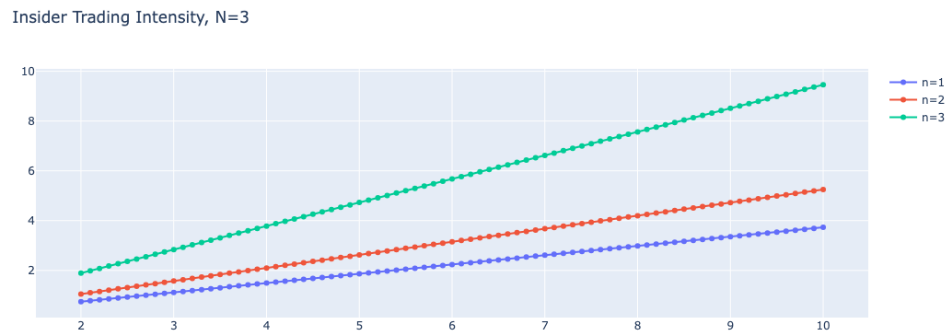

Figure 10: β_n vs σ_u.

Figure 10 shows that the insider's trading intensity, β_n, increases as noise trading increases. This is consistent with the idea that informed traders can trade more aggressively when there is more noise to hide their activity.

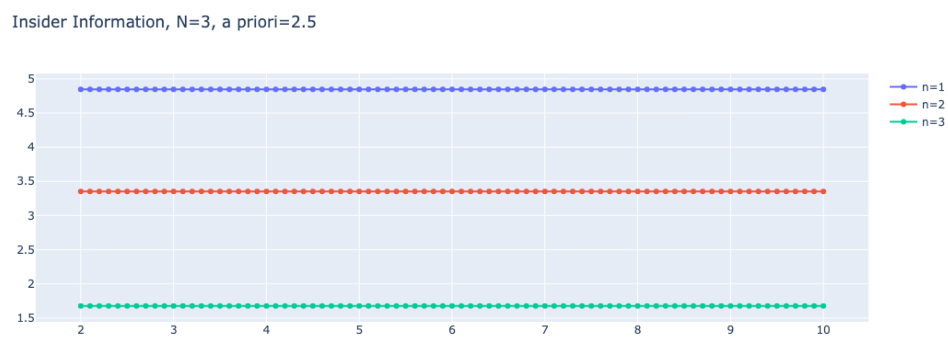

Figure 11: Σ_n vs σ_u.

Figure 11 reveals that Σ_n, which is related to the informativeness of trades, does not depend on noise trading volatility (σ_u).

5 Multi-Period Kyle Model with Multiple Informed Traders

Building upon the multi-period framework, Holden and Subrahmanyam (1992) proposed an extension that introduces M informed traders to the market. The core setup remains similar to the monopolistic multi-period Kyle model, but with crucial adjustments to account for the strategic interaction and competition among multiple informed agents.

5.1 Model Definition and Structure

In this extended model, the market structure is largely maintained, with auctions occurring sequentially over multiple periods. The key difference lies in the total order flow, Δx_n, which now represents the aggregate order of all informed traders at the n-th auction. A unique linear equilibrium is also found to exist for this model.

The equilibrium is defined by the following equations:

$$\Delta x_n = M\beta_n(v - p_{n-1})\Delta t_n \quad (14)$$

$$\Delta p_n = \lambda_n(\Delta x_n + \Delta u_n) \quad (15)$$

$$\Sigma_n = Var\{v|x_1 + u_1, \ldots, x_n + u_n\} \quad (16)$$

5.2 Model Solution

This model cannot be solved with the "guessing" approach used for the monopolistic multi-period Kyle model. Instead, Holden and Subrahmanyam found an explicit method that solves the model for any $M \geq 1$, including the case of $M = 1$ which reverts to the Kyle multi-period model.

The solution involves a difference equation system, with boundary conditions $\alpha_N = 0$ and the second order condition $\lambda_n(1 - \alpha_n\lambda_n) > 0$:

$$\beta_n = \frac{1 - 2\alpha_n\lambda_n}{\lambda_n\Delta t_n(M(1 - 2\alpha_n\lambda_n) + 1)} \quad (17)$$

$$\lambda_n = \frac{M\beta_n\Sigma_n}{\sigma_u^2} \quad (18)$$

$$\alpha_{n-1} = \frac{1 - \alpha_n\lambda_n}{\lambda_n(M(1 - 2\alpha_n\lambda_n) + 1)^2} \quad (19)$$

$$\Sigma_{n-1} = \frac{\Sigma_n}{1 - M\beta_n\lambda_n\Delta t_n} \quad (20)$$

The method is explicit and does not involve guessing a terminal Σ_N. It proceeds as follows:

5.2.1 Backward Solution for q_n

Define $q_n = \alpha_n\lambda_n$. Starting from $q_N = 0$, the system is solved in a backward manner for every n. The cubic equation for q_{n-1} is:

$$2M\frac{\Delta t_{n-1}}{\Delta t_n}q_{n-1}^3 - (M + 1)\frac{\Delta t_{n-1}}{\Delta t_n}q_{n-1}^2 - 2k_nq_{n-1} + k_n = 0 \quad (21)$$

where

$$k_n = \frac{(1-q_n)^2}{(1-2q_n)(M(1-2q_n)+1)^2} \quad (22)$$

For each n, the unique root that lies in the interval $(0, \frac{1}{2})$ is chosen.

5.2.2 Forward Calculation of Parameters

After determining the sequence of q_n values, starting from Σ_0, the remaining parameters are calculated iteratively forward for each n:

$$\Sigma_n = \frac{1}{M(1-2q_n)+1}\Sigma_{n-1} \quad (23)$$

$$\lambda_n = \left(\frac{M\Sigma_n(1-2q_n)}{\Delta t\sigma_u^2(M(1-2q_n)+1)}\right)^{1/2} \quad (24)$$

$$\beta_n = \left(\frac{(1-2q_n)\sigma_u^2}{\Sigma_n\Delta t_n(M(1-2q_n)+1)}\right)^{1/2} \quad (25)$$

5.3 Model Analysis and Simulation Results

In this section, we explore how the multi-period model behaves as more informed traders (M) are introduced, focusing on the evolution of λ and β and their impact on price discovery. The goal is to observe if adding more informed traders accelerates information incorporation into prices.

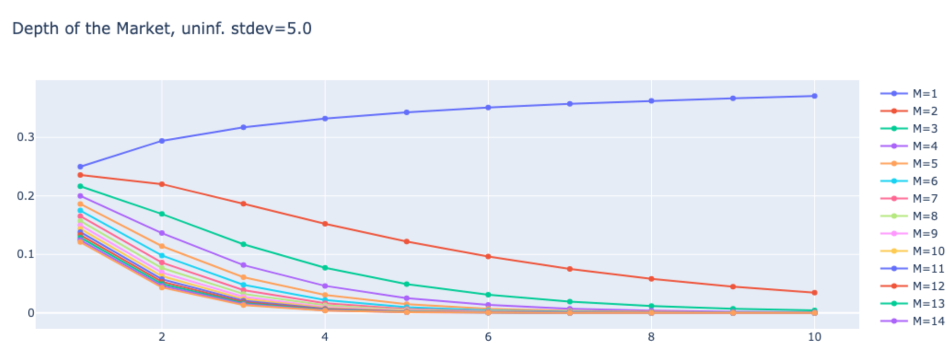

Figure 12: λ_N vs N for different M.

Figure 12 shows how the terminal market depth (λ_N) changes with the number of auctions (N) for varying numbers of informed traders (M). It highlights that as soon as competition is added ($M > 1$), market depth increases substantially with each additional informed trader, indicating a more liquid market.

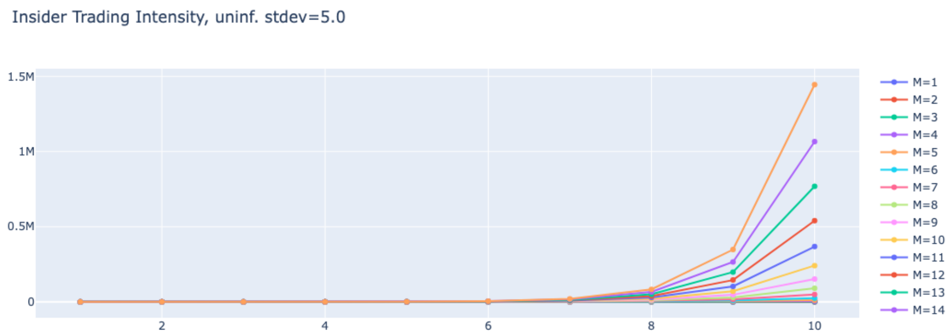

Figure 13: β_N vs N for different M.

As depicted in Figure 13, the trading intensity (β_N) experiences a considerable increase when multiple informed traders are present compared to the monopolistic case ($M = 1$). This suggests that informed traders are much more aggressive when competing with each other.

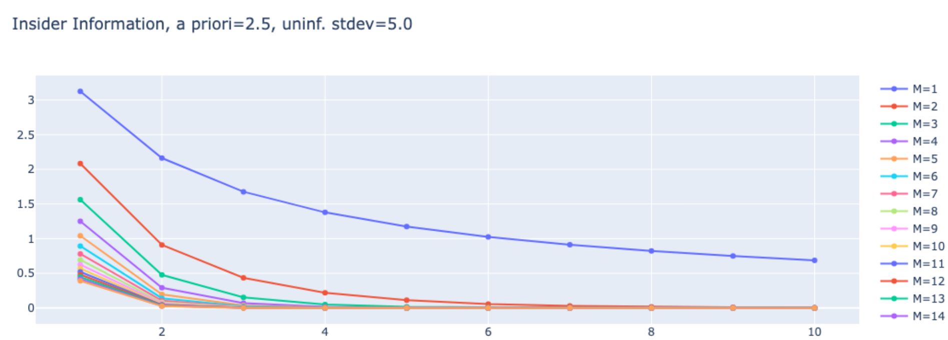

Figure 14: Σ_N vs N for different M.

Figure 14 illustrates the evolution of Σ_N with N for different M. It shows that private information is incorporated into prices very rapidly when there are multiple informed traders, indicating high informational efficiency.

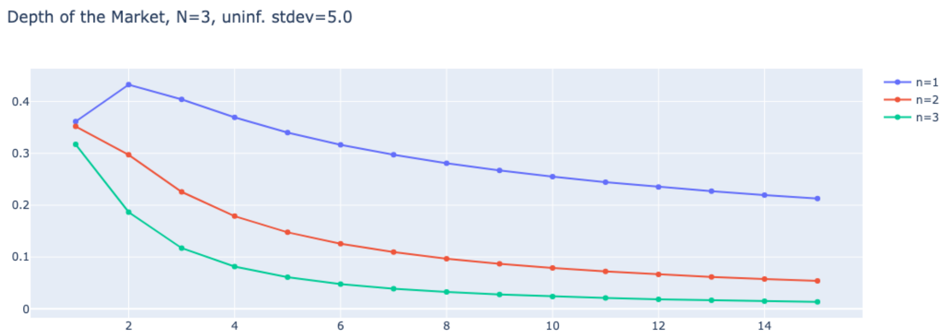

Figure 15: λ_n vs M for $N = 3$.

Figure 15 focuses on λ_n versus M for a fixed number of auctions $(N = 3)$. It highlights that the market becomes very liquid at the final auction as the number of informed traders (M) increases.

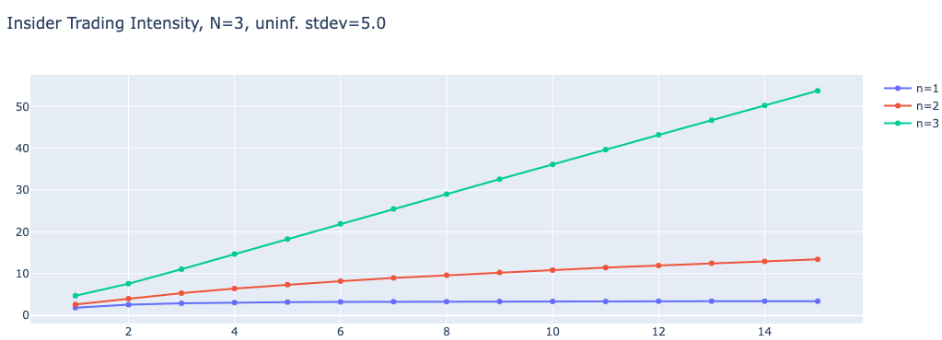

Figure 16: β_n vs M for $N = 3$.

Figure 16 illustrates that the final auction trading intensity (β_n) rapidly increases with an increasing number of informed traders (M).

Figure 17 shows that for $N = 3$, when the number of informed traders (M) reaches approximately 4 or more, there is virtually no hidden information left to trade on $(\Sigma_n$ approaches zero), signifying near-complete price discovery.

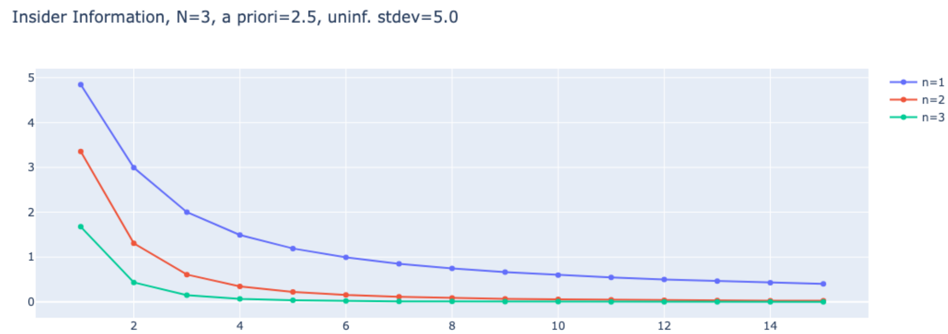

Figure 17: Σ_n vs M for $N = 3$.

6 Results and Conclusions

This report investigated various market microstructure models, building from the foundational single-period Kyle model to more complex multi-period and multi-informed trader extensions. The simulations and theoretical analyses provide significant insights into price discovery, market liquidity, and the strategic behavior of market participants under information asymmetry.

6.1 Results

6.1.1 Single-Period Kyle Model

The single-period Kyle model simulation demonstrated the fundamental process of price discovery. Bid and ask prices, initially set with a spread around a prior mean, gradually converge towards the true asset value as information is revealed through order flow. The model highlights the crucial role of Kyle's lambda (λ) and beta (β), which quantify market illiquidity and informed trader aggressiveness, respectively. The lambda surface analysis (Figure 2) clearly illustrated that market illiquidity increases with greater uncertainty about the asset's true value and decreases with higher levels of noise trading, confirming the theoretical inverse relationship with uninformed trading noise and direct relationship with asset uncertainty. Furthermore, the exploration of non-linear price adjustment functions (Figure 3) revealed that market makers' pricing rules significantly influence the dynamics of price convergence. Convex adjustments (e.g., squared, exponential) tended to accelerate price discovery but could introduce higher volatility, while concave adjustments (e.g., square root, logarithmic) often led to smoother price paths by dampening the impact of large trades.

6.1.2 Multi-Informed Traders in Single-Period Kyle Model

Extending the single-period Kyle framework to include multiple informed traders, as in Holden and Subrahmanyam (1992), highlights the profound impact of competition among insiders. As the number of informed traders (M) increases, market liquidity generally improves (reflected in a decrease in λ). However, this relationship does not hold uniformly in the single-period setting. Individual informed traders tend to become less aggressive in their trading strategies, as indicated by a decrease in β (Figure 5). Simulations (Figure 4) reveal that markets with multiple informed traders typically experience faster price discovery and quicker convergence of prices to the asset's true value. This acceleration often comes at the cost of slightly increased price volatility, driven by strategic competition as insiders attempt to front-run one another. Notably, the marginal benefit of adding more informed traders diminishes beyond a certain point, suggesting a saturation effect in the efficiency gains from insider competition.

6.1.3 Multi-Period Monopolistic Kyle Model

The multi-period model introduces a dynamic dimension to information revelation. In this setting with a single informed trader, private information is incorporated into prices gradually over time through a sequence of auctions. Simulations reveal that market depth, measured as the inverse of the price impact coefficient $1/\lambda_n$, generally increases over successive auctions (i.e., λ_n decreases with n) (Figure 6). This reflects a reduction in price sensitivity to order flow over time as more information is incorporated into prices.

Concurrently, the insider's trading aggressiveness, captured by β_n, tends to increase as auctions progress, albeit gradually (Figure 7). The conditional variance Σ_n, which represents the market makers' uncertainty about the asset's true value given observed order flow, declines with each auction (Figure 8), confirming a smooth and continuous information revelation process.

The analysis further shows that increasing the volatility of noise trading (higher σ_u) improves market depth (i.e., lowers λ_n; see Figure 9) and encourages more aggressive insider trading (Figure 10). Interestingly, despite these changes, the informativeness of prices — as reflected in the evolution of Σ_n — remains largely unaffected by the level of noise trading volatility (Figure 11).

6.1.4 Multi-Period Kyle Model with Multiple Informed Traders

The most comprehensive model explored was the multi-period setting with multiple informed traders. This framework highlighted a significant acceleration in price discovery compared to the monopolistic multi-period case (Figure 12). When multiple informed traders were present, market depth increased substantially with each additional trader, leading to a much more liquid market (Figure 15). Concurrently, individual trading intensity (β_n) among informed traders increased dramatically under competition (Figure 13, Figure 16), reflecting their aggression in exploiting private information. The simulations demonstrated that private information is incorporated into prices very rapidly in this competitive, dynamic environment (Figure 14). Indeed, with a sufficiently high number of informed traders (e.g., around 4 or more in some simulations, Figure 17), virtually all hidden information could be revealed, leading to near-complete price discovery.

6.2 Conclusions

The various Kyle-type models consistently demonstrate that private information enters prices gradually as informed traders exploit their advantage while avoiding detection, and prices adjust as market makers interpret total order flow. This process highlights the fundamental role of information asymmetry in market dynamics.

A key takeaway is the endogeneity of market liquidity. The price impact parameter (λ) is not static but is endogenously determined in equilibrium. It is inversely related to the market depth: a lower λ (indicating greater liquidity) encourages more aggressive trading, while a higher λ induces caution. Although market makers do not set λ directly, it emerges from their pricing strategy based on how informative they expect the order flow to be. Informed traders, in turn, adjust their aggressiveness in response to λ. This mutual dependence between pricing and trading behavior defines the market equilibrium.

The comparison between single-period and multi-period models underscores the temporal dimension of information revelation. In the single-period Kyle model, only partial information is revealed, with the market maker observing one noisy signal and adjusting price partially toward the true value. In contrast, in the multi-period model, the insider spreads trades over time. Prices are updated sequentially based on cumulative order flow, improving accuracy with each period. Price updates follow Bayesian learning, with the conditional variance Σ_n decreasing as more order flow is observed. In the multi-period model, λ_n typically decreases over time, as early trades are easier to disguise within noise, while later trades are more informative and move prices more sharply.

The introduction of multiple informed traders fundamentally accelerates price discovery. In the Holden and Subrahmanyam (1992) extension of the Kyle model, strategic competition among insiders compels them to reveal their private information more rapidly—particularly in the early stages of trading. As a result, markets become more efficient, with prices incorporating private information at a much faster pace. When the number of informed traders is sufficiently large, this process can lead to near-complete information revelation in just a few rounds. These dynamics underscore the critical role of competition in promoting market efficiency and enhancing informational transparency.