~~... ...~~

H 9.

H 10.

1). a) $\{0\}$

b) $\emptyset$

2) $f: \mathbb{R} \to \mathbb{R}$, $f(x) = x - \cos x$

f - hat eine Nullstelle zwischen $(0, \frac{\pi}{2})$

$\Leftrightarrow$ ~~f(x) =~~ $0 \; \exists \; x_0 \in (0, \frac{\pi}{2})$ s. d. $f(x_0) =$

$$\Rightarrow \left| \begin{array}{l} x_0 - \cos x_0 = 0 \\ x_0 = \cos x_0 \\ x_0 \in \left(0, \frac{\pi}{2}\right) \end{array} \right| \Rightarrow$$

$f(x) = x - \cos x$ – ist ~~wachsend~~ streng wachsend. weil

$\cos x \in [-1, 1]$ (3)

In unser Intervall $\left(0, \frac{\pi}{2}\right) \Rightarrow \cos$ hat nur positive Werte

$$f\left(\frac{\pi}{6}\right) = \frac{\pi}{6} - \cos\frac{\pi}{6} = \frac{\pi}{6} - \frac{\sqrt{3}}{2} = \cancel{\frac{2\pi - 6\sqrt{3}}{12}}$$

$$= \frac{\pi - 3\sqrt{3}}{6} \Rightarrow \underset{(\simeq 3,14)}{\pi} - 3 \cdot 1,73 < 0 \quad (1)$$

$\sqrt{3} \simeq 1,73$

~~$f(0) = 0 - 1$~~

$$f\left(\frac{\pi}{4}\right) = \frac{\pi}{4} - \frac{\sqrt{2}}{2} = \frac{\pi - 2\sqrt{2}}{2} \Rightarrow \pi - 2\sqrt{2} > 0 \quad (2)$$

$\sqrt{2} \simeq 1,4.$

$(1), (2), (3) \Rightarrow \exists\, x_0 \in \left(0, \frac{\pi}{2}\right)$ a.d. $f(x_0) = 0$

3) $\lim_{x\to 2} \left|\frac{f(x)-f(2)}{x-2}\right| = f'(2) \in \mathbb{R}$

$\Rightarrow f$ - este continuă în $x_0 = 2$

$\Rightarrow \exists\, l_s(2) = l_d(2) = f(2)$

$l_s(2) = \lim_{x \nearrow 2} f(x) = \lim_{x\nearrow 2} x^2 + a = 4 + a$

$l_d(2) = \lim_{x \searrow 2} f(2) = \lim_{x \searrow 2} ax + b = 2a + b$

$f(2) = 4 + a$

$\Rightarrow 4 + a = 2a + b = 4 + a$

$\Rightarrow 4 + a = 2a + b$

~~$4 - b = 3a$~~ $a + b = 4$

$\lim_{x\nearrow 2} \frac{f(x) - f(2)}{x-2} = \lim_{x\nearrow 2} \frac{x^2 + a - (4+a)}{x-2} =$

$= \lim_{x\nearrow 2} \frac{x^2 + a - 4 - a}{x-2} = \lim_{x\nearrow 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{(x-2)(x+2)}{(x-2)}$

$= 4$

$\lim_{x\searrow 2} \frac{f(x) - f(2)}{x-2} = \lim_{x\searrow 2} \frac{ax + b - (4+a)}{x-2} = \lim_{x\searrow 2} \frac{a(x-1) + b - 4}{x-2}$

$\Rightarrow$ ~~dies~~ wir nehmen nicht dieses benutzen

Aber

$\cancel{f_d}$ $\lim\limits_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} = f_d'(x_0) = (ax+b)'$

$= a$

$\Rightarrow f_0' = f_d' \Rightarrow a = 4$

$\Rightarrow \begin{cases} a = 4 \\ a + b = 4 \end{cases} \Leftrightarrow b = 0$