Numerical Methods for Differential Equations

University of Houston, Spring 2023

MATH 4365 (Dr. He)

Contents

1 Introduction

Definition 1.1 (Lipschitz continuous)**.**
A real-valued function $f : \mathbb{R} \to \mathbb{R}$ is **Lipschitz continuous** if there exists a positive real constant $L[f]$ such that for all $x_1, x_2 \in \mathbb{R}$,

$$\|f(x_1) - f(x_2)\| \leq L[f]\|x_1 - x_2\|$$

Definition 1.2 (Initial value problem)**.**

Theorem 1.1 (Picard-Lindelohf)**.**

2 Euler's method

3 Models

3.1 Predator-prey

3.2 Three-body problem

3.3 Chemical kinetics

4 Euler methods

5 Runge-Kutta

6 Multistep methods

7 Finite difference approximation

7.1 Laplace equation on the square

Solve $u : \mathbb{R}^2 \to \mathbb{R}$ on the unit square $\Omega = [0, 1]$ with Dirichlect boundary condition.