HSG 9 - RAZVAN POSTESCU

Tuesday, December 1, 2020 9:30 AM

H14. $(x^k)_{k\in\mathbb{N}^*}$ in $\mathbb{R}^3$, $x_k = \left(\left(\frac{\sqrt{k}}{1+\sqrt{k}}\right)^{\sqrt{k}}, \left(-\frac{1}{5}\right)^k, \frac{1+2^2+3^3+\dots+k^k}{k^k}\right)$

$$\lim_{k\to\infty}\left(\frac{\sqrt{k}}{1+\sqrt{k}}\right)^{\sqrt{k}} = \lim_{k\to\infty}\left(\frac{\sqrt{k}+1-1}{1+\sqrt{k}}\right)^{\sqrt{k}} = \lim_{k\to\infty}\left(\frac{\sqrt{k}+1}{1+\sqrt{k}} - \frac{1}{1+\sqrt{k}}\right)^{\sqrt{k}} =$$

$$= \lim_{k\to\infty}\left(1-\frac{1}{1+\sqrt{k}}\right)^{\sqrt{k}} = \lim_{k\to\infty}\left\{\left[1+\left(-\frac{1}{1+\sqrt{k}}\right)\right]^{-\sqrt{k}-1}\right\}^{\frac{\sqrt{k}}{-\sqrt{k}-1}} =$$

$$= \lim_{k\to\infty} e^{-\frac{\sqrt{k}}{\sqrt{k}+1}} = \lim_{k\to\infty} e^{-\frac{\cancel{\sqrt{k}}}{\cancel{\sqrt{k}}(1+\frac{1}{\sqrt{k}})}} = e^{-1} = \frac{1}{e}$$

$$\lim_{k\to\infty}\left(-\frac{1}{5}\right)^k = \lim_{k\to\infty} \pm\frac{1}{5^k} = 0$$

$$\lim_{k\to\infty}\frac{1+2^2+3^3+\dots+k^k}{k^k} =$$

Sei $y_k = k^k$, $k\in\mathbb{N}$ - ein streng wachsende Folge mit

$\lim_{k\to\infty} y_k = \lim_{k\to\infty} k^k = \infty$.

Sei $x_k = 1+2^2+3^3+\dots+k^k$, x_n - streng wachsend

$$\Rightarrow \lim_{k\to\infty}\frac{x_k}{y_k} = \lim_{k\to\infty}\frac{x_{k+1}-x_k}{y_{k+1}-y_k} = \lim_{k\to\infty}\frac{(1+2^2+3^3+\dots+k^k+(k+1)^{k+1})-(1+2^2+3^3+\dots+k^k)}{(k+1)^{k+1}-k^k} =$$

$$= \lim_{k\to\infty}\frac{(k+1)^{k+1}}{(k+1)^{(k+1)}-k^k} = \lim_{k\to\infty}\frac{\cancel{(k+1)^{k+1}}}{\cancel{(k+1)^{(k+1)}}\left(1-\frac{k^k}{(k+1)^{(k+1)}}\right)} =$$

$$= \frac{1}{1-0} = 1$$

$$\lim_{k\to\infty}\frac{k^k}{(k+1)^{(k+1)}} = \lim_{k\to\infty}\frac{k^k}{(k+1)^k(k+1)} = \lim_{k\to\infty}\left(\frac{k}{k+1}\right)^k\cdot\frac{1}{(k+1)} =$$

$$= \lim_{k\to\infty}\left(\frac{\cancel{k}}{\cancel{k}(1+\frac{1}{k})}\right)^k\cdot\frac{1}{(k+1)} = \lim_{k\to\infty}\left(\frac{1}{1+\frac{1}{k}}\right)^k\cdot\frac{1}{(k+1)} = 1\cdot 0 = 0$$

$\Rightarrow x^k$ - ist Konvergent und $\lim_{k\to\infty} x^k = \left(\frac{1}{e}, 0, 0\right)$

H15.

a) $A' = [-\infty, 1] \times \mathbb{R}$

b) $A' = \mathbb{R} \times \mathbb{R}$

c) $A' = \mathbb{N} \times \mathbb{R}$

H16. $f: \mathbb{R}^2 \to \mathbb{R}$, $f(x,y) = \begin{cases} \dfrac{x^4 - y^4}{2(x^4+y^4)}, & (x,y) \neq O_2 \\ 0, & (x,y) = O_2 \end{cases}$

$a^k = \left(\frac{1}{k}, 0\right)$, $k \in \mathbb{N}^*$, a^k - Glied eines Folge

$\lim\limits_{k\to\infty} a^k = (0,0) = O_2$

$\lim\limits_{k\to\infty} f(a^k) = \lim\limits_{k\to\infty} \dfrac{\left(\frac{1}{k}\right)^4 - 0^4}{2\left(\left(\frac{1}{k}\right)^4 + 0^4\right)} = \lim\limits_{k\to\infty} \dfrac{\left(\frac{1}{k}\right)^4 - 0^4}{\left(\frac{1}{k}\right)^4 + 0^4} \cdot \frac{1}{2} =$

$= \lim\limits_{k\to\infty} \dfrac{\cancel{\left(\frac{1}{k}\right)^4} \left(1 - \frac{0^4}{\left(\frac{1}{k}\right)^4}\right)}{\cancel{\left(\frac{1}{k}\right)^4} \left(1 + \frac{0^4}{\left(\frac{1}{k}\right)^4}\right)} \cdot \frac{1}{2} = \frac{1}{2}$

Für die Folge mit dem allgemeinen Glied $b^k = \left(\frac{1}{k}, \frac{1}{k}\right)$, $k \in \mathbb{N}^*$

$\Rightarrow \lim\limits_{k\to\infty} b^k = (0,0) = O_2$

$\lim\limits_{k\to\infty} f(b^k) = \dfrac{\overbrace{\left(\frac{1}{k}\right)^4 - \left(\frac{1}{k}\right)^4}^{=0}}{2\left(\left(\frac{1}{k}\right)^4 + \left(\frac{1}{k}\right)^4\right)} = 0$

$\lim\limits_{k\to\infty} f(b^k) \neq f\lim\limits_{k\to\infty} (a^k) \overset{Th\,S}{\Rightarrow} f$ - hat ~~kein~~ kein Grenzwert bei O_2 $\Rightarrow$ f - nicht stetig in O_2