清华大学2022春季学期

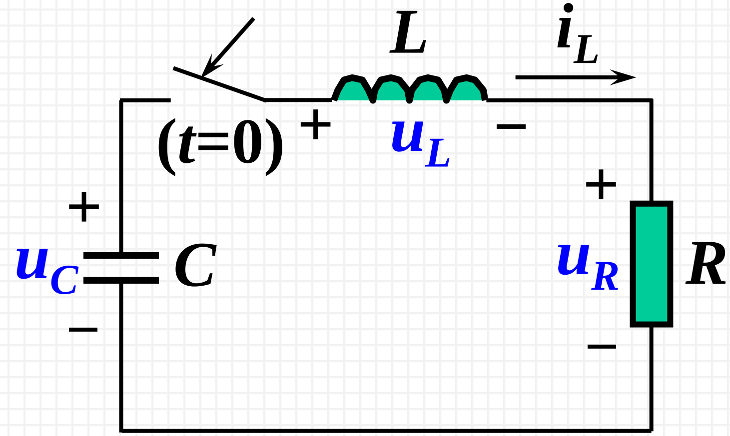
电路原理C

第11讲

二阶动态电路

内容

1、RLC串联二阶电路



零输入RLC串联

(1) 列方程

$$\begin{cases} u_C = L\dfrac{\mathrm{d}i_L}{\mathrm{d}t} + Ri_L \\ i_L = -C\dfrac{\mathrm{d}u_C}{\mathrm{d}t} \end{cases}$$

i_L 代入上式 →

$$\frac{\mathrm{d}^2 u_C}{\mathrm{d}t^2} + \frac{R}{L}\frac{\mathrm{d}u_C}{\mathrm{d}t} + \frac{1}{LC}u_C = 0$$

u_C 代入下式 →

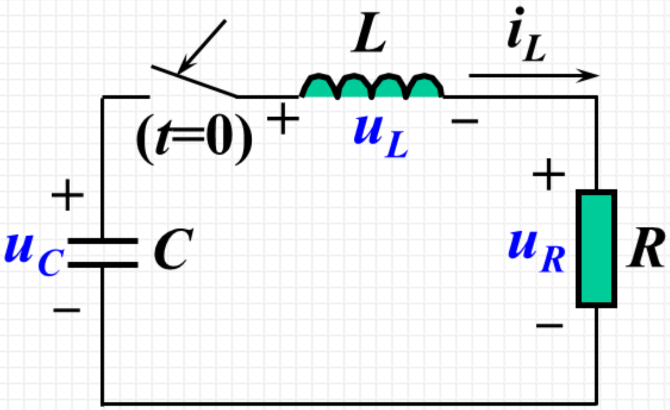
$$\frac{\mathrm{d}^2 i_L}{\mathrm{d}t^2} + \frac{R}{L}\frac{\mathrm{d}i_L}{\mathrm{d}t} + \frac{1}{LC}i_L = 0$$

衰减系数 α　2α　ω_0^2　自由振荡角频率/自然角频率 ω_0

课外练习：

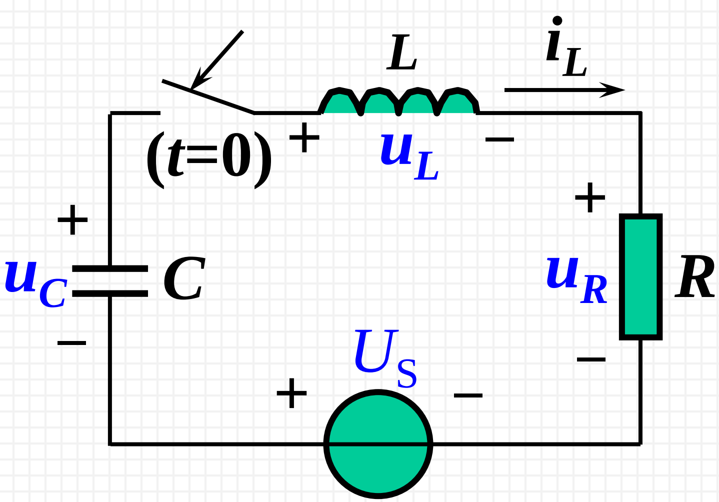
以u_R、u_L为变量列写微分方程。　**结果：特征方程都一样**

零输入RLC串联



以不同的变量列写方程，
得到的特征方程相同。

有输入RLC串联



$$\frac{d^2 i_L}{dt^2}+2\alpha\frac{di_L}{dt}+\omega_0^2 i_L=0$$

$$\frac{d^2 u_C}{dt^2}+2\alpha\frac{du_C}{dt}+\omega_0^2 u_C=0$$

$$\frac{d^2 u_L}{dt^2}+2\alpha\frac{du_L}{dt}+\omega_0^2 u_L=0$$

$$\frac{d^2 u_R}{dt^2}+2\alpha\frac{du_R}{dt}+\omega_0^2 u_R=0$$

可先列写零输入电路方程，求得特征根。

有独立源电路和零输入电路的特征方程相同。

$$\frac{d^2 i_L}{dt^2}+2\alpha\frac{di_L}{dt}+\omega_0^2 i_L=0$$

(2) 求自由分量

*LC*参数不变，随*R*增加，状态怎么变？

$$\frac{\mathrm{d}^2 u_C}{\mathrm{d}t^2} + 2\alpha\frac{\mathrm{d}u_C}{\mathrm{d}t} + \omega_0^2 u_C = 0$$

$$2\alpha = \frac{R}{L} \qquad \omega_0^2 = \frac{1}{LC}$$

$$p^2 + 2\alpha p + \omega_0^2 = 0$$

$\alpha^2 > \omega_0^2$

$$p_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

过阻尼

$$u_C = Ae^{p_1 t} + Be^{p_2 t}$$

$\alpha^2 = \omega_0^2$

$$p_1 = p_2 = -\alpha$$

临界阻尼

$$u_C = Ae^{p_1 t} + Bte^{p_2 t}$$

$\alpha^2 < \omega_0^2$

$$p_{1,2} = -\alpha \pm \mathrm{j}\sqrt{\omega_0^2 - \alpha^2}$$

欠阻尼

$$u_C = Ke^{-\alpha t}\sin\left(\omega_\mathrm{d} t + \theta\right)$$

$\alpha = 0$

$$p_{1,2} = \pm\mathrm{j}\omega_0$$

无阻尼

$$u_C = K\sin\left(\omega_0 t + \theta\right)$$

有关RLC串联欠阻尼3个参数的讨论

$$\frac{\mathrm{d}^2u_C}{\mathrm{d}t^2}+\frac{R}{L}\frac{\mathrm{d}u_C}{\mathrm{d}t}+\frac{1}{LC}u_C=0$$

2α $\qquad$ ω_0^2

衰减系数 α

自由振荡角频率/
自然角频率 ω_0

$$\frac{\mathrm{d}^2u_C}{\mathrm{d}t^2}+2\alpha\frac{\mathrm{d}u_C}{\mathrm{d}t}+\omega_0^2u_C=0$$

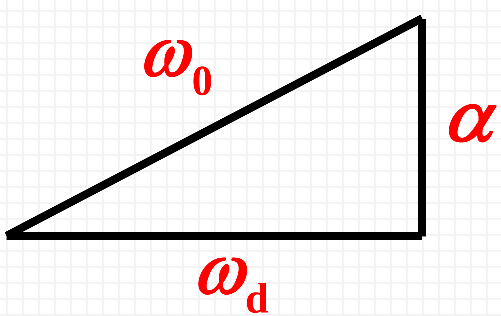
$$b^2-4ac<0$$

$$p_{1,2}=\frac{-2\alpha\pm \mathrm{j}2\sqrt{\omega_0^2-\alpha^2}}{2}$$

$$=-\alpha\pm \mathrm{j}\sqrt{\omega_0^2-\alpha^2}$$

$$=-\alpha\pm \mathrm{j}\omega_\mathrm{d}$$

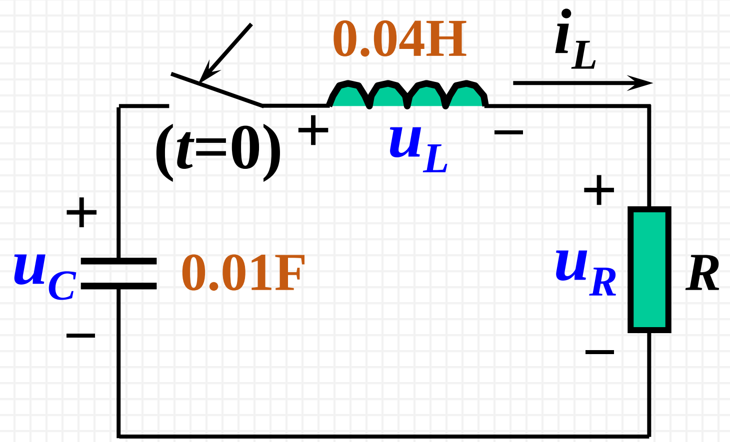
$$\omega_0^2=\omega_\mathrm{d}^2+\alpha^2$$



衰减振荡角频率 ω_d

数值例子 R分别为5Ω、4Ω、1Ω、0Ω时求$u_C(t)$、$i_L(t)$，$t \geq 0$

$$\frac{\mathrm{d}^2 u_C}{\mathrm{d}t^2} + \frac{R}{L}\frac{\mathrm{d}u_C}{\mathrm{d}t} + \frac{1}{LC}u_C = 0$$



$u_C(0^-) = 3\text{V}$，$i_L(0^-) = 0$

$$\frac{\mathrm{d}^2 u_C}{\mathrm{d}t^2} + 25R\frac{\mathrm{d}u_C}{\mathrm{d}t} + 2500u_C = 0$$

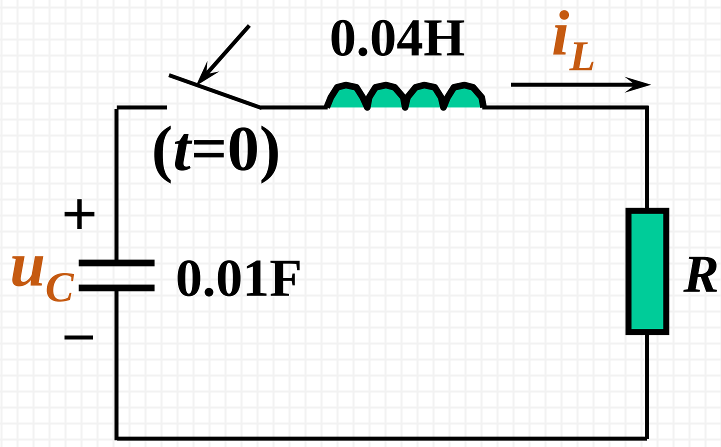
特征方程

$$p^2 + 25Rp + 2500 = 0$$

$$b^2 - 4ac = 625R^2 - 10000$$

$$p^2 + 25Rp + 2500 = 0$$

$$b^2 - 4ac = 625R^2 - 10000$$



$u_C(0^-) = 3\text{V},\ i_L(0^-) = 0$

$R = 5\Omega$

$$\begin{cases} b^2 - 4ac = 5625 > 0 \\ p_1 = -25 \quad p_2 = -100 \\ u_C(t) = A_1 e^{-25t} + A_2 e^{-100t} \end{cases}$$

$R = 4\Omega$

$$\begin{cases} b^2 - 4ac = 0 \\ p_1 = p_2 = -50 \\ u_C(t) = A_1 e^{-50t} + A_2 t e^{-50t} \end{cases}$$

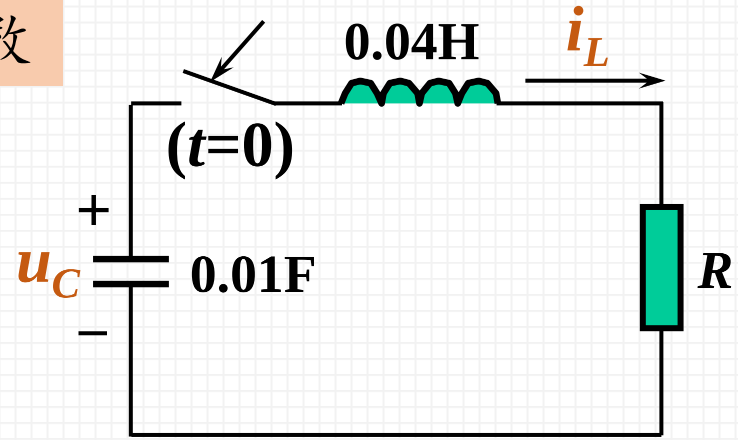
$R = 1\Omega$

$$\begin{cases} b^2 - 4ac = -9375 < 0 \\ p_{1,2} = -12.5 \pm \mathrm{j}48.4 \\ u_C(t) = K e^{-12.5t} \sin(48.4t + \theta) \end{cases}$$

$R = 0\Omega$

$$\begin{cases} p_{1,2} = \pm \mathrm{j}50 \\ u_C(t) = K \sin(50t + \theta) \end{cases}$$

(3) 用初值确定待定系数



$u_C(0^-) = 3\text{V}$

$i_L(0^-) = 0$

$$\left.\frac{\mathrm{d}u_C}{\mathrm{d}t}\right|_{0^+} = 0$$

$R=5\Omega$

$$\begin{cases} u_C(t) = A_1 \mathrm{e}^{-25t} + A_2\,\mathrm{e}^{-100t} \\ A_1 + A_2 = 3 \\ -25A_1 - 100A_2 = 0 \end{cases} \rightarrow \quad A_1 = 4 \quad A_2 = -1$$

$$u_C(t) = 4\mathrm{e}^{-25t} - \mathrm{e}^{-100t}\ \text{V} \qquad t > 0^+$$

如何直接求i_L?

$$C\frac{\mathrm{d}u_C}{\mathrm{d}t} = -i_L$$

$$i_L(t) = \mathrm{e}^{-25t} - \mathrm{e}^{-100t}\,\text{A} \qquad t > 0^+$$

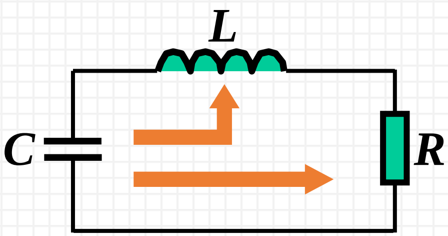
波形图和能量转换关系

$R=5\Omega$

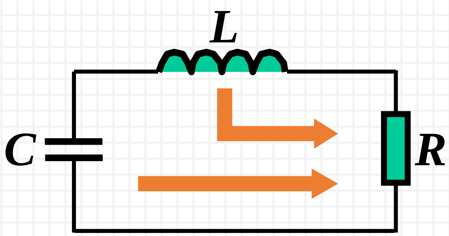
$$u_C(t)=4e^{-25t}-e^{-100t}\text{V}\quad t>0^+$$

$$i_L(t)=e^{-25t}-e^{-100t}\text{A}\quad t>0^+$$

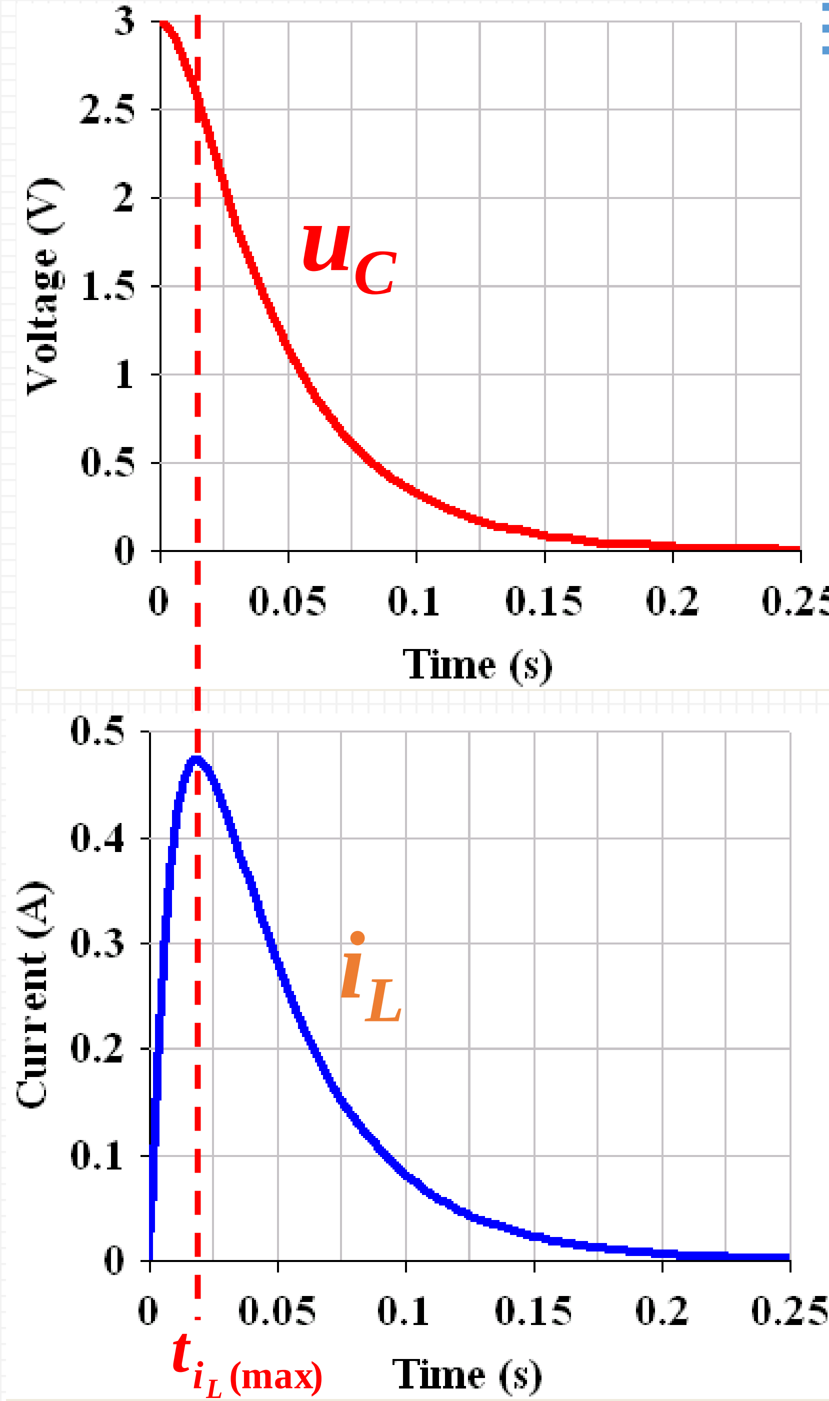
$0<t<t_{i_L(\max)}$ u_C 减小，i_L增大。

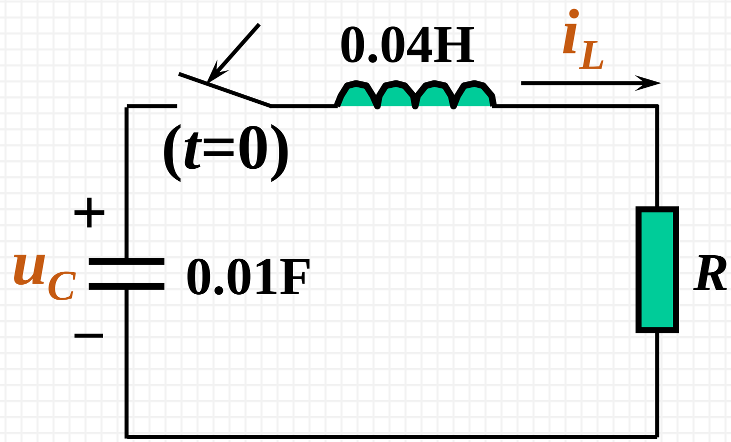


$t>t_{i_L(\max)}$ u_C 减小，i_L减小。



非振荡放电 **过阻尼**





$$u_C(0) = 3\text{V}$$

$$\left.\frac{\mathrm{d}u_C}{\mathrm{d}t}\right|_{t=0^+} = 0$$

$R = 4\Omega$

$$\begin{cases} u_C(t) = A_1 e^{-50t} + A_2 t\, e^{-50t} \\ A_1 = 3 \\ -50A_1 + A_2 = 0 \end{cases}$$

$\rightarrow$ $A_1 = 3 \quad A_2 = 150$

$$\boxed{u_C(t) = 3e^{-50t}(1 + 50t)\text{V} \quad t > 0^+}$$

$$\boxed{C\frac{\mathrm{d}u_C}{\mathrm{d}t} = -i_L}$$

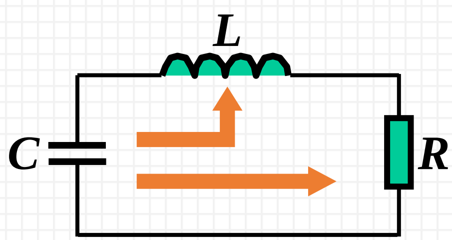
$$\boxed{i_L(t) = 75te^{-50t}\,\text{A} \quad t > 0^+}$$

$R = 4\Omega$

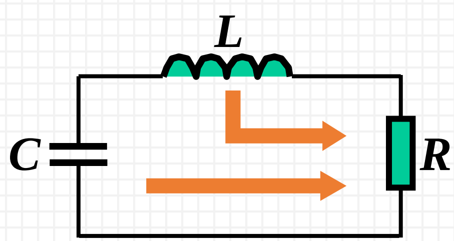
$$u_C(t) = 3e^{-50t}(1+50t)\text{V} \quad t > 0^+$$

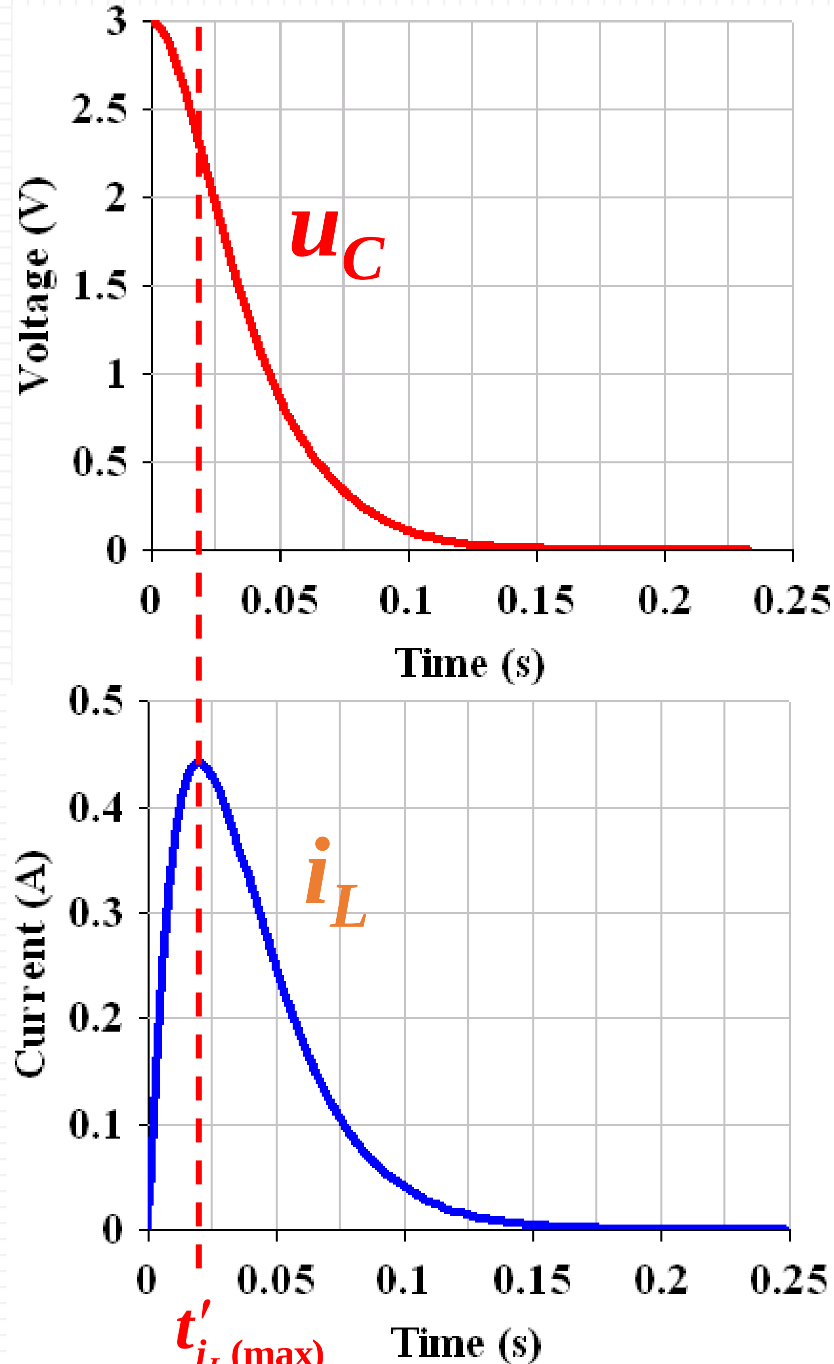
$$i_L(t) = 75te^{-50t}\text{A} \quad t > 0^+$$

$0 < t < t'_{i_L(\max)}$ u_C 减小，i_L增大。

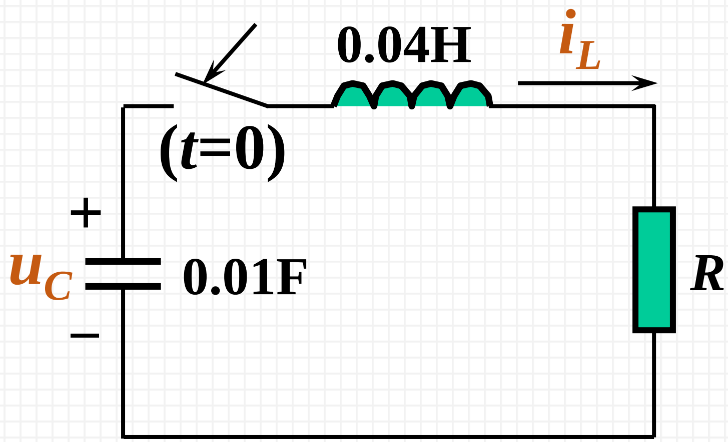


$t > t'_{i_L(\max)}$ u_C 减小，i_L减小。





非振荡放电 **临界阻尼**



$$u_C(0)=3\text{V}$$

$$\left.\frac{\mathrm{d}u_C}{\mathrm{d}t}\right|_{t=0^+}=0$$

$R=1\Omega$

$$\begin{cases} u_C(t)=K\mathrm{e}^{-12.5t}\sin(48.4t+\theta) \\ K\sin\theta=3 \\ -12.5K\sin\theta+48.4K\cos\theta=0 \end{cases}$$

$$K=3.1 \quad \theta=75.5^{\circ}$$

$$u_C(t)=3.10\mathrm{e}^{-12.5t}\sin(48.4t+75.5^{\circ})\text{V} \quad t>0^+$$

$$C\frac{\mathrm{d}u_C}{\mathrm{d}t}=-i_L$$

$$i_L(t)=1.55\mathrm{e}^{-12.5t}\sin(48.4t)\text{A} \quad t>0^+$$

$R=1\Omega$

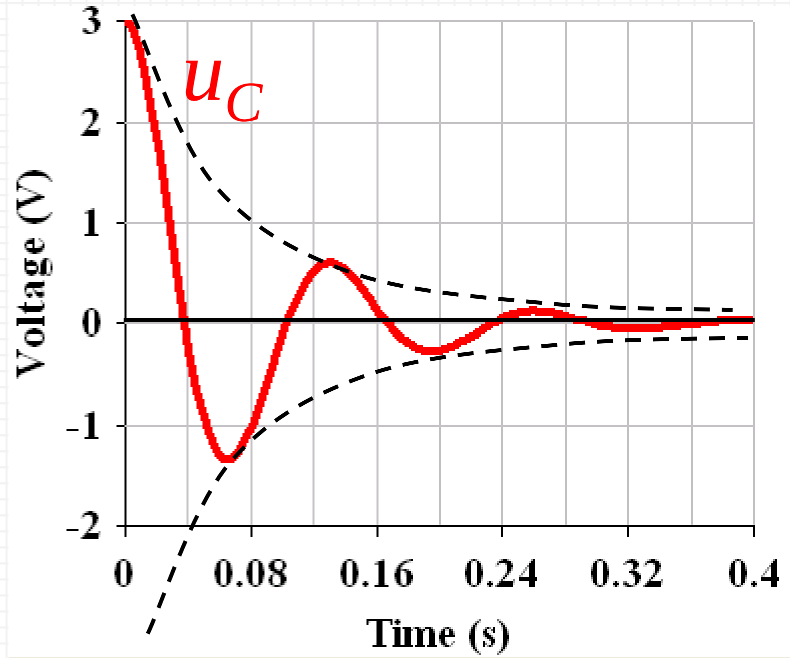
$$u_C(t)=3.10\mathrm{e}^{-12.5t}\sin(48.4t+75.5^{\circ})\mathrm{V}$$

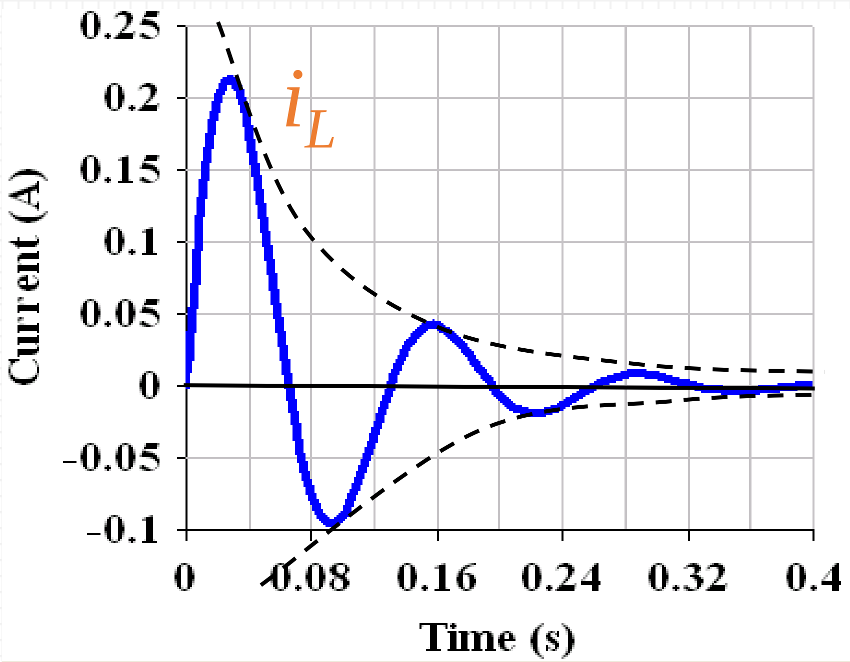
$$t>0^{+}$$

$$i_L(t)=1.55\mathrm{e}^{-12.5t}\sin 48.4t\mathrm{A}$$

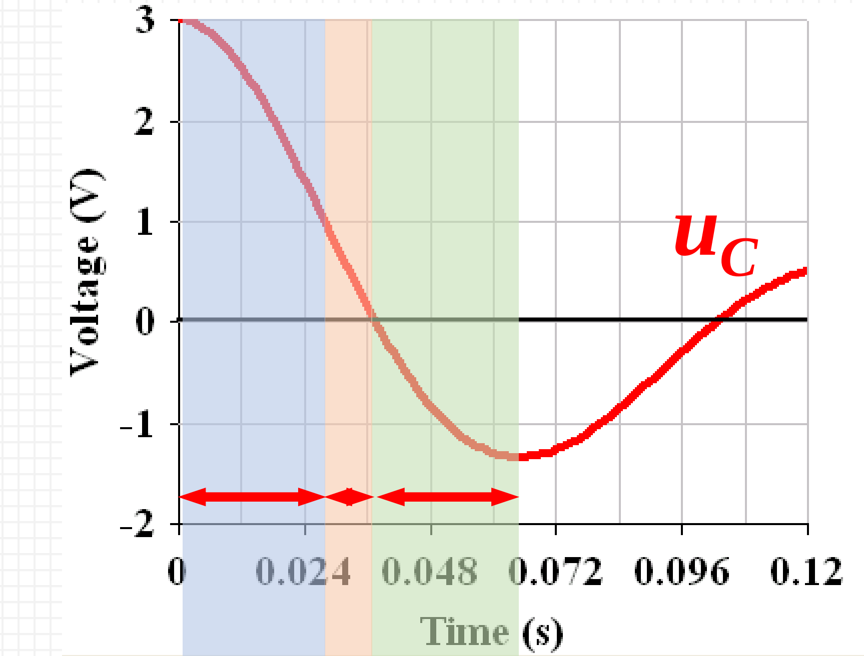
$$t>0^{+}$$

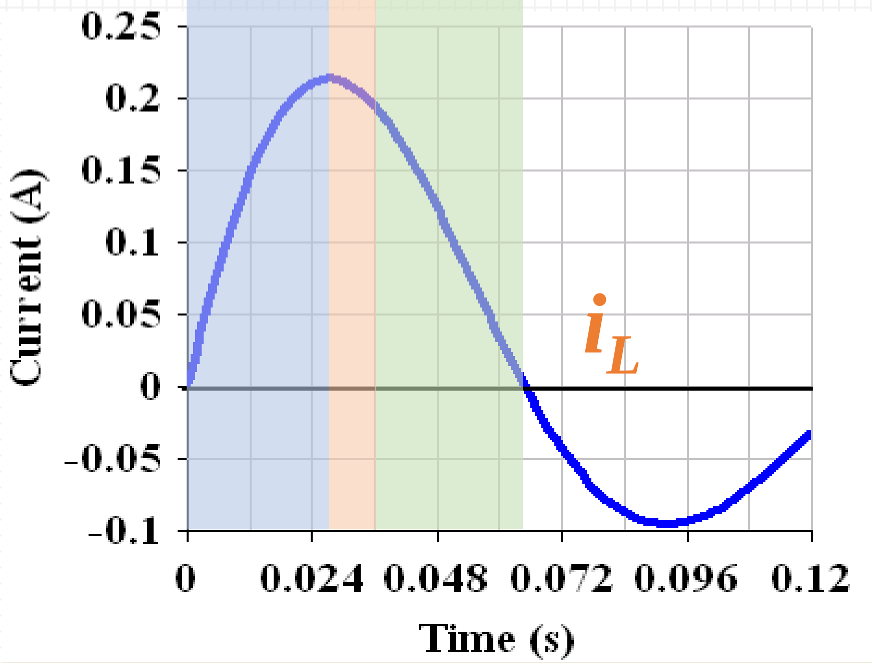
衰减振荡　**欠阻尼**





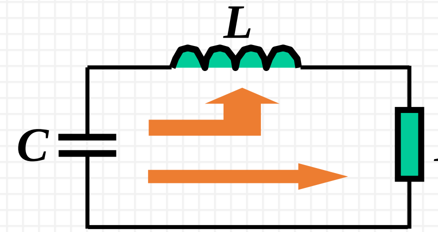
讨论半个周期中能量的关系





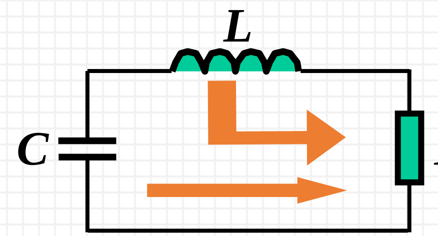
$0 \le 48.4t \le 75.5^{\circ}$

u_C减小，i_L增大



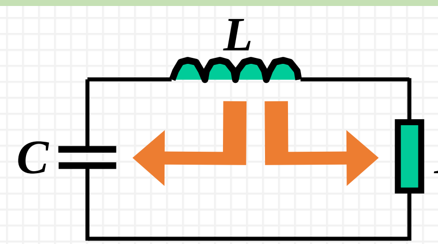
$75.5^{\circ} \le 48.4t \le 104.5^{\circ}$

u_C减小，i_L减小



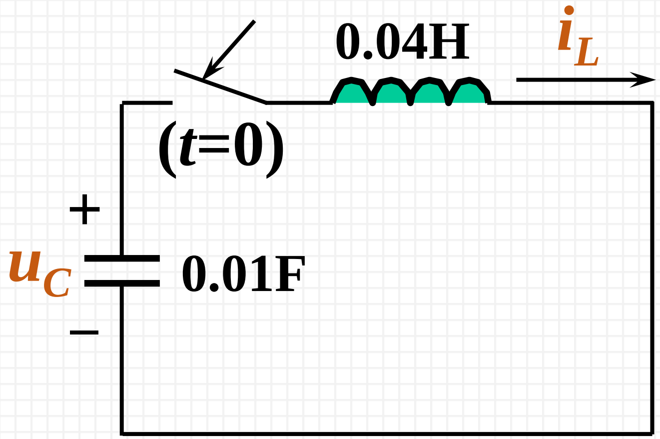
$104.5^{\circ} \le 48.4t \le 180^{\circ}$

$|u_C|$增大，i_L减小



周而复始，电阻不断消耗能量，u_C i_L衰减到零

$R=0$



$$LC\frac{\mathrm{d}^2u_C}{\mathrm{d}t}+u_C=0$$

$$p^2+2500=0 \qquad p=\pm \mathrm{j}50$$

$$u_C(t)=K\sin(50\,t+\theta)$$

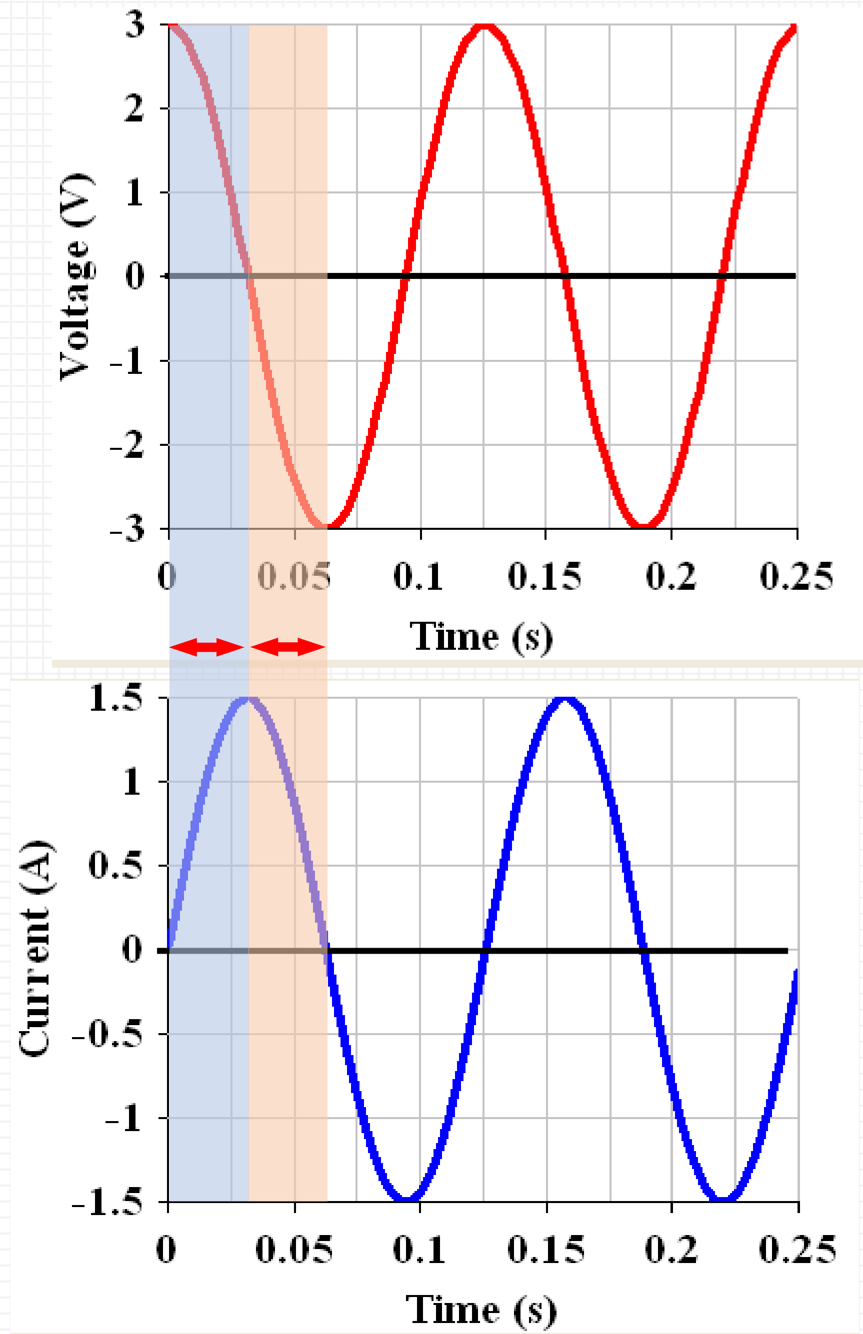
$$u_C(0)=3 \qquad \left.\frac{\mathrm{d}u_C}{\mathrm{d}t}\right|_{t=0^+}=0$$

$$K=3 \quad \theta=90^{\circ}$$

$$u_C(t)=3\cos 50t \text{ V} \quad t>0^+$$

$$i_L(t)=1.5\sin 50t \; A \quad t>0^+$$

等幅振荡　**无阻尼**

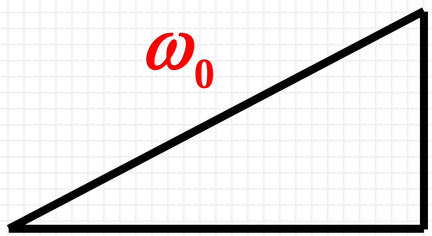


RLC串联二阶

$$p^2 + 2\alpha p + \omega_0^2 = p^2 + \frac{R}{L}p + \frac{1}{LC} = 0$$

衰减系数

$$\alpha = \frac{R}{2L}$$



自由振荡角频率/自然角频率

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

衰减振荡角频率

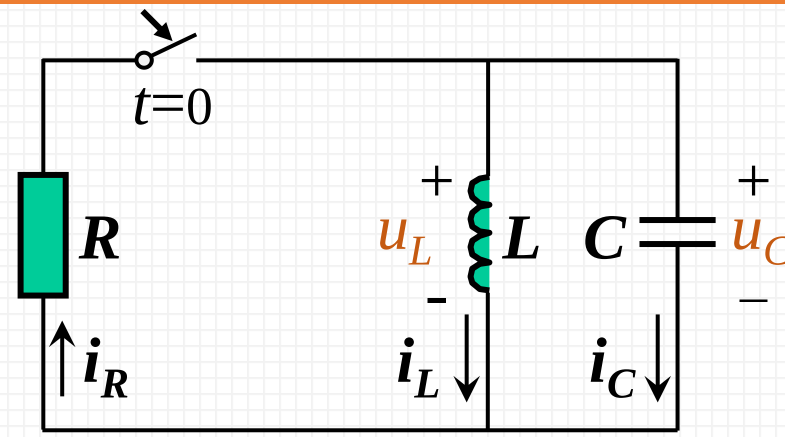
$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

$$u_C = K e^{-\alpha t}\sin\left(\omega_d t + \theta\right)$$

$\alpha^2 > \omega_0^2$	$\alpha^2 = \omega_0^2$	$\alpha^2 < \omega_0^2$	$\alpha = 0$
过阻尼	**临界阻尼**	**欠阻尼**	**无阻尼**

2、RLC并联二阶电路

零输入RLC并联



$$\begin{cases} i_R = i_L + C\dfrac{\mathrm{d}u_C}{\mathrm{d}t} \\ u_C = L\dfrac{\mathrm{d}i_L}{\mathrm{d}t} \\ i_R = -\dfrac{u_C}{R} \end{cases}$$

$$\frac{\mathrm{d}^2 i_L}{\mathrm{d}t^2} + \frac{1}{RC}\frac{\mathrm{d}i_L}{\mathrm{d}t} + \frac{1}{LC} i_L = 0$$

$$2\alpha = \frac{1}{RC}$$

$$\omega_0^2 = \frac{1}{LC}$$

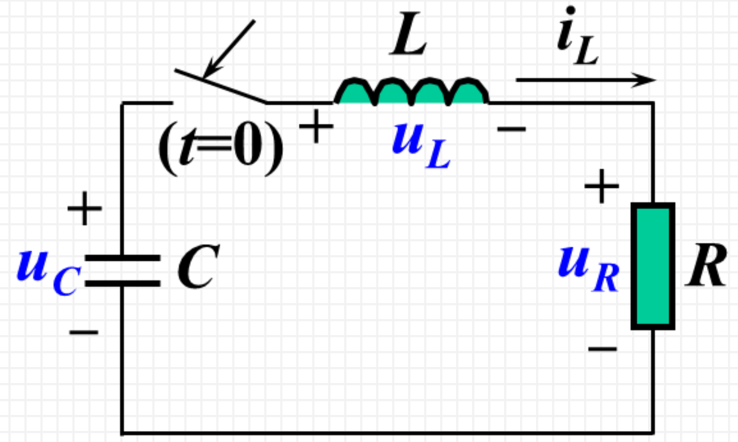
$$\frac{\mathrm{d}^2 i_L}{\mathrm{d}t^2} + 2\alpha\frac{\mathrm{d}i_L}{\mathrm{d}t} + \omega_0^2 i_L = 0$$

……

RLC串联

$$2\alpha = \frac{R}{L}$$

$$\omega_0^2 = \frac{1}{LC}$$



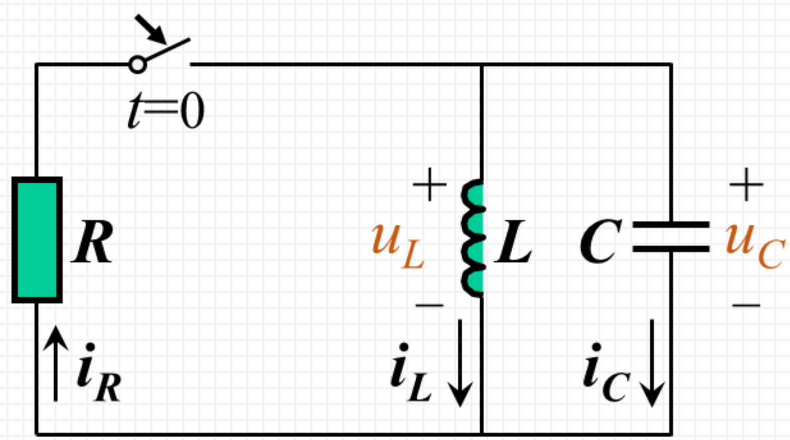
对偶!

$$p^2 + 2\alpha p + \omega_0^2 = 0$$

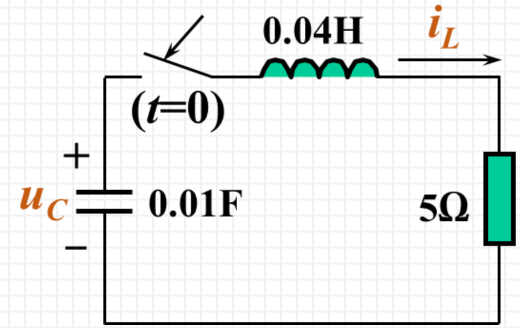
RLC并联

$$2\alpha = \frac{1}{RC}$$

$$\omega_0^2 = \frac{1}{LC}$$



3、二阶电路的直觉解法



不求待定系数定性画支路量的变化曲线

$$p^2+\frac{R}{L}p+\frac{1}{LC}=0$$

$u_C(0^-) = 3V$

$i_L(0^-) = 0$

(1) 过阻尼或临界阻尼（无振荡衰减）

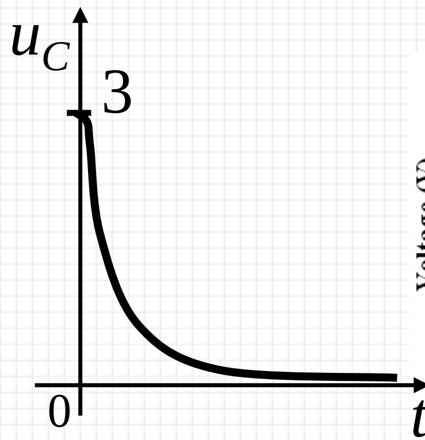
以过阻尼为例

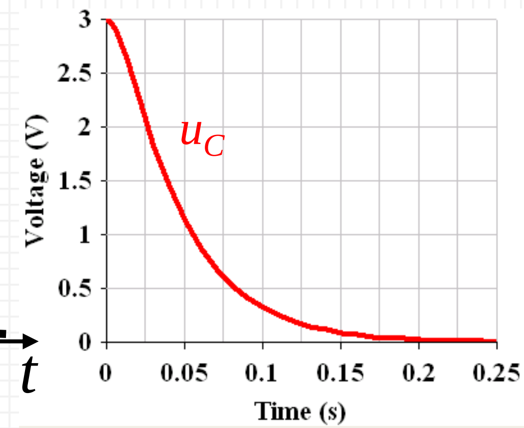
$$p_1=-25 \quad p_2=-100$$

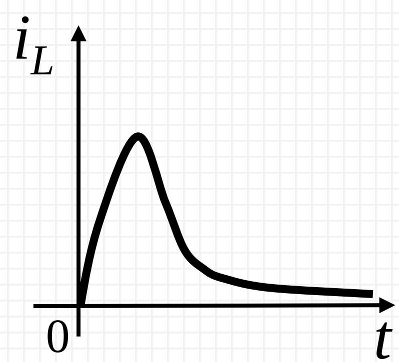
$$\begin{cases} u_C(0^+)=3V \\ \left.\dfrac{du_C}{dt}\right|_{t=0^+}=0 \end{cases}$$

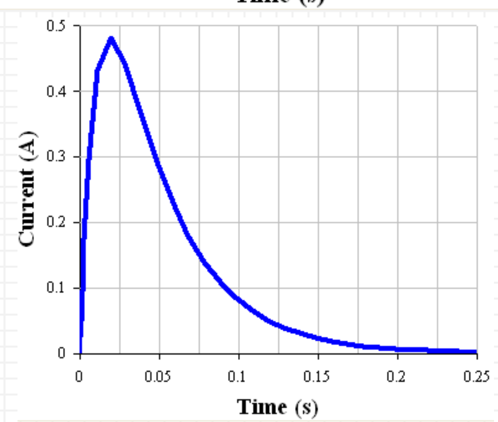
- 初值
- 导数初值
- 终值

$$\begin{cases} \left.i_L\right|_{0^+}=0 \\ \left.\dfrac{di_L}{dt}\right|_{0^+}=\dfrac{1}{L}\left.u_L\right|_{0^+}=\dfrac{3}{L} \end{cases}$$





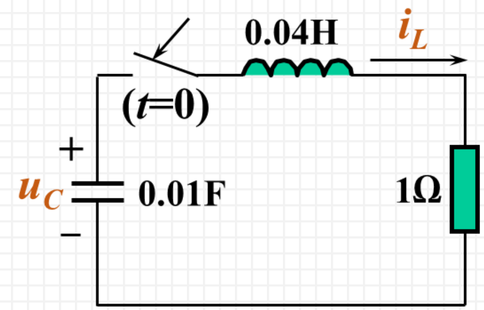




过（临界）阻尼，无振荡放电

(2) 欠阻尼（衰减振荡）

$$p^2+\frac{R}{L}p+\frac{1}{LC}=0$$



$$p_{1,2}=-12.5\pm j48.4$$

衰减振荡角频率 ω_d

衰减系数 α

$$\begin{cases} u_C(0^+)=3\text{V} \\ \left.\dfrac{du_C}{dt}\right|_{t=0^+}=0 \end{cases}$$

- 初值
- 导数初值
- 终值
- 经过多少周期振荡衰减完毕

$u_C(0^-)=3\text{V}$

$i_L(0^-)=0$

回忆一阶电路中的时间常数τ： 3～5 τ 后过渡过程结束

$$3\times\frac{1}{\alpha}=\frac{3}{12.5}=0.24\text{ s}$$

$$5\times\frac{1}{\alpha}=\frac{5}{12.5}=0.4\text{ s}$$

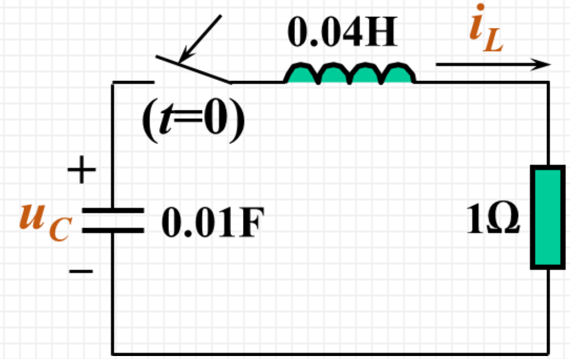
过渡过程结束

振荡周期为

$$T=\frac{2\pi}{\omega_d}=\frac{2\pi}{48.4}=0.13\text{ s}$$

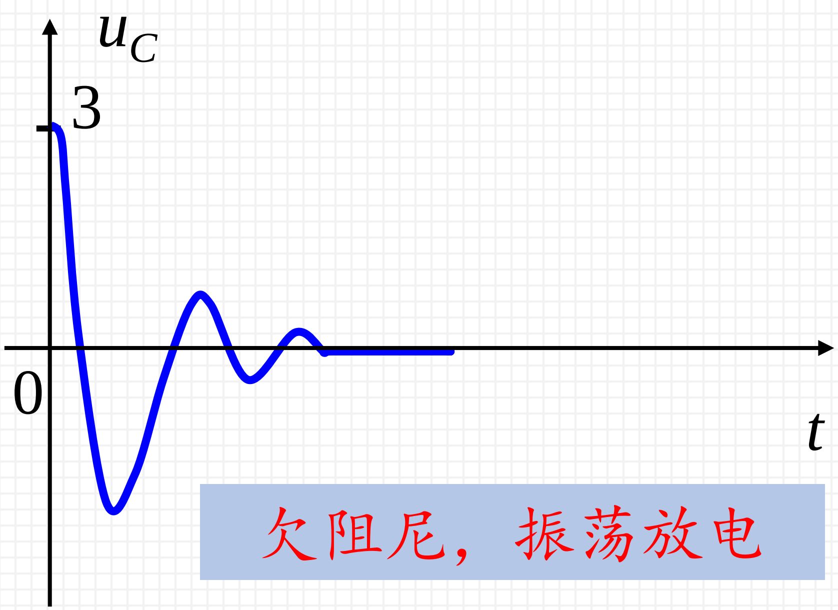
$$\lambda_{1,2} = -12.5 \pm j48.4$$

$$\begin{cases} u_C(0^+) = 3\text{V} \\ \left.\dfrac{\mathrm{d}u_C}{\mathrm{d}t}\right|_{t=0^+} = 0 \end{cases}$$

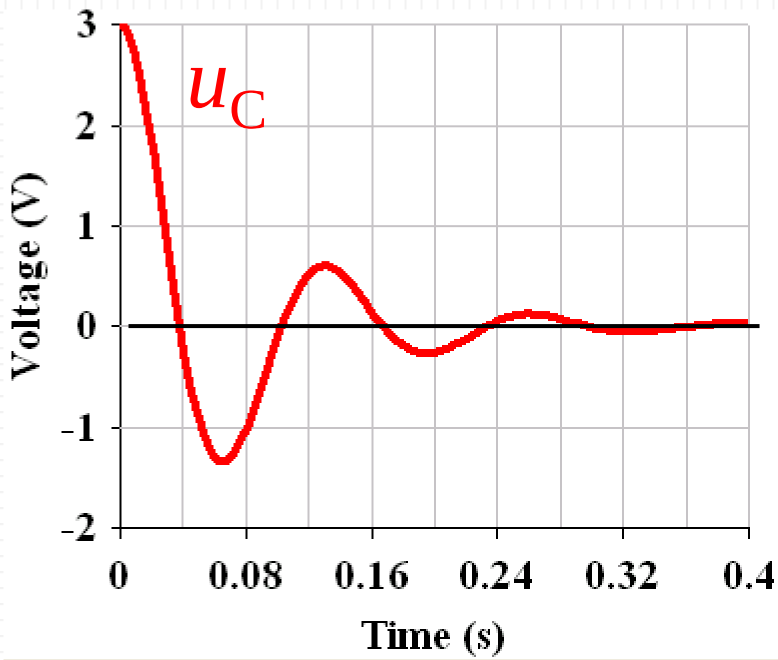


衰减过程中有
0.24/0.13≈2次振荡
或0.4/0.13≈3次振荡

- 初值
- 导数初值
- 终值
- 经过多少周期振荡衰减完毕



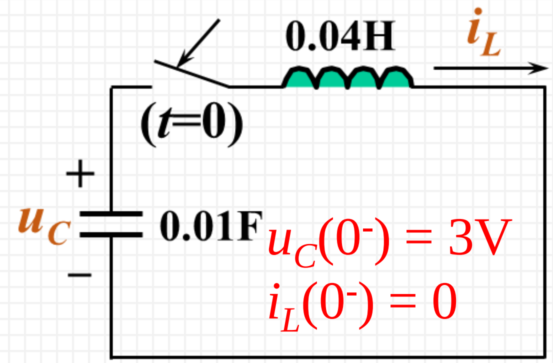
欠阻尼，振荡放电



(3) 无阻尼

$$p_{1,2}=\pm \mathrm{j}50$$

- 初值
- 导数初值
- 最大值

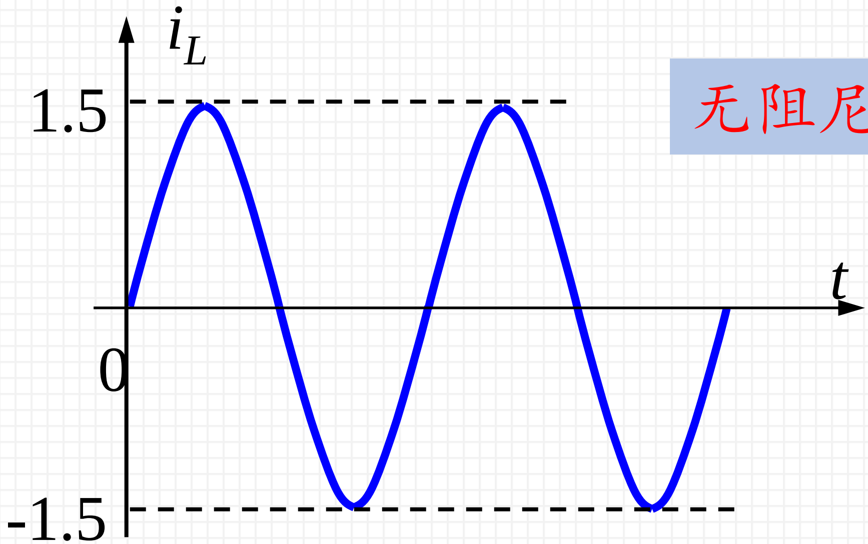


$$\begin{cases} i_L\big|_{0^+} = 0 \\ \dfrac{\mathrm{d}i_L}{\mathrm{d}t}\Big|_{0^+} = \dfrac{1}{L}u_L = \dfrac{3}{L} \end{cases}$$

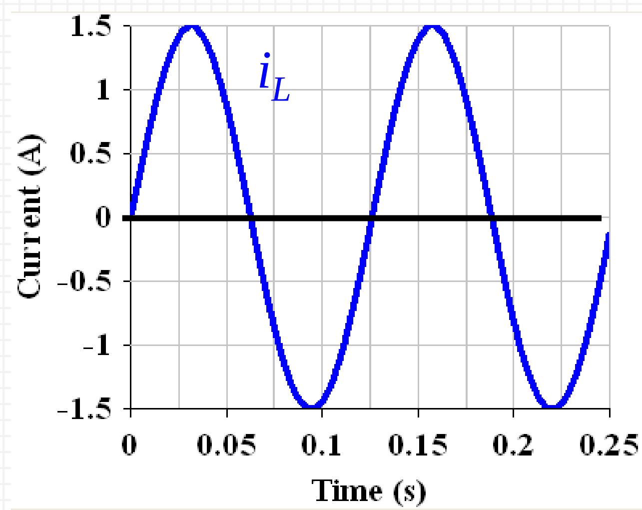
因为无阻尼，所以无能量损失

$$\frac{1}{2}Cu_C^2(0)+\frac{1}{2}Li_L^2(0)=\frac{1}{2}Cu_C^2(t)+\frac{1}{2}Li_L^2(t)$$

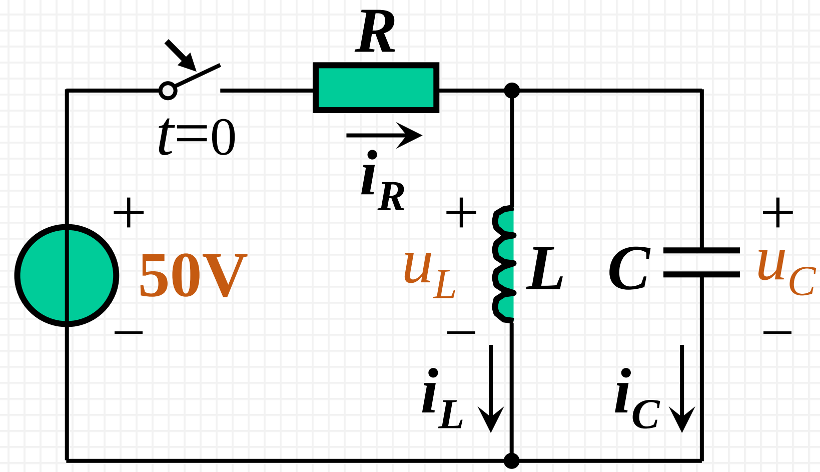
i_L取最大值时，$u_C=0$，因此 $i_{L,\max}=\sqrt{\dfrac{C}{L}}u_C(0)=1.5\text{A}$



无阻尼振荡



例 已知 $i_L(0)=2\text{A}$ $u_C(0)=0$
$R=50\Omega$, $L=0.5\text{H}$, $C=100\mu\text{F}$。
求：$i_R(t)$ 。



法1：列u_C的微分方程先求u_C再求i_R
法2：列i_R的微分方程求解
法3：通过求解一系列电阻电路求i_R

Step 1 由零输入电路得响应形式

零输入RLC并联

$$p^2+2\alpha p+\omega_0^2=0 \qquad 2\alpha=\frac{1}{RC}=200 \qquad \omega_0^2=\frac{1}{LC}=20000$$

$$p_{1,2}=-100\pm \text{j}100$$

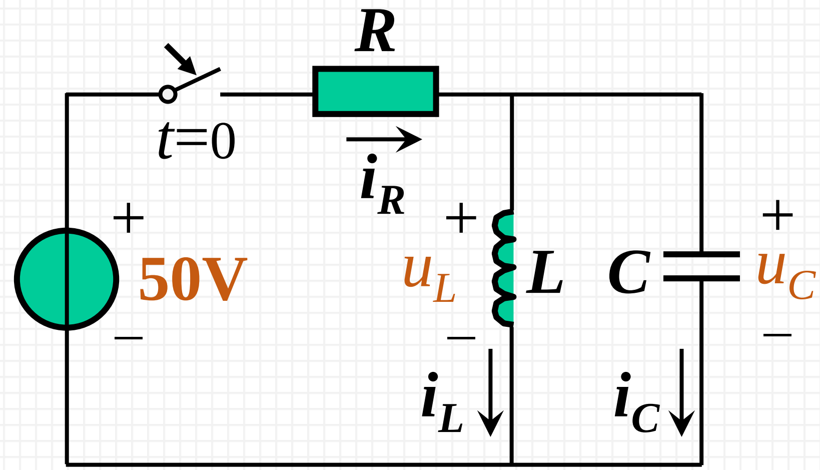
响应形式

$$i_R=i_R(\infty)+Ke^{-100t}\sin(100t+\theta)$$

已知 $i_L(0)=2\text{A}$ $u_C(0)=0$

$R=50\Omega$, $L=0.5\text{H}$, $C=100\mu\text{F}$。

求：$i_R(t)$ 。

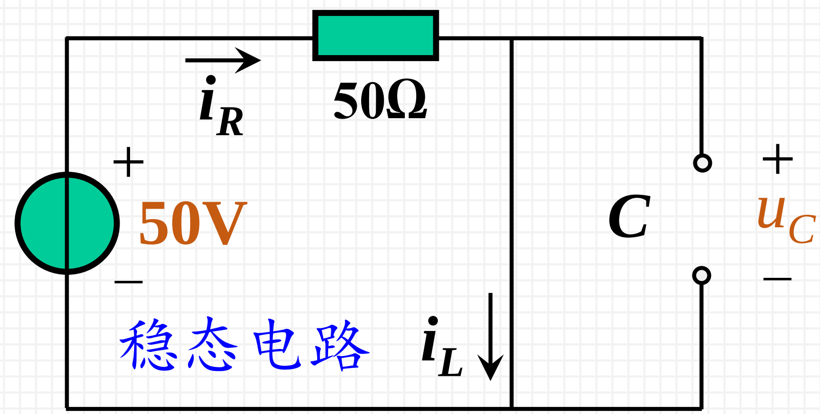


Step2 求稳态解

$$i_R(\infty)=1\text{A}$$

通解

$$i_R=1+Ke^{-100t}\sin\left(100t+\theta\right)$$

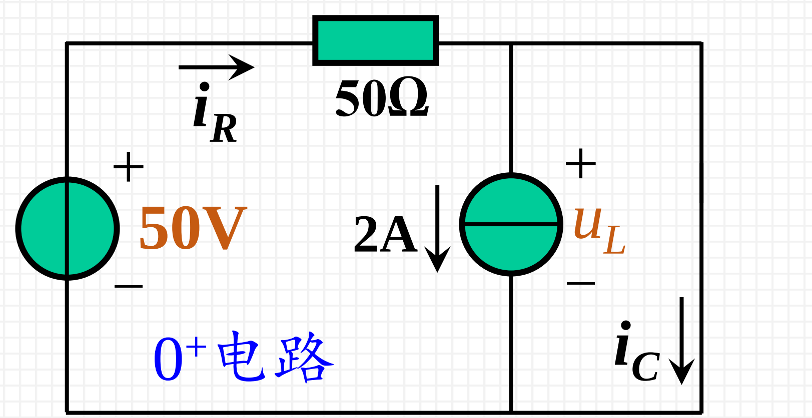


Step3 求初值 $i_L(0)=2\text{A}$ $u_C(0)=0$

$$i_R(0^+)=\frac{50-u_C(0^+)}{50}=1\text{A}$$

怎么求？

$$\left.\frac{\text{d}i_R}{\text{d}t}\right|_{t=0^+}$$



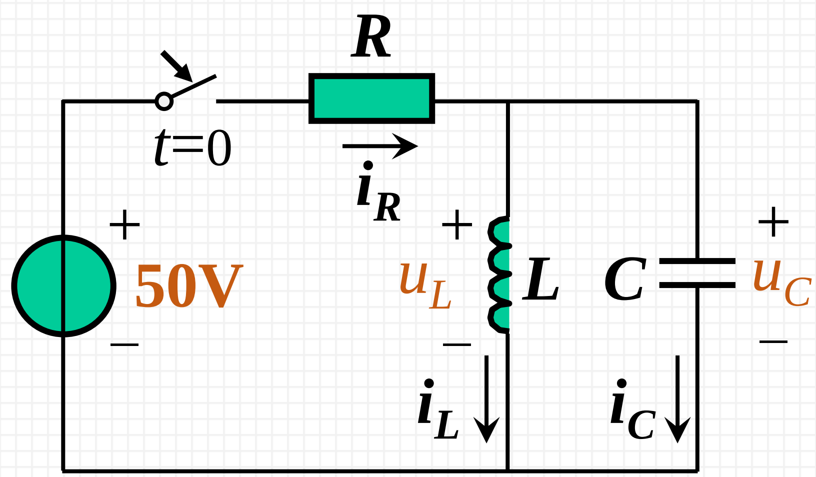
思路：用电源、u_C和i_L来表示i_R

$$i_R = \frac{50-u_C}{R}$$

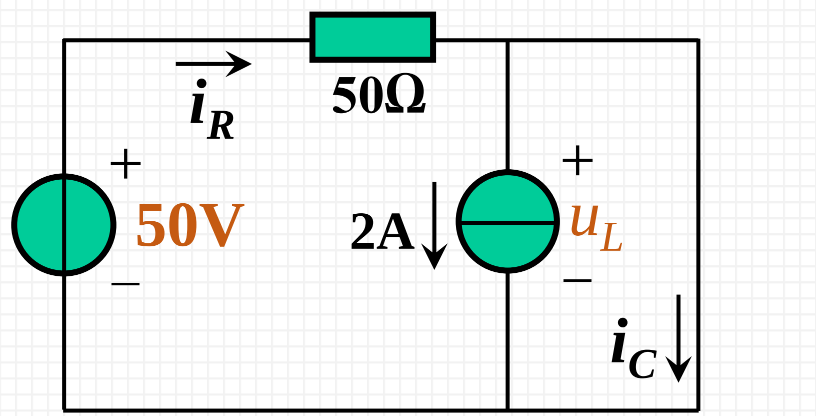
$$\frac{\mathrm{d}i_R}{\mathrm{d}t}\bigg|_{0+} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{50-u_C}{R}\right)\bigg|_{0+} = -\frac{1}{R}\frac{\mathrm{d}u_C}{\mathrm{d}t}\bigg|_{0+}$$

$$= -\frac{1}{RC}i_C(0^+)$$

$$= -\frac{-1}{50\times100\times10^{-6}} = 200\ \mathrm{A/s}$$



0^+电路

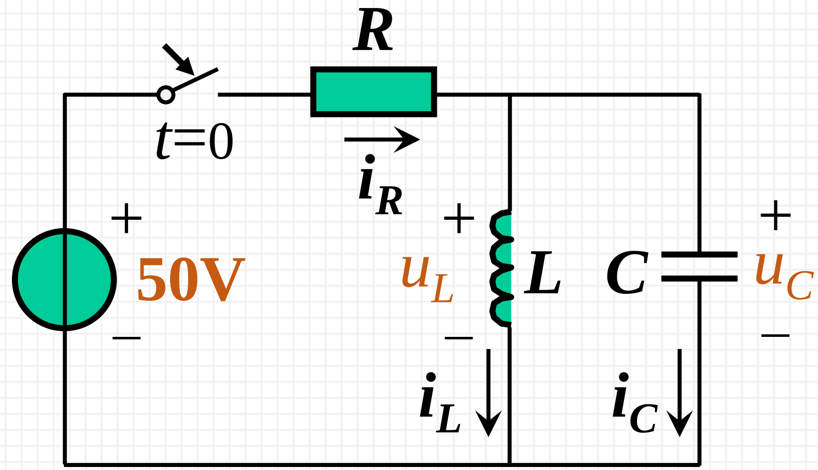


$$i_C(0^+) = -1\mathrm{A}$$

已知： $i_L(0)=2\text{A}$ $u_C(0)=0$

$R=50\Omega$, $L=0.5\text{H}$, $C=100\mu\text{F}$。

求： $i_R(t)$ 。



Step4 求待定系数

通解 $$i_R = 1 + Ke^{-100t}\sin(100t+\theta)$$

$$\begin{cases} i_R(0^+) = 1\text{A} \\ \left.\dfrac{\mathrm{d}i_R}{\mathrm{d}t}\right|_{0^+} = 200\ \text{A}/\text{s} \end{cases}$$

$$i_R(t) = 1 + 2e^{-100t}\sin 100t\ \text{A} \qquad t > 0^+$$

总结二阶电路的求解

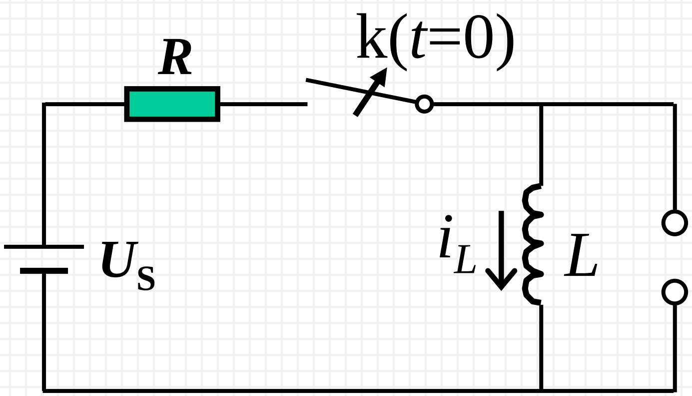
- 求响应形式
  - RLC串联、RLC并联 → 直接得到特征方程
  - ZIR非RLC串并联怎么办? → L12（根据状态方程）
- 求稳态值 → 得通解表达式
  - 电阻电路
- 求初值
  - 0^-电阻电路 → 换路定理得0^+电阻电路 → 初值
- 求导数初值
  - 将支路量用独立源、u_C、i_L来表示→0^+电路求i_C、u_L
  - L12（根据输出方程和状态方程）
- 用初值和导数初值确定通解待定系数

为什么一个动态电路中任意支路量都有相同的变化性质？（L12）

4、二阶电路的应用

（1）汽车点火系统

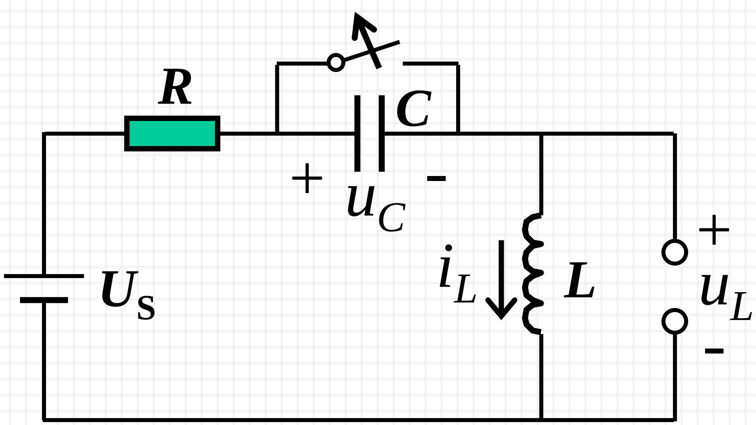
开关和气隙在搏命



一阶点火电路的问题：

开路开关和火花塞承受相同的无穷大电压

开关的设计耐压高于气隙的击穿电压



二阶点火电路的好处：

开路开关的电压被电容钳位

可通过电路参数控制开关电压

（2）电磁轨道炮电源

Dahlgren Surface Warfare Center

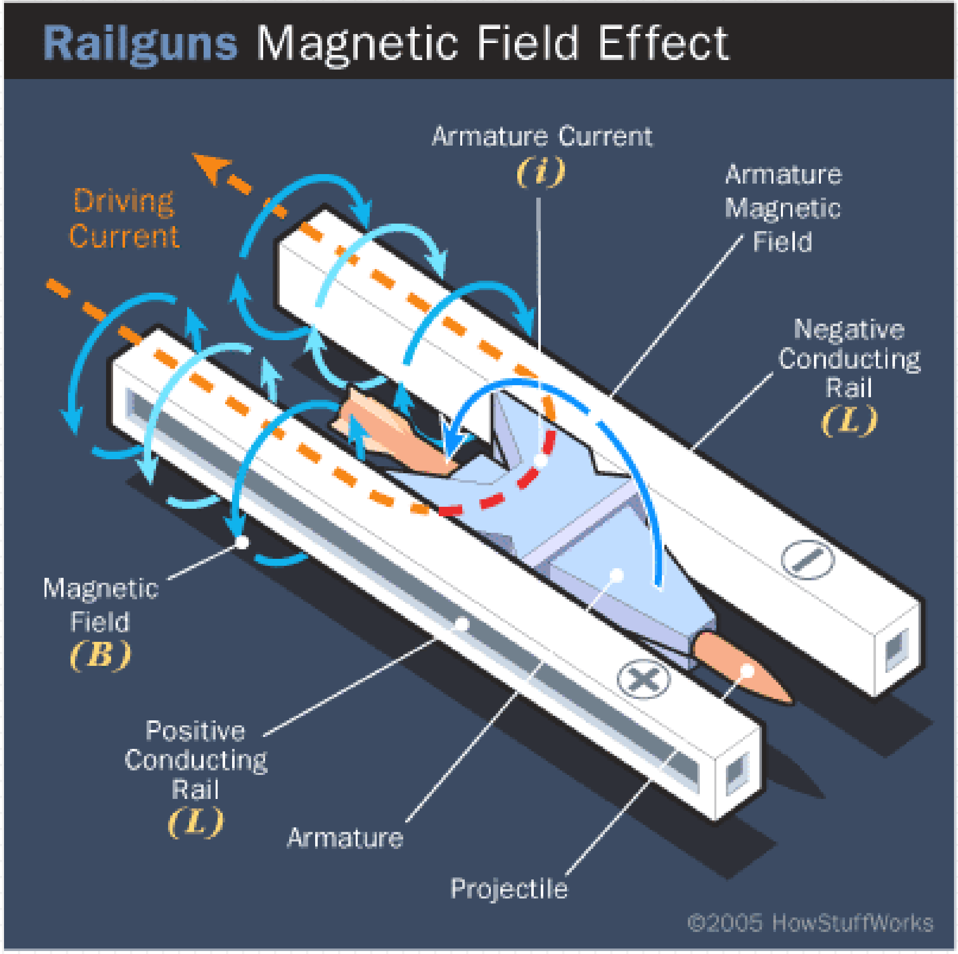
2012

电磁轨道炮原理

关键是

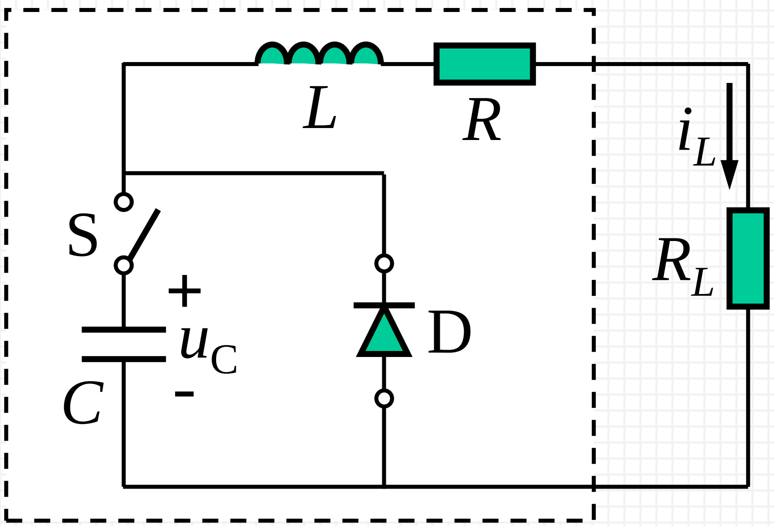
可控的脉冲大电流

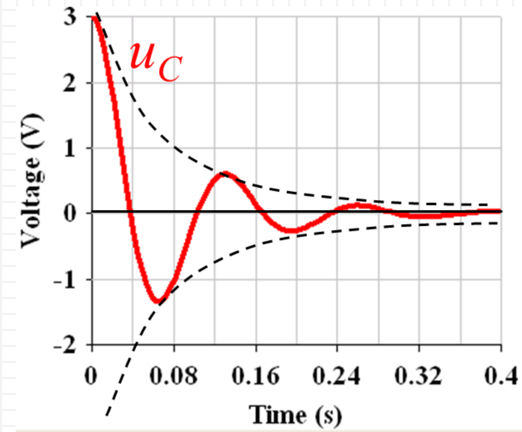
$$F=0.5L'i^2$$

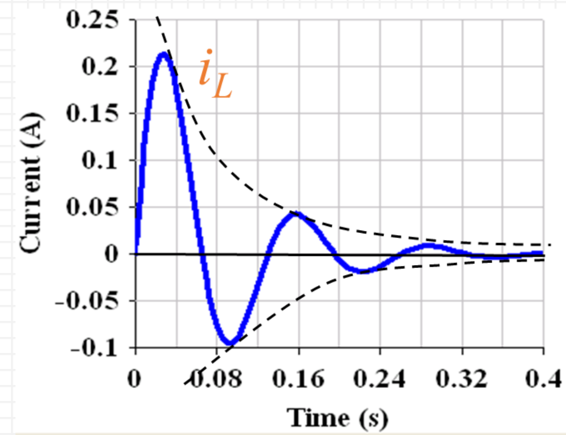


电磁轨道炮脉冲电源的基本电路（PFU）

（Pulse Forming Unit）

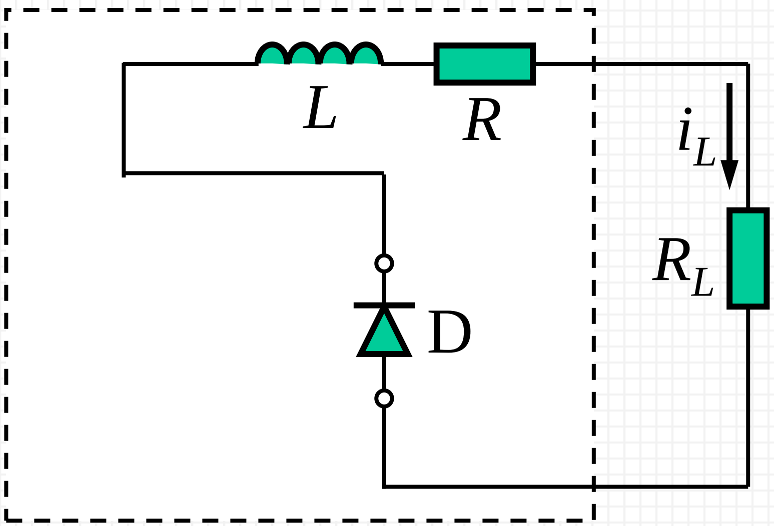






衰减振荡 **欠阻尼**

电磁轨道炮脉冲电源的基本电路（PFU）
（Pulse Forming Unit）



一阶RL放电电路

4个PFU并联