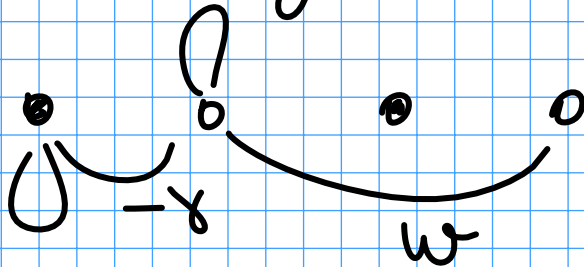


# Контрольная работа по теоретической физике твёрдого тела



Масса иона:  $M$

Масса электрона:  $m$

$$A_{12}(0) = -\gamma, \quad A_{12}(-a) = -\gamma$$

$$A_{21}(0) = -\gamma, \quad A_{21}(a) = -\gamma$$

$$A_{21}(0) = w, \quad A_{22}(-a) = w$$

Известно, что  $\sum_{jn} A_{ji}(n) = 0$

$$\sum_{jn} A_{j1}(n) = \sum_n A_{11}(n) + \sum_n A_{21}(n) = 0$$

$$A_{11}(0) + A_{21}(0) + A_{21}(a) = 0$$

$$A_{11}(0) = -A_{21}(0) - A_{21}(a) = 2\gamma$$

$$\begin{aligned} \sum_{jn} A_{j2}(n) &= \sum_n A_{12}(n) + \sum_n A_{22}(n) = \\ &= 0 \end{aligned}$$

$$A_{12}(0) + A_{12}(-a) + A_{22}(0) + A_{22}(a) + A_{22}(-a) = 0$$

$$A_{22}(0) = 2\gamma - 2w$$

$$C_{ij} = \frac{1}{\sqrt{M_i M_j}} \sum_n A_{ij}(n) e^{-inl}$$

$$C_{11} = \frac{1}{M} \cdot A_{11}(0) = \frac{2\gamma}{M}$$

$$\begin{aligned} C_{22} &= \frac{1}{m} (A_{22}(0) + A_{22}(a)e^{-i\alpha l} + A_{22}(-a)e^{i\alpha l}) \\ &= \frac{1}{m} (2\gamma - 2w + 2w \cos \frac{la}{2}) = \\ &= \frac{2\gamma}{m} \left( 1 - 2\alpha \sin^2 \frac{la}{2} \right), \end{aligned}$$

$$\text{где } \alpha = \frac{w}{\gamma}$$

$$\begin{aligned} C_{12} &= \frac{1}{\sqrt{Mm}} (A_{12}(0) + A_{12}(-a)e^{i\alpha l}) = \\ &= \frac{1}{\sqrt{Mm}} (-\gamma - \gamma e^{i\alpha l}) = -\frac{\gamma}{\sqrt{Mm}} (1 + e^{i\alpha l}) \end{aligned}$$

$$\begin{aligned} C_{21} &= \frac{1}{\sqrt{Mm}} (A_{21}(0) + A_{21}(a)e^{-i\alpha l}) = \\ &= -\frac{\gamma}{\sqrt{Mm}} (1 + e^{-i\alpha l}) \end{aligned}$$

Получим образы,

$$C_{11} = \frac{2\gamma}{M}, \quad C_{22} = \frac{2\gamma}{m} \left( 1 - 2\alpha \sin^2 \frac{la}{2} \right)$$

$$C_{12} = -\frac{\gamma}{\sqrt{Mm}} (1 + e^{i\alpha l}), \quad C_{21} = -\frac{\gamma}{\sqrt{Mm}} (1 + e^{-i\alpha l})$$

Найти собственные значения  
матрицы  $C$ , то есть  $\omega^2$ :

$$\begin{vmatrix} C_{11} - \omega^2 & C_{12} \\ C_{21} & C_{22} - \omega^2 \end{vmatrix} = (C_{11} - \omega^2)(C_{22} - \omega^2)$$

$$-C_{12}C_{21} = \omega^4 - \omega^2(C_{11} + C_{22}) + C_{11}C_{22} - C_{12}C_{21} = 0$$

Итак:

$$\omega^2 = \frac{C_{11} + C_{22}}{2} \pm \sqrt{\left(\frac{C_{11} + C_{22}}{2}\right)^2 - (C_{11}C_{22} - C_{12}C_{21})}$$

Подстановка  $C_{ij}$ .

$$C_{12}C_{21} = \frac{1}{Mm} \gamma^2 (1 + e^{i\frac{1}{2}ka}) (1 + e^{-i\frac{1}{2}ka}) =$$

$$= \frac{1}{Mm} \gamma^2 (2 + e^{i\frac{1}{2}ka} + e^{-i\frac{1}{2}ka}) = \frac{1}{Mm} \gamma^2 (2 +$$

$$+ 2\cos \frac{ka}{2}) = \frac{4\gamma^2}{Mm} \cos^2 \frac{ka}{2}$$

$$C_{11}C_{22} = \frac{4\gamma^2}{Mm} (1 - 2\alpha \sin^2 \frac{ka}{2})$$

$$C_{11}C_{22} - C_{12}C_{21} = \frac{4\gamma^2}{Mm} (1 - 2\alpha \sin^2 \frac{ka}{2} - \cos^2 \frac{ka}{2})$$

$$= \frac{4\gamma^2}{Mm} (1 - 2\alpha) \sin^2 \frac{ka}{2}$$

$$\frac{C_{11} + C_{22}}{2} = \frac{\gamma}{M} + \frac{\gamma}{m} - \frac{2\alpha\gamma}{m} \sin^2 \frac{ka}{2} =$$

$$= \frac{\gamma}{m} \left( \frac{m}{M} + 1 - 2\alpha \sin^2 \frac{f_a}{2} \right)$$

$U_{max,1}$

$$\omega^2 = \frac{\gamma}{m} \left( \frac{m}{M} + 1 - 2\alpha \sin^2 \frac{f_a}{2} \right) \pm \sqrt{\gamma^2 \left( \frac{m}{M} + 1 - 2\alpha \sin^2 \frac{f_a}{2} \right)^2 - \frac{4\gamma^2}{Mm} (1-2\alpha) \sin^2 \frac{f_a}{2}}$$

Можно записать в такой форме:

$$\omega^2 = \frac{\gamma}{m} \left( \frac{m}{M} + 1 - 2\alpha \sin^2 \frac{f_a}{2} \right) \cdot \left( 1 \pm \sqrt{1 - 4 \frac{m/M (1-2\alpha) \sin^2 \frac{f_a}{2}}{\left( \frac{m}{M} + 1 - 2\alpha \sin^2 \frac{f_a}{2} \right)^2}} \right)$$

Приступим ко второй части задачи.  
Пусть  $f_a \ll 1$ . Тогда:  
 $\sin^2 \frac{f_a}{2} \approx \left( \frac{f_a}{2} \right)^2$

$$\omega^2 \approx \frac{\gamma}{m} \left( \frac{m}{M} + 1 - 2\alpha \left( \frac{f_a}{2} \right)^2 \right) \cdot \left( 1 \pm \sqrt{1 - 4 \frac{(m/M)(1-2\alpha) \left( \frac{f_a}{2} \right)^2}{\left( \frac{m}{M} + 1 \right)^2}} \right) \approx$$

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