Laboratory Work

Numeric Differentiation

Given function (choose a variant):

Var.	Function	Reference point	First derivate at x_0	Second derivate at x_0
1	$f_1(x) = x^3 - 2x$	$x_0 = 1$	1.0	6.0
2	$f_2(x) = \sin x$	$x_0 = \pi / 3$	0.5	-0.8660254037
3	$f_3(x) = e^x$	$x_0 = 0$	1.0	1.0

Fill the table with calculation errors between numerical derivative and real value:

$f'(x)$	h	5.1.8. $\frac{f_1 - f_{-1}}{2h}$	5.1.9. $\frac{-f_2 + 8f_1 - 8f_{-1} + f_{-2}}{12h}$
	0.1	1.66750e-03	3.33730e-06
	0.01	1.66667e-05	3.33344e-10
$f''(x)$	h	5.3.1. $\frac{f_1 - 2f_0 + f_{-1}}{h^2}$	5.3.2. $\frac{-f_2 + 16f_1 - 30f_0 + 16f_{-1} - f_{-2}}{12h^2}$
	0.1	8.33611e-04	1.11210e-06
	0.01	8.33336e-06	1.11410e-10

2. Given function by data points (choose a variant):

	Var. 1		Var. 2		Var. 3	
	x	$f_1(x)$	x	$f_2(x)$	x	$f_3(x)$
x_1	0.8	−1.0880	0.8472	0.7494	−0.2	1.2214
x_2	0.9	−1.0710	0.9472	0.8118	−0.1	1.1052
x_3	1.0	−1.0000	1.0472	0.8660	0.0	1.0000
x_4	1.1	−0.8690	1.1472	0.9116	0.1	0.9048
x_5	1.2	−0.6720	1.2472	0.9481	0.2	0.8187

Find derivates at point x_3:

First derivate according to (5.1.8)	1.00167e+00
First derivate according to (5.1.9)	9.99997e-01
Second derivate according to (5.3.1)	1.00083e+00
Second derivate according to (5.3.2)	9.99999e-01

Approximate given data points using Newtons polynomial and find derivates at middle point:

First derivative $l_2(x)$	-1.1620
First derivate $l_4(x)$	-0.1667
Second derivative $l_2(x)$	0.5500
Second derivative $l_4(x)$	0.0417

Numerical Integration

1. Given integral:

Var.	Integral
1.	$\int_0^2 \left(x^3 - 2x\right)dx = 0$
2.	$\int_0^{\pi/2} \sin x\, dx = 1$
3.	$\int_0^1 e^{-x}\, dx = 0.63212055883$

Fill table with integration errors::

N	Trapezoidal method	Simpson's method	Romberg's method
4			
8			

2. Given integral:

	Integral
1.	$\int_0^{\infty} \frac{\sin x}{x} dx \cong \int_0^{100} \frac{\sin x}{x} dx$

Calculate that integral using the symbolic method, and fill the table with integration errors. `N = 200`, `tol = 1e-4`, `IGL = 20`, lower bound `a = 0.001`:

Simpson's metodas	
Adaptive quadrature	
`quad()`	
`quadl()`	
Gauss- Legendre	