Hausaufgabe 8 - Răzvan Postesu

H12.

a) $t \in \mathbb{R}$ so dass $\langle (4t,-4,1),(t,t,3)\rangle = 2$

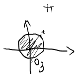
$\langle (4t,-4,1),(t,t,3)\rangle = 4t \cdot t - 4t + 3 = 4t^2 - 4t + 3$

$\Rightarrow 4t^2 - 4t + 3 = 2 \quad | -2$

$\Leftrightarrow 4t^2 - 4t + 1 = 0$

$\Delta = 16 - 16 = 0 \quad t_{1,2} = \frac{+4 \pm 0}{8} \Rightarrow t = \frac{4}{8} = \frac{1}{2} \in \mathbb{R}$

b) $t \in \mathbb{R}$, $x := (t-1,-3,-1) \in \mathbb{R}^3$. Alle $t \in \mathbb{R}^*$ so dass $x \notin B(0_3,|t|)$



$\|x - 0_3\| = \sqrt{\langle x-0_3, x-0_3\rangle} = \sqrt{\langle x, x-0_3\rangle - \langle 0_3, x-0_3\rangle}$

$= \sqrt{\langle x-0_3, x\rangle - \langle x-0_3, 0_3\rangle} = \sqrt{\langle x,x\rangle - \langle 0_3,x\rangle - (\langle x,0_3\rangle - \langle 0_3,0_3\rangle)}$

$= \sqrt{\|x\|^2 - 2\langle 0_3,x\rangle + \underbrace{\|0_3\|}_{0}} = \sqrt{\|x\|^2}$

$= \|x\| = \sqrt{(t-1)^2 + (-3)^2 + (-1)^2} = \sqrt{t^2 - 2t + 1 + 9 + 1} =$

$= \sqrt{t^2 - 2t + 11}$

$\Rightarrow \|x - 0_3\| \geq |t|$

$\Leftrightarrow \sqrt{t^2 - 2t + 11} \geq |t|$

$\sqrt{t^2 - 2t + 11} \geq |t| \quad | (\,)^2$

$\cancel{t^2} - 2t + 11 \geq \cancel{t^2}$

$-2t \geq -11 \quad | \cdot (-1)$

$2t \leq 11$

$t \leq \frac{11}{2}$

$t \in (-\infty, \frac{11}{2}]$

H13.

$f : (0,\infty) \to \mathbb{R}$, $f(x) = \frac{1}{x}$, $n \in \mathbb{N}$

a) $f^{(n)}(x)$, $x > 0$

$f'(x) = (x^{-1})' = -1 \cdot x^{-2}$

$f''(x) = (-1 \cdot x^{-2})' = 2x^{-3}$

$f'''(x) = (2x^{-3})' = -6x^{-4}$

$f^{(4)}(x) = (-6x^{-4})' = 24x^{-5}$

$1 \cdot 2 \cdot 3 = 6$

$1 \cdot 2 \cdot 3 \cdot 4 = 24$

$1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120$

$D(5)$

$^{IV}(x) = (-6x^{-4})' = 24\,x^{-5}$

$f^{(5)}(x) = (24\,x^{-5})' = -120\cdot x^{-6}$

Presupunem că $f^{(k)}(X) = (-1)^{k+1}\cdot k!\cdot X^{-(k+1)}$

Arătăm că $f^{(k+1)}(X) = \left(f^{(k)}(X)\right)'$

$\Leftrightarrow f^{(k+1)}(X) = \left(\underbrace{(-1)^{k+1}\cdot k!}_{\text{constant}}\cdot X^{-(k+1)}\right)'$

$\Leftrightarrow f^{(k+1)}(X) = 0 + 0 + (-1)^{k+1}\cdot k!\cdot\left(X^{-(k+1)}\right)'$

$\Leftrightarrow f^{(k+1)}(X) = (-1)^{k+1}\cdot k!\cdot(-(k+1))\cdot X^{-(k+1)-1}$

$\Leftrightarrow f^{(k+1)}(X) = (-1)^{k+1}\cdot k!\cdot(-1)(k+1)\cdot X^{-(k+2)}$

$\Leftrightarrow f^{(k+1)}(X) = (-1)^{k+2}\cdot(k+1)!\cdot X^{-(k+2)}$

$\Rightarrow f^{(n)}(X) = (-1)^{n+1}\cdot n!\cdot X^{-(n+1)}$

b) $T_n(x,1) = \sum_{k=0}^{n} \frac{f^{(k)}(1)}{k!}\cdot(x-1)^k = \frac{f^{0}(1)}{0!}\cdot(x-1)^0 + \frac{f'(1)}{1!}\cdot(x-1)^1 + \ldots +$

$f^{(k)}(1) = (-1)^{k+1}\cdot k!\cdot 1 \quad + \quad f^{(n)}(1)$

$$f^{(k)}(1) = (-1)^{k+1} \cdot k! \cdot 1$$

$$f^{(k)}(1) = (-1)^{k+1} \cdot k!$$

$$f^{(2k)}(1) = (-1) \cdot (k)!$$

$$f^{(2k-1)} = k!$$

$$+ \frac{f^{(n)}(1)}{n!} \cdot (x-1)^n \ldots$$

$$R_n(x,1) = \frac{f^{(n+1)}(c)}{(n+1)!}(x-1)^{n+1} = \frac{(-1)^{n+1} \cdot \cancel{n!} \cdot c^{-(n+1)}}{\cancel{(n+1)!}\,(n+1)} \cdot (x-1)^{n+1}$$

$$= \frac{(-1)^{n+1} \cdot c^{-(n+1)}}{(n+1)} \cdot (x-1)^{n+1}$$