Review

- 连续点与间断点

  初等函数的连续性

- 当$x \to 0$时, $\arcsin x \sim x$, $\arctan x \sim x$.

- 有界闭区间上连续函数的性质

  零点定理　　介值定理　　有界性定理

  最大最小值定理　一致连续性定理

- f在I上非一致连续$\Leftrightarrow$

$$\exists \varepsilon_0 > 0, \exists x_n, y_n \in I, \lim_{n\to\infty}(x_n - y_n) = 0, s.t. \left| f(x_n) - f(y_n) \right| \ge \varepsilon_0.$$

§ 1. 导数

Def.（导数, 左、右导数）

$$(1)\ f'(x_0) \triangleq \lim_{\Delta x \to 0} \frac{f(x_0+\Delta x)-f(x_0)}{\Delta x} = \lim_{x \to x_0} \frac{f(x)-f(x_0)}{x-x_0};$$

$$(2)\ f'_-(x_0) \triangleq \lim_{\Delta x \to 0^-} \frac{f(x_0+\Delta x)-f(x_0)}{\Delta x} = \lim_{x \to x_0^-} \frac{f(x)-f(x_0)}{x-x_0};$$

$$(3)\ f'_+(x_0) \triangleq \lim_{\Delta x \to 0^+} \frac{f(x_0+\Delta x)-f(x_0)}{\Delta x} = \lim_{x \to x_0^+} \frac{f(x)-f(x_0)}{x-x_0}.$$

Question. 导数的几何意义? 切线的斜率.

Question. 可导的几何意义? 光滑性

Remark. 导函数 $f'(x)$.

Ex. (1) $c' = 0$, (2) $(\sin x)' = \cos x$, (3) $(\cos x)' = -\sin x$,

(4) $(a^x)' = a^x \ln a$, (5) $(\log_a x)' = \dfrac{1}{x\ln a}$, (6) $(x^\alpha)' = \alpha x^{\alpha-1}$.

Proof. (1) $f(x) \equiv c$, 则

$$f'(x) = \lim_{\Delta x\to 0}\frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x\to 0}\frac{c-c}{\Delta x} = 0.$$

$$(2)(\sin x)' = \lim_{h\to 0}\frac{\sin(x+h) - \sin x}{h} = \lim_{h\to 0}\frac{2\sin\frac{h}{2}\cos(x+\frac{h}{2})}{h}$$

$$= \lim_{h\to 0}\frac{2\sin\frac{h}{2}}{h}\cdot\lim_{h\to 0}\cos(x+\frac{h}{2}) = 1\cdot\cos x = \cos x.$$

$$(3)(\cos x)' = \lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h}$$

$$= \lim_{h\to 0}\frac{-2\sin\frac{h}{2}\sin(x+\frac{h}{2})}{h}$$

$$= \lim_{h\to 0}\frac{-2\sin\frac{h}{2}}{h}\cdot\lim_{h\to 0}\sin(x+\frac{h}{2}) = -\sin x.$$

$$(4)(a^x)' = \lim_{h\to 0}\frac{a^{x+h}-a^x}{h} = a^x\lim_{h\to 0}\frac{a^h-1}{h} = a^x\ln a.$$

特别地, $(e^x)' = e^x$.

$$(5)(\log_a x)' = \lim_{h\to 0}\frac{\log_a(x+h)-\log_a x}{h}$$

$$= \frac{1}{\ln a}\lim_{h\to 0}\frac{\ln(x+h)-\ln x}{h}$$

$$= \frac{1}{x\ln a}\lim_{h\to 0}\frac{\ln(1+h/x)}{h/x} = \frac{1}{x\ln a}.$$

特别地, $(\ln x)' = \dfrac{1}{x}.$

$$(6)(x^{\alpha})' = \lim_{h\to 0}\frac{(x+h)^{\alpha}-x^{\alpha}}{h}$$

$$x\neq 0\text{时},(x^{\alpha})' = x^{\alpha}\lim_{h\to 0}\frac{(1+h/x)^{\alpha}-1}{h} = x^{\alpha}\lim_{h\to 0}\frac{\alpha h/x}{h} = \alpha x^{\alpha-1}.$$

$$x=0\text{时},\ f'(0) = \lim_{h\to 0}\frac{h^{\alpha}-0^{\alpha}}{h} = \lim_{h\to 0}h^{\alpha-1}$$

$$=\begin{cases} 1 & \alpha = 1 \\ \text{不存在} & \alpha < 1 \\ 0 & \alpha > 1\text{且}x^{\alpha-1}\text{在}(-\delta,\delta)\text{有定义} \\ \text{不存在} & \alpha > 1\text{且}x^{\alpha-1}\text{在}(-\delta,0)\text{无定义} \end{cases}$$

综上， $(x^{\alpha})' = \alpha x^{\alpha-1}$ （$x^{\alpha-1}$有意义时成立）.

Thm. $f'(x_0)$存在 $\Leftrightarrow f'_-(x_0), f'_+(x_0)$均存在且相等.

f在x_0可导时, $f'(x_0) = f'_-(x_0) = f'_+(x_0)$.

Ex. $f(x) = \begin{cases} x+1, & x \le 0, \\ e^x, & x > 0. \end{cases}$ 求$f'(0)$.

$$f'_-(0) = \lim_{x\to 0-} \frac{f(x)-f(0)}{x} = \lim_{x\to 0-} \frac{x+1-1}{x} = 1.$$

$$f'_+(0) = \lim_{x\to 0+} \frac{f(x)-f(0)}{x} = \lim_{x\to 0+} \frac{e^x-1}{x} = 1 = f'_-(0).$$

故$f'(0) = 1$.□

Thm. f在x_0可导 $\Rightarrow$ f在x_0连续

Proof. f在x_0可导, 则

$$\lim_{x\to x_0}\left(f(x)-f(x_0)\right)=\lim_{x\to x_0}\frac{f(x)-f(x_0)}{x-x_0}\cdot\lim_{x\to x_0}(x-x_0)$$

$$=f'(x_0)\cdot 0=0.$$

$$\lim_{x\to x_0}f(x)=\lim_{x\to x_0}\left(f(x)-f(x_0)\right)+\lim_{x\to x_0}f(x_0)$$

$$=0+f(x_0)=f(x_0),$$

故 f在x_0连续.□

Ex. $f(x)=x^2D(x)$的可导性质？$D(x)$为Dirichlet函数.

解：$f'(0)=\lim\limits_{x\to 0}\dfrac{x^2D(x)-0}{x}=0.$

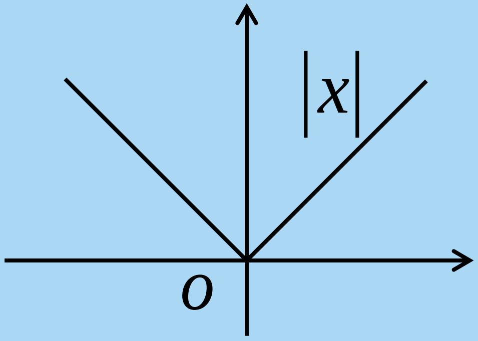
$f(x)$在任意$x_0\neq 0$处不连续, 因而不可导.□

Ex. $f(x)=|x|$在$x_0=0$是否可导？

连续$\nRightarrow$可导

$$\lim_{x\to 0^\pm}\frac{|x|-0}{x}=\pm 1,$$

$f(x)=|x|$在$x_0=0$不可导.



Remark. 利用级数可以构造处处连续处处不可导的例子.

Question. 导数的物理意义?

t	$f(t)$	$f'(t)$
时间	位移	速度
时间	速度	加速度

Def. 记$\Delta f(x_0) = f(x_0 + \Delta x) - f(x_0)$,若存在常数$\alpha$, s.t.

$$\Delta f(x_0) = \alpha \Delta x + o(\Delta x) \quad (\Delta x \to 0),$$

则称f在x_0可微,并称$\mathrm{d}f(x_0) \triangleq \alpha \Delta x \triangleq \alpha\,\mathrm{d}x$为$f$在点$x_0$的微分.

Thm. f在x_0可微 $\Leftrightarrow$ f在点x_0可导.

Proof. 设f在x_0可微, 则$\exists \alpha \in \mathbb{R}, s.t.$

$$\Delta f(x_0) = \alpha \Delta x + o(\Delta x) \quad (\Delta x \to 0).$$

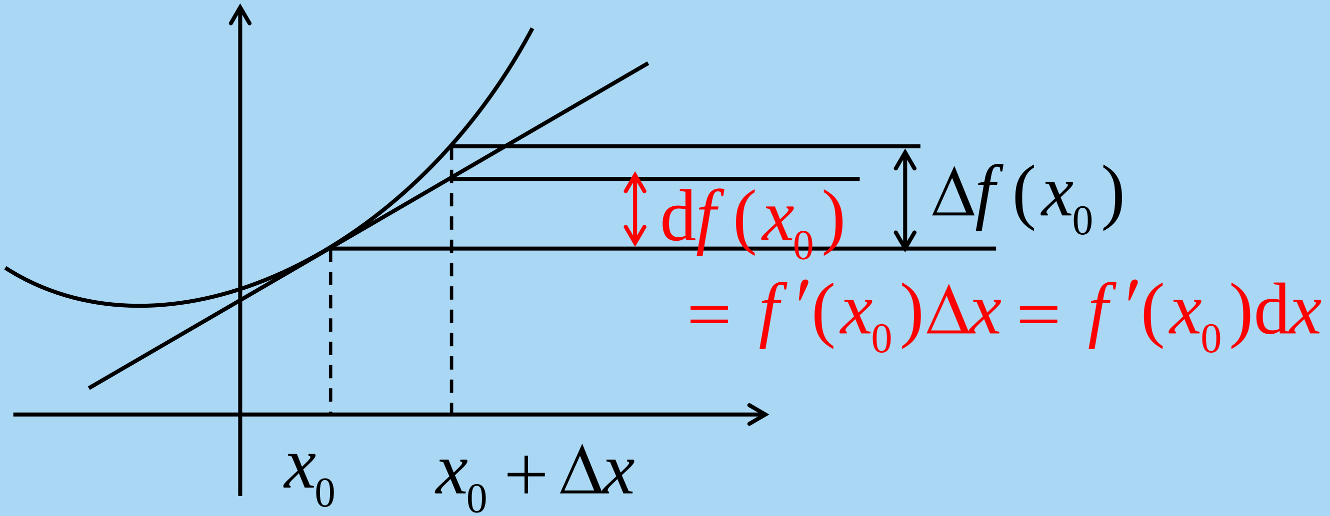
因此 $$f'(x_0) = \lim_{\Delta x \to 0} \frac{\Delta f(x_0)}{\Delta x} = \alpha + \lim_{\Delta x \to 0} \frac{o(\Delta x)}{\Delta x} = \alpha.$$

设f在x_0可导. $\rho(x) \triangleq \dfrac{f(x) - f(x_0)}{x - x_0} - f'(x_0)$, 则 $\lim\limits_{x \to x_0} \rho(x) = 0$,

$$f(x) - f(x_0) = f'(x_0)(x - x_0) + \rho(x)(x - x_0)$$

$$= f'(x_0)(x - x_0) + o(x - x_0) \quad (x \to x_0).$$

故f在x_0可微.□



Remark. $y = f(x)$在x_0可微,

$$\Delta f(x_0) = \mathrm{d}f(x_0) + o(\Delta x) \quad (\Delta x \to 0),$$

$$\mathrm{d}y = \mathrm{d}f(x_0) = \alpha\Delta x = \alpha\mathrm{d}x$$

则$f'(x_0) = \alpha = \dfrac{\mathrm{d}f}{\mathrm{d}x}(x_0) = \dfrac{\mathrm{d}y}{\mathrm{d}x}(x_0)$.

Remark. f在x_0可微, 则

$$f(x_0+\Delta x)\approx f(x_0)+f'(x_0)\Delta x.$$

Question. 可导与可微等价, 为什么需要给两个定义？

可微的概念是“以直代曲”, 便于推广到多元函数.

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§ 2. 求导法则

$$\mathrm{d}(f+g)=\mathrm{d}f+\mathrm{d}g$$

$$\mathrm{d}(cf)=c\,\mathrm{d}f$$

$$\mathrm{d}(fg)=g\,\mathrm{d}f+f\,\mathrm{d}g$$

$$\mathrm{d}\left(\frac{f}{g}\right)=\frac{g\,\mathrm{d}f-f\,\mathrm{d}g}{g^2}$$

Thm. f,g在x_0可导, $c\in\mathbb{R}$,则

(1) $(f+g)'(x_0)=f'(x_0)+g'(x_0)$;

(2) $(cf)'(x_0)=cf'(x_0)$;

(3) $(fg)'(x_0)=f'(x_0)g(x_0)+f(x_0)g'(x_0)$;

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(4) $g(x_0)\neq 0$时, $\left(\dfrac{f}{g}\right)'(x_0)=\dfrac{f'(x_0)g(x_0)-f(x_0)g'(x_0)}{g^2(x_0)}$.

特别地, $g(x_0)\neq 0$时, $\left(\dfrac{1}{g}\right)'(x_0)=\dfrac{-g'(x_0)}{g^2(x_0)}$.

Proof .(3)

$$(fg)'(x_0)=\lim_{h\to 0}\frac{f(x_0+h)g(x_0+h)-f(x_0)g(x_0)}{h}$$

$$=\lim_{h\to 0}\frac{f(x_0+h)g(x_0+h)-f(x_0+h)g(x_0)}{h}$$

$$+\lim_{h\to 0}\frac{f(x_0+h)g(x_0)-f(x_0)g(x_0)}{h}$$

$$=\lim_{h\to 0}f(x_0+h)\cdot\lim_{h\to 0}\frac{g(x_0+h)-g(x_0)}{h}$$

$$+g(x_0)\cdot\lim_{h\to 0}\frac{f(x_0+h)-f(x_0)}{h}$$

$$=f(x_0)g'(x_0)+f'(x_0)g(x_0).$$ （可导 $\Rightarrow$ 连续）

$$(4)\left(\frac{f}{g}\right)'(x_0)=\lim_{h\to 0}\left(\frac{f(x_0+h)}{g(x_0+h)}-\frac{f(x_0)}{g(x_0)}\right)\Big/h$$

$$=\lim_{h\to 0}\frac{f(x_0+h)g(x_0)-f(x_0)g(x_0+h)}{h}\cdot\frac{1}{g(x_0)g(x_0+h)}$$

$$=\frac{1}{g^2(x_0)}\lim_{h\to 0}\frac{f(x_0+h)g(x_0)-f(x_0)g(x_0+h)}{h}$$

$$=\frac{1}{g^2(x_0)}\left\{\lim_{h\to 0}\frac{f(x_0+h)g(x_0)-f(x_0)g(x_0)}{h}\right.$$

$$\left.+\lim_{h\to 0}\frac{f(x_0)g(x_0)-f(x_0)g(x_0+h)}{h}\right\}$$

$$=\frac{f'(x_0)g(x_0)-f(x_0)g'(x_0)}{g^2(x_0)}.\square$$

Ex. $(\tan x)' = \sec^2 x,$ $\qquad (\cot x)' = -\csc^2 x,$

$(\sec x)' = \sec x \tan x,$ $\qquad (\csc x)' = -\csc x \cot x.$

Proof. $(\tan x)' = \left(\dfrac{\sin x}{\cos x}\right)' = \dfrac{(\sin x)'\cos x - \sin x(\cos x)'}{\cos^2 x}$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x};$$

$$(\sec x)' = \left(\frac{1}{\cos x}\right)' = \frac{-(\cos x)'}{\cos^2 x} = \frac{\sin x}{\cos^2 x} = \sec x \tan x.$$

…… □

Thm.(复合函数求导的链式法则)$\varphi(x)$在x_0可导, $f(u)$在 $u_0=\varphi(x_0)$可导, 则$h(x)=f(\varphi(x))$在x_0可导, 且

$$h'(x_0)=f'(\varphi(x_0))\cdot\varphi'(x_0).$$

即$\mathrm{d}f(\varphi(x))=f'(\varphi(x))\mathrm{d}\varphi(x)=f'(\varphi(x))\cdot\varphi'(x)\mathrm{d}x.$

Proof.

令$g(u)=\begin{cases}\dfrac{f(u)-f(u_0)}{u-u_0}, & u\neq u_0,\\ f'(u_0), & u=u_0.\end{cases}$ 则$\lim\limits_{u\to u_0}g(u)=f'(u_0)$,

$$h'(x_0)=\lim_{x\to x_0}\frac{f(\varphi(x))-f(\varphi(x_0))}{x-x_0}=\lim_{x\to x_0}g(\varphi(x))\cdot\frac{\varphi(x)-\varphi(x_0)}{x-x_0}$$

$$=f'(\varphi(x_0))\cdot\varphi'(x_0).\square$$

Remark. $u=\varphi(x)$在x可导, $y=f(u)$在$u=\varphi(x)$可导, 则

$y=f(\varphi(x))$在x可导, 且

$$y'(x)=\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}u}\cdot\frac{\mathrm{d}u}{\mathrm{d}x}.$$

Remark.(一阶微分形式的不变性) $u=\varphi(x)$在x_0可微, $y=f(u)$

在$u_0=\varphi(x_0)$可微, 则$y=f(\varphi(x))$在x_0可微, 且

$$\mathrm{d}y=f'(\varphi(x_0))\varphi'(x_0)\mathrm{d}x=f'(u_0)\mathrm{d}u.$$

无论将u视为中间变量还是自变量, 都有$\mathrm{d}y=f'(u)\mathrm{d}u.$

Ex. $f(x)=\ln|x|$, 求$f'(x)$.

解. $x>0$时, $f(x)=\ln x$, $f'(x)=\dfrac{1}{x}$.

$x<0$时, $f(x)=\ln(-x)$, $f(x)$是$\ln u$与$u=-x$的复合函数.

由链式法则,

$$f'(x)=\frac{1}{-x}(-x)'=\frac{1}{x}.$$

综上, $(\ln|x|)'=\dfrac{1}{x}$. □

Ex. $f(x)=\left(\frac{x+1}{x-1}\right)^{3/2}$, 求$f'(x)$. $x\in(-\infty,-1]\cup(1,+\infty)$

解. 令$g(u)=u^{3/2}, h(x)=\frac{x+1}{x-1}$, 则$f(x)=g(h(x))$,

$$g'(u)=\frac{3}{2}u^{1/2},$$

$$h'(x)=\left(1+\frac{2}{x-1}\right)' =\frac{-2}{(x-1)^2}$$

$$f'(x)=g'(h(x))h'(x)=\frac{-3}{(x-1)^2}\left(\frac{x+1}{x-1}\right)^{1/2}.\square$$

Ex. $f(x)=\ln\left|x+\sqrt{x^2\pm a^2}\right|$, 求$f'(x)$.

解. $$f'(x)=\frac{\left(x+\sqrt{x^2\pm a^2}\right)'}{x+\sqrt{x^2\pm a^2}}=\frac{1+\dfrac{2x}{2\sqrt{x^2\pm a^2}}}{x+\sqrt{x^2\pm a^2}}=\frac{1}{\sqrt{x^2\pm a^2}}.\square$$

Ex. $f(x)=u(x)^{v(x)}$, $u(x)>0$, $u(x),v(x)$可导, 求$f'(x)$.

解. $$f'(x)=\left(e^{v(x)\ln u(x)}\right)'=e^{v(x)\ln u(x)}\cdot\left(v(x)\ln u(x)\right)'$$
$$=u(x)^{v(x)}\cdot\left(v'(x)\ln u(x)+v(x)\frac{u'(x)}{u(x)}\right)$$
$$=u(x)^{v(x)}\ln u(x)\cdot v'(x)+v(x)u(x)^{v(x)-1}u'(x).\square$$

Ex. $f(x)=f_1(x)f_2(x)\cdots f_n(x)$, 求 $f'(x)$.

对数求导法

解: $\ln|f(x)|=\ln|f_1(x)|+\ln|f_2(x)|+\cdots+\ln|f_n(x)|$,

两边对 x 求导, 得 $$\frac{f'(x)}{f(x)}=\frac{f_1'(x)}{f_1(x)}+\frac{f_2'(x)}{f_2(x)}+\cdots+\frac{f_n'(x)}{f_n(x)}.$$

$$f'(x)=f(x)\left(\frac{f_1'(x)}{f_1(x)}+\frac{f_2'(x)}{f_2(x)}+\cdots+\frac{f_n'(x)}{f_n(x)}\right)$$

$$=\sum_{k=1}^{n}f_1(x)\cdots f_{k-1}(x)f_k'(x)f_{k+1}(x)\cdots f_n(x).\square$$

Remark. 多个因子连乘的函数求导时先取对数再两端求导可简化计算. ($f(x_0)=0$ 时结论仍成立？如何处理?)

Ex. $f(x)=\begin{cases} x^2\sin\dfrac{1}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}$，求$f'(x)$.

解：$f'(0)=\lim\limits_{x\to 0}\dfrac{f(x)-f(0)}{x-0}=\lim\limits_{x\to 0}x\sin\dfrac{1}{x}=0.$

$$f'(x)=\begin{cases} 2x\sin\dfrac{1}{x}-\cos\dfrac{1}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}. \square$$

Question. (1) f在$[a,b]$可导，f'在$[a,b]$上一定连续吗？不一定.

(2) $f\in C[a,b]$，f在(a,b)可导，$f'_+(a)$与$f'_-(b)$一定存在？不一定.

反例？ (1)$x^2\sin\dfrac{1}{x}$， (2)$x\sin\dfrac{1}{x}$.

Thm.(反函数求导)设f在(a,b)严格单调且连续, $x_0\in(a,b)$, $f'(x_0)\neq 0$, 则$x=f^{-1}(y)$在$y_0=f(x_0)$处可导, 且

$$(f^{-1})'(y_0)=1/f'(x_0).$$

Proof. f在(a,b)严格单调且连续, 则其反函数$x=f^{-1}(y)$也严格单调且连续. 当$y\neq y_0, y\to y_0$时, 有$x\neq x_0, x\to x_0$.

$$(f^{-1})'(y_0)=\lim_{y\to y_0}\frac{f^{-1}(y)-f^{-1}(y_0)}{y-y_0}=\lim_{x\to x_0}\frac{x-x_0}{f(x)-f(x_0)}$$

$$=\frac{1}{\lim\limits_{x\to x_0}\dfrac{f(x)-f(x_0)}{x-x_0}}=\frac{1}{f'(x_0)}.\square$$

Remark. $\dfrac{\mathrm{d}x}{\mathrm{d}y}=\dfrac{1}{\dfrac{\mathrm{d}y}{\mathrm{d}x}}$.

Ex. $(\arcsin x)' = \dfrac{1}{\sqrt{1-x^2}}$, $\arctan x = \dfrac{1}{1+x^2}$,

$(\arccos x)' = \dfrac{-1}{\sqrt{1-x^2}}$, $\operatorname{arc}\cot x = \dfrac{-1}{1+x^2}$.

解:(1) $y=\arcsin x$与$x=\sin y$互为反函数, 因此

$$(\arcsin x)' = \frac{1}{(\sin y)'} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-\sin^2 y}} = \frac{1}{\sqrt{1-x^2}}.$$

(2) $y=\arctan x$与$x=\tan y$互为反函数, 因此

$$(\arctan x)' = \frac{1}{(\tan y)'} = \frac{1}{\sec^2 y} = \frac{1}{1+\tan^2 y} = \frac{1}{1+x^2}.$$

Ex. 证明:1) $2^x = ax + 2(a \le 0)$确定隐函数$x = x(a)$;

2) $x(a)$在其定义域上连续;

3) $x(a)$在其定义域上可导,并求$x'(a)$.

证明:1) 由$2^x = ax + 2(a \le 0)$得

$$0 \ge a = a(x) = \frac{2^x - 2}{x},\ (0 < x \le 1).$$

$2 - 2^x$与$\frac{1}{x}$均在$(0,1]$上非负且严格单调递减, $a(x)$在$(0,1]$上非正且严格单调递增, 故$a = a(x)$有反函数$x = x(a)$,

即 $2^x = ax + 2(a \le 0)$确定隐函数$x = x(a)$.

2) $a(x)=\dfrac{2^x-2}{x}$在$(0,1]$上严格单调递增且连续，值域为$(-\infty,0]$，其反函数 $x=x(a)$在$(-\infty,0]$上严格单调递增且值域为$(0,1]$，因此$x=x(a)$在$[0,+\infty)$上连续.

3) $a(x)$可导，且$a'(x)=\dfrac{2^x(x\ln 2-1)+2}{x^2}>0$，因此其反函数$x=x(a)$可导.在 $2^x=ax+2$中视$x=x(a)$，两边对a求导，得

$$2^x\ln 2\cdot x'(a)=x+ax'(a),\quad x'(a)=\frac{x}{2^x\ln 2-a}.\square$$

Ex. 求曲线$x^2+y\cos x-2e^{xy}=0$在点$M(0,2)$处的切线方程.

解: 视方程$x^2+y\cos x-2e^{xy}=0$中$y=y(x)$, 两边对x求导, 得

$$2x+y'\cos x-y\sin x-2(y+xy')e^{xy}=0,$$

$$y'=\frac{2ye^{xy}+y\sin x-2x}{\cos x-2xe^{xy}}.$$

将$x=0, y(0)=2$代入, 得 $y'(0)=4.$

故曲线在点$M(0,2)$处的切线方程为 $y=4x+2.$□

Remark. $x^2+y\cos x-2e^{xy}=0$在点$M(0,2)$附近隐函数$y(x)$的存在性与可微性, 需要用下学期的隐函数定理来证明.

Ex. $y=y(x)$由参数方程$\begin{cases} x=t+e^t \\ y=t^2+e^{2t} \end{cases}$确定,求$\dfrac{dy}{dx}$.

解:

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}\cdot\frac{\mathrm{d}t}{\mathrm{d}x}=\frac{\dfrac{\mathrm{d}y}{\mathrm{d}t}}{\dfrac{\mathrm{d}x}{\mathrm{d}t}}=\frac{2(t+e^{2t})}{1+e^t}.\square$$

Ex. $x=\varphi(t)$严格单调且连续,其反函数$t=\varphi^{-1}(x)$也严格单调且连续. 于是, 参数方程$\begin{cases} x=\varphi(t) \\ y=\psi(t) \end{cases}$确定了函数

$$y=\psi(t)=\psi(\varphi^{-1}(x)).$$

求$\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

解: 由复合函数求导的链式法则,

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}\cdot\frac{\mathrm{d}t}{\mathrm{d}x}=\psi'(t)\cdot(\varphi^{-1})'(x)=\frac{\psi'(t)}{\varphi'(t)}.\square$$

作业：

习题3.1 No. 5(4),9,14(1)

习题3.2 No. 4(3)(6)(8),6(1),7(1),
8(4),10(1),11(4)