$\forall x \in \mathbb{R}$, int. $\uparrow$

Hausaufgabe:

H7 a) $\sum_{n\geq 0} \frac{(-2)^n}{3^{n+1}} = \sum_{n\geq 0} \frac{(-2)^n}{3^n \cdot 3} = \sum_{n\geq 0} \left(\frac{-2}{3}\right)^n \cdot \frac{1}{3} = \frac{1}{3} \cdot \frac{(-\frac{2}{3})^0}{1+\frac{2}{3}}$

$= \frac{1}{3} \cdot \frac{1}{\frac{5}{3}} = \frac{1}{3} \cdot \frac{3}{5} = \frac{1}{5}$

b) $\sum_{n\geq 1} \frac{1}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{\frac{1}{2}}} = \infty$

c) $\sum_{n\geq 2} \frac{3n^2+3n+1}{n^3(n+1)^3} = \sum_{n=2} x_n = l - a_2 = 0 - \frac{1}{8} = -\frac{1}{8}$

$n^3 + 1 + 3n(n+1) =$
$= n^3 + 1 + 3n^2 + 3n$

$x_n = \frac{3n^2+3n+1}{n^3(n+1)^3} = \frac{(n+1)^3 - n^3}{n^3(n+1)^3} = \frac{\cancel{(n+1)^3}}{n^3\cancel{(n+1)^3}} - \frac{\cancel{n^3}}{\cancel{n^3}(n+1)^3}$

$= \frac{1}{n^3} - \frac{1}{(n+1)^3} = a_n - a_{n+1}$

d) $\sum_{n\geq 1} \frac{1}{(5n+1)(5n+6)} = \sum_{n\geq 1} \frac{\cancel{(5n+1)(5n+6)}}{}$

$= \sum_{n\geq 1} \frac{(5n+6)-(5n+1)}{(5n+1)(5n+6)\cdot 5}$

$x_n = \frac{(5n+6)-(5n+1)}{(5n+1)(5n+6)\cdot 5} = \frac{\cancel{5n+6}}{(5n+1)\cancel{(5n+6)}\cdot 5} - \frac{\cancel{5n+1}}{\cancel{(5n+1)}(5n+6)\cdot 5} =$

$= \frac{1}{(5n+1)\cdot 5} - \frac{1}{(5n+6)\cdot 5} = a_n - a_{n+1}$

$\Rightarrow \sum_{n\geq 1} \frac{1}{(5n+1)(5n+6)} = l - a_1 = 0 - \frac{1}{6\cdot 5} = -\frac{1}{30}$

e) $\sum_{n\geq 0} \left(\frac{(-3)^{n+1}}{5^{n+2}} - \frac{5}{(n+2)!}\right)$

$$= \sum_{n\ge 0} \frac{(-3)^{n+1}}{5^{n+1}\cdot 5} - \sum_{n\ge 0} \frac{5}{(n+1)!} = \underbrace{\sum \left(\frac{-3}{5}\right)^{n+1}\cdot\frac{1}{5}} - 5\sum_{n\ge 0}\frac{1}{(n+1)!}$$

$$\frac{1}{5}\sum_{n\ge 0}\left(-\frac{3}{5}\right)^{n+1} = \frac{1}{5}\cdot\sum_{m=1}^{\infty}\left(-\frac{3}{5}\right)^{m} = \frac{1}{5}\cdot\frac{\left(-\frac{3}{5}\right)^{1}}{1+\frac{3}{5}} = \frac{1}{5}\cdot\frac{-\frac{3}{5}}{\frac{8}{5}} =$$

ind. $m = n+1$

$$= \frac{1}{5}\cdot\left(-\frac{3}{\cancel{5}}\cdot\frac{\cancel{5}}{8}\right) = \frac{1}{5}\left(-\frac{3}{8}\right) = -\frac{3}{40}$$

mit $n+1=n$

$$5\cdot\sum_{n\ge 0}\frac{1}{(n+1)!} \underset{n+1=m}{=} 5\cdot\sum_{m=1}^{\infty}\frac{1}{m!} = 5\left(\sum_{m=0}^{\infty}\frac{1}{m!} - \frac{1}{0!}\right) = 5(e-1) =$$

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$\Rightarrow$ Ergebnis: $-\frac{3}{40} - 5(e-1)$