Price Discovery and Information Revelation

Single- and Multi-Period Kyle Models

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To study how private information becomes incorporated into asset prices through the process of price discovery. This is done by analyzing the one-period Kyle (1985) model and extending it to a simplified multi-period setting with $N > 1$ periods.

Some key questions:

- How is private information incorporated into prices over time?
- How does the informed trader's strategy interact with market liquidity?
- How does the price impact parameter λ evolve across periods in a multi-period Kyle model?
- How does information incorporation in the multi-period case differ from the static, single-period model?

In this canonical version of Kyle's model, all trading occurs in a single round. The market structure is defined by asymmetric information and strategic interaction between three types of agents:

- **Informed trader:** knows the true future value of the asset.
- **Noise traders:** trade randomly due to liquidity needs.
- **Market maker:** sets the price based on the total observed order flow and updates their beliefs accordingly.

- At the start of the period, agents trade an asset with an uncertain end-of-period value $\tilde{v} \sim \mathcal{N}(p_0, \Sigma_0)$.
- Noise traders trade for exogenous reasons unrelated to information. They submit a random market order $\tilde{u} \sim \mathcal{N}(0, \sigma_u^2)$, independent of $\tilde{v}$.
- The informed trader is risk-neutral and chooses trade size x to maximize expected profit. The trader knows $\tilde{v}$, but not $\tilde{u}$.
- The market maker is risk-neutral and observes total order flow $y = x + u$, but cannot distinguish informed trades from noise (all orders are anonymous).

Under normality assumptions, the Kyle (1985) model admits a linear equilibrium, where both the informed trader's demand and the market maker's pricing rule are linear functions:

- **Informed Trader's Strategy:** The trader chooses demand based on the deviation of the asset's value from the prior mean: $x = \beta(\tilde{v} - p_0)$. The parameter β reflects the insider's trading aggressiveness.
- **Market Maker's Pricing Rule:** Observing total order flow $y = x + u$, the market maker sets: $p = p_0 + \lambda y$, where λ captures the price sensitivity to order flow (i.e., the price impact).

Solving the Informed Trader and Market Maker Conditions

- **Insider's strategy:** $\max_x \mathbb{E}[(v-(p_0+\lambda(x+u)))x \mid v] = (v-p_0-\lambda x)x$.
  FOC gives $x = \frac{v-p_0}{2\lambda}$, implying $\beta = \frac{1}{2\lambda}$.
- **Market maker sets:** $p = \mathbb{E}[v \mid y] = p_0 + \frac{\text{Cov}(v,y)}{\text{Var}(y)}(y - \mathbb{E}[y])$.
  Since $y = \beta(v-p_0)+u$, we get: $\mathbb{E}[y] = 0$, $\text{Cov}(v,y) = \beta\Sigma_0$, $\text{Var}(y) = \beta^2\Sigma_0 + \sigma_u^2$. Thus,

$$\lambda = \frac{\beta\Sigma_0}{\beta^2\Sigma_0 + \sigma_u^2}$$

- Substituting $\beta = \frac{1}{2\lambda}$ gives the equilibrium values:

$$\boxed{\lambda = \frac{1}{2}\sqrt{\frac{\Sigma_0}{\sigma_u^2}}, \quad \beta = \sqrt{\frac{\sigma_u^2}{\Sigma_0}}}$$

Summary of Equilibrium Conditions

- The impact is linear and liquidity increases with the amount of noise traders:

$$p = p_0 + \frac{1}{2}\sqrt{\frac{\Sigma_0}{\sigma_u^2}}\, y$$

- The informed trader increases trade size when their orders can be more easily hidden within noise:

$$x = (v - p_0)\sqrt{\frac{\sigma_u^2}{\Sigma_0}}$$

- The expected profit of the informed agent grows with noise trading:

$$\mathbb{E}[\pi] = \frac{(v - p_0)^2}{2}\sqrt{\frac{\sigma_u^2}{\Sigma_0}}$$

- Noise traders lose money; market makers break even on average.

We simulated the model to visualize how the theoretical equilibrium plays out in practice. This included examining how the price impact parameter λ changes with true asset value uncertainty Σ_0 and noise trader order uncertainty σ_u^2.
We also tested different non-linear price adjustment functions to observe how these affect the evolution of market maker quotes.

Baseline parameters:

- Prior mean of asset value: $p_0 = 100.0$
- Standard deviation of true asset value: $\sigma_v = 2.5$
- Standard deviation of noise trader orders: $\sigma_u = 5.0$

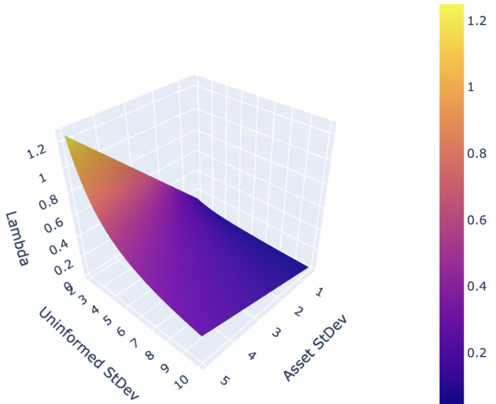

Figure 1: λ increases with asset uncertainty (insider's information is more valuable), and decreases with noise (order flow is less informative).

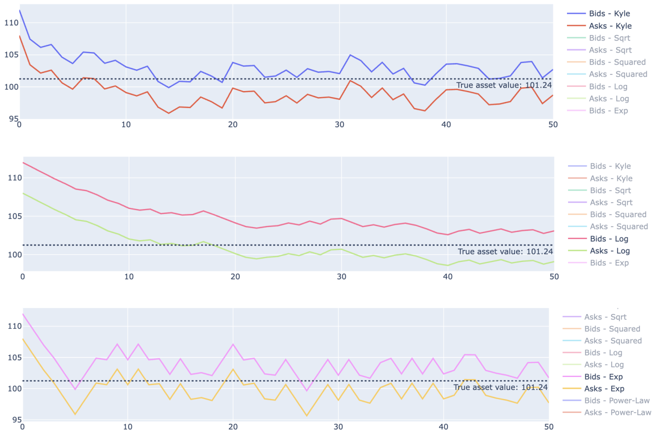

Figure 2: Concave adjustments (e.g., log, sqrt) reduce the effect of large trades, leading to more stable price paths than the linear Kyle. Convex adjustments (e.g., squared, exp) react more strongly to large trades, accelerating convergence but increasing volatility.

- Holden and Subrahmanyam (1992) extend Kyle's single-period model by allowing for $M > 1$ informed traders who compete strategically in the market.
- Although the overall structure remains largely intact, the presence of multiple insiders changes how private information is incorporated into prices.
- A competitive, risk-neutral market maker observes only total order flow $y = \sum x_i + u$ and sets the price $p = \mathbb{E}[v \mid y]$.
- Traders follow linear strategies $x_i = \beta(v - p)$, and the market maker uses a pricing rule $p = \mu + \lambda y$. A unique linear equilibrium exists.

Insider's strategy (for each of M insiders):

Each informed trader maximizes their expected profit, assuming the others follow the same linear strategy. The maximization function becomes:

$$\max_{x_i} \mathbb{E}[(v - p)x_i \mid v] = (v - \mu - \lambda(x_i + \sum_{j \neq i} x_j + u))x_i$$

Assuming symmetric strategies $x_j = \beta(v - \mu)$, this simplifies to:

$$\max_{x_i}(v - \mu - \lambda(x_i + (M - 1)\beta(v - \mu)))x_i$$

Solving the FOC gives:

$$x_i = \frac{1 - \lambda\beta(M - 1)}{2\lambda}(v - \mu)$$

From symmetry, $x_i = \beta(v - \mu)$, so:

$$\beta = \frac{1}{\lambda(M + 1)}$$

Market maker sets price:

The market maker observes total order flow $y = \sum_{i=1}^{M} x_i + u = M\beta(v - p_0) + u$, and sets the price using conditional expectation:

$$p = \mathbb{E}[v \mid y] = p_0 + \frac{\mathrm{Cov}(v, y)}{\mathrm{Var}(y)}(y - \mathbb{E}[y])$$

Since $y = M\beta(v - p_0) + u$, we compute:

$$\mathbb{E}[y] = 0, \quad \mathrm{Cov}(v, y) = M\beta\Sigma_0, \quad \mathrm{Var}(y) = M^2\beta^2\Sigma_0 + \sigma_u^2$$

Thus,

$$\lambda = \frac{M\beta\Sigma_0}{M^2\beta^2\Sigma_0 + \sigma_u^2}$$

Substituting $\beta = \frac{1}{\lambda(M+1)}$, we get the closed-form equilibrium:

$$\Rightarrow \boxed{\lambda = \frac{\sqrt{M\Sigma_0}}{\sigma_u\sqrt{M+1}}, \quad \beta = \frac{1}{\lambda(M+1)}}$$

- In this section, we explore how the model behaves as more informed traders are introduced.
- We focus on how:
  - λ and β respond as M increases.
  - The market maker adjusts prices toward the true asset value.
- The goal is to see if adding more informed traders speeds up price discovery.

Figure 3: Both models drive prices toward the true asset value, but the HS model adjusts faster initially. With multiple informed traders, prices show slightly higher volatility due to more aggressive competition. Quotes also begin closer to fair value, reflecting greater pricing efficiency. Overall, more informed traders make the market react faster to information. Additionally, as more informed traders are introduced, their incremental impact on price discovery diminishes.

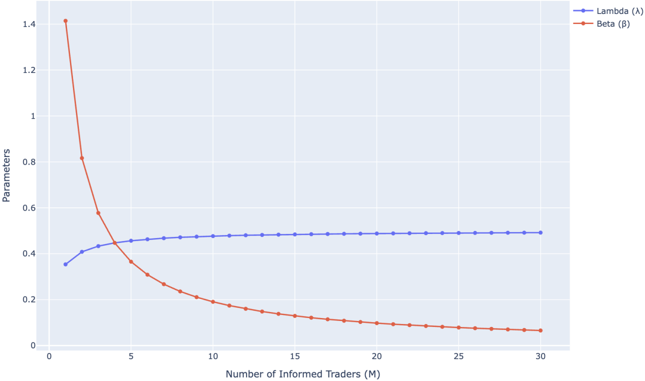

Figure 4: As the number of informed traders (M) increases, λ rises but more slowly — prices become less sensitive to each additional trader. β drops quickly, showing that each trader becomes less aggressive.

- One informed trader (a monopolist)
- There are N auctions happening at times t_k, $0 = t_0 < t_1 < \cdots < t_N = 1$.
- For implementation purposes, we assume equally spaced intervals: $\Delta t_k = t_k - t_{k-1} = \frac{1}{N}$.
- The liquidation value of the asset $v \sim \mathcal{N}(p_0, \Sigma_0)$.

- Quantity traded by noise traders is a *Brownian motion process* $u_n = u(t_n)$.
- $\Delta u_n = u_n - u_{n-1}$ is normally distributed with zero mean and variance $\sigma_t^2 \Delta t_n$.
- Quantities traded at one auction are independent of the quantities traded at other auctions.

- x_n is the aggregate position of the insider after n-th auction, so that $\Delta x_n = x_n - x_{n-1}$ is the quantity traded at n-th auction.
- p_n is the market clearing price at the n-th auction.
- When deciding what quantity to trade, the insider uses the liquidation value v and past prices:

$$x_n = X_n(p_1, \dots, p_{n-1}, v) \tag{1}$$

- Then, market makers set a market clearing price using current and all previous order flows:

$$p_n = P_n(x_1 + u_1, \dots, x_n + u_n) \tag{2}$$

- Profit of the insider on the positions acquired at auctions $n, \dots, N$:

$$\pi_n = \sum_{k=n}^{N} (v - p_k) x_k \tag{3}$$

A *sequential auction equilibrium* is defined such that:

- *profit maximization* is achieved, i.e. we find such trading rules $X = (X_1, \dots, X_N)$ that the expected profit is maximized:

$$\mathbb{E}\{\pi_n(X, P) \mid p_1, \dots, p_{n-1}, v\} \tag{4}$$

- *market efficiency* condition holds:

$$p_n = \mathbb{E}\{v \mid x_1 + u_1, \dots, x_n + u_n\} \tag{5}$$

Kyle considers a *recursive linear equilibrium*, where X_k and P_k are linear and:

$$\Delta x_n = \beta_n(v - p_{n-1})\Delta t_n \tag{6}$$

$$\Delta p_n = \lambda_n(\Delta x_n + \Delta u_n) \tag{7}$$

In this equilibrium, price increments are normally and independently distributed. Thus, the distribution function for the pricing process is characterized by a sequence of variance parameters measuring the volatility of price fluctuations:

$$\Sigma_n = Var\{v \mid x_1 + u_1, \dots, x_n + u_n\} \tag{8}$$

The equilibrium defined above *exists* and is *unique*. Given Σ_0, the values of λ_n, β_n and Σ_n are a unique solution to the difference equation system, subject to $\alpha_N = 0$ and second order condition $\lambda_n(1 - \alpha_n\lambda_n) > 0$:

$$\beta_n = \frac{1 - 2\alpha_n\lambda_n}{2\lambda_n\Delta t_n(1 - \alpha_n\lambda_n)} \tag{9}$$

$$\lambda_n = \frac{\beta_n\Sigma_n}{\sigma_u^2} \tag{10}$$

$$\alpha_{n-1} = \frac{1}{4\lambda_n(1 - \alpha_n\lambda_n)} \tag{11}$$

$$\Sigma_{n-1} = \frac{\Sigma_n}{1 - \beta_n\lambda_n\Delta t_n} \tag{12}$$

The system is solved backwards: first we provide a "guess" for Σ_N, and then for each auction in the order $N, N-1, \ldots, 1$, Kyle showed that λ_n is the middle root of the cubic equation:

$$\left(1 - \frac{\lambda_n^2 \sigma_u^2 \Delta t_n}{\Sigma_n}\right)(1 - \alpha_n \lambda_n) = \frac{1}{2} \tag{13}$$

So, for each iteration n, we already have α_n and Σ_n calculated. We solve (13) for λ_n, and then calculate β_n, α_{n-1} and Σ_{n-1} from λ_n. At the end, we compare our computed Σ_0 with the true value. If they are close, we solved the system, otherwise our guess for Σ_N was incorrect.

To find a correct Σ_N, we use methods like bisection or Newton-Raphson and minimize the difference between true and computed Σ_0.

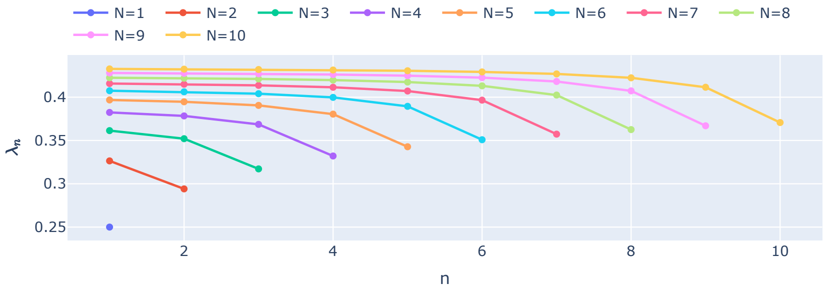

Figure 5: λ_n depending on number of auctions N.

Notice how terminal value λ_N increases with N. When we have a monopolistic informed trader, market depth decreases with more auctions held.

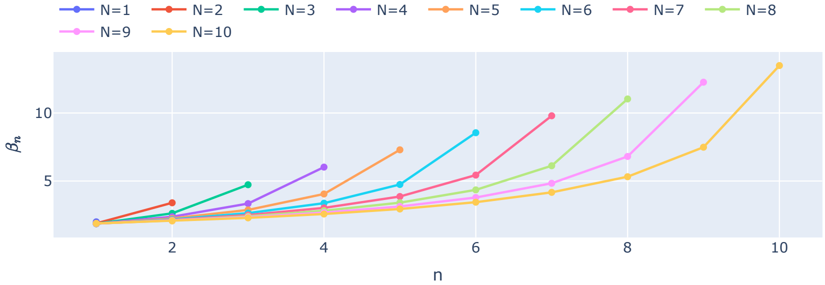

Figure 6: β_n depending on number of auctions N.

Insider trading intensity increases with N, but slowly.

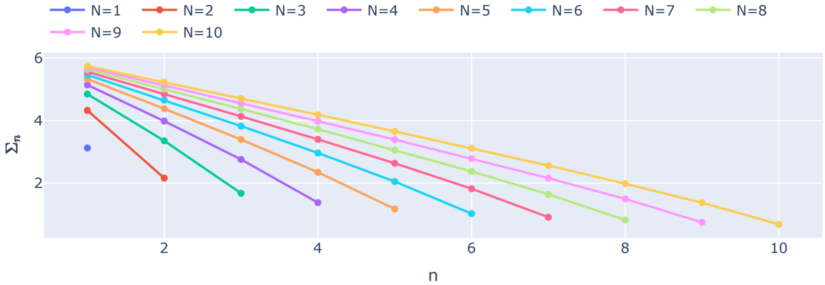

Figure 7: Σ_n depending on number of auctions N.

Variance of prices, or the measure of how much information is not yet incorporated into prices, is decreasing slowly.

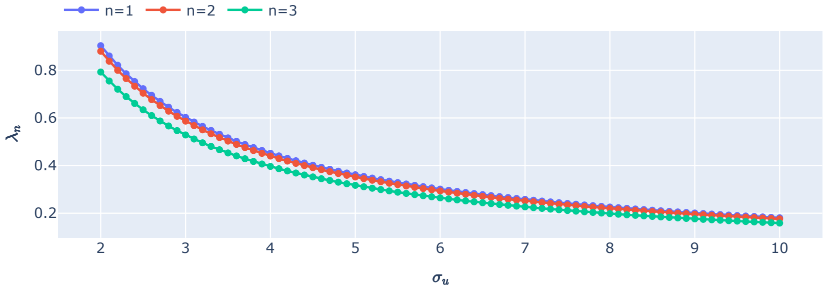

Figure 8: λ_n vs σ_u.

Increasing the amount of noise trading increases market depth.

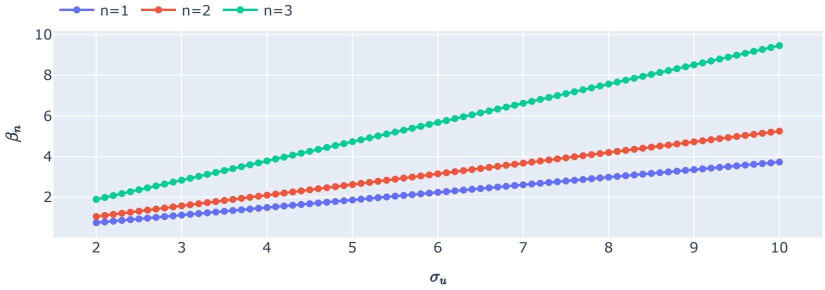

Figure 9: β_n vs σ_u.

Insider trading intensity increases with noise trading increasing.

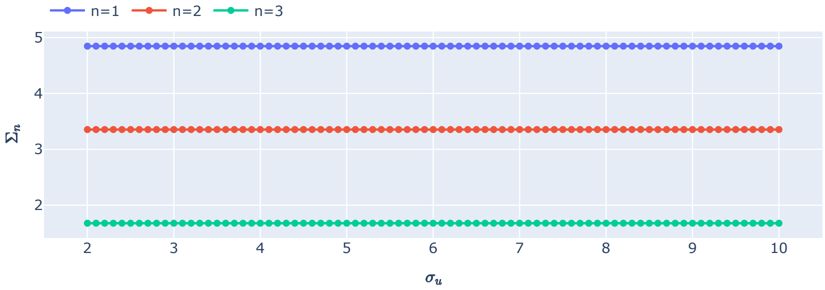

Figure 10: Σ_n vs σ_u.

Σ_n, informativeness of trades, does not depend on noise trading volatility.

- **Holden** and **Subrahmanyam** proposed an extension of multi-period Kyle model by introducing M informed traders.
- The model setup remains the same, except for Δx_n, which now means total order of *all* informed traders at n-th auction.
- A unique linear equilibrium also exists for such a model.

The equilibrium is defined by equations:

$$\Delta x_n = M\beta_n(v - p_{n-1})\Delta t_n \tag{14}$$

$$\Delta p_n = \lambda_n(\Delta x_n + \Delta u_n) \tag{15}$$

$$\Sigma_n = Var\{v \mid x_1 + u_1, \ldots, x_n + u_n\} \tag{16}$$

Difference equation system, with boundary conditions $\alpha_N = 0$ and second order condition $\lambda_n(1 - \alpha_n\lambda_n) > 0$:

$$\beta_n = \frac{1 - 2\alpha_n\lambda_n}{\lambda_n \Delta t_n (M(1 - 2\alpha_n\lambda_n) + 1)} \tag{17}$$

$$\lambda_n = \frac{M\beta_n\Sigma_n}{\sigma_u^2} \tag{18}$$

$$\alpha_{n-1} = \frac{1 - \alpha_n\lambda_n}{\lambda_n (M(1 - 2\alpha_n\lambda_n) + 1)^2} \tag{19}$$

$$\Sigma_{n-1} = \frac{\Sigma_n}{1 - M\beta_n\lambda_n\Delta t_n} \tag{20}$$

When $M = 1$, we get the Kyle multi-period model.

- This model cannot be solved with "guessing" approach, but Holden and Subrahmanyam found a better way, which solves model for M, including $M = 1$.
- The method is explicit, and does not involve "guessing" Σ_N.

Define $q_n = \alpha_n \lambda_n$. Starting from $q_N = 0$, solve in a backward manner, for every n, the cubic equation:

$$2M \frac{\Delta t_{n-1}}{\Delta t_n} q_{n-1}^3 - (M+1) \frac{\Delta t_{n-1}}{\Delta t_n} q_{n-1}^2 - 2k_n q_{n-1} + k_n = 0 \quad (21)$$

where

$$k_n = \frac{(1-q_n)^2}{(1-2q_n)(M(1-2q_n)+1)^2} \quad (22)$$

and choose the unique root that lies in the interval $(0, \frac{1}{2})$.

After that, starting from Σ_0, iterate forward and calculate parameters:

$$\Sigma_n = \frac{1}{M(1-2q_n)+1}\Sigma_{n-1} \tag{23}$$

$$\lambda_n = \left(\frac{M\Sigma_n(1-2q_n)}{\Delta_t\sigma_u^2(M(1-2q_n)+1)}\right)^{\frac{1}{2}} \tag{24}$$

$$\beta_n = \left(\frac{(1-2q_n)\sigma_u^2}{\Sigma_n\Delta t_n(M(1-2q_n)+1)}\right)^{\frac{1}{2}} \tag{25}$$

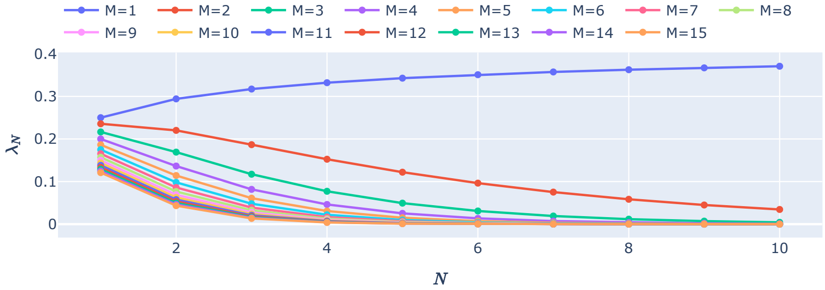

Figure 11: λ_N vs N for different M.

As soon as we add competition, market depth increases a lot with each added informed trader.

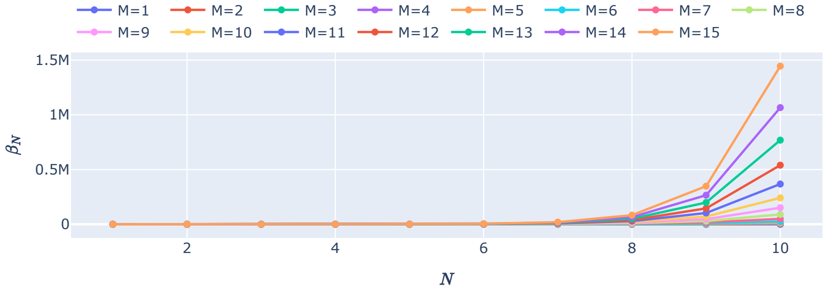

Figure 12: β_N vs N for different M.

Trading intensity explodes in comparison to $M = 1$ case. Informed traders are much more aggressive.

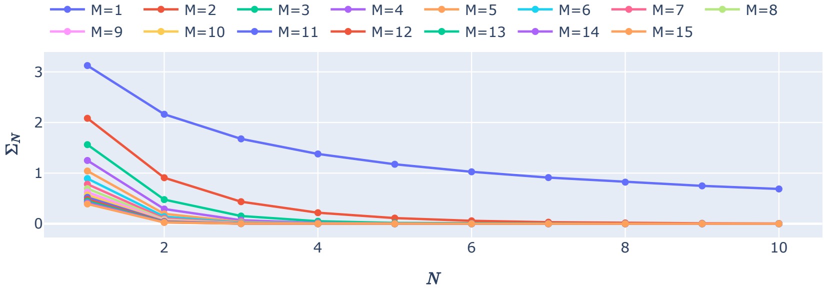

Figure 13: Σ_N vs N for different M.

Private information is incorporated very fast.

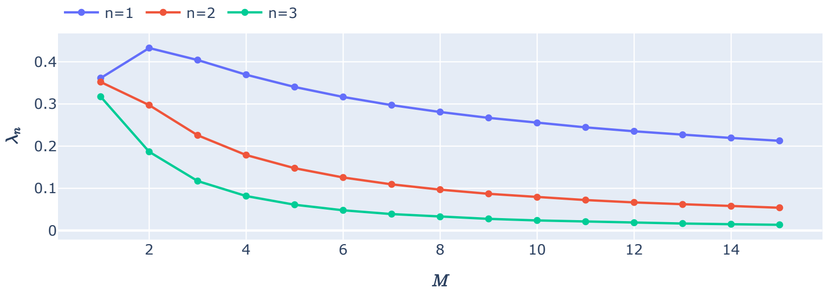

Figure 14: λ_n vs M, $N = 3$.

Market is very liquid at the final auction.

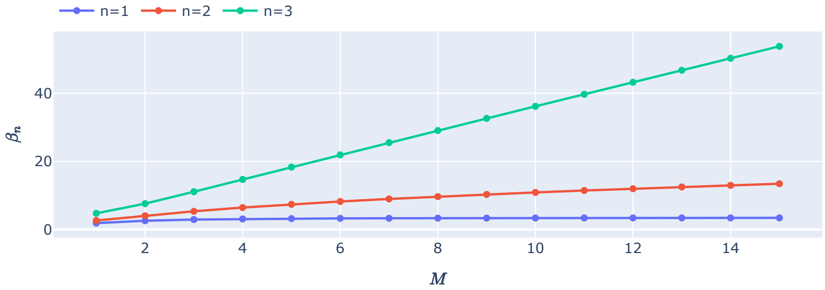

Figure 15: β_n vs M, $N = 3$.

Final auction trading intensity is rapidly increasing with increasing M.

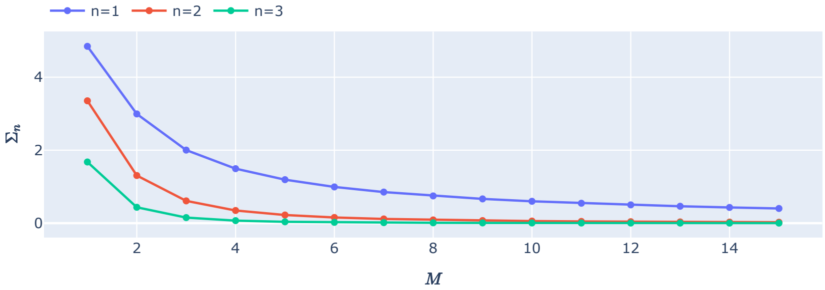

Figure 16: Σ_n vs M, $N = 3$.

Already when we have 4 competitors, there is no hidden information left to trade on.

- **Private information enters prices gradually** as informed traders exploit their advantage while avoiding detection. Prices adjust as market makers interpret total order flow.
- **In the single-period Kyle model**, only partial information is revealed. The market maker observes one noisy signal and adjusts price partially toward the true value.
- **In the multi-period model**, the insider spreads trades over time. Prices are updated sequentially based on cumulative order flow, improving accuracy with each period.
- **Price updates follow Bayesian learning**, with the conditional variance Σ_n decreasing as more order flow is observed.
- **Trading strategy depends on liquidity**, inversely related to the price impact λ. Lower λ (more liquidity) leads to more aggressive trades; higher λ leads to caution.

- **Liquidity and strategy are jointly determined.** The market maker sets the price impact λ based on how informative trades are expected to be. Informed traders, in turn, adjust their aggressiveness depending on λ. This mutual dependence defines the equilibrium.
- **In the multi-period model,** λ_n typically increases over time. Early trades are easier to disguise within noise; later trades are more informative and move prices more sharply.
- **Multiple informed traders accelerate price discovery.** In the Holden Subrahmanyam (1992) extension, strategic competition pushes traders to reveal information more quickly, especially early in the trading sequence.

Q&A