Задание 5.

$$\omega_f = \omega_0 - \omega_1 \sin^2 \frac{fa}{2} =$$

$$= \omega_0 - \omega_1 \left(\frac{1 - \cos fa}{2} \right) =$$

$$= \omega_0 - \frac{\omega_1}{2} + \frac{\omega_1}{2} \cos(fa)$$

$$g(\omega) = \frac{1}{3N} \sum_{f}{}' \delta\left(\omega_0 - \frac{\omega_1}{2} + \frac{\omega_1}{2}\cos(fa)\right.$$

$$\left. - \omega\right) =$$

$$= \frac{a}{3N\, 2\pi} \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} \delta\left(\omega_0 - \frac{\omega_1}{2} + \frac{\omega_1}{2}\cos(fa) - \omega\right) df$$

$$= \frac{1}{3N \cdot 2\pi} \int_{-\pi}^{\pi} \delta\left(\omega_0 - \frac{\omega_1}{2} + \frac{\omega_1}{2}\cos\xi - \omega\right) d\xi$$

$$= \frac{1}{3N(2\pi)^2} \int_{-\pi}^{\pi} d\xi \int_{-\infty}^{\infty} \exp\left(ip\left(\omega_0 - \frac{\omega_1}{2} + \frac{\omega_1}{2}\cos\xi -\right.\right.$$

$$\left.\left. - \omega\right)dp = \frac{1}{3N(2\pi)^2} \int_{-\infty}^{\infty} dp\, \exp\left(ip\left(\omega_0 - \frac{\omega_1}{2} - \omega\right)\right)$$

$$\cdot \int_{-\pi}^{\pi} \exp\left(ip\frac{\omega_1}{2}\cos\xi\right) d\xi =$$

$$= \frac{1}{3N\cdot 2\pi}\int_{-\infty}^{\infty} dp\, \exp\left(ip\left(\omega_0 - \frac{\omega_1}{2} - \omega\right)\right) J_0\left(p\frac{\omega_1}{2}\right)$$

$$= \frac{1}{3\pi N}\int_{0}^{\infty} dp\, \cos\left(p\left(\omega_0 - \frac{\omega_1}{2} - \omega\right)\right) J_0\left(p\frac{\omega_1}{2}\right)$$

Если $\omega_0 - \frac{\omega_1}{2} - \omega < \frac{\omega_1}{2}$, то

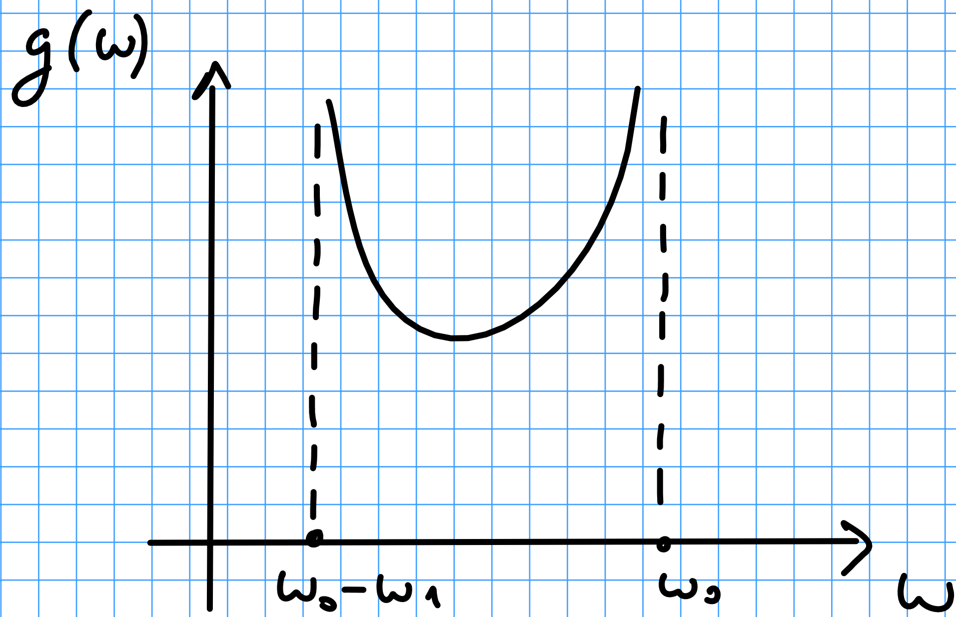
если если $\omega_0 - \omega_1 < \omega$, то

$$g(\omega) = \frac{1}{3\pi N}\cdot\frac{1}{\sqrt{\left(\frac{\omega_1}{2}\right)^2 - \left(\omega_0 - \frac{\omega_1}{2} - \omega\right)^2}}$$

$$= \frac{1}{3\pi N}\,\frac{1}{\sqrt{\left(\frac{\omega_1}{2}\right)^2 - \left(\frac{\omega_1}{2}\right)^2 - (\omega_0-\omega)^2 + 2(\omega_0-\omega)\frac{\omega_1}{2}}}$$

$$= \frac{1}{3\pi N}\,\frac{1}{\sqrt{(\omega_0-\omega)(\omega_1-\omega_0+\omega)}}$$

В противном случае получим ноль.



g(ω)
ω₀-ω₁
ω₀
ω