Hausaufgabe 5

H8. a) $\sum\limits_{n\geq 1} \frac{n\cdot 4^n}{(n+4)!}$

Wir benutzen Quotientkriterium:

$$Q_n = \frac{x_{n+1}}{x_n} = \frac{\frac{(n+1)\cdot 4^{n+1}}{(n+5)!}}{\frac{n\cdot 4^n}{(n+4)!}} = \frac{(n+1)\cdot 4^{n+1}}{(n+5)!}\cdot\frac{(n+4)!}{n\cdot 4^n} =$$

$$= \frac{(n+1)\cdot 4}{(n+5)\cdot n} = \frac{4n+4}{n^2+5n} < 1 \Rightarrow \sum x_n - \text{konv.}$$

b) $$\sum_{n\geq 1} \frac{\sqrt{n^2+4}-\sqrt{n^2+2}}{\sqrt{n^3+1}} = \sum_{n\geq 1} \frac{2}{(\sqrt{n^2+4}+\sqrt{n^2+2})\sqrt{n^3+1}}$$

$$= 2\sum \frac{1}{(\sqrt{n^2+4}+\sqrt{n^2+2})\cdot\sqrt{n^3+1}}$$

Sei $x_n = \frac{1}{(\sqrt{n^2+4}+\sqrt{n^2+2})\cdot\sqrt{n^3+1}}$, $y_n = \frac{1}{\cancel{\sqrt{n^3+1}}\,(\sqrt{n^2+4}+\sqrt{n^2+2})}$?

$n\in\mathbb{N}$

$$\Rightarrow \lim_{n\to\infty}\frac{x_n}{y_n} = \lim_{n\to\infty} \cancel{\frac{1}{\sqrt{n^2+4}+\sqrt{n^2+2}}}\ \frac{1}{\sqrt{n^3+1}} = 0$$

harm. Reihe $\alpha > 1$ $(\alpha = \frac{3}{2})$

$\Rightarrow \sum$ - konv.

c) $$\sum_{n\geq 1}\left(1+\frac{1}{n^4+1}\right)^{5n^4+n^3+n}$$

Wurzelkriterium:

$$\lim_{n\to\infty}\sqrt[n]{\left(1+\frac{1}{n^4+1}\right)^{5n^4+n^3+n}} = \lim_{n\to\infty}\sqrt[n]{e^{\frac{5n^4+n^3+n}{n^4+1}}}$$

$$= e^{\lim_{n\to\infty}\sqrt[n]{\frac{5n^4+n^3+n}{n^4+1}}} = e^{\lim_{n\to\infty}\sqrt[n]{5}} = e^{5^{\frac{1}{\infty}}} = e^{5^0} =$$

$$= e \Rightarrow \sum_{n\geq 1}\left(1+\frac{1}{n^4+1}\right) - \text{div.}$$

d) $\sum_{n\geq 1} \left(\frac{n^2+n+1}{n^2} \cdot a\right)^n, \ a>0$

Wurzel Kriterium

$$\lim_{n\to\infty} \sqrt[n]{\left(\frac{n^2+n+1}{n^2}\cdot a\right)^n} = \lim_{n\to\infty} \left\| \frac{n^2+n+1}{n^2} \cdot a \right\|$$

$$= \lim_{n\to\infty} |a| \cdot \left(\frac{n^2+n+1}{n^2}\right) = |a| = a$$
$(a>0)$

? ..

H 9.

a) $\sum_{n\geq 1} (-1)^n \frac{\pi^n}{\cos^2(n^2+2)+5^n}$

Absolute Konvergent: $\sum_{n\geq 1} \left|(-1)^n \frac{\pi^n}{\cos^2(n^2+2)+5^n}\right| =$

$$= \sum_{n\geq 1} \left|\frac{\pi^n}{\cos^2(n^2+2)+5^n}\right|$$

Vergleichskriterium:

$$\frac{\pi^n}{\cos^2(n^2+2)+5^n} < \frac{\pi^n}{5^n} = \left(\frac{\pi}{5}\right)^n \Leftrightarrow \frac{\left(\frac{\pi}{5}\right)^1}{5/1-\frac{\pi}{5}} = \frac{\frac{\pi}{5}}{\frac{5-\pi}{5}} =$$

$$= \frac{\pi}{5} \cdot \frac{5}{5-\pi} = \frac{\pi}{5-\pi} \Rightarrow \text{Reihe ist div}$$
>1

$\Rightarrow$ es ist nicht absolute Konvergent.

.... ?

b) $\sum_{n \geq 1} (-1)^n \left(\sqrt{n+4} - \sqrt{n+2} \right)$

<u>Abs. konv:</u>

$\sum_{n \geq 1} \left| (-1)^n \left(\sqrt{n+4} - \sqrt{n+2} \right) \right| = \sum_{n \geq 1} \sqrt{n+4} - \sqrt{n+2} \cdot \frac{\sqrt{n+4} + \sqrt{n+2}}{\sqrt{n+4} + \sqrt{n+2}}$

$= \sum_{n \geq 1} \frac{2}{\sqrt{n+4} + \sqrt{n+2}}$

...