Laboratory Work

Numeric Differenciation

1. Given function (**variant – function number**):

Function	Reference point	First derivate at x_0	Second derivate at x_0
$f_1(x) = x^3 - 2x$	$x_0 = 1$	1.0	6.0
$f_2(x) = \sin x$	$x_0 = \pi/3$	0.5	-0.8660254037
$f_3(x) = e^x$	$x_0 = 0$	1.0	1.0

Using MATLAB function `difapx()`, fill table with calculation errors:

$f'(x)$	h	$\frac{f_1 - f_{-1}}{2h}$	$\frac{-f_2 + 8f_1 - 8f_{-1} + f_{-2}}{12h}$
	0.1		
	0.01		
$f''(x)$	h	$\frac{f_1 - 2f_0 + f_{-1}}{h^2}$	$\frac{-f_2 + 16f_1 - 30f_0 + 16f_{-1} - f_{-2}}{12h}$
	0.1		
	0.01		

2. Given function by data points (**variant – function number**)::

x	$f_1(x)$	x	$f_2(x)$	x	$f_3(x)$
0.8	−1.0880	0.8472	0.7494	−0.2	1.2214
0.9	−1.0710	0.9472	0.8118	−0.1	1.1052
1.0	−1.0000	1.0472	0.8660	0.0	1.0000
1.1	−0.8690	1.1472	0.9116	0.1	0.9048
1.2	−0.6720	1.2472	0.9481	0.2	0.8187

Fill tables

Find derivates at point x_3:

First derivate according to (5.1.8)	
First derivate according to (5.1.9)	
Second derivate according to (5.3.1)	
Second derivate according to (5.3.2)	

Aproximate given data points using Newtons polynomial and find derivates at middle point:

First derivate $l_2(x)$	
First derivate $l_4(x)$	
Second derivate $l_2(x)$	
Second derivate $l_4(x)$	

Numerical Integration

1. Given integral:

Variant	Integralas
1.	$\int_0^2 \left(x^3 - 2x\right)dx = 0$
2.	$\int_0^{\pi/2} \sin x\, dx = 1$
3.	$\int_0^1 e^{-x}\, dx = 0.63212055883$

Fill table with integration errors:

N	Trapezoidal method	Simpsons method	Rombergs method
4	0.24999999999999986	1.3877787807814457e-16	2.216617274837823e-14
8	0.06249999999999986	1.3877787807814456e-16	2.216617274837823e-14

2. Given integral:

Variant	Integralas
1.	$\int_0^{\infty} \frac{\sin x}{x} dx \cong \int_0^{100} \frac{\sin x}{x} dx$
2.	$\int_0^{\infty} e^{-x^2}\, dx \cong \int_0^{100} e^{-x^2}\, dx$

Calculate that integral using symbolic method, and fill table with integration errors.
`N = 200`, `tol = 1e-4`, number ofgrid points `IGL = 20`:

Simpsonos method	Integral 1 - 1.75588506881041e-5 Integral 2 - 0.675980982585610
Adaptive quadrature	Integral 1 - 6.66133814775094e-16 Integral 2 - 0
`quadl()`	Integral 1 - 6.66133814775094e-16 Integral 2 - 0
Gauss–Hermite method	Integral 1 - 1.56222546688906 Integral 2 - 0.367087211385694

Present MATLAB script file.