Algebra HSG

Wednesday, November 25, 2020 9:16 PM

3.1.31. $\mathbb{R}_+^* = (0,\infty)$ - $\mathbb{R}$ spațiu vectorial în raport cu adunarea vectorială

$\boxplus : \mathbb{R}_+^* \times \mathbb{R}_+^* \to \mathbb{R}_+^*$, $x \boxplus y = xy$, $\forall x, y \in \mathbb{R}_+^*$

Asociativitate:

$x, y, z \in \mathbb{R}_+^*$

$x \boxplus (y \boxplus z) = (x \boxplus y) \boxplus z$

$x \boxplus (yz) = (xy) \boxplus z$

$xyz = xyz$

Comutativitate

$x, y \in \mathbb{R}_+^*$ $x \boxplus y = y \boxplus x$

$xy = yx$ (A)

Elem. neutru:

$\exists e \in \mathbb{R}_+^*$ a.î.

$x \boxplus e = x$, $\forall x \in \mathbb{R}_+^*$

$\Leftrightarrow xe = x$

$\Leftrightarrow e = 1 \in \mathbb{R}_+^*$

Elem. simetrizabil:

$x \boxplus x' = e$

$x x' = e$

$x x' = 1$

$x' = \frac{1}{x}$

$x' \in \mathbb{R}_+^*$, $\forall x \in \mathbb{R}_+^*$

$\Rightarrow (\mathbb{R}_+^*, \boxplus)$ - grup abelian

$\boxdot$ $\mathbb{R} \times \mathbb{R}_+^* \to \mathbb{R}_+^*$, $\alpha \boxdot x = x^\alpha$, $\forall x \in \mathbb{R}_+^*$, $\alpha \in \mathbb{R}$

$x, y \in \mathbb{R}_+^*$

$\alpha$ $\alpha$

$= x^{\alpha} \cdot y^{\alpha} = \underline{(\alpha \boxdot x) \oplus (\alpha \boxdot y)}, \forall \alpha \in \mathbb{R}$

$\boxed{II}$ $(\alpha + \beta) \boxdot x = x^{(\alpha+\beta)} = x^{\alpha} \cdot x^{\beta} =$

$= x^{\alpha} \boxplus x^{\beta} = (\alpha \boxdot x) \boxplus (\beta \boxdot x), \quad \forall \alpha, \beta \in \mathbb{R}$

$\boxed{III}$ $\alpha \boxdot (\beta \boxdot x) = \alpha \boxdot (x^{\beta}) = (x^{\beta})^{\alpha} = x^{\beta \cdot \alpha} =$

$x^{\alpha \cdot \beta} = (\alpha \cdot \beta) \boxdot x, \quad \forall \alpha, \beta \in \mathbb{R}$

$(IV)$ $1 \boxdot x = x^{1} = x$

$\Rightarrow (\mathbb{R}_{+}^{*}, \boxplus)$ ? ă un ... ură un

$\mathbb{R}$ - spațiu vect

<u>3.1.33</u>

• $A = \{[x_1, x_2, x_3] \in \mathbb{R}^3 \mid 2x_1 + x_2 - x_3 = 0\}$

i) $A \neq \emptyset$