MNIST Classification with Quantum Phase Estimation

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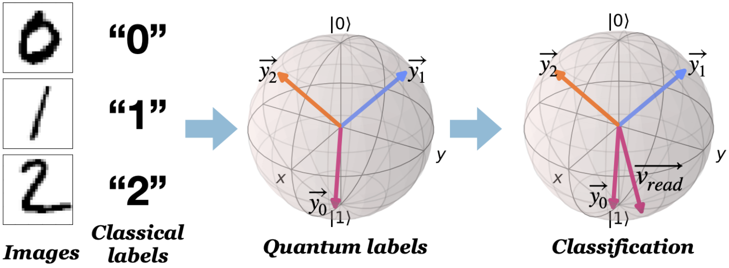
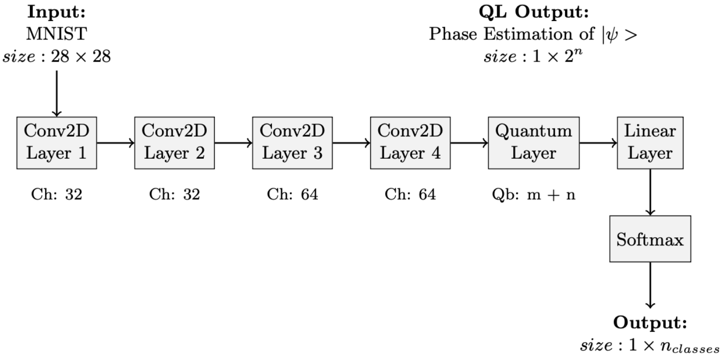
MORE adopts the same variational ansatz as binary classifiers while performing multi-classification by fully utilizing the quantum information of a single readout qubit.



Neural Network

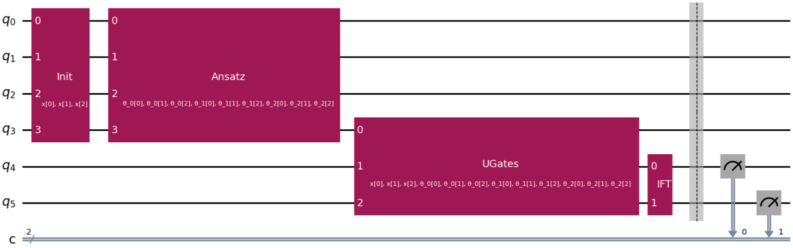
2 NN



The Conv2D layers extract features from images and build their embedding to pass to the quantum layer.

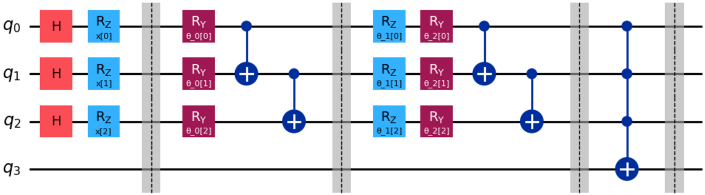
Quantum Layer structure

The final quantum layer is made of four main blocks:



Initialization + Ansatz

It is made of n qubits and has the below architecture:



The number of iterations ($R_z + R_y$ gates on each qubit) can be arbitrary set. The state on the last qubit can be written as:

$$|\psi >= a|1 > +b|0 > \tag{1}$$

U Matrix

In order to apply the QPE, it is crucial to build a unitary gate such that:

$$U|\psi >= e^{-i2\pi\phi}|\psi > \tag{2}$$

$$U = V\Lambda V^{\dagger} \quad \text{where} \quad V = \begin{bmatrix} b & -a^* \\ a & b^* \end{bmatrix}, \quad \Lambda = \begin{bmatrix} e^{-i2\pi\phi} & 0 \\ 0 & e^{i2\pi\phi} \end{bmatrix} \tag{3}$$

$$U = \begin{bmatrix} \cos(2\pi\phi) - ic\sin(2\pi\phi) & 2ia^*b\sin(2\pi\phi) \\ 2iab^*\sin(2\pi\phi) & \cos(2\pi\phi) + ic\sin(2\pi\phi) \end{bmatrix}, \quad c = |b|^2 - |a|^2 = \frac{2^m - 2}{2^m} \tag{4}$$

U Matrix

3 Quantum Layer

U can be written as a composition of standard gates, $R_z + U_3$:

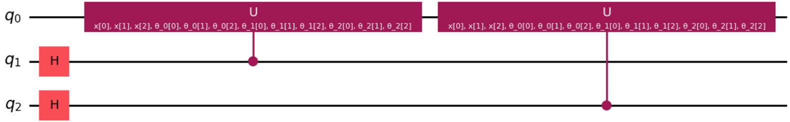
$$U = U_3 R_z = \begin{bmatrix} e^{-i\alpha}\cos\theta & -e^{i(\alpha+\lambda)}\sin\theta \\ e^{i(\beta-\alpha)}\sin\theta & e^{i(\alpha+\beta+\lambda)}\cos\theta \end{bmatrix} \tag{5}$$

Comparing equations 4 and 5, all the angles can be written as:

$$\begin{aligned} &\alpha = \arctan\left[c\tan(2\pi\phi)\right], \quad \theta = \arccos\left(\frac{\cos(2\pi\phi)}{\cos\alpha}\right) \\ &\lambda = -\alpha + \arctan(\cot\phi), \quad \beta = -\lambda \end{aligned} \tag{6}$$

U Gate

Knowing U allows us to build the UGate block; it is made of m qubits.



An arbitrary function can be chosen for the global phase ψ. Here:

$$\phi = \sum_{weights} \omega_i^2 + \sum_{inputs} x_i^2 \tag{7}$$

QPE Gate

At the end of the circuit, a IFT gate is applied to measure the global phase ϕ. This gate produces a state (on the last m qubits) of the form:

$$|\psi_{IFT}>=\sum_x \alpha_x|x> \quad \textit{where} \quad \alpha_x = \frac{1}{2^m}\sum_k e^{-i2\pi k(\phi-\phi_x)} \tag{8}$$

The probability of measuring a state $|x>$ is then:

$$P_x = \frac{1}{2^{2m}}\left(\frac{\sin(2^m\pi(\phi-\phi_x))}{\sin(\pi(\phi-\phi_x))}\right)^2 \tag{9}$$

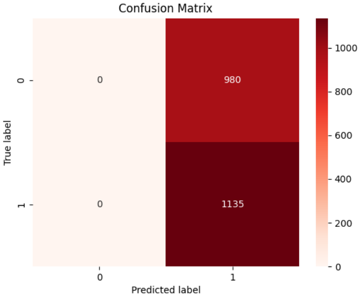
Training loss and Configurations

The model is trained using **cross-entropy** loss between its output and the image label, thereby forcing it to map each input to one of the available quantized phases ($\mathbb{R}^{2^n}$) corresponding to a single label.

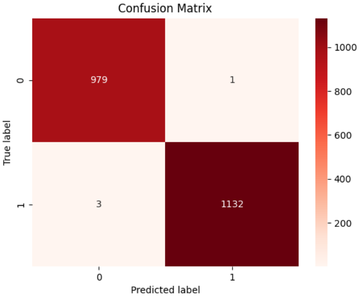
For each configuration $m = 5$ and $n\_iter = 5$ where n_iter is the number of repetitions of the block $R_z + R_y + CNOT$.

0-1 Digits

5 Results



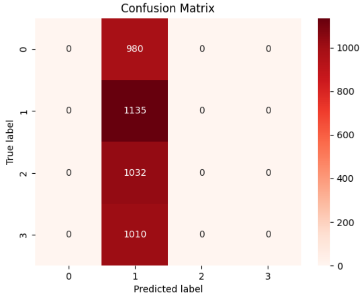
(a) Before Training



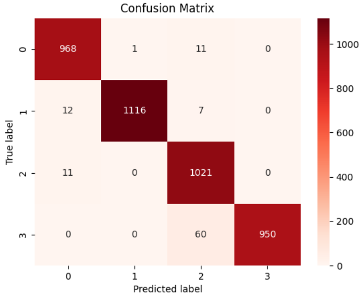
(b) After Training

Total accuracy: 0.998

0-3 Digits



(a) Before Training

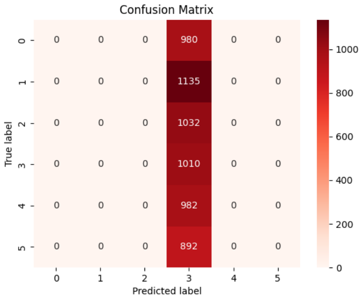


(b) After Training

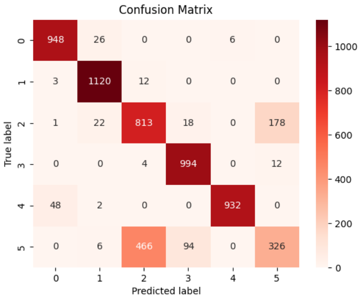
Total accuracy: 0.975

0-5 Digits

5 Results



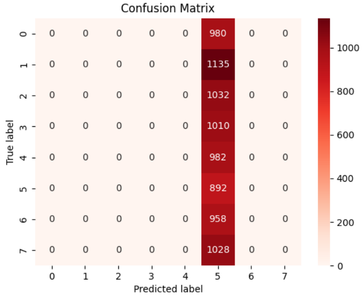
(a) Before Training



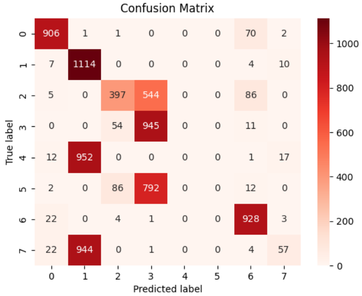
(b) After Training

Total accuracy: 0.851

0-7 Digits



(a) Before Training



(b) After Training

Total accuracy: 0.542

Thanks for your attention!