H12 - RAZVAN POSTESCU

Wednesday, January 6, 2021 6:45 PM

#22.

a) $f:[0,\infty)\to\mathbb{R},\ f(x)=\frac{1}{(x+3)(x+4)}$

$$f(x)=\frac{1}{(x+3)(x+4)}=\frac{(x+4)-(x+3)}{(x+3)(x+4)}=\frac{(x+4)}{(x+3)\cancel{(x+4)}}-\frac{\cancel{(x+3)}}{\cancel{(x+3)}(x+4)}=$$

$$=\frac{1}{(x+3)}-\frac{1}{(x+4)}$$

$$\int f(x)\,dx=\int\left(\frac{1}{x+3}-\frac{1}{x+4}\right)dx=\int\frac{1}{x+3}dx-\int\frac{1}{x+4}dx=$$

$$=\int\frac{(x+3)'}{(x+3)}dx-\int\frac{(x+4)'}{(x+4)}dx=\ln|x+3|-\ln|x+4|=$$

$$=\ln\frac{x+3}{x+4}+C\Rightarrow F:[0,\infty)\to\mathbb{R},\ F(x)=\ln\frac{x+3}{x+4}$$

Aus $\boxed{Th2}$ ü.n. $\Rightarrow f$ - ist uneigentlich Integrierbar $\Leftrightarrow$

$\lim_{x\to b}F(x)$ - endlich, wo $b=\infty$ $\Rightarrow$

$$\Rightarrow\lim_{x\to\infty}F(x)=\lim_{x\to\infty}\ln\frac{x+3}{x+4}=\ln\lim_{x\to\infty}\frac{x+3}{x+4}=\ln\lim_{x\to\infty}\frac{\cancel{x}(1+\frac{3}{x})}{\cancel{x}(1+\frac{4}{x})}=$$

$=\ln 1=0\Rightarrow f$ - uneigentlich Integrierbar und

$$I=\lim_{x\to\infty}F(x)-F(0)=0-\ln\frac{3}{4}=-\ln\frac{3}{4}=\ln\frac{4}{3}$$

b) $f:(0,3]\to\mathbb{R},\ f(x)=\frac{1}{\sqrt[3]{x}+x}$

$$\int f(x)dx=\int\frac{1}{\sqrt[3]{x}+x}dx=\int\frac{1}{\sqrt[3]{x}\,\underbrace{(1+x^{\frac{2}{3}})}_{u}}dx$$

$u=x^{\frac{2}{3}}+1$

$du=\frac{2}{3}x^{-\frac{1}{3}}\,dx$

$du=\frac{2}{3\sqrt[3]{x}}dx \quad \Big|:\frac{2}{3\sqrt[3]{x}}$

$\frac{du}{\frac{2}{3\sqrt[3]{x}}}=dx$

$\frac{3\sqrt[3]{x}}{2}du=dx$

$$=\int\frac{1}{\cancel{\sqrt[3]{x}}\cdot u}\cdot\frac{3\cancel{\sqrt[3]{x}}}{2}du$$

$$=\frac{3}{2}\int\frac{1}{u}du$$

$$=\frac{3}{2}\ln(u)$$

$$=\frac{3}{2}\ln(x^{\frac{2}{3}}+1)+C$$

$F:(0,3]\to\mathbb{R},\ F(x)=\frac{3}{2}\ln(x^{\frac{2}{3}}+1)$

$\boxed{Th 3}$ 12 üs.

f- ist auf $(0, 3]$ uneigentlich integrierbar $\Leftrightarrow$

F hat eine endliche Grenzwert in 0:

$$\lim_{x \to 0} F(x) = \lim_{x \to 0} F(x) = \frac{3}{2} \ln(1) = \frac{3}{2} \cdot 0 = 0$$

$\Rightarrow$ f- uneigentlich integrierbar $\Rightarrow$

$$\Rightarrow I = F(b) - \lim_{x \to a} F(x) = \frac{3}{2} \ln(3^{\frac{2}{3}} + 1) - 0$$

$$= \frac{3}{2} \ln(\sqrt[3]{9} + 1)$$

$$(x^a)' = a \cdot x^{a-1}$$

c) $f : [2, \infty) \to \mathbb{R}$, $f(x) = \frac{1}{x(\ln x)^{\frac{3}{2}}}$

$$\int f(x)\, dx = \int \frac{1}{x(\ln x)^{\frac{3}{2}}} dx$$

$$\frac{1}{a+1} \int (a+1) x^a = \frac{1}{a+1} \int (x^a)'$$

$$\Rightarrow \frac{x^a}{a}$$

$\ln x = u$

$\frac{1}{x} dx = du$

$dx = x\, du$

$$= \int \frac{1}{u^{\frac{3}{2}}} du$$

$$= \int u^{-\frac{3}{2}} du$$

$$= \frac{u^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} + C = \frac{u^{-\frac{1}{2}}}{-\frac{1}{2}} = -2u^{-\frac{1}{2}} =$$

$$= \frac{-2}{\sqrt{u}} + C$$

$$\Rightarrow \int f(x)\, dx = \frac{-2}{\sqrt{\ln x}} + C$$

$$\Rightarrow F : [2, \infty) \to \mathbb{R}, \quad F(x) = -\frac{2}{\sqrt{\ln x}}$$

$\boxed{Th 2}$ 12 üs

f- ist auf $[2, \infty)$ - uneigentlich integrierbar $\Leftrightarrow$

F - hat einen endlichen Grenzwert in ∞

$$\Rightarrow \lim_{x \to \infty} F(x) = -\lim_{x \to \infty} \frac{2}{\sqrt{\ln \infty}} = -\frac{2}{\infty} = 0$$

$\Rightarrow$ f - uneigentlich integrierbar $\Rightarrow$

$$\Rightarrow I = \lim_{x \to \infty} F(x) - F(2) = 0 + \frac{2}{\sqrt{\ln 2}} = \frac{2}{\sqrt{\ln 2}}$$

$$\Rightarrow I = \lim_{x\to\infty} F(x) - F(2) = 0 + \frac{2}{\sqrt{\ln 2}} = \frac{2}{\sqrt{\ln 2}}$$

d) $f:[0,\infty) \to \mathbb{R}$, $f(x) = \dfrac{x}{x^4+2x^2+2}$

$$\int f(x)\,dx = \int \frac{x}{x^4+2x^2+2}\,dx$$

$u = x^2$
$du = 2x\,dx$
$dv = \frac{1}{2x}du$

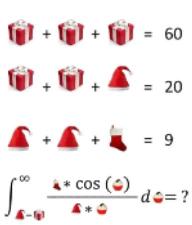
$$= \frac{1}{2}\int \frac{1}{u^2+2u+2}\,du$$
$$= \frac{1}{2}\int \frac{1}{(u+1)^2+1}\,du$$
$$= \frac{1}{2}\operatorname{arctg}(u+1) + C$$

$\Rightarrow F:[0,\infty) \to \mathbb{R}$, $F(x) = \frac{1}{2}\operatorname{arctg}(x^2+1)$

$\boxed{\text{Th 7}}$ 12 Üb 5 $\Rightarrow$ .... $\to \lim_{x\to\infty} F(x) = \lim_{x\to\infty} \frac{1}{2}\operatorname{arctg}(x^2+1) = \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

$= \frac{1}{2}\cdot\frac{\pi}{2} = \frac{\pi}{4} \to f$. uneigentlich Integrierbar $\Rightarrow$

$$\Rightarrow I = \lim_{x\to\infty} F(x) - F(0) = \frac{\pi}{4} - \frac{\pi}{4} = 0$$



$\Rightarrow$ 🎁 $= 20$

$\Rightarrow$ 🎅 $= -20$

$\Rightarrow$ 🧦 $= 49$

$$\Rightarrow \int_{-20-20}^{\infty} \frac{49\cdot\cos(x)}{-20\cdot x}\,dx = \frac{49}{-20}\int_{-40}^{\infty}\frac{\cos x}{x}\,dx =$$

Cosine Integral? Ich weiß nicht was das ist!