Fuzzy Sets Decision-making Theory and Its Application in Reallocation of Replaced Water of the Yellow River

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***Abstract*—**Reallocation of replaced water of the Yellow River is regarded as a complicated problem of multi-area and multi-objective decision-making, which has not only quantitative indexes but also qualitative indexes. Based on assigning the index system of water reallocation, in view of the deficiencies of analytical hierarchy process (AHP), unit systematic theory of the fuzzy sets decision-making can solve this problem effectively. A case study shows that this theory fully takes account of the influence of reallocation of replaced water of the Yellow River on every index and people's experience knowledge. The decision is rational and reliable.

Keywords-Fuzzy sets decision-making; Replaced water; Qualitative index; Quantitative index

I. INTRODUCTION

The south-to-north water transfer is a strategic measure that can solve water shortage in North china by diverting water from the Yangtze River valley. At present its planning includes east-route, mid-route and west-route. But the west-route won't have been finished in recent twenty years because of many problems. Considering the problem that water consumption proportion given to upstream areas of the Yellow River is on the low side [1], a part of water diverted from Yellow River to the areas served by the east-route and mid-route may be reallocated to the upstream areas after commissioning of the east-route and mid-route and before commissioning of the west-route. It helps to increase the water consumption proportion given to upstream and midstream areas of the Yellow River.

A complete set of indicators of water allocation was constituted in reference [2]. A method of combination of AHP with fuzzy decision-making theorem was developed for distribution of water right. This method is based on the system of indicators to distribute water right. The application of this method is advanced to some extent. The result is satisfactory. But AHP has some deficiencies: Simplification to lingual signification of AHP is reasonless because all two-dimensional comparison properties of elements come down to important comparison. In fact, there are various kinds of element's properties comparison, such as importance comparison, superiority comparison etc. These properties are different in many fields. The consistency of two-dimensional comparison matrix is the base that non-structured decision-making is reasonable and scientific. But AHP has not solved this problem in theory. Linear model is applied to calculation of the weights in every level, so it is difficult to reflect complicated non-linear problems. In view of these deficiencies, unit systematic theory of the fuzzy sets decision-making is proposed in this paper. This theory is based on the complementarity rule to two-dimensional comparison. The decision-making result is reasonable and efficient to application for reallocation of replaced water of the Yellow River.

II. UNIT SYSTEMATIC FIGURE OF MULTI-LEVEL AND MULTI-INDEX

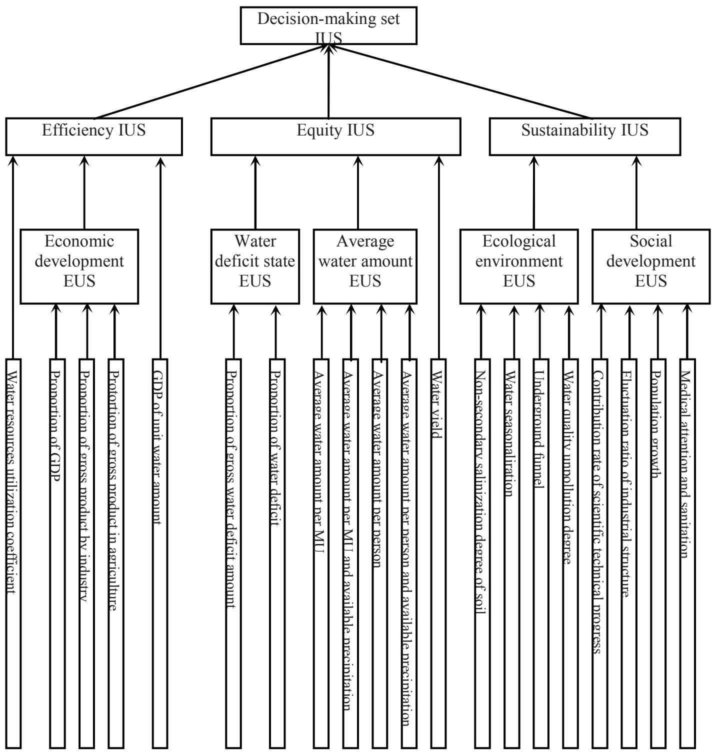

Figure 1 Unit systematic figure of multi-level and multi-index
IUS—integrative unit system
EUS—elementary unit system

The index system in reference [2] has been with efficiency, equity and sustainability as criterion. From the

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point of the right to use of water, it has roundly considered various kinds of factors influencing reallocation of the amount of water. This paper has built unit system of multi-level and multi-index for unit systematic theory of the fuzzy sets decision-making which structure is shown as figure 1. The complicated question of decision-making is changed to solve a set of unit systems from the lowest level to the highest level.

III. UNIT SYSTEMATIC THEORY OF THE FUZZY SETS DECISION-MAKING[3]

3.1 The principles of elementary unit system

3.1.1 The formula of quantitative index's relative membership degree on superiority

Quantitative index always can be divided into two types, one is benefit, and the other is cost. The relative membership degree on superiority can be calculated by different formula. Assume that the number of objects is n and the number of indexes is m,

Where x_{ij} —the characteristics of object j at index i

r_{ij} —the relative membership degree on superiority of object j at index i on fuzzy characteristics

1. For the benefit type

$$r_{ij} = \frac{x_{ij}}{x_{i\max}} \tag{1}$$

2. For the cost type

$$r_{ij} = \begin{cases} \dfrac{x_{i\min}}{x_{ij}} & x_{i\min} \neq 0 \\ 1 - \dfrac{x_{ij}}{x_{i\max}} & x_{i\min} = 0 \end{cases} \tag{2}$$

3.1.2 The formula of qualitative index's relative membership degree on superiority

Assume that the decision making set of elementary unit system is composed of n samples,

$$d_1, d_2, \cdots, d_n \tag{3}$$

Where n—the total of alternatives.

To index c_i , relative membership degree on superiority vector of the decision set is:

$$_i r = ({}_i r_1, {}_i r_2, \cdots, {}_i r_n) \qquad i = 1,2,\cdots,m \tag{4}$$

1. Qualitative compositor of the decision set on superiority (to index c_i)

Two-dimensional comparison are made on excellence between d_k and d_l . Given a scale ${}_i e_{kl} = 0$, 0.5 or 1.

If d_k in front of d_l , then ${}_i e_{kl} = 1, {}_i e_{lk} = 0$;

If d_l in front of d_k , then ${}_i e_{kl} = 0, {}_i e_{lk} = 1$;

If d_k and d_l make no distinction, then ${}_i e_{kl} = {}_i e_{lk} = 0.5$

The cases mentioned above satisfy the condition:

$${}_i e_{kl} + {}_i e_{lk} = 1, \ {}_i e_{kk} = {}_i e_{ll} = 0.5 \tag{5}$$

Make consistency scale matrix of compositor on excellence ${}_i E$

$${}_i E = \begin{bmatrix} {}_i e_{11} & {}_i e_{12} & \cdots & {}_i e_{1n} \\ {}_i e_{21} & {}_i e_{22} & \cdots & {}_i e_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ {}_i e_{n1} & {}_i e_{n2} & \cdots & {}_i e_{nn} \end{bmatrix} = ({}_i e_{kl}) \tag{6}$$

$$k = 1,2,\cdots,n \ ; \ l = 1,2,\cdots,n$$

Satisfy the following conditions:

$$\left.\begin{array}{ll} {}_i e_{hk} > {}_i e_{hl} & {}_i e_{kl} = 0 \\ {}_i e_{hk} < {}_i e_{hl} & {}_i e_{kl} = 1 \\ {}_i e_{hk} = {}_i e_{hl} = 0.5 & {}_i e_{kl} = 0.5 \end{array}\right\}$$

The compositor of row sum of matrix ${}_i E$ is qualitative compositor of the decision set on superiority. If two rows have the same value 0.5, their compositors are same too. We obtain

$$d_1', d_2', \cdots, d_n' \tag{7}$$

d_1' is the first to excellent alternative、 d_2' the second to excellent alternative、 ⋯

2 、 Quantitative calculation of the decision set on superiority (to index c_i)

Based on qualitative compositor, the formula of relative membership degree (RMD) on superiority is as follows:

$${}_i r_j = \frac{1 - {}_i a_{1j}}{{}_i a_{1j}}, 0.5 \le {}_i a_{1j} \le 1 \tag{8}$$

${}_i a_{1j}$ ——the quantitative scale on excellence of alternative d_1' to d_j' when d_1' is compared with d_j' regarding to qualitative factor c_i . We can get the value of ${}_i a_{1j}$ from table 1.

Table 1 Linear relation between mood operator and quantitative scale

Mood operator	**Equal**	**Slight**	**Somewhat**	**Rather**	**Obvious**	**Remarkable**
Quantitative scale	0.50	0.55	0.60	0.65	0.70	0.75
RMD	1.0	0.818	0.667	0.538	0.429	0.333
Mood operator	**Very**	**Extra**	**Exceeding**	**Extreme**	**Incomparable**	
Quantitative scale	0.80	0.85	0.90	0.95	1	
RMD	0.25	0.176	0.111	0.053	0	

After the above two steps, we have relative membership degree matrix of unit system as follows (qualitative index: ${}_i r_j = r_{ij}$):

$$R_{m\times n} = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix} = (r_{ij}) \tag{9}$$

3.1.3 Calculation of the weights of indexes

The input of unit system has been obtained. Before we compute the output of unit system, we must solve the weight vector of indexes. According to the method of qualitative index's relative membership degree on superiority, we can use the similar method to solve the weight vector of indexes.

$$w = (w_1, w_2, \cdots, w_m) \tag{10}$$

1. Qualitative compositor of importance

Assume that two-dimensional comparison are made on excellence between c_k and c_l. Given a scale $f_{kl} = 0$, 0.5 or 1

If f_k in front of f_l, then $f_{kl} = 1, f_{lk} = 0$;

If f_l in front of f_k, then $f_{kl} = 0, f_{lk} = 1$;

If f_k and f_l make no distinction, then $f_{kl} = f_{lk} = 0.5$.

The cases mentioned above satisfy the condition:

$$f_{kl} + f_{lk} = 1,\ f_{kk} = f_{ll} = 0.5$$

Make consistency scale matrix of compositor on excellence F

$$F = \begin{Bmatrix} f_{11} & f_{12} & \cdots & f_{1m} \\ f_{21} & f_{22} & \cdots & f_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ f_{m1} & f_{m1} & \cdots & f_{mm} \end{Bmatrix} = (f_{kl}) \tag{11}$$

$$k = 1,2,\cdots,m,\quad l = 1,2,\cdots,m$$

Satisfy the following condition:

$$\left.\begin{matrix} f_{hk} > f_{hl} & f_{kl} = 0 \\ f_{hk} < f_{hl} & f_{kl} = 1 \\ f_{hk} = f_{hl} = 0.5 & f_{kl} = 0.5 \end{matrix}\right\}$$

The compositor of row sum of matrix F is qualitative compositor of the decision set on superiority. If two rows have the same value 0.5, their compositors are same too. We obtain

$$f_1, f_2, \cdots, f_m \tag{12}$$

f_1 is the first to excellent alternative、 f_2 the second to excellent alternative、 ⋯

2、Quantitative calculation of the weights of indexes

$$w=(w_1, w_2, \cdots, w_m)= \left(\frac{\frac{1-g_{11}}{g_{11}}}{\sum_{i=1}^{m}\frac{1-g_{1i}}{g_{1i}}}, \frac{\frac{1-g_{12}}{g_{12}}}{\sum_{i=1}^{m}\frac{1-g_{1i}}{g_{1i}}}, \cdots, \frac{\frac{1-g_{1m}}{g_{1m}}}{\sum_{i=1}^{m}\frac{1-g_{1i}}{g_{1i}}} \right) \tag{13}$$

Satisfy the following condition:

$$\sum_{i=1}^{m} w_i = 1 \tag{14}$$

$$w_i = \frac{\frac{1-g_{1i}}{g_{1i}}}{\sum_{i=1}^{m}\frac{1-g_{1i}}{g_{1i}}}, 0.5 \le g_{1i} \le 1 \tag{15}$$

$$i = 1,2,\cdots,m$$

g_{1i} ——the quantitative scale on excellence of alternative c_1 to c_i when c_1 is compared with c_i. We can get the value of g_{1i} from table 1.

3.1.4 The output of unit system

A fuzzy optimum model can be obtained in reference [4]

$$u_j = \frac{1}{1+\left\{\frac{\sum_{i=1}^{m}[w_i(r_{ij}-1)]^p}{\sum_{i=1}^{m}(w_i r_{ij})^p}\right\}^{\frac{\alpha}{p}}} \tag{16}$$

u_j —the relative membership degree on superiority of index j.

p=2 or 1, a=2 or 1.

The output of unit system is obtained:

$$u = (u_1, u_2, \cdots, u_n) \quad j = 1,2,\cdots,n \tag{17}$$

3.2 Calculation of the integrative unit system

Assume that the number of unit systems in the lowest level (level 1) is m and the number of outputs is m after solving every unit system. So the outputs matrix is obtained as follows:

$${}_i u = \begin{bmatrix} {}_1u_1 & {}_1u_2 & \cdots & {}_1u_n \\ {}_2u_1 & {}_2u_2 & \cdots & {}_2u_n \\ \cdots & \cdots & \cdots & \cdots \\ {}_mu_1 & {}_mu_2 & \cdots & {}_mu_n \end{bmatrix} = ({}_iu_j) \tag{18}$$

$$i = 1,2,\cdots,m;\ j = 1,2,\cdots,n$$

Matrix ${}_iu$ can be regarded as an output matrix of any higher integrative unit system:

$$R=\begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix}=(r_{ij}) \tag{19}$$

The weight vector of the integrative unit system can be solved by the same method of unit system:

$$w=(w_1,w_2,\cdots,w_m)\,,\quad \sum_{i=1}^{m} w_i=1 \tag{20}$$

According to equation (17), the output of the integrative unit system is obtained—relative membership degree vector of decision-making set in higher lever.

By this method, we can solve the unit system (elementary unit system and integrative unit system) from the lowest level to the highest level. The output vector of the highest level is obtained as follows:

$$u=(u_1,u_2,\cdots,u_n) \tag{21}$$

The normalized formula is as follows:

$$u_i^{'}=u_i \Big/ \sum_{i=1}^{n} u_i \tag{22}$$

We have

$$u^{'}=(u_1^{'},u_2^{'},\cdots,u_n^{'}) \tag{23}$$

This vector can be regarded as the reallocation proportion of replaced water of the Yellow River.

IV. CALCULATION FOR REALLOCATION OF REPLACED WATER OF THE YELLOW RIVER

To the first level, qualitative index's relative membership degree on superiority is obtained by the method of this paper. The results are shown in table 2:

Table 2 Qualitative indexes of reallocation of replaced water of the Yellow River

Area	Water yield	Non-secondary salinization degree of soil	Water seasonaliration	Underground funnel	Water quality unpollution degree
Qing Hai	1.0	0.818	0	0	0.905
Gan Su	0.905	0.818	0	0	1.0
Ning Xia	0.29	0.212	0.25	0.6	0.667
Nei Meng	0.212	0.818	0.667	0.538	0.905
Shan Xi	0.739	0.905	0.905	0.818	0.667
Shan Xi	0.333	1.0	1.0	1.0	1.0
He Nan	0.111	0.538	0.818	0.739	0.212

Area	Contribution rate of scientific technical progress	Fluctuation ratio of industrial structure	Population growth	Medical attention and sanitation
Qing Hai	0.111	0.111	0.333	0.818
Gan Su	0.29	0.29	0.379	0.818
Ning Xia	0.379	0.379	0.333	0.818
Nei Meng	0.429	0.429	0.538	0.818
Shan Xi	0.818	1.0	0.905	1.0
Shan Xi	1.0	0.905	1.0	1.0
He Nan	0.905	0.538	0.111	0.111

Quantitative index's original data are shown in table 3:

Table 3 Quantitative indexes of reallocation of replaced water of the Yellow River

Area	Water resources utilization coefficient	Proportion of GDP	Proportion of gross product by industry	Proportion of gross product in agriculture	GDP of unit water amount	Average water amount per MU
Qing Hai	0.38	3.9	2.4	2.5	10.1	511.4
Gan Su	0.46	13	13.5	13.7	15.3	453.9
Ning Xia	0.32	4.7	4	10	2.7	665.8
Nei Meng	0.37	12.2	12.8	11	7	400.4
Shan Xi	0.46	20.8	25.7	18.4	29.5	351
Shan Xi	0.48	26.8	21.2	30	24.6	232.4
He Nan	0.46	18.6	20.5	13.4	39.8	354.4

Area	Average water amount per MU and available precipitation	Average water amount per capita	Average water amount per capita and available precipitation	Proportion of gross water deficit amount	Proportion of gross water deficit amount
Qing Hai	582.8	310.9	389.8	0	0.01
Gan Su	499.1	169.2	227.5	7.5	7.02
Ning Xia	605	736.3	867.7	0.9	0.41
Nei Meng	469.5	722.8	932.4	1.1	0.54
Shan Xi	389.4	206	333	31	26.7
Shan Xi	305.7	137.1	264.8	49.5	27.84
He Nan	379	155.7	274.8	10	13.6

According to Figure 1, we solve economic development unit system at first.

There are three indexes: the proportion of GDP c_1, the proportion of gross product by industry c_2, the proportion of gross product in agricultural c_3. They all belong to the benefit type of quantitative index. According to equation (1) the input matrix is obtained:

$$R=\begin{bmatrix} 0.146 & 0.485 & 0.175 & 0.455 & 0.776 & 1.0 & 0.694 \\ 0.093 & 0.525 & 0.156 & 0.498 & 1.0 & 0.805 & 0.798 \\ 0.083 & 0.457 & 0.333 & 0.367 & 0.613 & 1.0 & 0.447 \end{bmatrix}$$

Make consistency scale matrix of compositor on excellence before the weights of indexes.

$$\begin{array}{cc} & \begin{array}{ccc} c_1 & c_2 & c_3 \end{array} \\ \begin{array}{c} c_1 \\ c_2 \\ c_3 \end{array} & F = \begin{bmatrix} 0.5 & 1 & 1 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \end{bmatrix} \end{array} \quad \begin{array}{c} \text{consistency} \\ (1) \\ (2) \\ (2) \end{array}$$

We take c_1, whose ranking is 1, as comparison standard, and get under consideration: c_1 is more "rather" important than c_2 and c_3 but not "obvious" important than c_2 and c_3, so the scale value is 0.675 according to table 1. According to equation (13), the relative membership degree vector on importance of index c_1, c_2 and c_3 is obtained as follows:

$$w = (0.5115, 0.2442, 0.2442)$$

Set $p = 2, \quad \alpha = 2$, according to equation (16), we have the relative membership degree vector on superiority:

$$u_{\text{eco}} = \{0.0219, 0.4738, 0.0611, 0.3976, 0.9157, 0.9950, 0.7957\}$$

Similarly, we can calculate the other unit system from lowest level to highest level. Finally we have

$$u = [0.1893, 0.5026, 0.2075, 0.4414, 0.8898, 0.8412, 0.7444]$$

The normalized result is as follows:

$$u' = [0.050, 0.132, 0.054, 0.116, 0.233, 0.220, 0.195]$$

This proportion can be used for the decision-making of reallocation of replaced water of the Yellow River.

V. CONCLUSIONS

This paper studies reallocation of replaced water of the Yellow River's downstream to upstream and midstream areas when east-route and mid-route of the south-to-north water transfer supply water. This issue is of great practical significance because it relates to many reasons in lots of aspects. The unit systematic theory of the fuzzy sets decision-making is applied to reallocation of replaced water of the Yellow River, which transforms the complicated problem of decision-making into a series of calculations of the unit systems. This method takes account of both quantitative indexes (such as economic develop, water deficit, etc.) and qualitative indexes (such as science, population, medical attention, etc.). The decision-making plan is comprehensive, intuitionistic and according with real demands. A case study shows that this method is of valid and feasible.

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