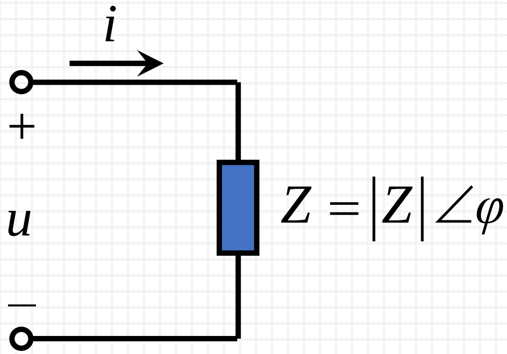
3 电路原理习题课

0.1



电路如图所示，已知电压和电流为

$$u(t) = 10\sin(400\pi t + 60^\circ)\ \mathbf{V}$$

$$i(t) = -\frac{1}{\sqrt{2}}\cos(400\pi t - 150^\circ)\ \mathbf{A}$$

(a) ω = 400π rad/s，f = 200Hz，T= 0.005s.

(b) 有效值 U= 7.07V，有效值 I= 0.5A.

(c) u 和 i 的相位差 $\psi_u - \psi_i$ = -60°.

(d)负载是 容性，|Z|= 14.14Ω，φ = -60°.

比较相位时，要将时域表达式转换成标准正弦形式

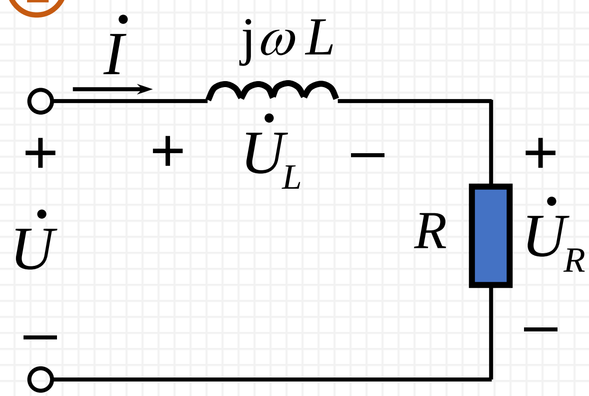
$$i(t) = \frac{1}{\sqrt{2}}\cos(400\pi t - 150^\circ + 180^\circ) = \frac{1}{\sqrt{2}}\cos(400\pi t + 30^\circ)$$

$$= \frac{1}{\sqrt{2}}\sin(400\pi t + 30^\circ + 90^\circ) = \frac{1}{\sqrt{2}}\sin(400\pi t + 120^\circ)A$$

$\varphi = \psi_u - \psi_i = 60^\circ - 120^\circ = -60^\circ$

0.2 下列哪些表达式是正确的，哪些是错误的，并改正。

①



(1) $\dot I = \dfrac{\dot U}{R + j\omega L}$

(2) $I = \dfrac{U}{\sqrt{R^2 + (\omega L)^2}}$

✓ (3) $u = u_R + u_L$

$\dot U = \dot U_R + \dot U_L$

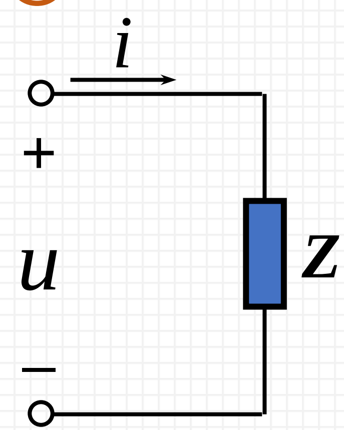
(4) $U^2 = U_L^2 + U_R^2$

(5) $P = \dfrac{U_R^2}{R}$

✓ (6) $P = I^2 R$

(7) $|Z| = \sqrt{R^2 + (\omega L)^2}$

②



如果 $u(t) = 311\sin(\omega t + 45°)\mathrm{V}$, $Z = 25\angle 60°\ \Omega$

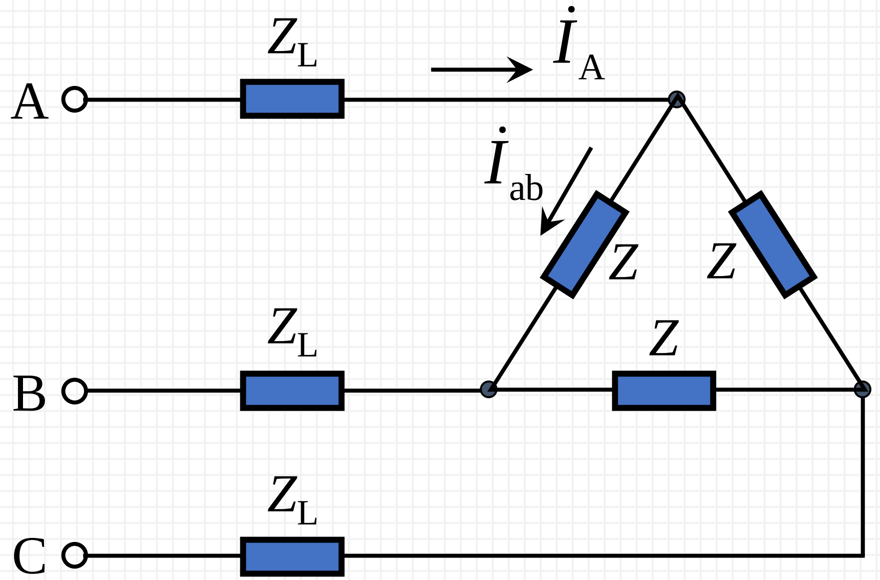
那么 $i \neq \dfrac{u}{Z} \neq \dfrac{311\sin(\omega t + 45°)}{25\angle 60°} \neq 12.44\sin(\omega t + 45° - 60°)\mathrm{A}$

$$\dot I = \frac{\dot U}{Z} = \frac{\frac{311}{\sqrt{2}}\angle 45^\circ}{25\angle 60^\circ} = 8.8\angle -15^\circ \ \ \mathrm{A}$$

相量 $\neq$ 正弦量

$$i = 8.8\sqrt{2}\sin(\omega t - 15^\circ)\ \ \mathrm{A}$$

0.3 平衡三相电路中，$\dot{U}_{AB}$ 是线电压，$\dot{U}_{AN}$ 是相电压.



(1) $\dot{I}_{ab} \cancel{=} \dfrac{\dot{U}_{AB}}{Z}$

(2) $\dot{I}_{ab} \cancel{=} \dfrac{\dot{U}_{AB}}{2Z_L + Z}$

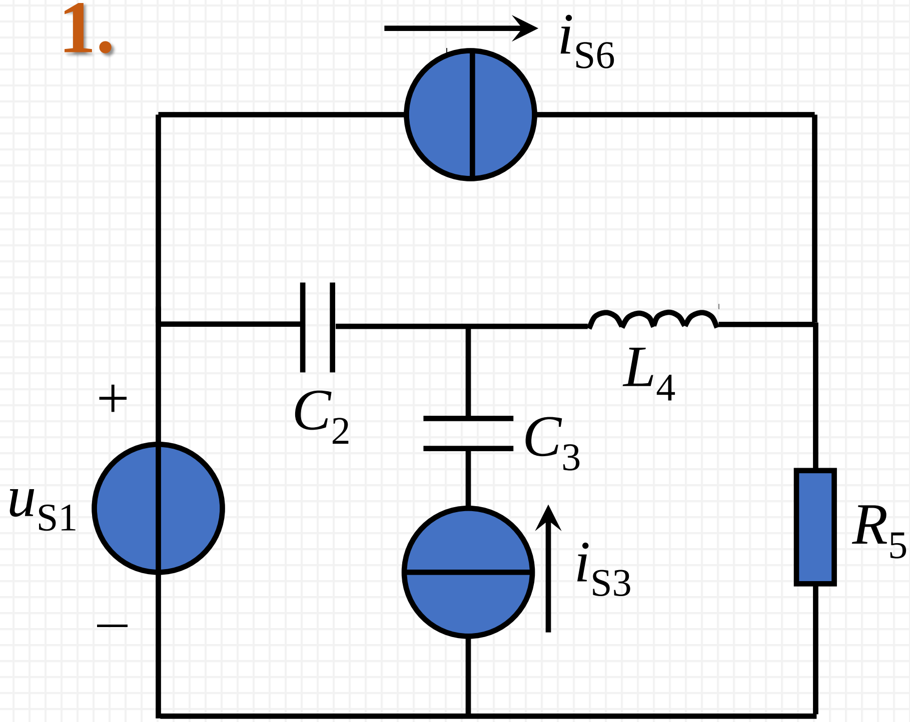
(3) $\dot{I}_{ab} \cancel{=} \dfrac{\dot{U}_{AN}}{Z_L + Z/3}$

(4) $\dot{I}_A \cancel{=} \dfrac{\dot{U}_{AB}}{2Z_L + Z}$

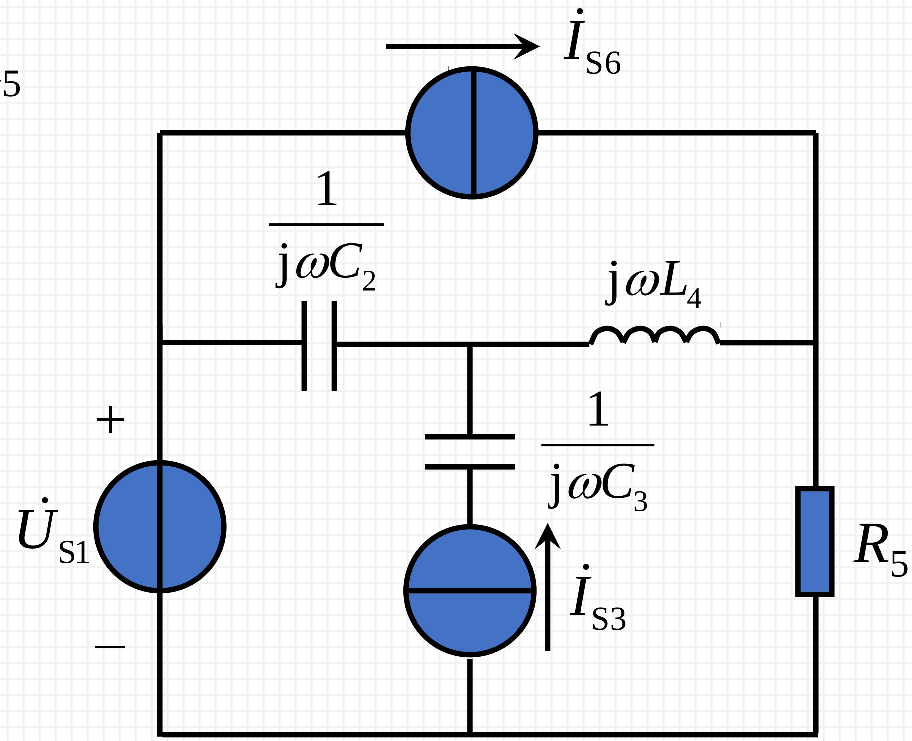
(5) $\dot{I}_A \cancel{=} \dfrac{\dot{U}_{AB}}{Z_L + Z}$

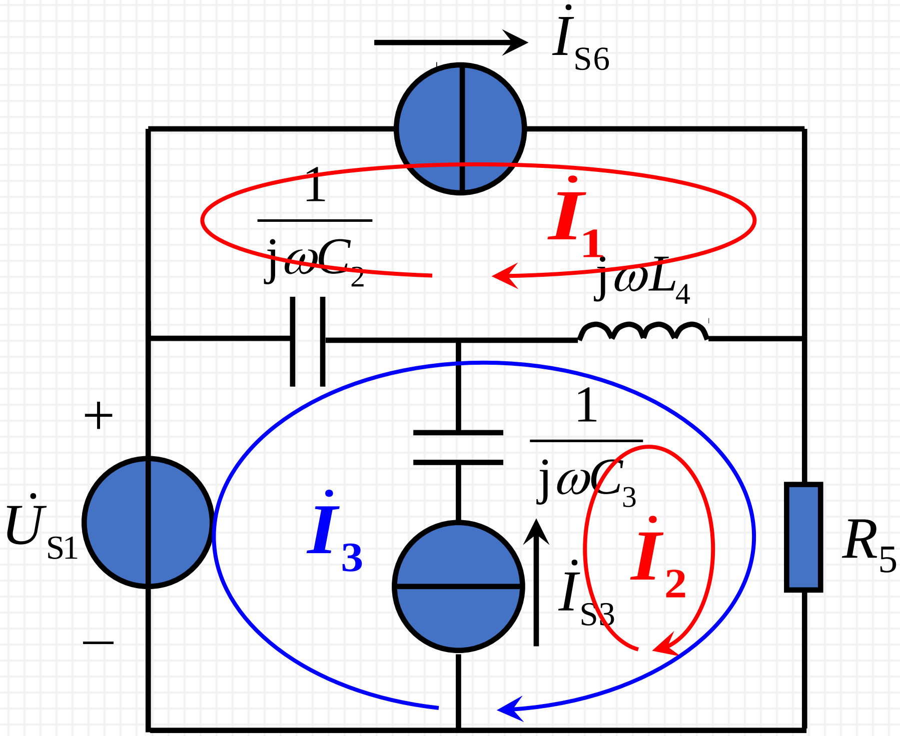
(6) $\dot{I}_A = \dfrac{\dot{U}_{AN}}{Z_L + Z/3}$

1.



电路如图所示。写出其相量形式的节点方程和回路方程。





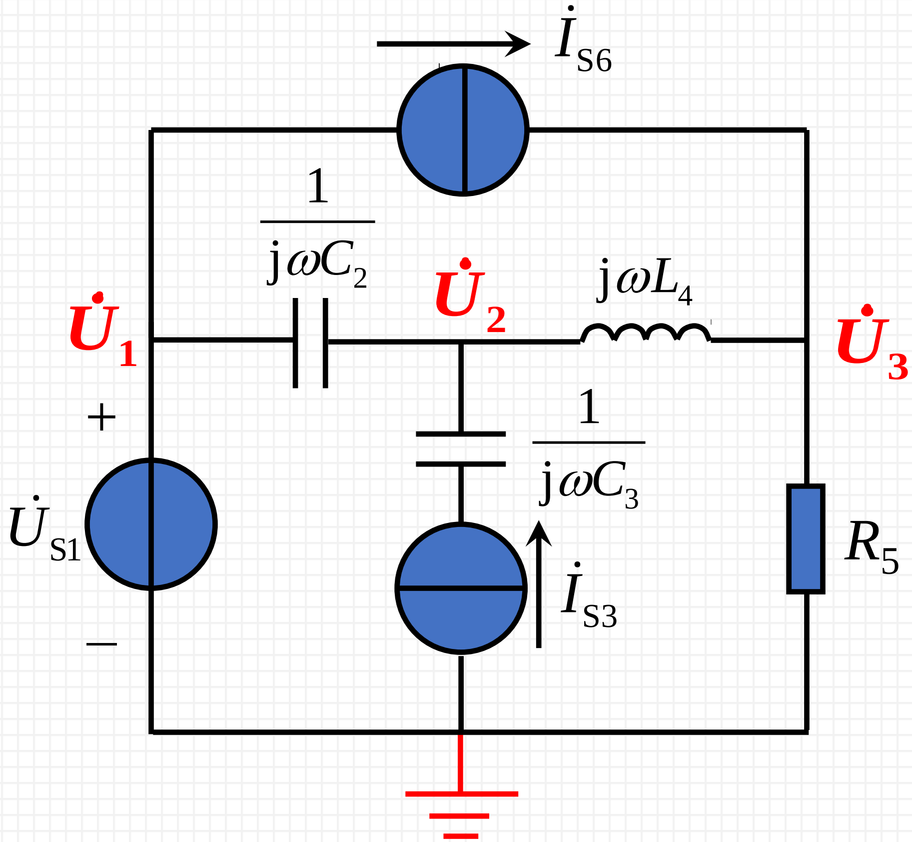
回路法：

$$\dot{I}_1 = \dot{I}_{S6} \qquad ①$$

$$\dot{I}_2 = \dot{I}_{S3} \qquad ②$$

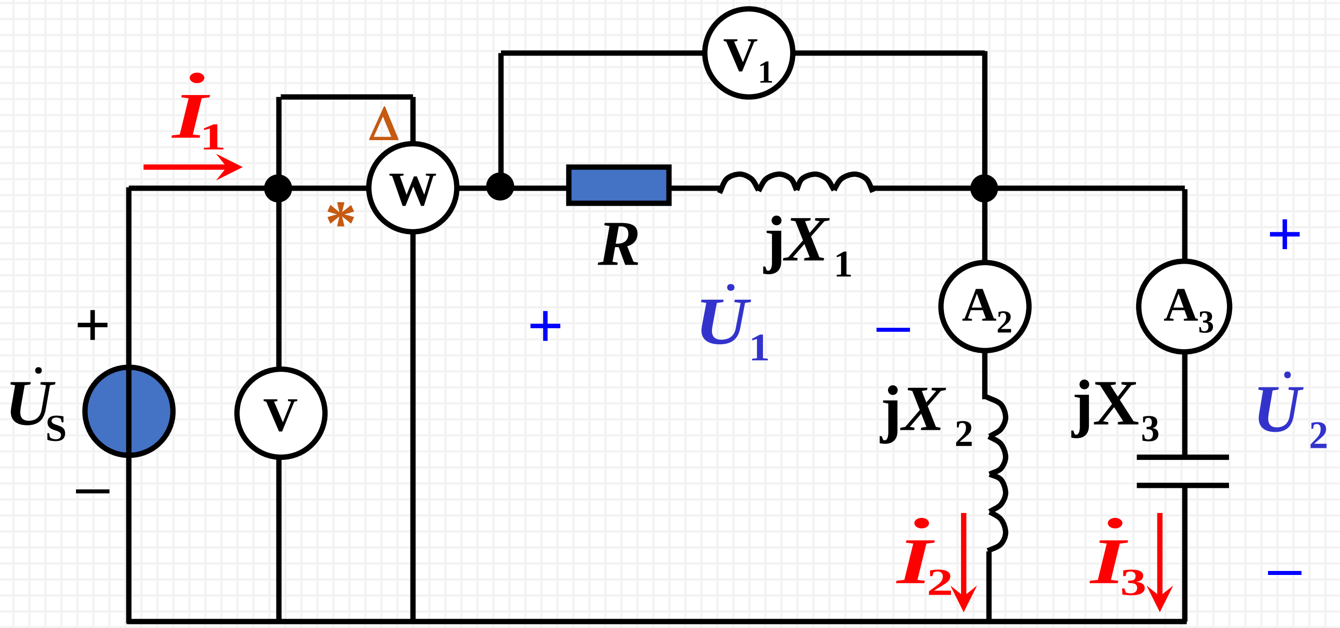
$$\left(\frac{1}{j\omega C_2} + j\omega L_4 + R_5\right)\dot{I}_3 - \left(\frac{1}{j\omega C_2} + j\omega L_4\right)\dot{I}_1 + (j\omega L_4 + R_5)\dot{I}_2 = \dot{U}_{S1} \quad ③$$

节点法:

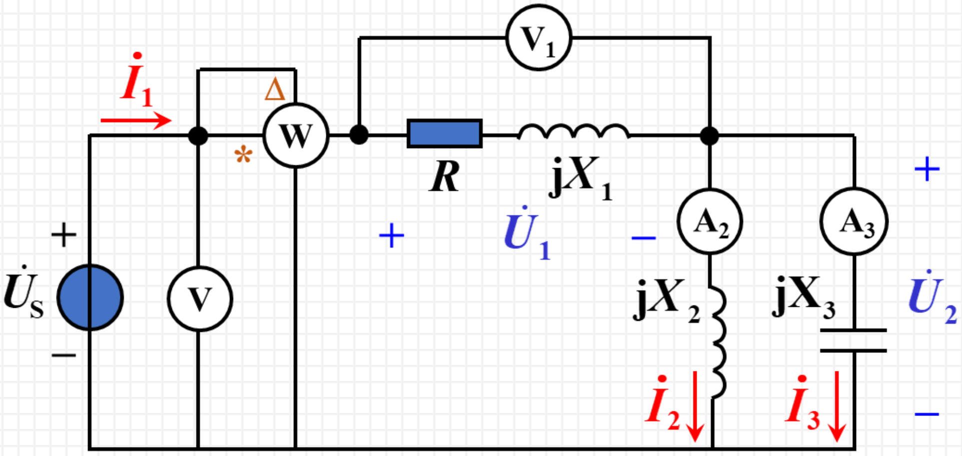


$$
\begin{cases}
\dot{U}_1 = \dot{U}_{S1} \\
-j\omega C_2 \dot{U}_1 + (j\omega C_2 + \dfrac{1}{j\omega L_4})\dot{U}_2 - \dfrac{1}{j\omega L_4}\dot{U}_3 = \dot{I}_{S3} \\
-\dfrac{1}{j\omega L_4}\dot{U}_2 + (\dfrac{1}{j\omega L_4} + \dfrac{1}{R_5})\dot{U}_3 = \dot{I}_{S6}
\end{cases}
$$

2.



电路如图所示。电压表 V 的读数是 220V，V_1 的读数是$100\sqrt{2}$ V，A_2的读数是30A，A_3的读数是20A，功率表的读数是 1000W (有功功率)。求参数 R、X_1、X_2 和 X_3。



设: $\dot{U}_2 = U_2\angle 0^\circ\ \mathrm{V}$

那么: $\dot{I}_2 = -\mathrm{j}30\mathrm{A}$

$\dot{I}_3 = \mathrm{j}20\mathrm{A}$

$\dot{I}_1 = \dot{I}_2 + \dot{I}_3 = -\mathrm{j}10\mathrm{A}$

$P = I_1^{\,2} R$

$R = P / I_1^{\,2} = 1000/10^2 = 10\Omega$

设: $Z_1 = R + \mathrm{j}X_1 = |Z_1|\angle\varphi_1$ 则 $|Z_1| = \dfrac{U_1}{I_1} = \dfrac{100\sqrt{2}}{10} = 10\sqrt{2}\ \Omega$

$X_1 = \sqrt{|Z_1|^2 - R^2} = \sqrt{(10\sqrt{2})^2 - 10^2} = 10\,\Omega$

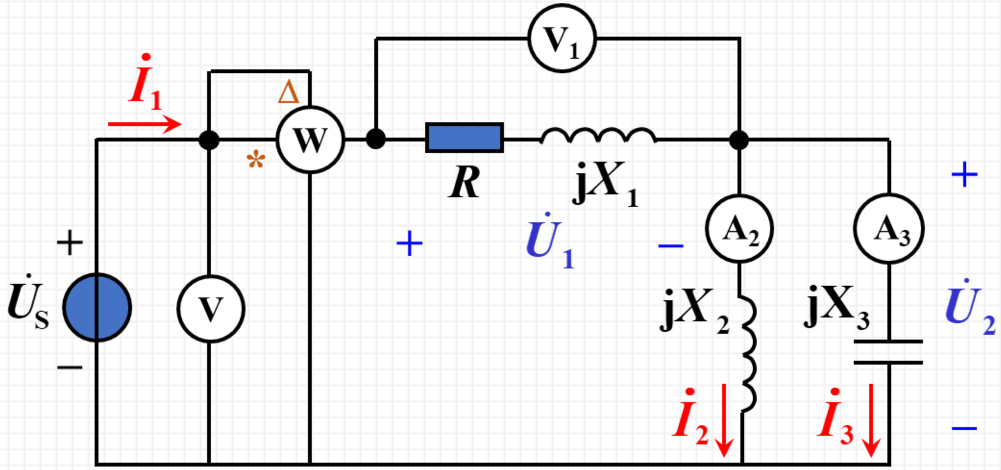
$\varphi_1 = \mathrm{arctg}\dfrac{X_1}{R} = 45^\circ$

$\because \dot{I}_1 = -\mathrm{j}10 = 10\angle -90^\circ\ \mathrm{A} \quad \therefore \dot{U}_1 = 100\sqrt{2}\angle -45^\circ\ \mathrm{V}$

$\dot{U}_\mathrm{S} = \dot{U}_1 + \dot{U}_2 = 100 - \mathrm{j}100 + U_2 = 100 + U_2 - \mathrm{j}100$

$U_\mathrm{S}^2 = (100 + U_2)^2 + 100^2, \quad U_2 = \sqrt{220^2 - 100^2} - 100 = 96\mathrm{V}$

$X_2 = U_2 / I_2 = 96/30 = 3.2\Omega, \quad X_3 = -U_2 / I_3 = -96/20 = -4.8\Omega$

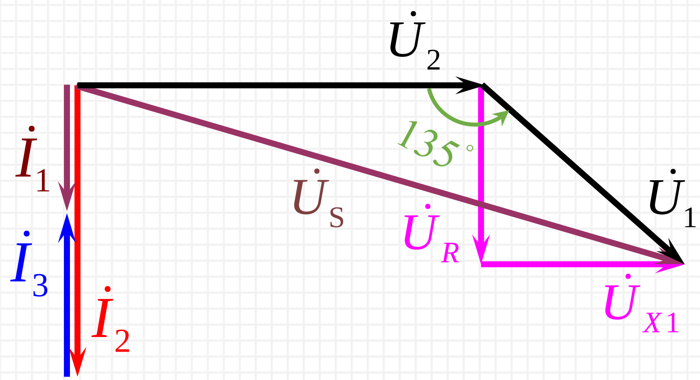


U_2也可以用相量图求出.

$$\dot{U}_S = \dot{U}_1 + \dot{U}_2$$

$$\dot{U}_1 = \dot{U}_R + \dot{U}_{X1}$$

$$\dot{I}_1 = \dot{I}_2 + \dot{I}_3$$



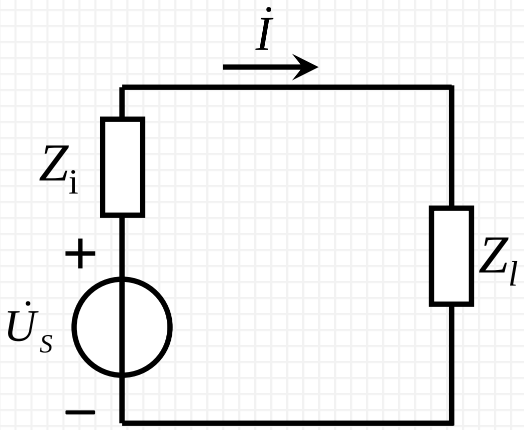
$$U_S^2 = U_2^2 + U_1^2 - 2U_1U_2\cos 135°$$

$$U_2^2 - 2\times 100\sqrt{2}\times\left(-\frac{\sqrt{2}}{2}\right)U_2 + (100\sqrt{2})^2 - 220^2 = 0$$

$U_2^2 + 200U_2 - 28400 = 0,\qquad U_2 = 96\text{V}$ （忽略负值）

交流电路最大功率传输

讨论正弦电流电路中负载获得最大功率P_{max}的条件。



$Z_i = R_i + jX_i$， $Z_l = R_l + jX_l$

$$\dot{I} = \frac{\dot{U}_S}{Z_i + Z_l}, \quad I = \frac{U_S}{\sqrt{(R_i + R_l)^2 + (X_i + X_l)^2}}$$

$Z_l = R_l + jX_l$ 可任意改变时

$$Z_l = Z_i^*, \text{ 即 } \begin{cases} R_l = R_i \\ X_l = -X_i \end{cases} \qquad P_{max} = \frac{U_S^2}{4R_i}$$

(2) 若$Z_l = R_l + \mathrm{j}X_l$只允许$X_l$改变

此时获得最大功率的条件$X_\mathrm{i} + X_l = 0$，即$X_l = -X_\mathrm{i}$。

最大功率为 $$P_{\max} = \frac{R_l U_\mathrm{S}^2}{(R_\mathrm{i} + R_l)^2}$$

(3) 若$Z_l = R_l + \mathrm{j}X_l = |Z_l|\angle\varphi$，$R_l$、 X_l均可改变，但X_l / R_l不变

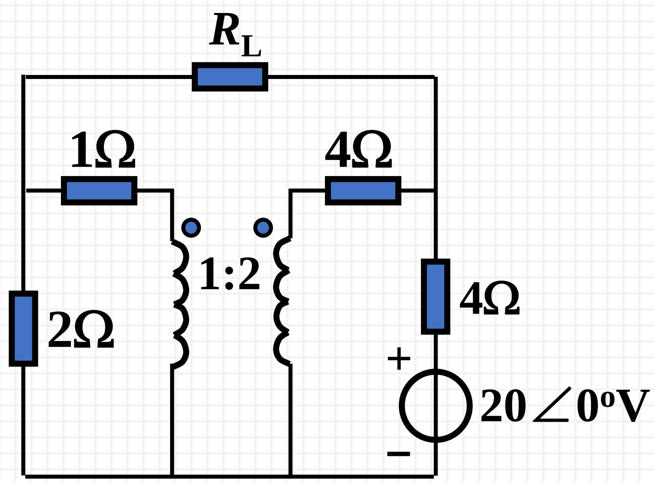
(即$|Z_l|$可变，φ不变)

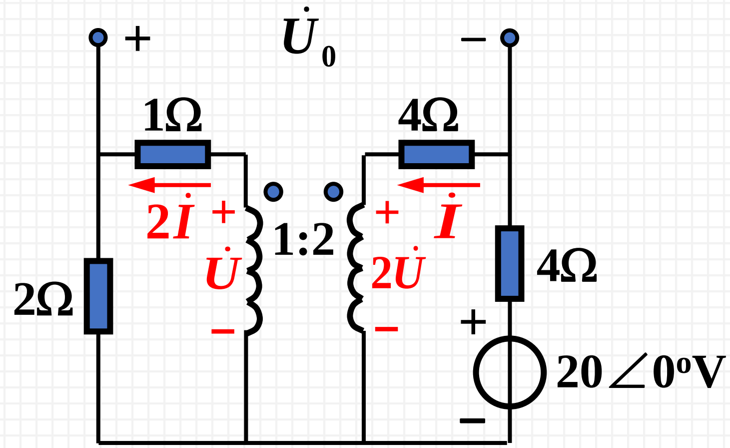
模匹配

此时获得最大功率的条件 $|Z_l| = |Z_\mathrm{i}|$ 。

最大功率为 $$P_{\max} = \frac{\cos\phi\ U_\mathrm{S}^2}{2|Z_\mathrm{i}| + 2(R_\mathrm{i}\cos\phi + X_\mathrm{i}\sin\phi)}$$

3. R_L取值为多大时获得最大功率？最大功率是多少？





解法：戴维南定理＋理想变压器

求开路电压

左：$\dot{U} = 3\times 2\dot{I}$

右：$20\angle 0^\circ = 8\dot{I} + 2\dot{U}$

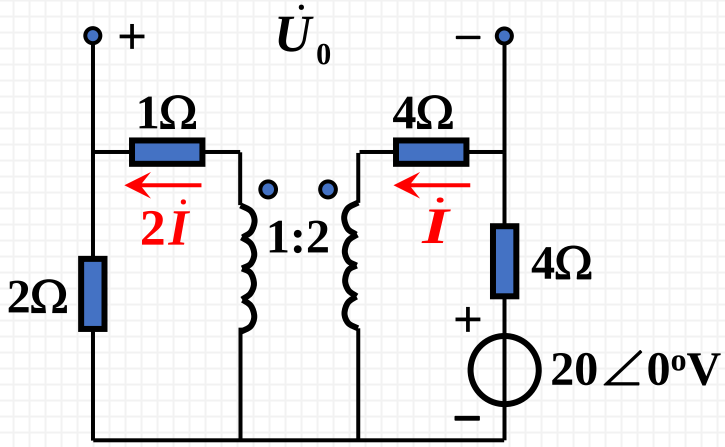
$$\dot{U} = 6\angle 0^\circ$$

$$\dot{I} = 1\angle 0^\circ$$

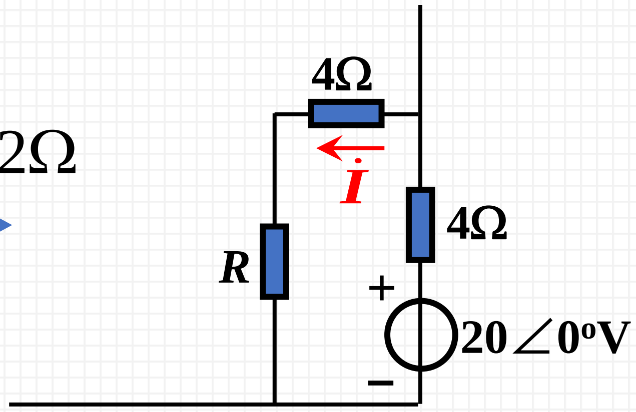
$$\dot{U}_0 = 2\times 2\dot{I} - 20\angle 0^\circ + 4\dot{I} = 12\angle 180^\circ \text{ V}$$

求开路电压

法二：



$R=2^2\times3=12\Omega$

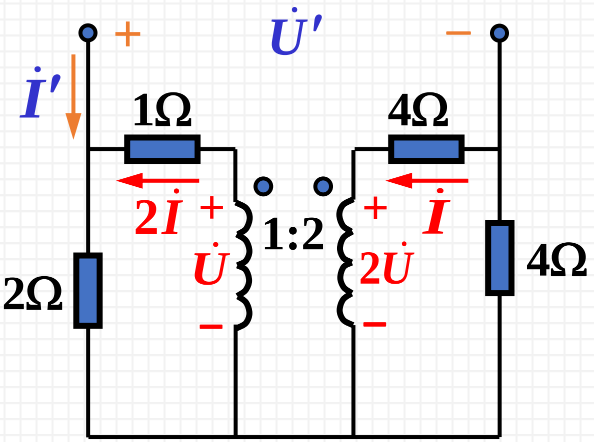


$$\dot{I}=20\angle0^\circ/(R+4+4)=1\angle0^\circ\,\text{A}$$

$$\dot{U}_0=2\times2\dot{I}-20\angle0^\circ+4\dot{I}=12\angle180^\circ\,\text{V}$$

并不简便

加压求流求内阻



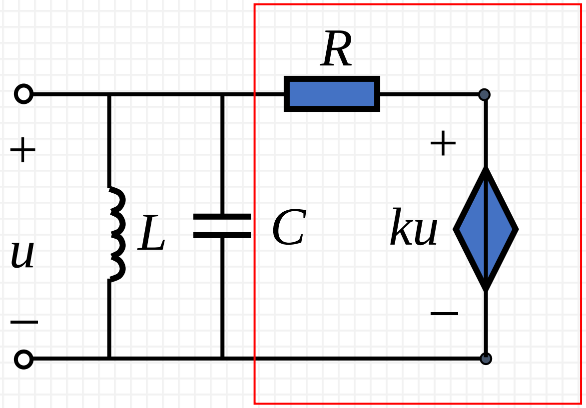
$$\begin{cases} \dot{U} = 2\dot{I} + 2(2\dot{I} + \dot{I}') \\ 4\dot{I} + 2\dot{U} + 4(\dot{I} + \dot{I}') = 0 \\ \dot{U}' = 2(2\dot{I} + \dot{I}') + 4(\dot{I} + \dot{I}') \end{cases} \Rightarrow \dot{U}' = 2.8\dot{I}'$$

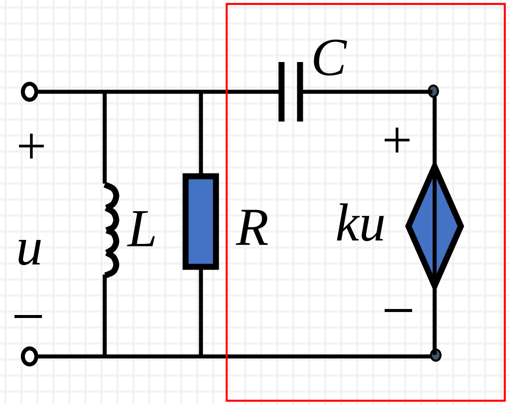
$R = 2.8\Omega$

则 $R_L = 2.8\Omega$ 时获得最大功率.

最大功率 $P = 12^2/(4\times 2.8) = 12.9W$

4. 求图示电路的谐振频率以及在谐振时的入端阻抗 (0 < k < 1)





$$R_{等} = \frac{u}{(1-k)u/R} = \frac{R}{1-k}$$

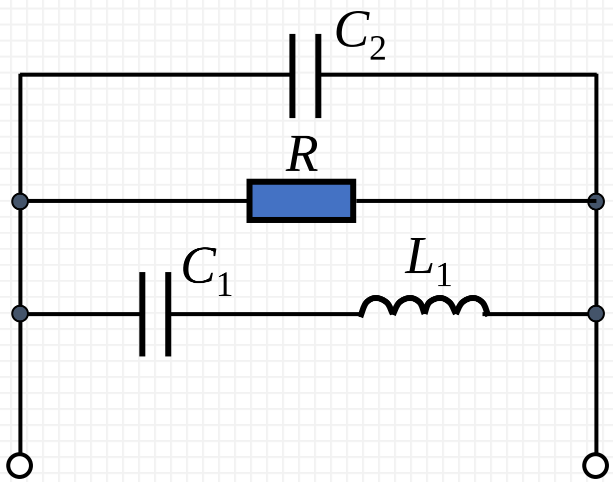
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

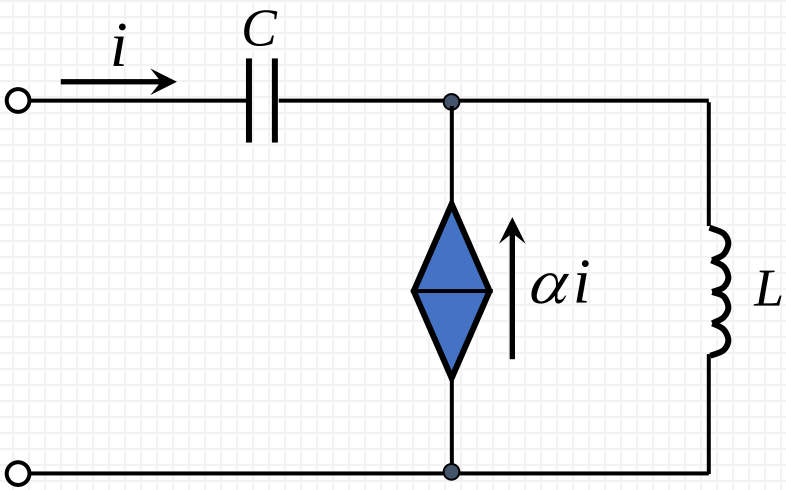
$$Z(\omega_0) = \frac{R}{1-k}$$

$$Z_{等} = \frac{\dot{U}}{(1-k)\dot{U}\mathrm{j}\omega C} = \frac{1}{(1-k)\mathrm{j}\omega C}$$

$$\omega_0 = \frac{1}{\sqrt{LC(1-k)}}$$

$$Z(\omega_0) = R$$





$$\omega_{01}=\frac{1}{\sqrt{L_1C_1}}$$

$$\omega_{02}=\frac{1}{\sqrt{L_1\frac{C_1C_2}{C_1+C_2}}}$$

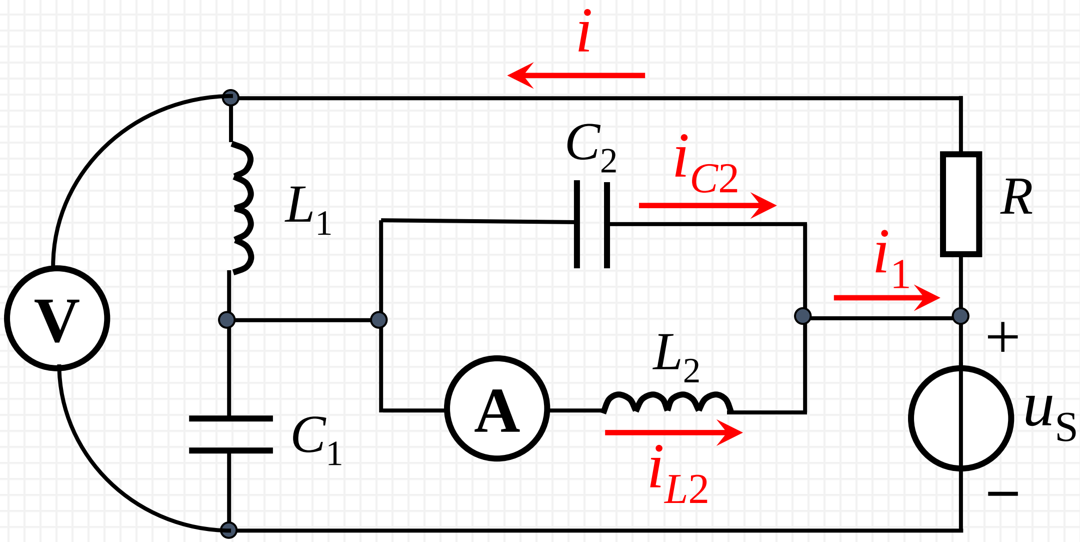
$$\dot{U}=-\mathrm{j}\frac{1}{\omega C}\dot{I}+\mathrm{j}\omega L\dot{I}(1+\alpha)$$

$$Z=\frac{\dot{U}}{\dot{I}}=-\mathrm{j}\frac{1}{\omega C}+\mathrm{j}(1+\alpha)\omega L$$

$$\omega_0=\frac{1}{\sqrt{(1+\alpha)LC}}$$

5. 电路如图所示. $u_S(t)=\sin t$ V，$L_1=L_2=1$H，$C_1=C_2=1$F，$R=1\Omega$。

求电压表和电流表的读数 (rms).



解： 设 $\dot{U}_S=\dfrac{1}{\sqrt{2}}\angle 0^\circ \text{V}$

$$=0.707\angle 0^\circ \text{V}$$

$$\omega L_2=\frac{1}{\omega C_2}=1\Omega$$

(并联谐振) $\dot{I}_1=0$

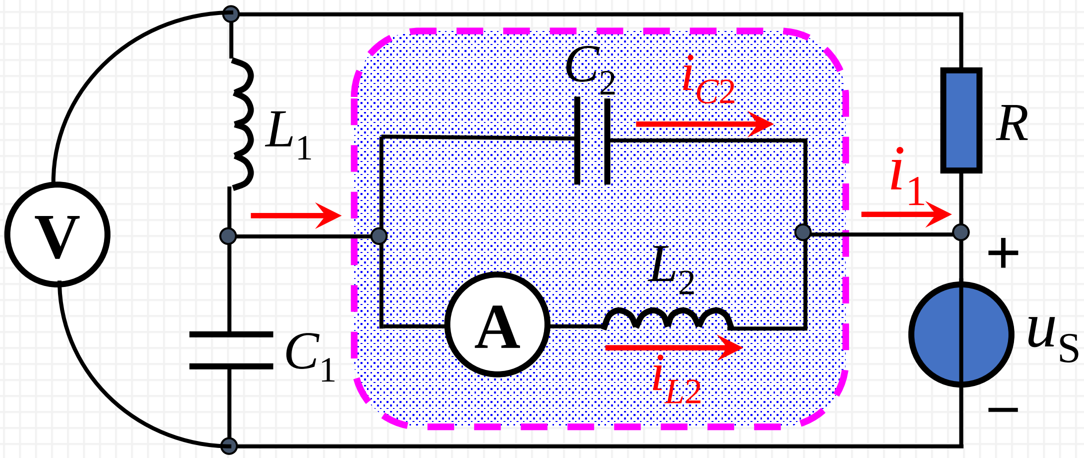
$$\omega L_1=\frac{1}{\omega C_1}=1\Omega$$ (串联谐振), V = 0

$$\dot{I}=\frac{\dot{U}_S}{R}=\frac{0.707\angle 0^\circ}{1}=0.707\angle 0^\circ \text{A}$$

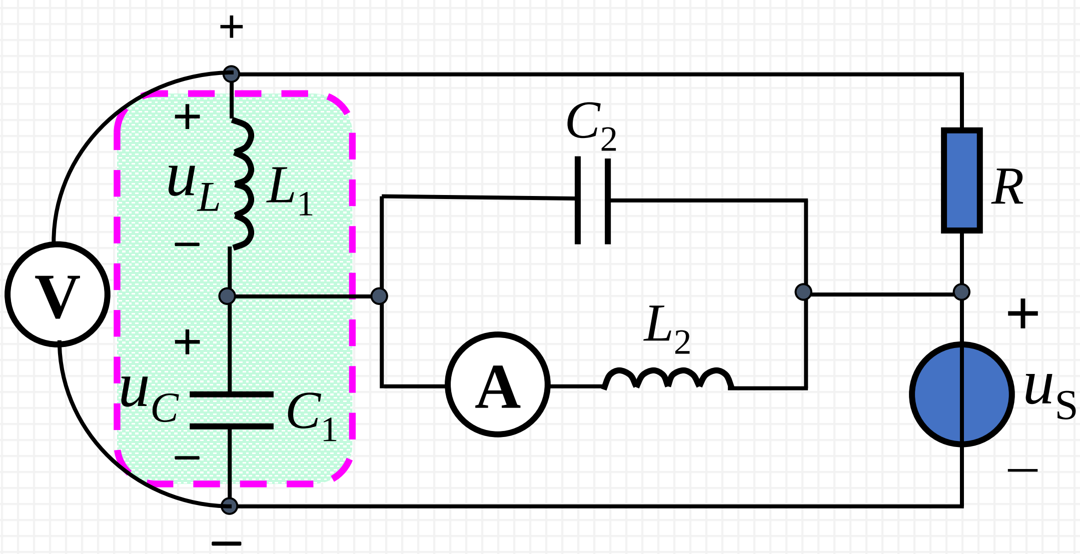
$$\dot{I}_{L2}=\frac{\dfrac{1}{\text{j}\omega C_1}\dot{I}-\dot{U}_S}{\text{j}\omega L_2}=\frac{-\text{j}0.707-0.707}{\text{j}1}=-0.707+\text{j}0.707=1.00\angle 135^\circ \text{A}$$

A = 1A

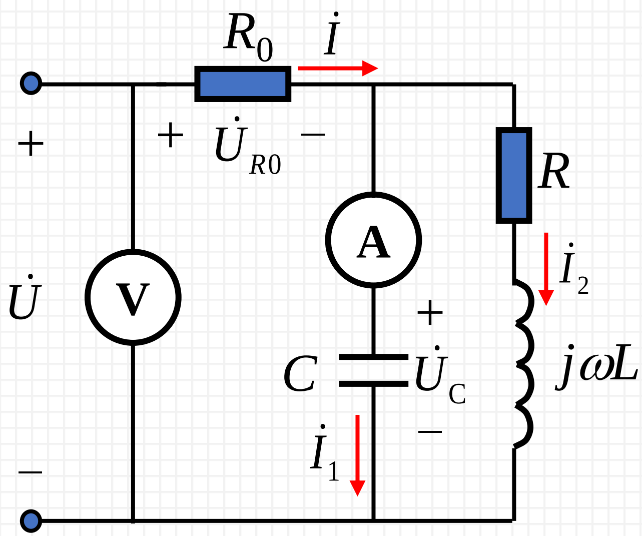
将发生并联谐振的电路看成一个二端网络 (或一条广义的支路), 则流进或流出端口的电流 (或通过该广义支路的电流)为零, 但网络内部的各个支路的电流 并不一定为零。



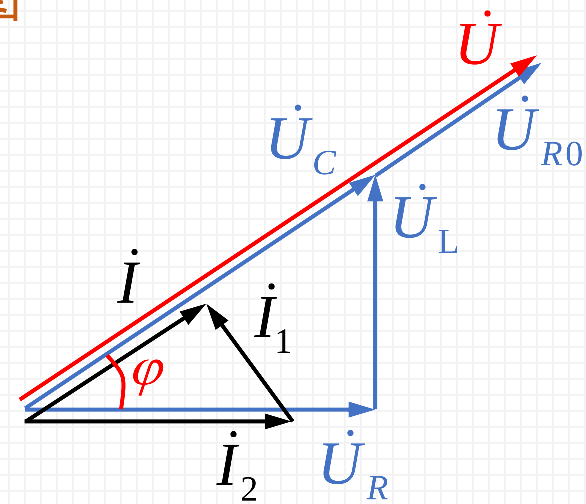
将发生串联谐振的电路看成一个二端网络(或一条广义的支路),则两个端钮之间的电压(或广义支路两端的电压)为零,但网络内部的各个支路上的电压并不一定为零。



6. $\omega = 1000\text{rad/s}$时, 电路发生谐振。 $R_0 = 25\Omega$, $C = 16\mu\text{F}$, 电压表的读数是 100V, 电流表的读数是 1.2A， 求 R 和 L.



解: 相量图



$$I_1 = 1.2\text{A}$$

$$U_C = 1.2 \Big/ (1000 \times 1.6 \times 10^{-6}) = 75\text{V}$$

谐振时:

$$U_{R0} = 25\text{V} \qquad I = 1\text{A}$$

$$I_2 = \sqrt{I_1^2 + I^2} = 1.562\text{A}$$

$$\phi = \arctan(I_1 / I) = 50.2^\circ$$

$$|Z| = U_C \Big/ I_2 = 75 \Big/ 1.562 = 48.01\Omega$$

$$R = 48.01\cos 50.2^\circ = 30.7\Omega$$

$$L = 48.01\sin 50.2^\circ \Big/ 1000 = 36.9\text{mH}$$

解二

设 $\dot{U}=100\angle 0^{\circ}\,\text{V}$

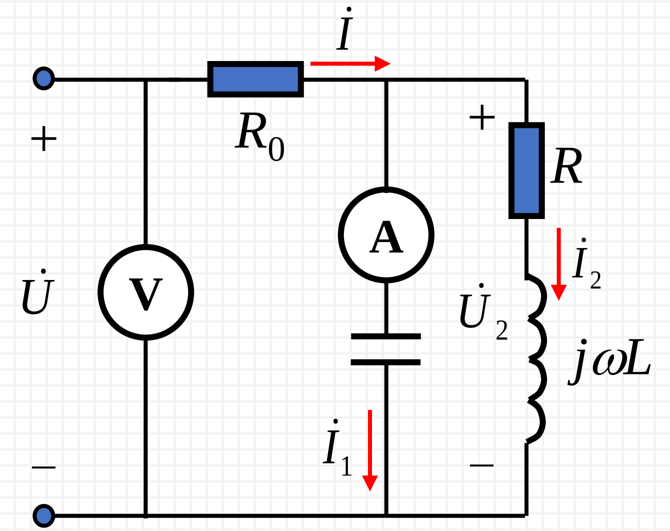
分析可知u、i、u_2同相，则i_1领先u_2 90度

$$\dot{I}_1=\text{j}1.2\text{A}$$

$$\dot{U}_2=\dot{I}_1*\frac{1}{\text{j}\omega C}=75\angle 0^{\circ}\,\text{V}$$

$$\dot{I}=\frac{\dot{U}-\dot{U}_2}{R_0}=1\text{A}$$

$$\dot{I}_2=\dot{I}-\dot{I}_1=1-j1.2=1.562\angle 50.2^{\circ}\,\text{A}$$



由于谐振，无功在电感与电容间交换，则

$$Q_L=I_2^2\omega L \qquad Q_C=-I_1^2\frac{1}{\omega C}$$

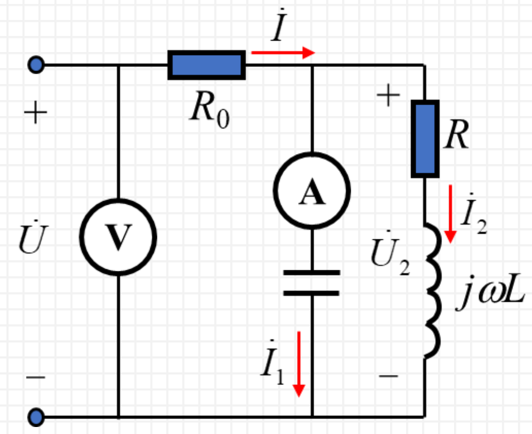
$$I_1^2\frac{1}{\omega C}=I_2^2\omega L$$

$$\Rightarrow L=\frac{I_1^2}{\omega^2 C I_2^2}=0.0369\text{H} \qquad R=\sqrt{(\frac{U_2}{I_2})^2-(\omega L)^2}=\sqrt{(\frac{75}{1.562})^2-(39.6)^2}=30.7\Omega$$

解三

设 $\dot{U}=100\angle 0°\text{V}$

则因为电容电流 $\dot{I}_C$ 相量领先电压相量 $\dot{U}_C$ $90°$，有：



$$\dot{I}_C=\text{j}1.2\text{A}\quad \text{j}X_C=\frac{1}{\text{j}\omega C}=\frac{1}{\text{j}\times 1000\times 16\times 10^{-6}}=-\text{j}62.5\Omega$$

$$\dot{U}_C=\dot{I}_C\text{j}X_C=75\text{V}\qquad \dot{U}_{R0}=\dot{U}-\dot{U}_C=100-75=25\text{V}$$

$$\dot{I}=\frac{\dot{U}_{R0}}{R_0}=\frac{25\angle 0^\circ}{25}=1\angle 0^\circ\text{A}$$

所以R，L，C 并联支路等效为一个 $\dfrac{\dot{U}_C}{\dot{I}}=75\Omega$ 的电阻。即：

$$\frac{\dfrac{1}{\text{j}\omega C}\times(R+\text{j}\omega L)}{\dfrac{1}{\text{j}\omega C}+(R+\text{j}\omega L)}=75\Omega$$

设 $R+\text{j}\omega L=Z$

则有：$\dfrac{-\text{j}62.5\times Z}{-\text{j}62.5+Z}=75$

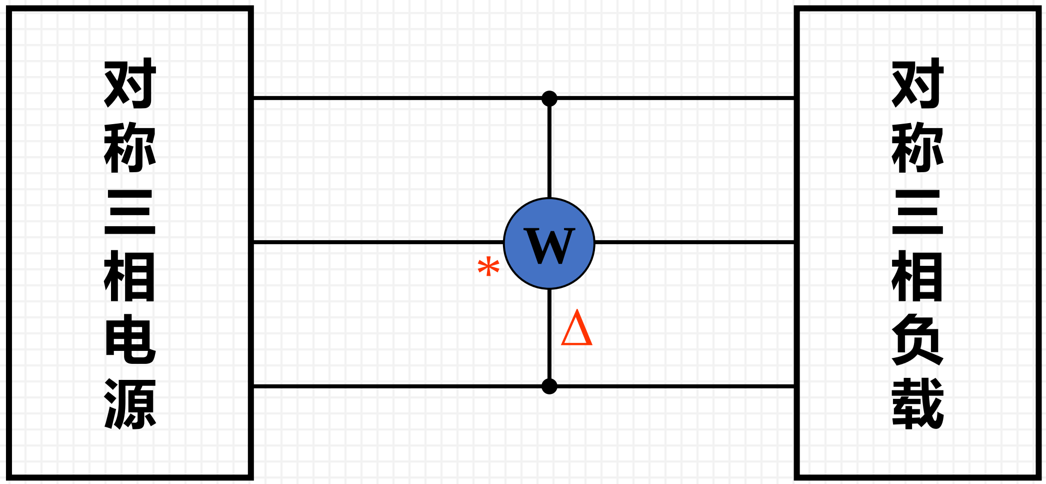
$$Z=30.7+\text{j}36.9\Omega$$

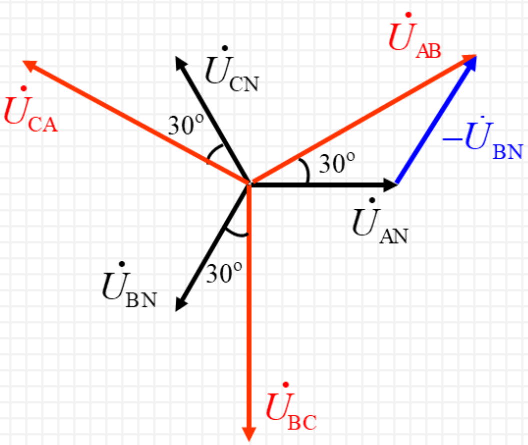
$$R=30.7\Omega$$

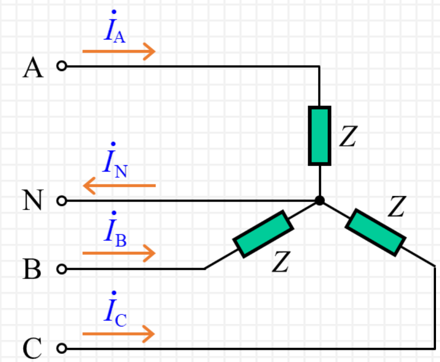
$$\omega L=36.9\Omega$$

$$L=36.9\text{mH}$$

7. W的读数有何物理意义







设 $\dot U_{AN} = U_P \angle 0^\circ$

$$\boxed{\mathrm{W}} = U_{CA} I_B \cos\left(\phi_{u_{CA}} - \phi_{i_B}\right)$$

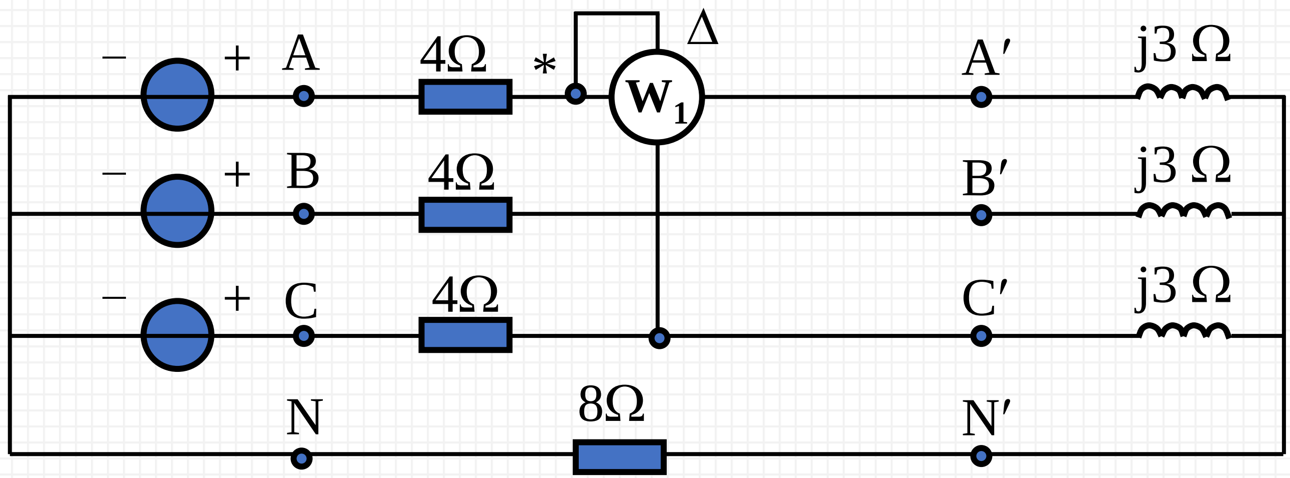
$$= \sqrt{3} U_P I_P \cos\left(\left(150^\circ\right) - \left(-120^\circ - \phi_P\right)\right)$$

$$= \sqrt{3} U_P I_P \cos\left(270^\circ + \phi_P\right)$$

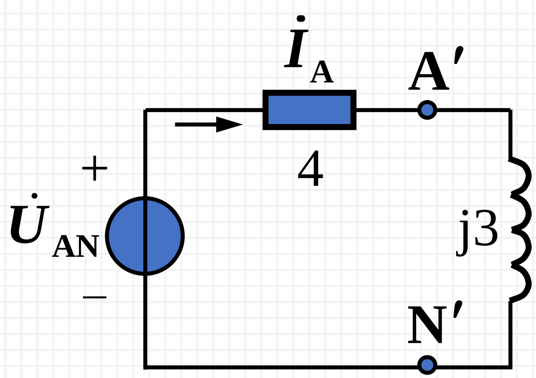
$$= \sqrt{3} U_P I_P \sin \phi_P$$

测量对称三相负载吸收的无功功率 $\times \sqrt{3}$

8. 平衡三相电路的相电压是 220V。求：(1)线电流和通过中线的的电流； (2) 求功率表的读数； (3)电源发出的有功功率和无功功率； (4)能用两表法测量负载吸收的功率吗？如果能，画出另一块表，求读数。



解：(1) 抽单相:



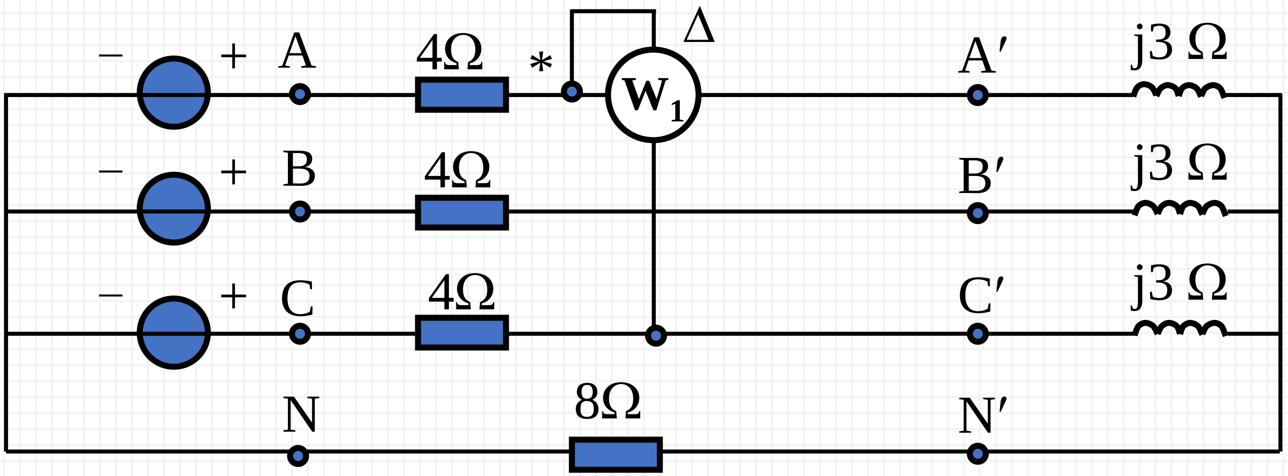
$$\dot{U}_{AN}=220\angle 0^{\circ}\,\mathrm{V}$$

$$\dot{I}_A=\frac{220\angle 0^{\circ}}{4+\mathrm{j}3}=44\angle -36.9^{\circ}\,\mathrm{A}$$

$$I_l=44\mathrm{A} \qquad I_N=0$$

(2) 求功率表的读数

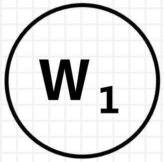
$\dot{I}_A = 44\angle -36.9^\circ$ A



$$\dot{U}_{A'N} = j3\dot{I}_A = 132\angle 53.1^\circ\,\text{V}$$

$$\dot{U}_{A'B'} = \sqrt{3}\,132\angle\left(30^\circ + 53.1^\circ\right) = \sqrt{3}\,132\angle 83.1^\circ\,\text{V}$$

$$\dot{U}_{A'C'} = -\dot{U}_{C'A'} = -\sqrt{3}\,132\angle\left(120^\circ + 83.1^\circ\right) = \sqrt{3}\,132\angle 23.1^\circ\,\text{V}$$



$$U_{A'C'}I_A\cos[23.1^\circ - (-36.9^\circ)]$$

$$= \sqrt{3}132\times 44\cos 60^\circ = 5029\text{W}$$

(3) 电源发出的功率:

$$\dot{U}_{AN} = 220\angle 0^{o}\,\mathrm{V}$$

$$\dot{I}_{A} = 44\angle -36.9^{o}\,\mathrm{A}$$

$$P = 3U_{p}I_{p}\cos\phi_{p} = 3\times 220\times 44\cos 36.9^{o} = 23.2\mathrm{kW}$$

OR $$P = \sqrt{3}U_{l}I_{l}\cos\phi_{p} = \sqrt{3}\times\sqrt{3}\times 220\times 44\cos 36.9^{o} = 23.2\mathrm{kW}$$

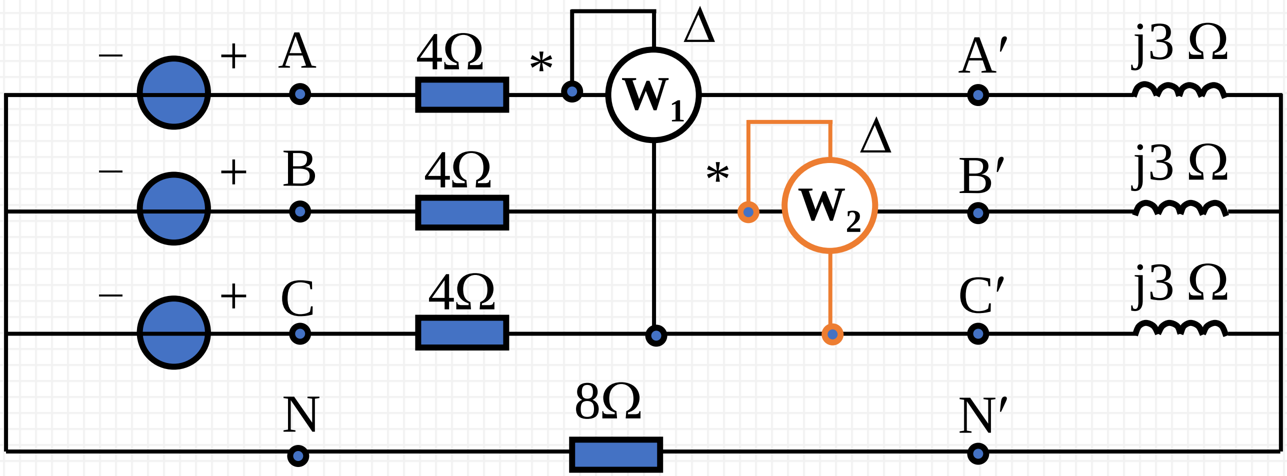
$$Q = 3U_{p}I_{p}\sin\phi_{p} = 3\times 220\times 44\sin 36.9^{o} = 17.4\mathrm{kvar}$$

另一种求法:

$$P = 3I_{l}^{2}R = 3\times 44^{2}\times 4 = 23.2\mathrm{kW}$$

$$Q = 3I_{l}^{2}X = 3\times 44^{2}\times 3 = 17.4\mathrm{kvar}$$

（4）可以用两表法测负载功率



$$\dot{U}_{A'B'} = \sqrt{3}\ 132\angle 83.1^{\mathrm{o}}\,\mathrm{V} \qquad \dot{I}_{A} = 44\angle -36.9^{\mathrm{o}}\,\mathrm{A}$$

$$\dot{U}_{B'C'} = \sqrt{3}\ 132\angle -36.9^{\mathrm{o}}\,\mathrm{V} \qquad \dot{I}_{B} = 44\angle -156.9^{\mathrm{o}}\,\mathrm{A}$$

W_2

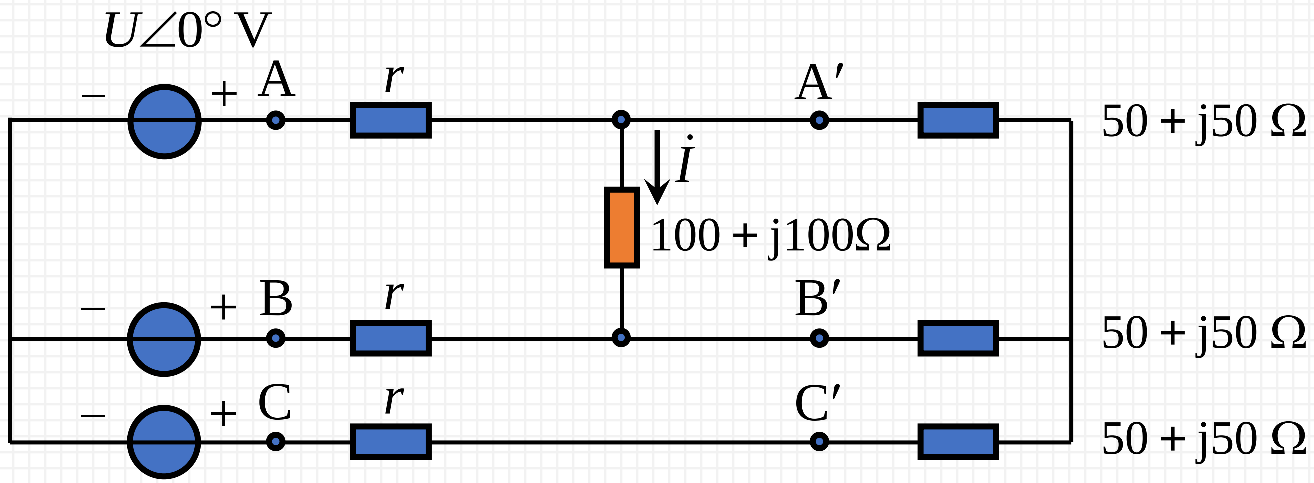
$$U_{B'C'} I_{B} \cos[(-36.9^{\mathrm{o}}) - (-156.9^{\mathrm{o}})]$$

$$= \sqrt{3}132 \times 44 \cos 120^{\mathrm{o}} = -5029\mathrm{W}$$

W_1

$$= 5029\mathrm{W}$$

9. 电源三相对称，r 分别为0和10Ω时求 $\dot{I}$ 。



解: (1) $r=0$ $\quad \dot{I}=\dfrac{\dot{U}_{AB}}{100+\mathrm{j}100}=0.0122U\angle -15^\circ$

(2) $r=10\ \Omega$ 戴维南等效

开路电压：抽单相 $\dot{U}_{OC}=1.56U\angle 35.2^\circ$

等效内阻抗：交流电桥平衡 $Z_{eq}=18.03+\mathrm{j}1.64\Omega$

$$\dot{I}=\frac{\dot{U}_{OC}}{Z_{eq}+100+\mathrm{j}100}=0.01U\angle -5.53^\circ$$