Hausaufgabe 7

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H11.

1) $f: \mathbb{R} \to \mathbb{R}$, $f(-3) = 2$, $f'(-3) = 0$, $f''(-3) = f^{(3)}(-3) = -6$, $f^{(4)}(-3) = 72$

Aus (2) 7. Vorlesung $\Rightarrow$

$$\Rightarrow f(x) = f(-3) + \frac{f'(-3)}{1!}(x+3) + \frac{f''(-3)}{2!}(x+3)^2 + \frac{f^{(3)}(-3)}{3!}(x+3)^3 + \frac{f^{(4)}(-3)}{4!}(x+3)^4$$

$$= 2 + 0 + \frac{-6}{2}(x+3)^2 + \frac{-6}{6}(x+3)^3 + \frac{72}{24}(x+3)^4$$

$$= 2 - 3(x+3)^2 - (x+3)^3 + 3(x+3)^4$$

2. $f, g: (-1, \infty) \to \mathbb{R}$, $f(x) = \frac{1}{\sqrt{x+1}}$, $g(x) = f(x) \cdot \sin x$

a) $T_2(x, 0)$

$$T_2(x, 0) = \sum_{k=0}^{2} \frac{f^{(k)}(0)}{k!}(x)^k$$

$$f^{(1)}(x) = \left(\frac{1}{\sqrt{x+1}}\right)' = \frac{1' \cdot \sqrt{x+1} - 1 \cdot (\sqrt{x+1})'}{(\sqrt{x+1})^2} = \frac{0 - \frac{1}{2\sqrt{x+1}} \cdot (x+1)'}{x+1} =$$

$$= \frac{-\frac{1}{2\sqrt{x+1}}}{x+1} = -\frac{1}{2} \cdot \frac{1}{\sqrt{x+1} \cdot (x+1)} = -\frac{1}{2} \cdot \frac{1}{(x+1)^{\frac{1}{2}} \cdot (x+1)} =$$

$$= -\frac{1}{2} \cdot \frac{1}{(x+1)^{\frac{3}{2}}}$$

$$f^{(2)}(x) = (f^{(1)}(x))' = \left(-\frac{1}{2} \cdot \frac{1}{(x+1)^{\frac{3}{2}}}\right)' = 0 + \left(-\frac{1}{2}\right) \cdot \left[(x+1)^{-\frac{3}{2}}\right]' =$$

$$= -\frac{1}{2} \cdot \left(-\frac{3}{2}\right) \cdot (x+1)^{-\frac{3}{2}-1} \cdot (x+1)' = -\frac{1}{2} \cdot \left(-\frac{3}{2}\right) \cdot (x+1)^{-\frac{5}{2}} =$$

$$= \frac{3}{4} \cdot (x+1)^{-\frac{5}{2}}$$

$$\Rightarrow T_2(x, 0) = \sum_{k=0}^{2} \frac{f^{(k)}(0)}{k!}(x)^k = \frac{f(0)}{0!} \cdot 1 + \frac{f'(0)}{1!} \cdot (x) + \frac{f''(0)}{2!}(x)^2$$

$$= \frac{1}{\sqrt{x+1}} - \frac{1}{2} \cdot \frac{x}{(x+1)^{\frac{3}{2}}} + \frac{3}{4} \cdot \frac{x^2}{(x+1)^{\frac{5}{2}}}$$

$$= \frac{1}{(x+1)^{\frac{1}{2}}}\left[\overset{4(x+1)^2}{1} - \frac{\overset{2(x+1)}{x}}{2(x+1)} + \frac{3x^2}{4(x+1)^2}\right]$$

$$= \frac{1}{(x+1)^{\frac{1}{2}}}\left[\frac{4(x+1)^2 - 2x(x+1) + 3x^2}{4(x+1)^2}\right]$$

$$= \frac{1}{(x+1)^{\frac{1}{2}}}\left[\frac{4(x^2+2x+1) - 2x^2 - 2x + 3x^2}{4(x+1)^2}\right]$$

$$= \frac{1}{(x+1)^{\frac{1}{2}}}\left[\frac{4x^2 + 8x + 4 - 2x^2 - 2x + 3x^2}{4(x+1)^2}\right]$$

$$= \frac{1}{(x+1)^{\frac{1}{2}}}\left[\frac{5x^2 + 6x + 4}{4(x+1)^2}\right]$$

b) $R_2(x, 0)$, $x \in (-1, \infty) \setminus \{0\}$

$$R_n(x, x_0) = \frac{f^{(n+1)}(c)}{(n+1)!}(x - x_0)^{n+1}$$

$(n+1)!$

$$\Rightarrow R_2(x,0) = \frac{f^{(3)}(c)}{3!} x^3$$

$$f^{(3)}(x) = \left(\frac{3}{4}\cdot(x+1)^{-\frac{5}{2}}\right)' = -\frac{3}{4}\cdot\left(-\frac{5}{2}\right)\cdot(x+1)^{-}$$

$$= \frac{15}{8}\cdot(x+1)^{-\frac{7}{2}}$$

$$\Rightarrow R_2(x,0) = \frac{\frac{15}{8}\cdot(c+1)^{-\frac{7}{2}}}{3!}\cdot x^3$$

c) $f^{(n)}(x)$, $x \in (-1,\infty)$, $n \in \mathbb{N}$

$$f^{(1)}(x) = -\frac{1}{2}\cdot\frac{1}{(x+1)^{\frac{3}{2}}}$$

$$f^{(2)}(x) = \frac{3}{4}\cdot\frac{1}{(x+1)^{\frac{5}{2}}}$$

$$f^{(3)}(x) = \frac{15}{8}\cdot\frac{1}{(x+1)^{\frac{7}{2}}}$$

Induktion: $g^{(n)}(x) =$

d) $g^{(n)}(0) = 0$?

$\frac{7}{2} \cdot (x+1)' =$

$c \cdot \frac{1}{(x+1)^{n+1}}$

$\Rightarrow 0 - $ ist konstant