#17.

a) $\operatorname{int} A = (-3, 0) \cup (5, 7)$

a_2) $\operatorname{int} A = (-\infty, 3) \cap \mathbb{Q}$

b_1) $\operatorname{int} B = \mathbb{Z} \times \mathbb{Z}$

b_2) $(1, 3) \times \mathbb{R}$

#18.

$f : \mathbb{R}^3 \to \mathbb{R}, \; f(x,y,z) = x^2 + 2x + \frac{y}{1+y^2} e^x \cdot z$

$$\frac{\partial f}{\partial x}(x,y,z) = \left(x^2 + 2x + \frac{y}{1+y^2} \cdot e^x \cdot z\right)'_x =$$

$$= 2x + 2 + \frac{y}{1+y^2} \cdot e^x \cdot z$$

$$\frac{\partial f}{\partial y}(x,y,z) = \left(x^2 + 2x + \frac{y}{1+y^2} \cdot e^x \cdot z\right)'_y =$$

$$= \left(\frac{y}{1+y^2}\right)' \cdot e^x \cdot z = \frac{(y)' \cdot (1+y^2) - y \cdot (1+y^2)'}{(1+y^2)^2} \cdot e^x \cdot z =$$

$$= \frac{(1+y^2) - y \cdot (2y)}{(1+y^2)^2} \cdot e^x \cdot z = \frac{1+y^2-2y^2}{(1+y^2)^2} = \frac{1-y^2}{(1+y^2)^2}$$

$$\frac{\partial f}{\partial z}(x,y,z) = \left(x^2 + 2x + \frac{y}{1+y^2} \cdot e^x \cdot z\right)'_z = \frac{y}{1+y^2} \cdot e^x$$

$$\nabla f(x,y,z) = \left(\frac{\partial f}{\partial x}(x,y,z), \frac{\partial f}{\partial y}(x,y,z), \frac{\partial f}{\partial z}(x,y,z)\right) =$$

$$= \left(2x + 2 + \frac{y}{1+y^2} \cdot e^x \cdot z, \; \frac{1-y^2}{(1+y^2)^2}, \; \frac{y}{1+y^2} \cdot e^x\right)$$

b) $u := \nabla f(1,1,1) = \left(2 + 2 + \frac{1}{2} \cdot e \cdot 1, \; 0, \; \frac{1}{2} \cdot e\right) =$

$= \left(4 + \frac{e}{2}, 0, \frac{e}{2}\right)$

$v := \nabla f(2,0,2) = (4+2, \; 1, \; 0) = (6, 1, 0)$

$$\|u - v\| = \sqrt{\langle u-v, u-v \rangle} = \sqrt{\langle u, u-v \rangle - \langle v, u-v \rangle} =$$

$$= \sqrt{\langle u-v, u \rangle - \langle u-v, v \rangle} = \sqrt{\langle u,u \rangle - \langle v,u \rangle - (\langle u,v \rangle - \langle v,v \rangle)}$$

$$= \sqrt{\langle u,u \rangle - \langle v,u \rangle - \langle v,u \rangle + \langle v,v \rangle} = \sqrt{\langle u,u \rangle - 2\langle v,u \rangle + \langle v,v \rangle}$$

$$= \sqrt{\left(4+\frac{e}{2}\right)^2 + \left(\frac{e}{2}\right)^2 - 2(24 + 3e) + 36 + 1} =$$

$$= \sqrt{16 + 4e + \left(\frac{e}{2}\right)^2 + \left(\frac{e}{2}\right)^2 - 48 + 6e + 36 + 1}$$

$$= \sqrt{10e + 2\left(\frac{e}{2}\right)^2 + 1} = \sqrt{10e + \frac{e^2}{2} + 1}$$

$$\langle u, v \rangle = 24 + 31$$

Hilg.

$f:\mathbb{R}^2 \to \mathbb{R}$, $f(x,y) = \begin{cases} \dfrac{x\sin y - y\sin x}{x^2+y^2}, & (x,y) \neq 0_2 \\ 0, & (x,y) = 0_2 \end{cases}$

$$\frac{\partial f}{\partial x}(x,y) = \left(\frac{x\sin y - y\sin x}{x^2+y^2}\right)'_x = \frac{(x\sin y - y\sin x)'_x \cdot (x^2+y^2) - (x\sin y - y\sin x)\cdot (x^2+y^2)'_x}{(x^2+y^2)^2} =$$

$$= \frac{(x\sin y - y\cos x)\cdot(x^2+y^2) - (x\sin y - y\sin x y)\cdot 2x}{(x^2+y^2)^2}$$

$$\frac{\partial f}{\partial y}(x,y) = \left(\frac{x\sin y - y\sin x}{x^2+y^2}\right)'_y = \frac{(x\sin y - y\sin x)'_y \cdot (x^2+y^2) - (x\sin y - y\sin x)\cdot(x^2+y^2)'}{(x^2+y^2)^2}$$

$$= \frac{(x\cos y - \sin x)\cdot(x^2+y^2) - 2y(x\sin y - y\sin x)}{(x^2+y^2)^2}$$

Wir untersuchen die partielle Differenzierbarkeit in 0_2:

- nach x: $\lim\limits_{x \to 0} \dfrac{f(x,0) - f(0,0)}{x - 0} = \dfrac{0 - 0}{x} = \lim\limits_{x\to 0} \dfrac{0-0}{x} = 0$

$$\Rightarrow \frac{\partial f}{\partial x}(0,0) = 0$$

- nach y: $\lim\limits_{y \to 0} \dfrac{f(0,y) - f(0,0)}{y - 0} = \lim\limits_{y\to 0} \dfrac{0-0}{y} = 0$

$$\Rightarrow \frac{\partial f}{\partial y}(x,y) = 0$$

$$\frac{\partial f}{\partial x}(x,y) = \begin{cases} \dfrac{(x\sin y - y\cos x)\cdot(x^2+y^2) - (x\sin y - y\sin x y)\cdot 2x}{(x^2+y^2)^2}, & (x,y) \neq 0_2 \\ 0, & (x,y) = 0_2 \end{cases}$$

$$\frac{\partial f}{\partial y}(x,y) = \begin{cases} \dfrac{(x\cos y - \sin x)\cdot(x^2+y^2) - 2y(x\sin y - y\sin x)}{(x^2+y^2)^2}, & (x,y) \neq 0_2 \\ 0, & (x,y) = 0_2 \end{cases}$$