Metamodels for complex structured objects classification

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Abstract

The development and proliferation of various portable sensors poses new challenges for analyzing and finding meaning in this data. In our work we investigate classification of complex structured objects. One of the main problems in this task is to generate meaningful and relatively small set of features. We compare several approaches for feature extraction such as expertly defined features, autoregression model and SSA. We propose a new feature generation algorithm, based on local spline approximation. The experiment is conducted on two datasets for human activity recognition using accelerometer.

Introduction

This project is dedicated to multiclass classification of complex structured objects (with features of different scale, length and/or type). The problems in this area include human activity recognition from accelerometer time series [1–3], multimedia indexing [4] and recognition of people activity in smart homes [5].

We investigate classification of accelerometer time series. New methods in this field range from topological data analysis [6] to using convolutional neural networks [7]. The extensive survey of methods and datasets for this problem can be found in [8]. In our work the data is time series of acceleration from three axis, which is sensed by mobile phone or other portable device with accelerometer. The problem is to predict the activity a person is performing. List of activities includes walking, running, sitting or walking up/down stairs. In this setup time series are regarded as complex structured objects without explicit feature description. This is reasonable because we can't operate with original features as time series might be of different size, not aligned or even multiscaled [9].

The problem of classifying complex structured objects is split in two distinctive procedures. First, we need to extract informative features, and then we use those features as input to some classifier to obtain final model. For simplicity, we assume that these two procedures can be built and analyzed separately. In our project we focused mainly on comparing different methods of feature generation [10, 11].

The first approach for feature generation is calculating expertly defined functions of time series [12]. These functions include average value, standard deviation, mean absolute deviation and distribution for each component. We consider this approach a baseline, as it is the simplest method we use.

We compare baseline with more sophisticated parametric feature generation methods, in which we build approximation models and use their parameters as our final features for classification. In this paper we propose using local spline approximation for feature extraction. In this setup features are knots and parameters of optimal cubic splines approximating our data. We compare this method with other well-known methods for extracting features from time series. One of them is autoregressive model [13]. For each time series we build parametric model and use those parameters as features for classification. Another approach is the model of singular spectrum analysis of time series [14]. We use eigenvalues of trajectory matrix as features for building classifier.

The experiment was conducted on two real accelerometer datasets [15, 16]. We compared the performance of stated feature extraction methods, as well as different classification algorithms. The latter include logistic regression, random forest and SVM.

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Problem Statement

Let $\mathcal{S}$ be a space of complex structured objects (i.e. we don't have feature representation suitable for direct classification), Y is a finite set of class labels. Denote by $\mathfrak{D} = \{(s_i, y_i)\}_{i=1}^m$ a given sample, where $s_i \in \mathcal{S}$ and $y_i \in Y$. We consider the problem of recovering the function $f : \mathcal{S} \to Y$

$$\hat{y} = f(s).$$

или убрать

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y|X-?

Let $L(f, \mathfrak{D})$ be an error function, which expresses the classification error of the function f over the sample $\mathfrak{D}$. The goal is to determine function f^* which minimizes the error

$$f^* = \arg\min_f L(f, \mathfrak{D}). \tag{1}$$

(*) L

We assume that the target function f^* belongs to the class of function compositions $f = g \circ \boldsymbol{h}$, where кратко про задачи классиф. мат.

Feature generation

- $\boldsymbol{h} : \mathcal{S} \to H$ is a map from the original space $\mathcal{S}$ to the feature space $H \subset \mathbb{R}^n$;

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- $g : H \times \Theta \to Y$ is a parametric map from the feature space H to the space of class labels Y. The function g is parametrized by a vector parameter $\boldsymbol{\theta} \in \Theta$.

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The determining of the function f^* is equivalent to determining the functions $\boldsymbol{h}^*$ and g^*.

In this paper we consider the following ways of generating feature space H.

- expert functions based on prior knowledge of the original objects. These functions can be expressed as a set of statistics $\{h_i\}_{i=1}^n$, where $h_i : \mathcal{S} \to \mathbb{R}$. Thus, the description $\boldsymbol{h}^*(s)$ of the object s is the value of these statistics on the object

$$\boldsymbol{h}^*(s) = (h_1(s), \dots, h_n(s)).$$

- Local approximation models. In this case the features are the estimated parameters the model, approximating our data. Let $S(s, \boldsymbol{h}, \boldsymbol{\lambda})$ be the error function of approximation, e.g. one could define the function S as negative log-likelihood function [17]. The optimal feature map $\boldsymbol{h}^*(s)$ is obtained by

$$\boldsymbol{h}^*(s) = \arg\min_{\boldsymbol{h}} S(s, \boldsymbol{h}, \boldsymbol{\lambda}). \tag{2}$$

The parameter $\boldsymbol{\lambda}$ is external structural parameter for the function S. The equation (2) determines the feature map $\boldsymbol{h}^*$ for each object $s \in \mathcal{S}$.

Given appropriate feature space H and feature map $\boldsymbol{h}$ we transform our original sample $\mathfrak{D} = \{s_i, y_i\}_{i=1}^m$ with complex structured objects to the new sample $\mathfrak{D}_H = \{\mathbf{h}_i, y_i\}_{i=1}^m$, where $\mathbf{h}_i = \boldsymbol{h}(s_i) \in H$. The function $g(\mathbf{h}, \boldsymbol{\theta})$ is defined by its

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parameter vector $\boldsymbol{\theta} \in \Theta$. The optimal parameters $\boldsymbol{\theta}^*$ are given by

*

$$\boldsymbol{\theta}^* = \arg\min_{\boldsymbol{\theta}} L(\boldsymbol{\theta}, \mathfrak{D}_H, \boldsymbol{\mu}), \tag{3}$$

where $L(\cdot,\cdot,\cdot)$ is an analogue of the function (1). Here the vector $\boldsymbol{\mu}$ is a external parameters of the particular classification model.

We consider the accuracy score to be the negative of main quality measure function L. This choice is based on our wish to compare our results with previous articles [10, 11] and this measure is easy to interpret. Accuracy score is a relation correctly labeled objects and the total amount of objects in dataset:

**

$$\text{accuracy}(y, \hat{y}) = \frac{1}{m}\sum_{i=1}^{m}[y_i = \hat{y}_i],$$

where $\hat{y}_i$ is a prediction of the classifier.

In our project we consider accelerometer time series as complex structured objects. Time series is represented in the following way:

~~s-матрице~~.

$$s = \left[x_1, \ldots, x_T\right]^T \in \mathcal{S},$$

s = x-вектор
как в Табл1?

where T denotes the length of time series.

Feature generation

The main focus of this paper is to compare different approaches for feature generation. In this section we provide analysis and motivation behind each of the methods.

Expert functions

[illegible]

в пост. задачи

Given a set of complex objects $\{s_i\}_{i=1}^m$ we extract features in a non-parametric way with a set of expert functions $\{h_j\}_{j=1}^n$. We list commonly used expert functions for time series in table (1). The main drawback of this approach is that we are restricted by our choice of the expert functions and these functions might be impossible to derive for some types of data.

в Экспер
Discussion

Function description	Formula
Mean	$\bar{\mathbf{x}} = \frac{1}{T}\sum_{t=1}^{T}\mathbf{x}_t$
Standard deviation	$\sqrt{\frac{1}{T}\sum_{t=1}^{T}(\mathbf{x}_t - \bar{\mathbf{x}})^2}$
Mean absolute deviation	$\frac{1}{T}\sum_{t=1}^{T}\lvert\mathbf{x}_t - \bar{\mathbf{x}}\rvert$
Distribution	Number of points in each histogram bin

Table 1: Expert functions

i=t

Это не вектор

Autoregressive model

In this method we assume autoregressive model [13] of the order n as a hypothesis for generation of time series s. Each component of the object s is assumed as a linear combination of the previous n components

s

$$x_t = w_0 + \sum_{j=1}^{n} w_j x_{t-j} + \varepsilon_t,$$

where ε_t is a random noise. Prediction of the autoregressive model is defined by

$$\hat{x}_t = w_0 + \sum_{j=1}^{n} w_j x_{t-j}. \tag{4}$$

For this method n is a structural parameter and $\boldsymbol{\lambda} = n$.

The Feature map $\boldsymbol{h}(s)$ is given by optimal parameters of autoregressive model $\mathbf{w}^* = \{w_j^*\}_{j=0}^{n}$ for time series s. The ~~hypothesis~~ error function (2) in this case is the squared error between the original object s and its prediction of the model (4).

Local approx err
classif err.

$$\boldsymbol{h}(s) = \mathbf{w}^* = \underset{\mathbf{w}\in\mathbb{R}^{n+1}}{\arg\min}\, S(s, \mathbf{w}, \boldsymbol{\lambda}) = \underset{\mathbf{w}\in\mathbb{R}^{n+1}}{\arg\min}\left(\sum_{t=n+1}^{T} \|x_t - \hat{x}_t\|^2\right). \tag{5}$$

The problem (5) ~~could be easily converted~~ is to the linear regression problem. Hence, for each initial time series s = we have to solve linear regression problem with n predictors. The example of approximation using autoregressive model is demonstrated on the Figure 1.

x

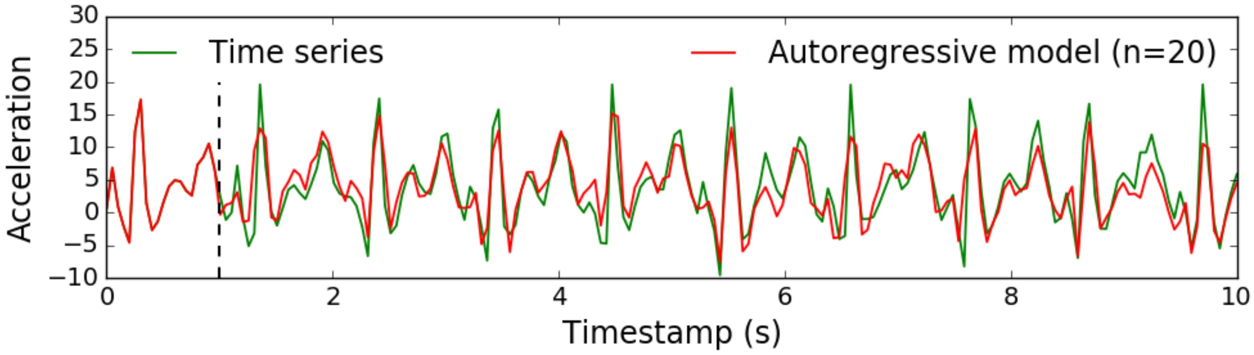

Figure 1: Time series approximation using autoregressive model with $n = 20$

Singular spectrum decomposition

Alternative hypothesis for generation of time series is SSA (Singular Spectrum Analysis) model [14]. We construct trajectory matrix for each time series $s = (x_1, \dots x_T)$:

X =

$$\mathbf{X} = \begin{pmatrix} x_1 & x_2 & \dots & x_n \\ x_2 & x_3 & \dots & x_{n+1} \\ \dots & \dots & \dots & \dots \\ x_{T-n+1} & x_{T-n+2} & \dots & x_T \end{pmatrix}.$$

Here n, called the window width, is an external structural parameter. ~~Let find~~ the singular decomposition [18] of the matrix $\mathbf{X}^{\mathsf{T}}\mathbf{X}$: is

$$\mathbf{X}^{\mathsf{T}}\mathbf{X} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^{\mathsf{T}},$$

where $\mathbf{U}$ is a unitary matrix and $\Lambda = \mathrm{diag}(\lambda_1, \dots, \lambda_n)$ whose entries λ_i are eigenvalues of $\mathbf{X}^{\mathsf{T}}\mathbf{X}$. In this case we use the spectrum of the matrix $\mathbf{X}^{\mathsf{T}}\mathbf{X}$ as feature description of the object s

$$\boldsymbol{h}(s) = (\lambda_1, \dots, \lambda_n).$$

Splines

+ ?

Approximate of time series ~~can be done~~ by splines [19]. The spline is defined by its parameters:

- $\{\xi_\ell\}_{\ell=0}^{L}$ — T~~t~~he set of knots. Knots can be uniform ~~or nonuniform~~ are uniformly distributed over time scale. To get the adequate result ~~we~~ normalize~~d~~ the knots for each time series.

- $\{\mathbf{w}_\ell\}_{\ell=1}^L$ — parameters of the models are built on the interval $[\xi_{\ell-1}; \xi_\ell]$. The dimension of the each parameter vector $\mathbf{w}_\ell$ depends on the spline order.

In order to find the optimal spline parameters $\mathbf{w}$, ^-? ~~one need to~~ solve the system of equations with additional constraints of equality of derivatives up to second order on the edges of intervals. ~~If we~~ Denote each spline segment as $p_i(t)$ and spline as a whole as $S(t)$, ~~we can~~ write these equations in ~~a~~ the following way:

$$S(x) = \begin{cases} p_1(x) = w_{10} + w_{11}t + w_{12}t^2 + w_{13}t^3, & t \in [\xi_0, \xi_1] \\ p_2(x) = w_{20} + w_{21}t + w_{22}t^2 + w_{23}t^3, & t \in [\xi_1, \xi_2] \\ \dots \\ p_L(x) = w_{L0} + w_{L1}t + w_{L2}t^2 + w_{L3}t^3, & t \in [\xi_{L-1}, \xi_L] \end{cases}$$

$$S(t) = x_t \quad t = 1, \dots, T,$$

$$p'_i(\xi_i) = p'_{i+1}(\xi_i), \quad p''_i(\xi_i) = p''_{i+1}(\xi_i), \quad i = 1, \dots, L-1 .$$

The feature description of the time series ~~could be assumed~~ is as a union of these parameters:

$$\boldsymbol{h}(s) = (\xi_0, \dots, \xi_L, \mathbf{w}_1, \dots, \mathbf{w}_L).$$

⊕ ? This approach gives another approximation of the time series. In the Figure 2 one could find the result of time series approximation given by splines. Compared to the autoregressive model, the splines method gives smoother approximation using almost the same number of parameters.

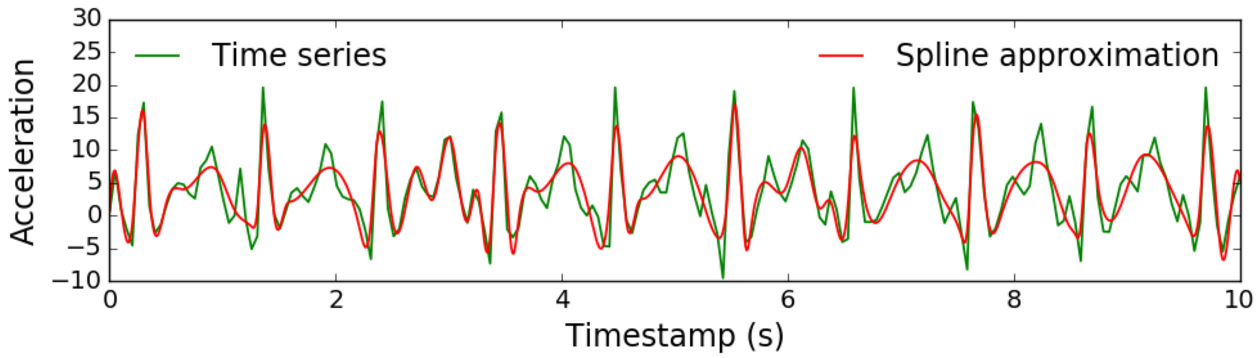

Figure 2: Time series approximation using splines

Classification

Multiclass classification

As we had numerous labels in our datasets we had to choose one of the multiclass approaches to classification. We decided to use one-vs-rest classification as a simple, yet effective approach. The main idea is that we train binary classifiers for each class label and then, on the prediction step, we classify new object according to the most confident classifier. In this section we will describe our approach to classification of time series using newly generated features. We use three different classification models: logistic regression, SVM and random forest.

Classification methods

Logistic regression

наст время

The first approach to classification ~~we played with was~~ regularized logistic regression model. The optimal model parameters (3) is determined by minimising the following error function

$$L(\boldsymbol{\theta}, \mathfrak{D}_H, \mu) = \sum_{i=1}^{m} \log\left(1 + \exp(-y_i \langle \boldsymbol{\theta}, \mathbf{h}_i \rangle)\right) + \frac{\mu}{2} \|\boldsymbol{\theta}\|^2 ,$$

$$\boldsymbol{\theta}^* = \arg\min_{\theta} L(\boldsymbol{\theta}, \mathfrak{D}_H, \mu) .$$

The classification rule $g(\mathbf{h}, \boldsymbol{\theta})$ is given by sign of the linear combination for the object description $\mathbf{h}$ and parameters $\boldsymbol{\theta}^*$

$$\hat{y} = g(\mathbf{h}, \boldsymbol{\theta}^*) = \text{sgn}\langle \boldsymbol{\theta}^*, \mathbf{h} \rangle .$$

SVM

The

~~We also used~~ binary SVM model. The problem in this case can be formulated in a following way:

$$\boldsymbol{\theta}^* = \begin{pmatrix} \boldsymbol{w}^* \\ b^* \\ \boldsymbol{\xi}^* \end{pmatrix} = \underset{\boldsymbol{w},b,\boldsymbol{\xi}}{\arg\min} \frac{1}{2}\|\boldsymbol{w}\|^2 + C\sum_{i=1}^{m} \xi_i \quad ,$$

$$\text{subject to} \quad y_i(\langle \boldsymbol{w}, \mathbf{h}_i\rangle + b) \geqslant 1 - \xi_i \quad ,$$

$$\xi_i \geqslant 0, \quad 1 \leqslant i \leqslant m.$$

For new objects ~~we can~~ make a prediction

$$\hat{y} = \operatorname{sgn}(\hat{\mathbf{w}}^T\mathbf{h} + b) .$$

^\mathsf{T}
base

Прояснить решение чего.

Random Forest

The Random forest is an algorithm, which exploits the idea of bagging. This is an approach of building many random weak classifiers and aggregating their predictions. This method works especially well if as base models we select models with low bias and high variance (due to aggregating variance is reduced). In case of random forest decision trees take the role of base models, also not only objects are used for bagging, but also features. In this case we make the prediction for each new object as the mean of the predictions of single trees:

$$\hat{y}_i = \frac{1}{B}\sum_{i=1}^{B} g(\mathbf{h}_i),$$

where B is an amount of trees used for bagging.

Experiment

In this paper we consider two different smart phone based datasets: WISDM [15] and USC-HAD [16]. ~~Data from~~ The smart phone accelerometer ~~consists of information about~~ measures acceleration along ~~each of~~ three axis. ~~Time difference~~ between measurements equals 50 ms. The WISDM dataset consists of 4321 objects and each time series belongs to one of the six activities : Standing, Walking, Upstairs, Sitting, Jogging, Downstairs. The USC-HAD dataset contains 13620 objects with one of the twelve

sample rate

class labels: Standing, Elevator-up,Walking-forward, Sitting, Walking-downstairs, Sleeping, Elevator-down, Walking-upstairs, Jumping, Walking-right, Walking-left, Running. The distributions of time series activities for each datasets are presented in Table 2. The length of each time series equals 200 which accounts 10 second. In the Figure 3 the example of the time series for one activity of the specific person is given.

classes

Table 2: Activities distributions of the

(a) WISDM

	Activity	**# objects**	
1	Standing	229	5.30 %
2	Walking	1917	44.36 %
3	Upstairs	466	10.78 %
4	Sitting	277	6.41 %
5	Jogging	1075	24.88 %
6	Downstairs	357	8.26 %
	Total	4321	

(b) USC-HAD

	Activity	**# objects**	
1	Standing	1167	8.57 %
2	Elevator-up	764	5.61 %
3	Walking-forward	1874	13.76 %
4	Sitting	1294	9.50 %
5	Walking-downstairs	951	6.98 %
6	Sleeping	1860	13.66 %
7	Elevator-down	763	5.60 %
8	Walking-upstairs	1018	7.47 %
9	Jumping	495	3.63 %
10	Walking-right	1305	9.58 %
11	Walking-left	1280	9.40 %
12	Running	849	6.23 %
	Total	13620	

the experiment procedure.

For each dataset we applied the feature generation approaches described above: expert functions, autoregressive model, SSA, splines. We used ~~three different widely used~~ three classification model ~~for each generated feature description~~: logistic regression, support vector machine and random forest. The external structural e parameters $\boldsymbol{\lambda}$ (2) for feature generation procedures, ~~such as~~ the length n for autoregression, the window width n for SSA and the number of splines knots L, were tuned using 3-fold cross validation procedure. (1) The hyperparameters $\boldsymbol{\mu}$ for classification models were ~~also~~ tuned using the same cross validation procedure (1)

~~In~~ paper [12] ~~the authors proposed to use~~ the following expert functions for time series classification:

- ~~(3)~~ average acceleration for each axis;
- ~~(3)~~ standard deviation for each axis;

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$\lambda_2 = \underset{\text{arg min}}{} CV = \dots$

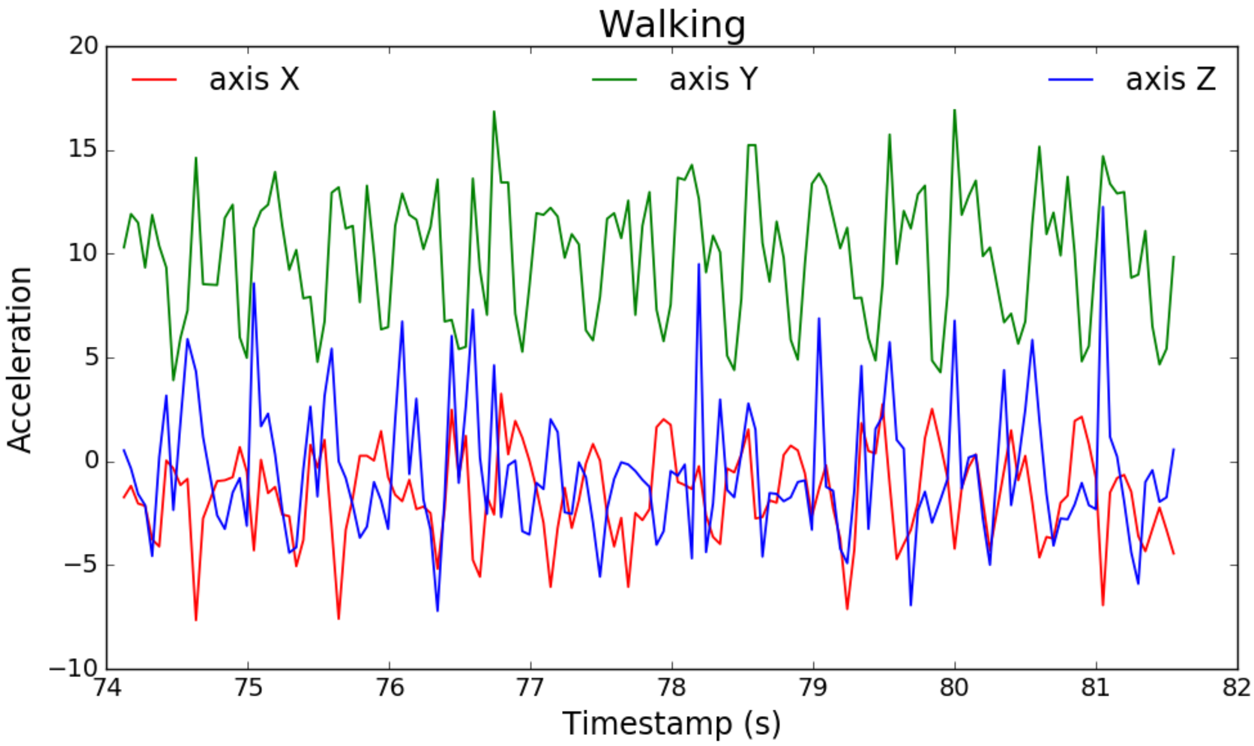

Figure 3: Time series example

- ~~(3)~~ average absolute difference for each axis;
- ~~(1)~~ average resultant acceleration;
- ~~(30)~~ values of histogram with 10 bins for each axis.

таблица или перечисление

Hence feature extraction procedure gives us the feature description of time series $\boldsymbol{h}(s) \in \mathbb{R}^{40}$.

The Autoregressive model(s) were tuned to find the optimal length n. The Cross validation procedure(s) gives optimal value $n = 20$ for both datasets.

The Singular spectrum analysis were tuned in the same way(,) to find the optimal window width n. ~~Analogously to~~ autoregressive model the cross validation procedure gives the same value $n = 20$. Authors assume that it means that time series has memory of this size.

as the prev(.)

We fit the (.) splines for time series using *scipy* python library(:) This software fits 3-order B-splines [19]. The knots $\{\xi_\ell\}_{\ell=1}^{L}$ for splines were distributed uniformly. The number L were chosen implicitly by choosing the proper smoothing parameter s. The less the value of s, the larger the number of knots L. The fitting was constructed as follows. Firstly, there was the initialization step to find appro-

of knots

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priate bounds for smoothing parameter. The next step is finding the smoothing parameter in this interval using bi-search approach.

The feature extraction methods gives the following number of features for both datasets: в строчку или табл

- expert features: 40;
- autoregressive model: 63;
- singular spectrum analysis: 60;
- splines: 33.

The results of the experiments for the both datasets is presented in Figure 4. For WISDM dataset the worst result is obtained by splines parameters. The results for expert functions, autoregressive model and SSA is roughly identical. For USC-HAD dataset the results highly depend on the classification model. For both datasets logistic regression shows the worst quality, while the accuracy for support vector machine and random forest are strongly correlated.

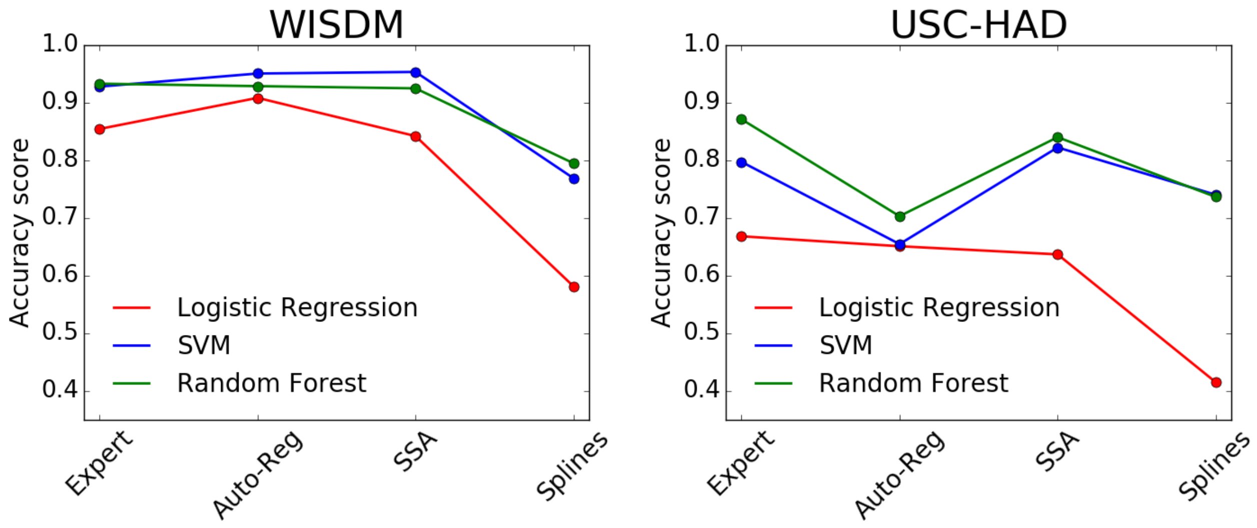

Figure 4: Multiclass accuracy score

All results with classification accuracy scores for each class are represented in Table 3 and Table 4. The first row of these tables introduces the multiclass accuracy score for each classification model and each feature extraction procedure. Next rows are related to binary accuracy scores for each class. For WISDM dataset the best scores have the least active classes such as Standing and Sitting. For USC-HAD dataset all classes have the similar accuracy scores.

Table 3: Binary accuracy scores for WISDM using different feature generation methods: EX — Expert, AR — Auto-Reg, SSA and SPL for Splines

	Logistic Regression				**Random Forest**				**SVM**			
	EX	AR	SSA	SPL	EX	AR	SSA	SPL	EX	AR	SSA	SPL
All	0.85	0.91	0.84	0.58	0.93	0.93	0.92	0.79	0.93	0.95	0.95	0.77
Standing	0.99	0.98	1.00	0.95	1.00	0.99	1.00	0.99	0.99	0.98	1.00	0.96
Walking	0.91	0.96	0.86	0.61	0.96	0.97	0.95	0.86	0.96	0.98	0.98	0.84
Upstairs	0.91	0.95	0.91	0.89	0.96	0.96	0.96	0.90	0.96	0.98	0.97	0.89
Sitting	0.99	0.98	1.00	0.99	1.00	0.99	1.00	1.00	0.99	0.98	1.00	1.00
Jogging	0.98	0.99	0.99	0.80	0.99	0.99	0.99	0.92	0.99	0.99	0.99	0.93
Downstairs	0.93	0.96	0.94	0.92	0.96	0.97	0.96	0.92	0.96	0.98	0.97	0.92

Table 4: Binary accuracy scores for USC-HAD using different feature generation methods: EX — Expert, AR — Auto-Reg, SSA and SPL for Splines

	Logistic Regression				**Random Forest**				**SVM**			
	EX	AR	SSA	SPL	EX	AR	SSA	SPL	EX	AR	SSA	SPL
All	0.67	0.65	0.64	0.41	0.87	0.70	0.84	0.74	0.80	0.65	0.82	0.74
Standing	0.94	0.94	0.92	0.89	0.98	0.94	0.97	0.98	0.95	0.94	0.97	0.96
Elevator-up	0.94	0.94	0.93	0.92	0.95	0.95	0.95	0.95	0.93	0.94	0.94	0.93
Walking-forward	0.87	0.87	0.89	0.70	0.97	0.89	0.96	0.88	0.95	0.87	0.97	0.91
Sitting	0.98	0.95	0.94	0.96	0.99	0.96	0.98	0.99	0.98	0.96	0.99	0.99
Walking-downstairs	0.95	0.93	0.93	0.90	0.99	0.96	0.98	0.95	0.98	0.93	0.98	0.96
Sleeping	1.00	0.98	0.99	1.00	1.00	0.98	1.00	1.00	1.00	0.98	1.00	1.00
Elevator-down	0.94	0.94	0.94	0.91	0.95	0.95	0.95	0.95	0.93	0.94	0.94	0.93
Walking-upstairs	0.94	0.95	0.93	0.92	0.98	0.95	0.98	0.96	0.98	0.95	0.98	0.96
Jumping	0.99	0.99	1.00	0.97	1.00	0.99	1.00	0.99	1.00	0.99	0.97	0.99
Walking-right	0.91	0.90	0.91	0.86	0.97	0.92	0.96	0.92	0.96	0.90	0.97	0.93
Walking-left	0.89	0.91	0.90	0.88	0.97	0.93	0.97	0.93	0.95	0.91	0.97	0.93
Running	0.99	0.99	0.99	0.92	1.00	0.99	1.00	0.97	1.00	1.00	0.95	0.98

We also carried out the experiment for union of all 196 generated features. The results are demonstrated on the Figure 5. In the Table 2 one can see class labels, that are represented on the corresponding histograms. As expected, the accuracy scores in this case are higher in all cases. All binary accuracy scores for WISDM datasets is larger than 97% for each classification model. These numbers for USC-HAD dataset is larger than 93%.

Conclusion

The research

The problem of complex structured objects classification ~~were~~ considered. We investigated the different approaches of feature extraction, particularly the

the paper

expert functions and data generation hypothesis. The experiment on the real data from smart phone accelerometer were carried out. We compared different feature descriptions and different classification models. The results show that obtained features allows to recover the class label with the high quality.

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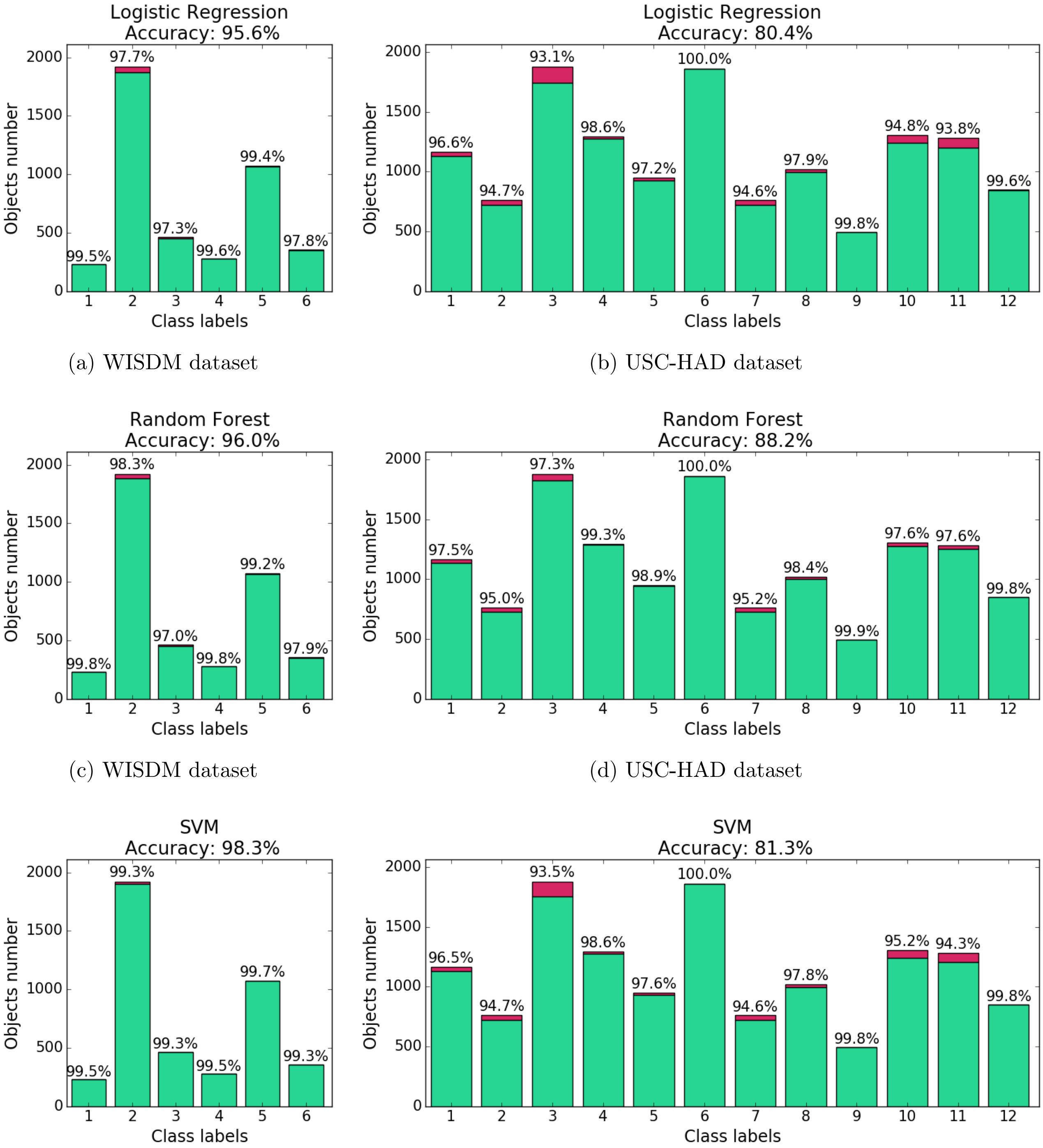

(a) WISDM dataset

(b) USC-HAD dataset

(c) WISDM dataset

(d) USC-HAD dataset

(e) WISDM dataset

(f) USC-HAD dataset

Figure 5: Accuracy scores of classification of each class using all features