LOGIK U11

Thursday, December 10, 2020 6:00 PM

G1.

	NNF	bereinigt	Pränexnormalform	Skolemnormalform
$(\exists p(x,y)) \to (\forall y p(y,a))$	✗	✗	✗	✗
$(\forall x p(a,x)) \land (\exists y p(y,a))$	✓	✓	✗	✓
$(\forall x p(x,y)) \lor (\exists y p(y,y))$	✓	✗	✗	✗
$\forall x \exists y (p(a,x) \land p(x,y))$	✓	✓	✓	✗
$\forall x \exists z \forall y \neg(p(x,y) \lor p(x,z))$	✗	✓	✗	✗
$\forall x \forall y (p(x,a) \lor \neg p(x,y))$	✓	✓	✓	✓
$\neg(p(x,x) \land p(x,y))$	✗	✓	✗	✗
$\neg p(y,x) \lor p(x,y)$	✓	✓	✓	✓

G2.

$F = \forall x \exists y (p(y,a,x) \leftrightarrow \neg\exists z p(z,y,x))$ (Elim $\leftrightarrow$)

$\equiv \forall x \exists y (p(y,a,x) \to \neg\exists z p(z,y,x) \land \neg\exists z p(z,y,x) \to p(y,a,x))$ (Elim $\to$)

$\equiv \forall x \exists y ((\neg p(y,a,x) \lor \neg\exists z p(z,y,x)) \land (\exists z p(z,y,x) \lor p(y,a,x)))$ (De Morgan, $\neg E \equiv V$)

$\equiv \forall x \exists y ((\neg p(y,a,x) \lor \forall z \neg p(z,y,x)) \land (\exists z p(z,y,x) \lor p(y,a,x)))$ (Bereinigung)

$\equiv \forall x \exists y ((\neg p(y,a,x) \lor \forall z_1 \neg p(z_1,y,x)) \land (\exists z_2\, p(z_2,y,x) \lor p(y,a,x)))$ (Pränex)

$\equiv \forall x \exists y \forall z_1 \exists z_2 ((\neg p(y,a,x) \lor \neg p(z_1,y,x)) \land (p(z_2,y,x) \lor p(y,a,x)))$

G3.

a) Negationsnormalform

$\neq$

$= \neg\forall x \exists y (p(y) \to (\neg q(a,x) \land \neg q(x,y)))$ (Elim $\to$)

$\equiv \neg\forall x \exists y (\neg p(y) \lor (\neg q(a,x) \land \neg q(x,y)))$ ($\neg\forall \equiv \exists$, DM, $\neg\exists = \forall$)

$\equiv \exists x \forall y \neg(\neg p(y) \lor \neg(q(a,x) \lor q(x,y)))$ (DM)

$\equiv \exists x \forall y (p(y) \land (q(a,x) \lor q(x,y)))$ (NNF)

b) Bereinigt.

$\neq$

$\forall x \exists y ((\exists z \exists x (q(a,x) \to r(z,w,y))) \leftrightarrow (q(a,z) \land \neg(\exists w \exists z\, r(x,w,z))))$

$\forall x_1 \exists y ((\exists z_1 \exists x_2 (q(a,x_2) \to r(z_1,w,y))) \leftrightarrow (q(a,z) \land \neg(\exists w_1 \exists z_2\, r(x_1,w_1,z_2))))$

c) Pränexnormalform

$\neq$

$(\forall w q(a,w)) \lor (\exists x \forall y (\neg r(a,x,y) \land \exists z \neg r(x,y,z)))$

$\forall w \exists x \forall y \exists z (q(a,w) \lor (\neg r(a,x,y) \land \neg r(x,y,z)))$

d) Skolemnormalform

$\neq$

$\exists u \forall u' \exists w \exists x \forall y \exists z (\neg r(f(u'),x,y) \land r(w,a,z) \land r(y,x,u))$

$u \mapsto sk_u$; $w \mapsto sk_w(u')$; $x \mapsto sk_x(u')$; $z \mapsto sk_z(u',y)$

$\exists u \forall u' \forall y (\neg r(f(u'), sk_x(u'), y) \land r(sk_w(u'), a, sk_z(u',y)) \land r(y, sk_x(u'), sk_u))$

$\ldots \neg r(f(u'), sk_x(u'), y) \wedge r(sk_w(u'), a, sk_z(u', y)) \wedge r(y, sk_x(u'), sk_u))$

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