

Задача 2:

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \quad P = Q(Q^T Q)^{-1} Q^T$$

$$\text{Тогда } P_A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0,5 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \end{pmatrix}$$

$$P_B = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \frac{5}{6} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{6} & \frac{2}{6} \\ -\frac{1}{3} & \frac{1}{3} \\ \frac{5}{6} & -\frac{1}{3} \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix} = \begin{pmatrix} \frac{5}{6} & \frac{1}{3} & \frac{1}{6} \\ \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} \\ \frac{1}{6} & -\frac{1}{3} & \frac{5}{6} \end{pmatrix}$$

$$v_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \text{ нормируем, получаем } \begin{pmatrix} \frac{\sqrt{2}}{2} \\ 0 \\ \frac{\sqrt{2}}{2} \end{pmatrix} \quad v_1 = v_1 - \hat{v}_1$$

$$v_2 = v_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \text{---} \quad \text{---} \quad \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad Q = \begin{pmatrix} \frac{\sqrt{2}}{2} & 0 \\ 0 & 1 \\ \frac{\sqrt{2}}{2} & 0 \end{pmatrix} \quad \hat{v}_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$A = QR \rightarrow Q^T A = Q^T QR$$

$$Q^T A = R \Rightarrow R = \begin{pmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} \sqrt{2} & 0 \\ 0 & 1 \end{pmatrix}$$

$$A = QR = \begin{pmatrix} \frac{\sqrt{2}}{2} & 0 \\ 0 & 1 \\ \frac{\sqrt{2}}{2} & 0 \end{pmatrix} \begin{pmatrix} \sqrt{2} & 0 \\ 0 & 1 \end{pmatrix}$$

$$v_1 = v_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad v_2 = v_2 - \hat{v}_2, \text{ где } \hat{v}_2 = \left(\frac{v_2 \cdot v_1}{v_1 \cdot v_1} \right) v_1 \quad \hat{v}_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$\text{Значит } v_2 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

$$Q = \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{3}}{3} \\ 0 & \frac{\sqrt{3}}{3} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{3}}{3} \end{pmatrix} \quad R = \begin{pmatrix} \sqrt{2} & \sqrt{2} \\ 0 & \sqrt{3} \end{pmatrix} \quad B = \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{3}}{3} \\ 0 & \frac{\sqrt{3}}{3} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{3}}{3} \end{pmatrix} \begin{pmatrix} \sqrt{2} & \sqrt{2} \\ 0 & \sqrt{3} \end{pmatrix}$$