Statistical Model Checking and Uppaal SMC

Formal Methods for Concurrent and Real-Time Systems, A.Y. 20/21

Livia Lestingi

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Why *Statistical* Model Checking?

(Some) Issues with numerical techniques:

- If the system has complex dynamics, basic problems may not have efficient solution methods
- They require an explicit description of system implementation
- They do not scale!

Why *Statistical* Model Checking?

Statistical techniques to the rescue!

- Applicable to any system, as long as its behavior is stochastic...
- ...and we have samples
- The state-space explosion problem can often be elided

Some facts about SMC:

- Research active for more than 15 years since 2002-2005 (but some key ideas can be traced back to a decade before)
- Many dedicated tools developed (14 surveyed by Agha and Palmskog[1])
- Areas of application ranging from *computer science* to *biology*

[1] Agha, Gul, and Karl Palmskog. "A survey of statistical model checking." ACM Transactions on Modeling and Computer Simulation (TOMACS) 28.1 (2018): 1-39.

SMC Workflow

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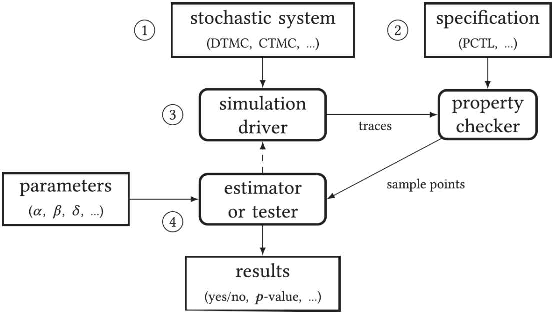
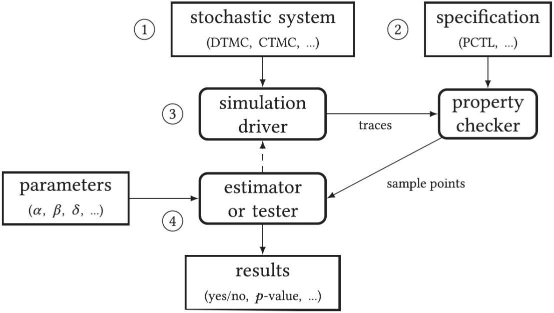


1. Input model of system

SMC Workflow



1. Input model of system
2. Formal property specification

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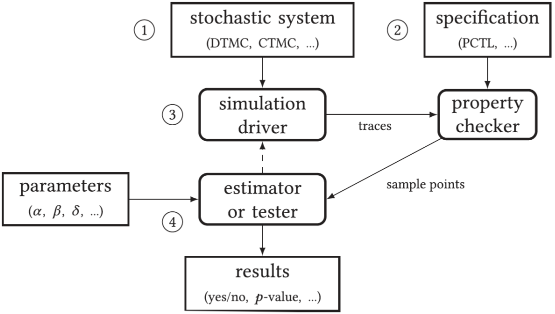

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3. *Trace*-generator (out-of-scope for this lecture)

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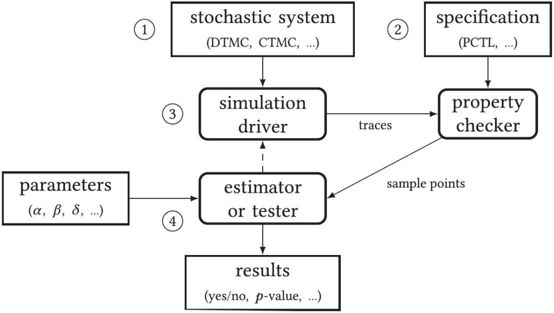
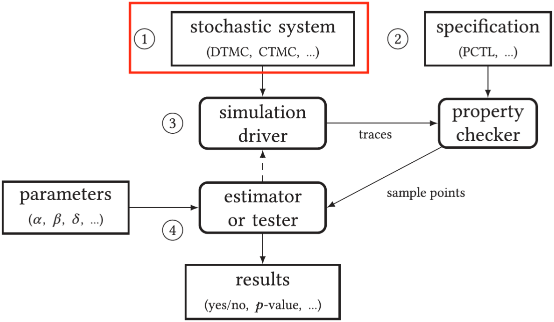


1. Input model of system
2. Formal property specification
3. *Trace*-generator (out-of-scope for this lecture)
4. Statistical technique



1
stochastic system
(DTMC, CTMC, ...)
2
specification
(PCTL, ...)
3
simulation driver
traces
property checker
sample points
parameters
(α, β, δ, ...)
4
estimator or tester
results
(yes/no, p-value, ...)

SMC is applicable to any process that can be considered a *stochastic discrete event system*: DTMCs (Discrete-Time Markov Chains) (Fig.1) are often used as a reference, but results carry over to other types of systems.

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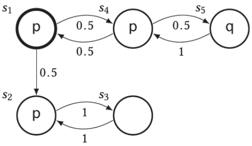

Fig.1. An example of DTMC.

Definition

Let AP be a finite set of atomic propositions. A DTMC is a tuple $\mathcal{M} = (S, s_i, M, L)$ where S is a finite set of states, $s_i \in S$ is the initial state, $M : S \times S \to [0, 1]$ is a transition probability function s.t. for all $s \in S$, $\sum_{s' \in S} M(s, s') = 1$, and $L : S \to 2^{AP}$ is a *labeling function* associating states with their respective true atomic propositions.

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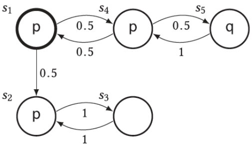

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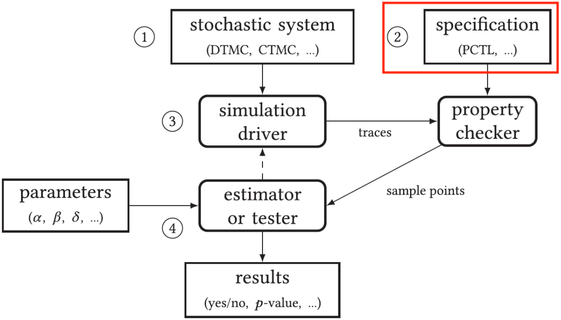
A *path* is an infinite sequence $s_1, s_2, \dots$ of elements of S, and a *trace* is a finite non-empty prefix of a path.

Stochastic Logics



- **Qualitative** properties of a system are expressed in *temporal logics* (typically Computational Tree Logic)

- If the process is stochastic, we require a logic with which we can express **quantitative** properties about time and probability

- We will see Probabilistic Computational Tree Logics (PCTL)

The semantics of a PCTL formula are defined w.r.t a DTMC M and a state $s \in S$. Syntax can be found in Fig.2.

$\phi ::= \top \mid a \mid \neg\phi \mid \phi \wedge \phi' \mid \mathrm{P}_{\geq\theta}(\psi)$	$a \in AP$	atomic proposition
$\psi ::= \phi \mid \mathrm{X}\,\phi \mid \phi\,\mathrm{U}^{\leq t}\,\phi' \mid \phi\,\mathrm{U}\,\phi'$	$\theta \in [0,1]$	probability bound
	$t \in \mathbb{Z}^{\geq 0}$	time bound

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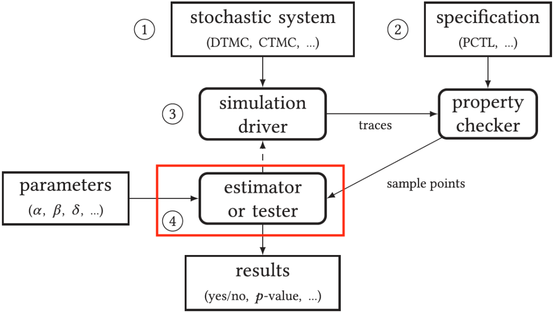
- $P_{\geq\theta}(\psi)$: true when the probability that ψ holds on paths starting from s is $\geq \theta$
- $\phi U^{\leq t}\phi'$: true for a path $\pi = s_1, s_2 ...$ if $\exists s_{k+1}, k+1 \leq t$ in π where ϕ' holds, and ϕ holds in all the states up to s_k

s	s_1	$...$	s_k	s_{k+1}	$\cdots$	s_t
ϕ	ϕ	$\cdots$	ϕ	ϕ'	$\cdots$	$\top$

Fig.3. A path where $\phi U^{\leq t}\phi'$ holds.



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- With SMC, the system is simulated multiple times and the property ψ is checked on every path prefix (trace): a checked trace constitutes a sample point of a Bernoulli variable X, with:

$$X = \begin{cases} 1, & \textit{if } \psi \textit{ holds} \\ 0, & \textit{if } !\psi \textit{ holds} \end{cases} \tag{1}$$

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- Therefore, $P(\psi) = E[X] = p$: the value p can be measured and compared with threshold $\theta \rightarrow$ Hypothesis Testing or Estimation

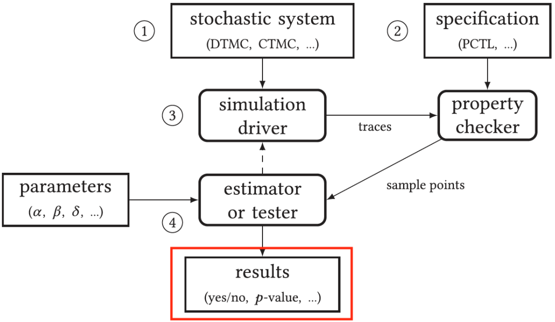
- *Hypothesis Testing*: Given two competing hypotheses $H_0 : p \geq \theta$ and $H_1 : p < \theta$, we evaluate the probability of obtaining such set of samples (the so-called p-value), assuming H_0 was true. If the p-value is smaller than a significance level α, we reject H_0, otherwise we accept it. If we reject H_0 when it is true we make a Type I error (probability α), in the mirrored case we make a Type II error (probability β), as in Fig.4

	Decision	
Truth	*accept* H_0, *reject* H_1	*reject* H_0, *accept* H_1
$p \geq \theta$: H_0 *true*, H_1 *false*	correct ($>1-\alpha$)	type I error ($\leq\alpha$)
$p < \theta$: H_0 *false*, H_1 *true*	type II error ($\leq\beta$)	correct ($>1-\beta$)

The conditions inside parentheses are on the probability for the given outcome.

Fig.4. Schema of errors in Hypothesis Testing.

- *Estimation*: Given a set of observations that represents a random sample, this can be used to estimate the parameters of the distribution, such as the value of p in a Bernoulli process. In this case, the proposed approximation, with m sample points, is $p' = \frac{\sum_{i=1}^{m} x_i}{m}$. Given a precision ϵ, p is guaranteed to belong to $[p' - \epsilon, p' + \epsilon]$.



- $P(\psi) = p \geq \theta$?
- Hypothesis Testing: accept $H_0 \rightarrow p \geq \theta$, reject $H_0 \rightarrow p < \theta$
- Estimation: approximate value of p and compare it with threshold θ

Uppaal SMC

Uppaal SMC

- Uppaal developers have recently (2012) released an Uppaal extension that allows us to model systems with stochastic features and run SMC experiments;
- Reading the Uppaal SMC Tutorial[2] is highly suggested if you choose to pursue the stochastic path for the project.
- If you do, make sure you download the latest build (**v.4.1**) that includes the SMC extension.

[2] https://link.springer.com/content/pdf/10.1007/s10009-014-0361-y.pdf

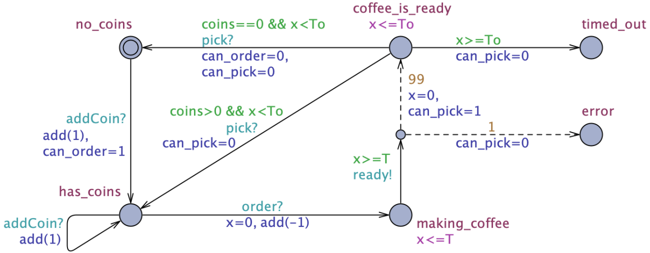
- Uppaal SMC extends the formalism underlying the tool to Stochastic Hybrid Automata (SHA):
  - *stochastic* features replace plain non-determinism with probabilistic choices;
  - *hybrid* systems are an extension of Timed Automata where clock rates can be given as general expressions (i.e., a differential equation)[3]

[3] No time to look into hybrid systems today (and they are not required by the project) but, if you are interested, we got theses!

Case Study: Stochastic Coffee Machine

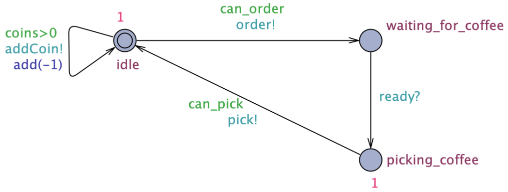
- We want to apply the following changes to previous coffee machine model:
  - When time T required to brew coffee elapses, with a 1% probability the machine shuts down and enters a deadlock *error* state;
  - When the user is idle, the probability of them performing an action is distributed according to an *exponential* distribution with fixed rate $\lambda = 1$.
  - When the coffee is ready, the probability of the user picking up the coffee is distributed according to an *exponential* distribution with fixed rate $\lambda = 1$.

- Stochastic Coffee Machine:



Case Study: Stochastic Coffee Machine

- Stochastic User:



- The model will be uploaded on Beep.

Uppaal SMC: Verifier

- Uppaal queries are also extended to include PCTL operators:
  - $\texttt{Pr}[<= \texttt{TAU}; \texttt{N}](\psi)$ is equivalent to PCTL formula $P^{\leq\tau}(\psi)$ and yields a probability range for formula ψ holding within time-bound τ. Optional parameter $\texttt{N}$ bounds the number of runs generated for the SMC experiment (decreasing its value might save you some time, but it may also reduce the degree of confidence!)

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  - $\texttt{simulate}[<= \texttt{TAU}; \texttt{N}]\{\texttt{E}_1, \ldots, \texttt{E}_\texttt{m}\}$ generates N runs of the system (that you can plot or export), each `TAU` time instants long, where $\texttt{E}_1, \ldots, \texttt{E}_\texttt{m}$ are the monitored expressions (this is a precious tool to test your system!).

Case Study: Stochastic Coffee Machine

- Let us run some queries on the updated coffee machine model:
  - `Pr[<= TAU](<> u.waiting_for_coffee)` yields a probability range for the user *eventually* ordering coffee;
  - `Pr[<= TAU](<> m.coffee_is_ready)` yields a probability range for the machine *eventually* producing a cup of coffee (play around to see how this changes in relation to the probability that the user will actually order coffee);
  - `Pr[<= TAU](<> m.error)` yields a probability range for the coffee machine *eventually* shutting down;

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  - $\texttt{Pr}[<= \texttt{TAU}](<> \texttt{m.error})$ yields a probability range for the coffee machine *eventually* shutting down;

  - $\texttt{simulate}[<= \texttt{TAU}; 1]\{\texttt{u.idle} * 2, \texttt{u.waiting\_for\_coffee} * 2, \texttt{u.picking\_coffee} * 2, \texttt{m.x}/\texttt{m.T}, \texttt{m.making\_coffee}, \texttt{m.coffee\_is\_ready}\}$ generates a trace monitoring user/machine states (the $*2$ is purely for visualization purposes).