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An Efficient Super Resolution Algorithm Using Simple Linear Regression

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***Abstract*—With the improvement in technology of thin-film-transistor liquid-crystal display (TFT-LCD), the resolution requirement of display becomes higher and higher. Super-resolution algorithms are used to enlarge original low-resolution (LR) images to meet the visual quality of the high-resolution (HR) display. In this research, an efficient super resolution algorithm is proposed. The proposed algorithm consists of two steps. First, the Lanczos interpolation is used for LR images to get the preliminary HR images. For solving the over-smoothing problems generally caused by interpolation, it needs to add texture information to refine the preliminary HR images. Subsequently, a refinement process based on simple linear regression and the self-similarity between a pair of LR and HR images is performed to provide proper information of textures. In the experimental results, the proposed algorithm not only performs well in the objective measurement such as PSNR, but also in visual qualities.**

***Index Terms*—super-resolution; detail enhancement; simple linear regression; self-similarity**

I. Introduction

Images with high resolution are desirable in many applications, such as high definition television broadcasting, remote surveillance, high quality video conference and wildlife sensor network. It is not always feasible to capture the high-resolution (HR) images even if the camera is capable of it since most applications are limited by memory, transmission bandwidth, power and camera cost. Therefore, algorithmic approaches to obtain HR images from low-resolution (LR) ones have been more and more important and have become a deeply concerned issue in recent years. The objective of super resolution (SR) techniques is to make HR images that are obtained from the LR ones look like they had been acquired with a sensor having the expected resolution or at least present natural textures.

Nearest, bilinear and bicubic interpolation are the most popular algorithms to enlarge images and are usually used as the pre-processing methods in many SR algorithms due to the computational simplicity and the flexible scalability of image resolution. In such conventional interpolation-based algorithms, the values of pixels are assumed to be continuous in an image. However, in near-edge regions, pixel continuity typically exists only along edge directions, and steep discontinuity may occur across edge directions and thus blurring artifacts are usually induced in the reconstructed HR image. In order to reduce those visual artifacts, improved algorithms are further proposed. An edge-directed interpolation algorithm for natural images was presented in [1], where the edge-directed property of covariance-based adaptation attributes to its capability of tuning the interpolation coefficients to match an arbitrarily oriented step edge, and a hybrid approach of switching between bilinear interpolation and covariance-based adaptive interpolation is proposed to reduce the overall computational complexity. Zhang [2] had introduced a soft-decision interpolation method which adaptively determined the parameters of an autoregressive model. An effective color image interpolation algorithm was proposed by Zhou [3], where the correlation among three color channels were considered and an edge detection algorithm was applied in conjunction with the one-dimensional cubic interpolation to well preserve the edges and textures.

Besides, another concept, back-projection, for enhancing the details by properly refine the HR image was also developed. The main concept of this category is to properly reconstruct the details of an HR image. Dai [4] and Dong [5] introduced an effective approach by iterative back-projection. It could minimize the reconstruction errors efficiently by an iterative process. In each iteration, the current reconstruction details are back-projected to adjust the image intensity. A fast fractal super resolution technique (OFSR), a special type of orthogonal fractal coding method, was proposed by Wee [6], where the fractal affine transform is determined by the range block mean and contrast scaling was utilized to quickly and efficiently reconstruct HR images. A significant drawback of most of the aforementioned algorithms except OFSR is the high computing complexity and thus decreases the practicability for the use in daily life. Though OFSR can performs very well in the representation of textures with computational simplicity, it usually induces jagged artifacts along edges.

Motivated by this, an efficient SR algorithm is proposed where the self-similarity between a pair of LR and HR images is used to properly refine the details in conjunction with the simple linear regression. Please note that the concepts of regression models were widely used in the SR algorithms in recent years [7]. Rather than other regression-based SR algorithms, what we concern are the details of "natural" HR images, that are supposed to be lost in the upscaling process. The method for obtaining details used in the proposed algorithm is similar to that in OFSR due to its good performance on reconstructing detailed textures. In the experimental results, the proposed algorithm not only performs well in the objective measurement such as PSNR, but also in visual qualities.

The remainder of this paper is organized as follows. Some

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relative works such as OFSR and simple linear regression are given in section II. In section III, we shall clearly and completely address our proposed SR algorithms based on simple linear regression. Then the simulation results and a further discussion are provided in section IV. Finally, a brief conclusion is drawn in section V.

II. Relative Work

In this section, some concepts of the methods such as Lanczos interpolation and simple linear regression used in the proposed algorithm for different purposes are briefly introduced

A. Lanczos Interpolation

Lanczos interpolation or Lanczos filter is used to smoothly interpolate the value of a digital signal from the given samples. It maps each of the given sample to a translated and scaled copy of the Lanczos kernel, which is a sinc function windowed by the central hump of a dilated sinc function. Eq. (1) is the mathematical expression of the Lanczos kernel where the parameter a is a positive integer, typically 2 or 3, which determines the size of the kernel.

$$L(x) = \begin{cases} sinc(x)sinc(x/a) & \text{for} \quad -a < x < a \ and \ x \neq 0 \\ 1 & \text{for} \quad x = 0 \\ 0 & \text{for} \quad otherwise. \end{cases} \tag{1}$$

B. Orthogonal Fractal Super Resolution (OFSR)

The motivation of using fractal coding is utilizing its self-similarity encoding. A single image is divided into non-overlapping range blocks R of size $k \times k$. Each R is correlated to a domain block D of size $2k \times 2k$, and D offers local similarity for R. By a contraction effect, the domain block D can be reduced to a block B of size $k \times k$. The contraction effect can be presented by

$$B = \mu\left(D\right), \tag{2}$$

where μ denotes the contraction function. It reduces the block B and transforms to approximate the range block $\overline{R}$ by the following equation

$$\overline{R} = s\overline{B} + g, \tag{3}$$

where s is the contrast scaling factor, which is smaller than 1, and g is the luminance offset between $\overline{R}$ and $s\overline{B}$, which are the means of the range block and the scaled contracted domain block, respectively. Then, orthogonal fractal decoding can be presented by

$$R^{(k)} = \overline{R}U + s\mu\left(D^{(k-1)} - \overline{D}^{(k-1)}U\right), \tag{4}$$

where $\overline{D}$ is the mean of the domain block and U is a unitary matrix.

Based on the concepts previously introduced, the implement of OFSR is then presented. Assuming that an LR image of size $m \times n$ with the size of range block being 2×2, orthogonal fractal decoding will construct an HR image of size $2m \times 2n$. The key to use OFSR well is the selection of a fine transformation that results in a good self-similarity. The most self-similar pixel in SR problem should be the pixel nearby the range block. Local similarity is utilized by taking a 4×4 domain block containing the range block at its center. The contracted value $b_{i,j}$ at $r_{i,j}$ is obtained by averaging the two pixel values neighboring $r_{i,j}$, and then averaging the result with the pixel value at $r_{i,j}$ as shown in Fig. 1. And the scaling factor s is set to be 0.75 which is observed by a large amount of experiments.

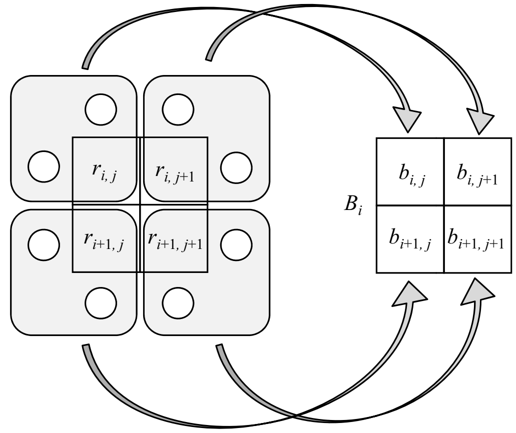


Fig. 1. The contraction operator

C. Multiple Regression

Multiple Regression is a causal-effect model for the case having multiple independent variables and single dependent variable. If there are d independent variables $x_1, x_2, , x_d$, and one dependent variable y. The regression model is

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_d x_d + \epsilon \tag{5}$$

with $\epsilon \overset{\text{iid}}{\sim} N(0, \sigma^2)$, that is, ϵ is normal distribution with mean 0 and variance σ^2. The sample size equals n and the ith observation is $(y_i, x_{i1}, x_{i2}, \cdots, x_{id})$. Let Y be the $n \times 1$ column vector containing the sample data of dependent variable y_i, $1 \leq i \leq n$; X be the $n \times (d+1)$ design matrix containing the sample data for independent variables $x_1, x_2, , x_d$ as presented in (6); β be the $(d+1) \times 1$ column vector containing the model parameters β_i, $0 \leq i \leq d$; ϵ be the $n \times 1$ column vector containing the error term ϵ_i, of the i^{th} observation, $1 \leq i \leq n$. And then the regression model in the matrix form is designated as (7).

$$X = \begin{bmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1d} \\ 1 & x_{21} & x_{22} & \cdots & x_{2d} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & x_{n2} & \cdots & x_{nd} \end{bmatrix} \tag{6}$$

$$Y = X\beta + \epsilon \tag{7}$$

The least-square estimate of β equals

$$\hat{\beta} = (X'X)^{-1}X'Y. \tag{8}$$

Therefore the estimated regression function becomes

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 + \cdots + \hat{\beta}_d x_d. \tag{9}$$

The simple linear regression is the easiest case of the multiple linear regression with only one independent variable.

III. The Proposed Algorithm

Actually, if the mathematical model of the aforementioned OFSR is reviewed, it can be explained as that the estimated HR image is taken as the summation of a preliminarily upscaled image and the properly enhanced details. In other words, the contraction process can be taken as detail acquirement. And thus the formula of the aforementioned OFSR can be rewritten as

$$HR_{estimation} = HR_{initial} + Ref(HR_{initial}) \times s, \tag{10}$$

where $HR_{initial}$ is the preliminary HR image by using general up-scaling method, and $Ref(\cdot)$ is the process to acquire the details for refinement and can be taken as the part $\mu(\cdot)$ in (4). In other words, the contraction effect in OFSR algorithm can be used to acquire details to properly reconstruct the HR image. And $HR_{estimation}$ represents the final result of the output HR image integrating the preliminary HR image and the proper details. In OFSR, the scaling factor s is set to be 0.75 and 3 iterations are suggested to achieve a stable performance. However, using a fixed scaling factor to decide the magnitude of details for various images is not suitable.

In the proposed algorithm, the contraction operator used in OFSR is adopted to acquire details. Different from OFSR, the scaling factor is determined adaptively according to the images by using simple linear regression and the self-similarity between a pair of LR and HR images.

In a large amount of experiments, Lanczos interpolation with the kernel size 3 achieves the best image quality and thus is used for obtaining the preliminary upscaled image $HR_{initial}$. By down-sampling the $HR_{initial}$, another LR image is obtained and denoted as LR_d. Let LR_{dif} be the difference of the input LR and LR_d . Then, the LR_{dif} can be presented as :

$$LR_{dif} = LR - LR_d, \tag{11}$$

and it can be taken as the residual image of LR. Since the LR_d is known and by using the fractal refinement process (contraction process), accroding to (10), LR_{dif} can be computed by the following equation:

$$LR = LR_d + LR_{dif} = LR_d + Ref(LR_d) \times s, \tag{12}$$

where $Ref(LR_d)$ is the LR_d with fractal refinement process.

Now, the scaling factor, s, can be taken as the relationship between the given LR_{dif} and $Ref(LR_d)$ and can be calculated by simple linear regression. Subsequently, if (10) is compared with (12), it is found that they are very similar. Indeed, there is usually self-similarity in a pair of LR and HR images, and thus the scaling factor s in (12) can be further used in (10). Hence, the details of HR image can be computed and added to the preliminary HR image, and the final estimation of the HR image can be obtained. In brief, the proposed algorithm contains the following steps:

1) initially up-scale the input image;
2) down-sample the preliminary HR image;
3) fractal refinement for the down-sampled image LR_d;
4) obtain the difference image between the down-sampled image LR_d and input LR image;
5) build a simple linear regression model for LR_{dif} and $Ref(LR_d)$ images;
6) obtain the scaling factor s;
7) fractal refinement to the preliminary HR image;
8) obtain the proper details by the aforementioned regression model;
9) integrate the preliminary HR image and the compensation image.

(a) N1 (b) N2 (c) P1 (d) P2

(e) P3 (f) A1 (g) A2 (h) A3

(i) A4 (j) A5 (k) A6 (l) A7

Fig. 2. Test images

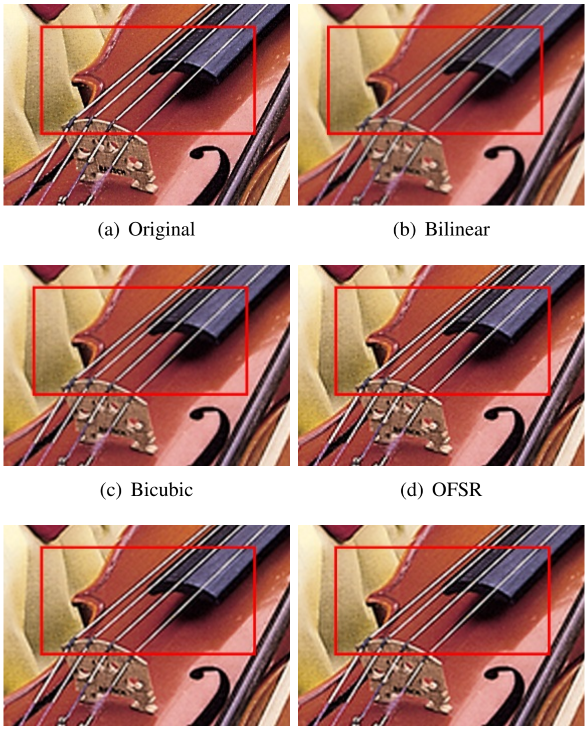
IV. Experimental Results

The experimental results of the proposed algorithm are exhibited in this section. The comparison of both objective and subjective performance of the results of the proposed algorithm and others is listed. Besides, the comparison of computation time is also exhibited. There are 12 test images in our experiment as shown in Fig. 2 where the resolution of the N-series images is 2560×2048; the resolution of the P-series images is 3840×2160; the resolution of the A-series images is 1920×1080.

Peak Signal-to-Noise Ratio (PSNR) is the most commonly used to measure the quality of the reconstruction image. In the experiment, PSNR is taken as the objective measurement, and is defined as

$$PSNR = 10 \cdot \log_{10}(\frac{MAX_I^2}{MSE}), \tag{13}$$

where MSE is the mean squared error and MAX_I is the maximum possible pixel value of the image. The experimental environment is given as follows. CPU: I7-3770@3.40GHz; RAM: 12GB; programming code: Matlab R2012a; Operating system: Windows 7. The significant advantages of the



(a) Original (b) Bilinear

(c) Bicubic (d) OFSR

(e) Lanczos (f) Proposed

Fig. 3. Visual quality of partial results in N2

proposed algorithm are presenting clear details, decreasing artifacts, flexible scalability of image resolution and, high computation speed. Therefore, the results of the proposed algorithm are compared with that of several fast SR algorithms such as bilinear interpolation, bicubic interpolation, Lanczos interpolation, and OFSR.

Fig. 3 is the comparison of visual quality of partial results in N2 where (b) and (c) are obviously more blurred than others. Besides, as aforementioned, the results of OFSR present clear textures but meanwhile suffer from jagged artifacts along the bowstrings. Oppositely, the proposed algorithm also preserves clear textures as OFSR and moreover, the edges along the bowstrings are clear. Besides, the textures and edges in (f) are also more clear than that in (e), and this indeed responses the performance of the refinement process in the proposed algorithm.

Table I is the PSNR comparison where it can be found that the results of the proposed algorithm are better than that of others and this is consistent with the performance of the visual quality. Besides, from Table II where the comparison of computation time is listed, it is obvious that the proposed algorithm can perform more efficient than OFSR. Actually, all the experimental results presented prove that the proposed SR algorithm is efficient and effective.

V. Conclusion

An efficient SR algorithm is proposed in this research. The concept of self-similarity between a pair of LR and HR images is used in conjunction with the simple linear regression to properly refine the HR image. It solves the blurring artifacts caused in the process of interpolation and meanwhile, avoids to induce jagged artifacts. Besides, the advantage of flexible scalability of image resolution is also well maintained. Both of its efficiency and effect are proved in the experimental results.

TABLE I
PSNR of test images (dB)

Test image	Bilinear	Bicubic	OFSR	Lanczos	Proposed
Resolution : 1920×1080					
A1	34.137	36.393	36.930	37.565	37.595
A2	29.036	31.099	31.762	32.119	32.117
A3	25.469	26.553	27.054	27.130	27.2
A4	20.561	21.346	21.806	21.694	21.728
A5	22.467	23.427	24.009	23.821	23.883
A6	19.997	20.719	21.170	21.042	21.081
A7	42.307	44.514	44.874	45.496	45.505
Resolution : 2560×2048					
N1	26.722	27.570	28.012	27.944	27.974
N2	24.331	25.202	25.686	25.607	25.653
Resolution : 3840×2160					
P1	34.141	35.832	36.390	36.700	36.738
P2	24.545	25.747	26.301	26.363	26.440
P3	33.488	35.418	36.092	36.466	36.508
average	**28.103**	**29.485**	**30.007**	**30.162**	**30.202**

TABLE II
Average computation time of different resolution (second)

Image resolution	Lanczos	OFSR	Proposed
1920×1080	0.07450	1.62440	0.63294
2560×2048	0.16443	4.15194	1.59250
3840×2160	0.28160	6.62733	2.92584

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