MNIST Classification with Quantum Phase Estimation

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04/23/2025

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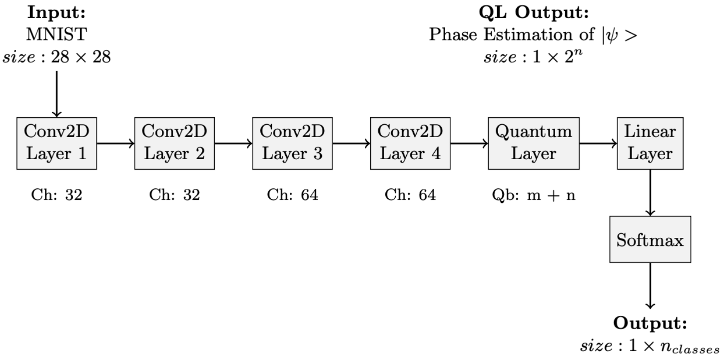
Neural Network

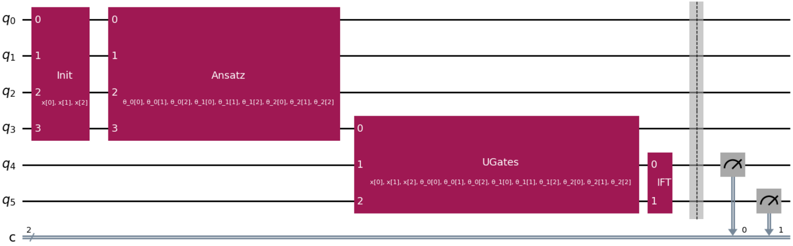
1 Introduction



The Conv2D layers extract features from images and build their embedding to pass to the quantum layer.

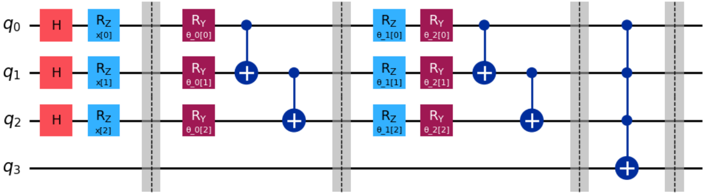
Quantum Layer structure

The final quantum layer is made of four main blocks:



Initialization + Ansatz

It is made of n qubits and has the below architecture:



The number of iterations ($R_z + R_y$ gates on each qubit) can be arbitrary set. The state on the last qubit can be written as:

$$|\psi >= a|1 > +b|0 > \tag{1}$$

U Matrix

In order to apply the QPE, it is crucial to build a unitary gate such that:

$$U|\psi >= e^{-i2\pi\phi}|\psi > \tag{2}$$

$$U = V\Lambda V^{\dagger} \quad \text{where} \quad V = \begin{bmatrix} b & -a^* \\ a & b^* \end{bmatrix}, \quad \Lambda = \begin{bmatrix} e^{-i2\pi\phi} & 0 \\ 0 & e^{i2\pi\phi} \end{bmatrix} \tag{3}$$

$$U = \begin{bmatrix} \cos(2\pi\phi) - ic\sin(2\pi\phi) & 2ia^*b\sin(2\pi\phi) \\ \\ 2iab^*\sin(2\pi\phi) & \cos(2\pi\phi) + ic\sin(2\pi\phi) \end{bmatrix}, \quad c = |b|^2 - |a|^2 = \frac{2^m - 2}{2^m} \tag{4}$$

U Matrix

U can be written as a composition of standard gates, $R_z + U_3$:

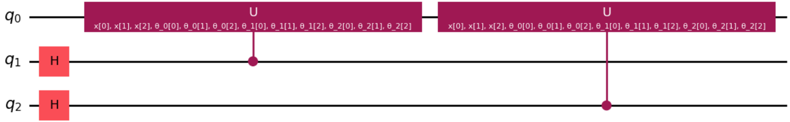
$$U = U_3 R_z = \begin{bmatrix} e^{-i\alpha}\cos\theta & -e^{i(\alpha+\lambda)}\sin\theta \\ e^{i(\beta-\alpha)}\sin\theta & e^{i(\alpha+\beta+\lambda)}\cos\theta \end{bmatrix} \tag{5}$$

Comparing equations 4 and 5, all the angles can be written as:

$$\begin{aligned} &\alpha = \arctan\left[c\tan(2\pi\phi)\right], \quad \theta = \arccos\left(\frac{\cos(2\pi\phi)}{\cos\alpha}\right) \\ &\lambda = -\alpha + \arctan(\cot\phi), \quad \beta = -\lambda \end{aligned} \tag{6}$$

U Gate

Knowing U allows us to build the UGate block; it is made of m qubits.



An arbitrary function can be chosen for the global phase ψ. Here:

$$\phi = \sum_{weights} \omega_i^2 + \sum_{inputs} x_i^2 \tag{7}$$

QPE Gate

At the end of the circuit, a IFT gate is applied to measure the global phase ϕ. This gate produces a state (on the last m qubits) of the form:

$$|\psi_{IFT}> = \sum_x \alpha_x |x> \quad \textit{where} \quad \alpha_x = \frac{1}{2^m} \sum_k e^{-i2\pi k(\phi - \phi_x)} \tag{8}$$

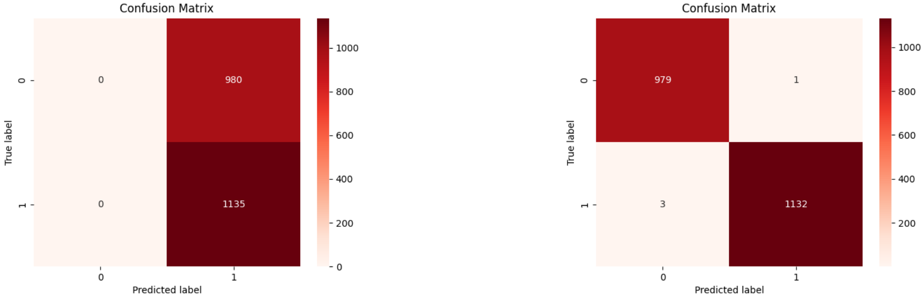
The probability of measuring a state $|x>$ is then:

$$P_x = \frac{1}{2^{2m}} \left(\frac{\sin(2^m \pi(\phi - \phi_x))}{\sin(\pi(\phi - \phi_x))} \right)^2 \tag{9}$$

Training loss and Configurations

The model is trained using **cross-entropy** loss between its output and the image label, thereby forcing it to map each input to one of the available quantized phases ($\mathbb{R}^{2^n}$) corresponding to a single label.

For each configuration $m = 5$ and $n\_iter = 5$ where n_iter is the number of repetitions of the block $R_z + R_y + CNOT$.

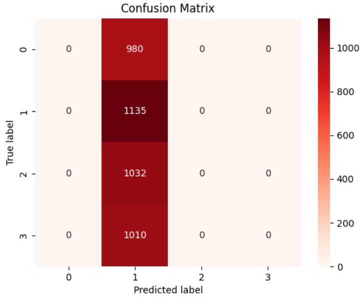


(a) Before Training (b) After Training

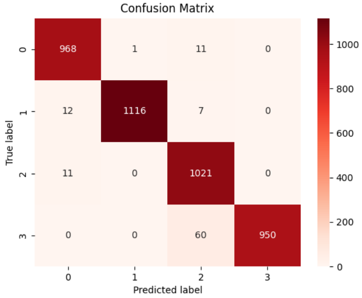
Total accuracy: 0.998

0-3 Digits

4 Results



(a) Before Training

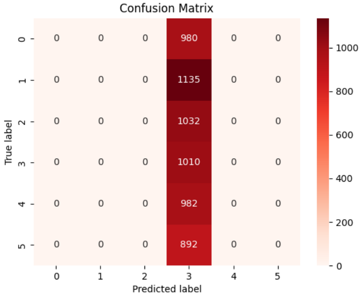


(b) After Training

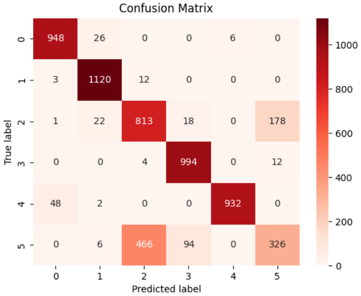
Total accuracy: 0.975

0-5 Digits

4 Results



(a) Before Training

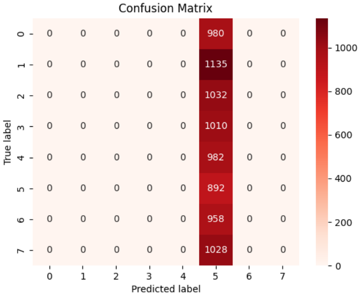


(b) After Training

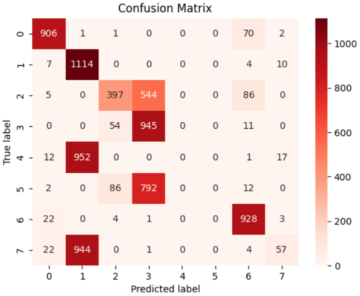
Total accuracy: 0.851

0-7 Digits

4 Results



(a) Before Training



(b) After Training

Total accuracy: 0.542

Thanks for your attention!