

2

电路原理习题课



求初始值的步骤

1. 由**换路前电路**（稳定状态）求 $u_C(0^-)$ 和 $i_L(0^-)$ 。

电阻电路1(直流)

2. 由**换路定律**得 $u_C(0^+)$ 和 $i_L(0^+)$ 。

3. 画出 **0^+ 时刻的等效电路**。

3.1 画换路后电路的拓扑结构；

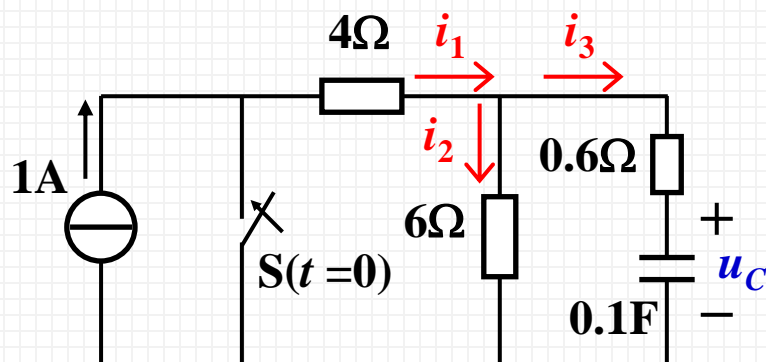
3.2 **电容**（**电感**）用**电压源**（**电流源**）替代。

取 **0^+ 时刻值**，方向同原假定的电容电压、
电感电流方向。

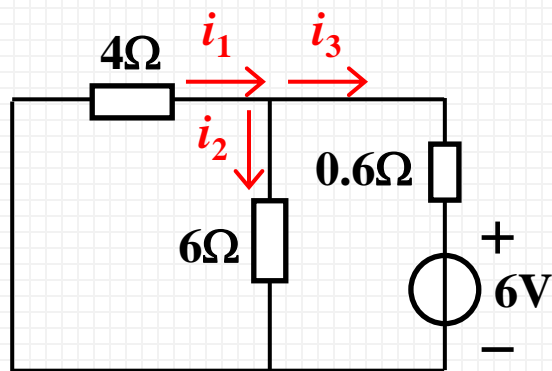
电阻电路2

4. 由 **0^+ 电路**求其它各变量的 **0^+ 值**。

1. 求: $i_1(0^+)$, $i_2(0^+)$, $i_3(0^+)$.



$t=0^+$ 时刻电路:



解:

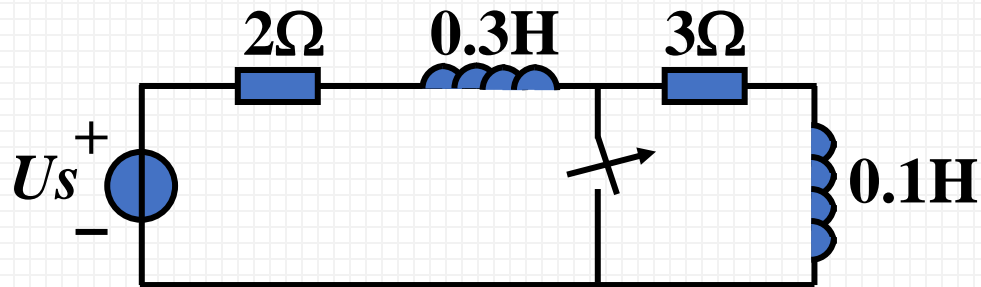
$$u_C(0^+) = u_C(0^-) = 6 \times 1 = 6 \text{ V}$$

$$i_3(0^+) = -\frac{6}{0.6 + 4 // 6} = -2 \text{ A}$$

$$i_1(0^+) = \frac{i_3(0^+) \times 6}{4 + 6} = -1.2 \text{ A}$$

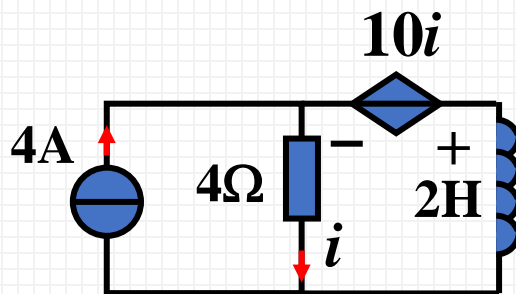
$$i_2(0^+) = -\frac{i_3(0^+) \times 4}{4 + 6} = 0.8 \text{ A}$$

2. ①求时间常数.



$$\tau = \frac{L}{R} = \frac{0.3+0.1}{2+3} = 0.08 \text{ s}$$

②求时间常数.



$$u = 10i + 4i = 14i$$

$$R_{\text{等}} = 14\Omega$$

$$\tau = 2/R_{\text{等}} = 2/14 = 0.143\text{s}$$

求解一阶电路的三要素法（适用直流与正弦激励）

$$i(t) = i(\infty) + [i(0^+) - i(\infty)] e^{-\frac{t}{\tau}} \quad t > 0$$

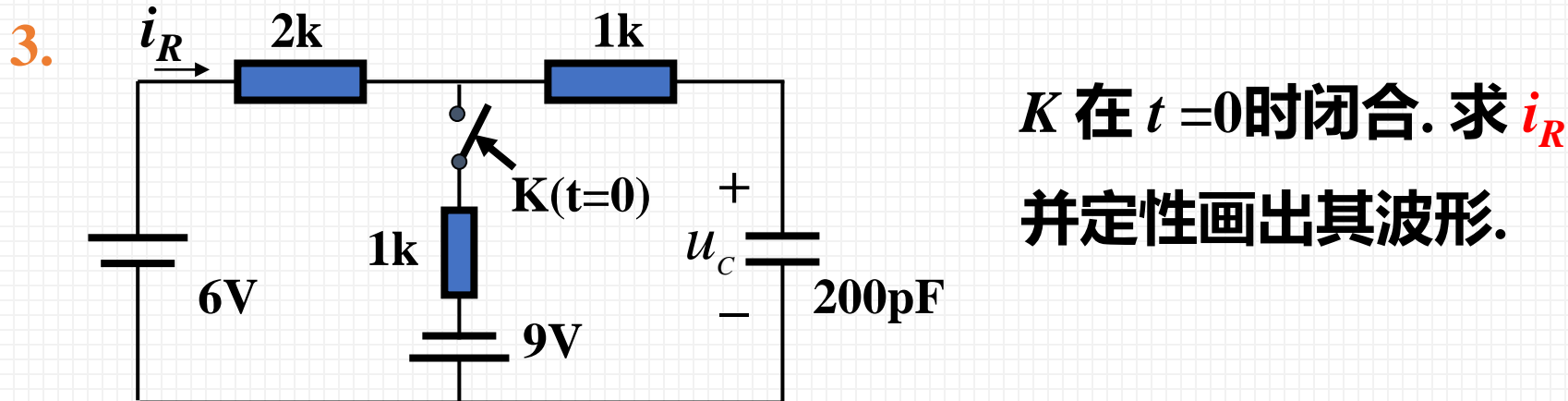
$i(\infty)$ 为 $i(t)|_{t \rightarrow \infty}$ 的简写

$t = 0$ 对正弦激励而言

$$\text{三要素} \begin{cases} f(\infty) & \text{稳态解} \\ f(0^+) & \text{起始值} \\ \tau & \text{时间常数} \end{cases}$$

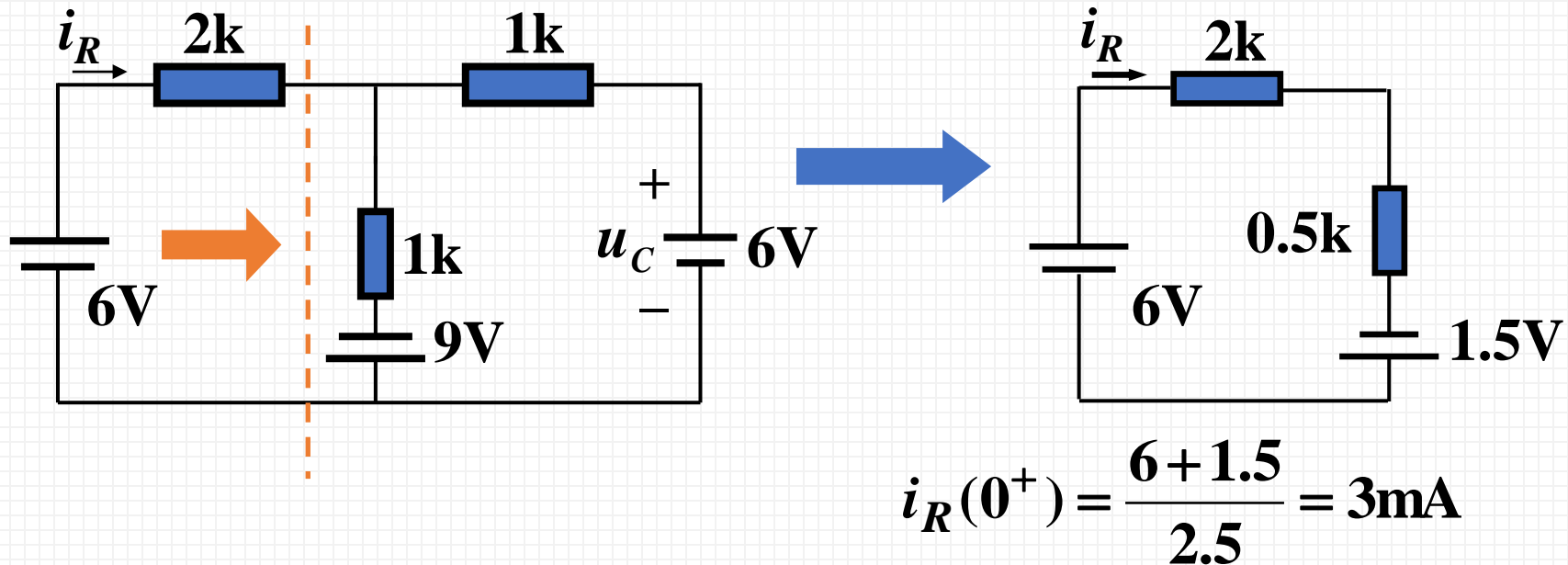
$$\text{全响应} = \text{零输入响应} + \text{零状态响应}$$

$$\text{全响应} = \begin{matrix} \text{通解} \\ \text{(自由分量)} \end{matrix} + \begin{matrix} \text{特解} \\ \text{(强制分量)} \end{matrix}$$



解: $u_C(0^+) = u_C(0^-) = 6V$

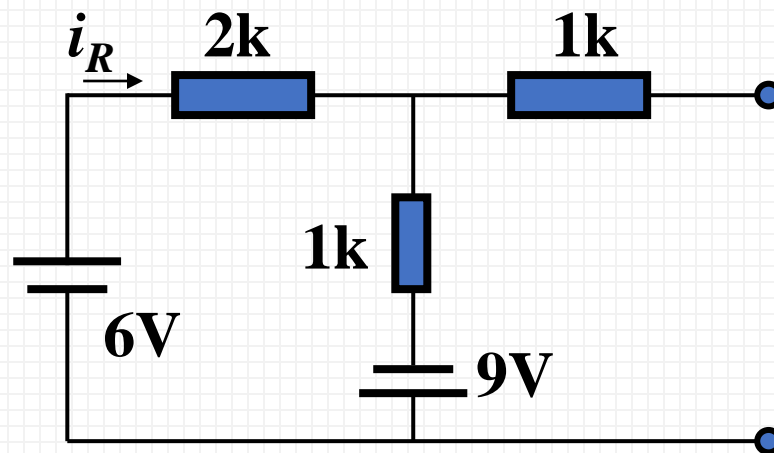
0^+ 时刻电路:



$$i_R(0^+) = 3\text{mA}$$

$t = \infty$ 时电路:

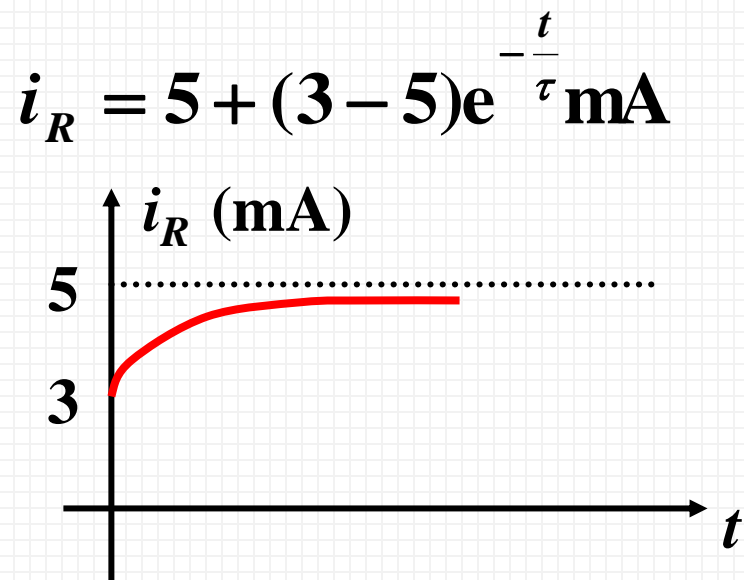
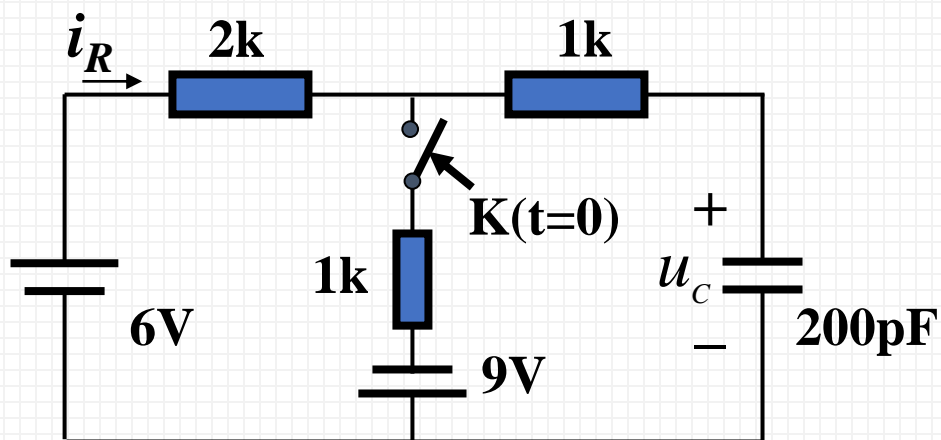
$$i_R(\infty) = \frac{6+9}{3} = 5\text{mA}$$



$$R_{\text{等}} = 1 + \frac{2 \times 1}{2 + 1} = \frac{5}{3} \text{K}\Omega$$

$$\tau = \frac{5}{3} \times 10^3 \times 200 \times 10^{-12} = \frac{1}{3} \times 10^{-6} \text{s}$$

$$i_R = 5 + (3 - 5)e^{-\frac{t}{\tau}} = 5 - 2e^{-\frac{t}{\tau}} \text{mA} \quad t \geq 0^+ \quad \underline{\underline{\quad \quad \quad}} \quad \bullet$$



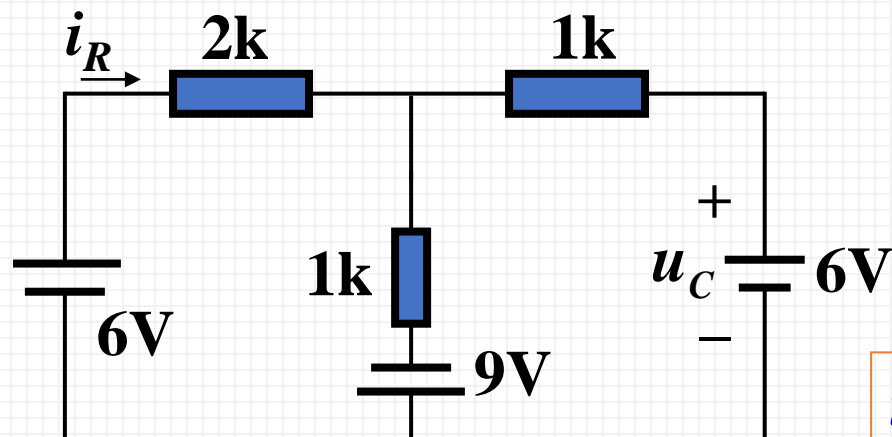
$$i_R(0^+) = 3\text{mA} \quad i_R(\infty) = 5\text{mA}$$

i_R 的零输入响应: ~~$i_{R\text{ZIR}} = 3e^{-\frac{t}{\tau}}$~~

i_R 的零状态响应: ~~$i_{R\text{ZSR}} = 5\left(1 - e^{-\frac{t}{\tau}}\right) \text{ mA}$~~

?

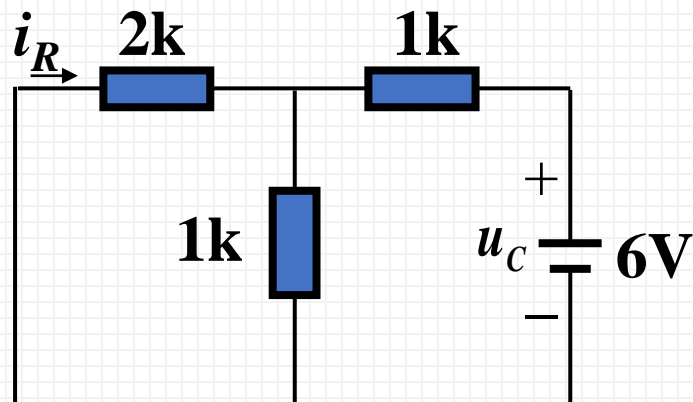
0⁺时刻电路:



$$i_R(0^+) = 3\text{mA}$$

i_R 的零输入响应:

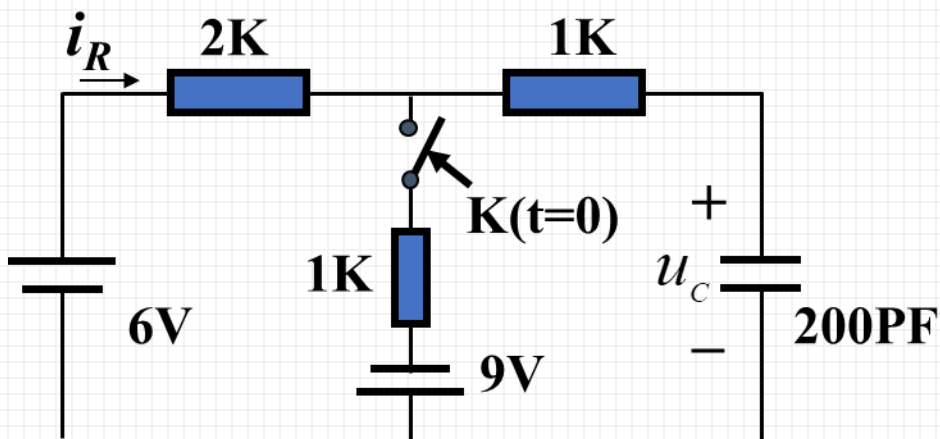
0⁺时刻电路:



i_L 和 u_C 在 $t = 0^+$ 时的值由 $t = 0^-$ 时刻的电路确定; 而其它电量在 $t = 0^+$ 的值则由 $t = 0^+$ 时刻的电路确定, 求全响应所用的 0⁺ 时刻电路与求零输入响应所用的 0⁺ 时刻电路时不一样的。

$$i_R(0^+) = -\frac{6}{1 + 2/3} \times \frac{1}{3} = -1.2\text{mA}$$

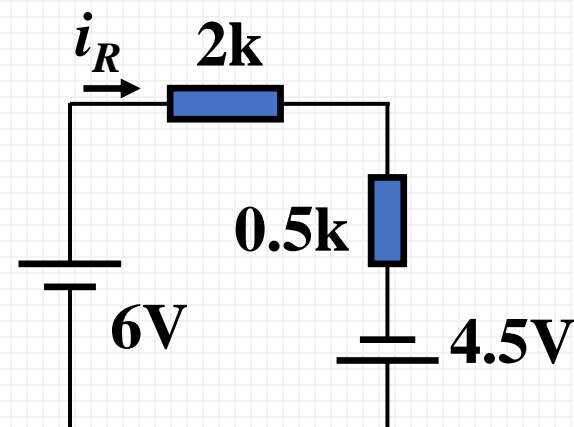
$$i_{R\text{ZIR}} = -1.2e^{-\frac{t}{\tau}}\text{mA} \quad t \geq 0^+$$



i_R 的零状态响应:

$$u_C(0^+) = 0$$

0^+ 时刻电路:



$$i_R(0^+) = \frac{6 + 4.5}{2.5} = 4.2\text{mA}$$

$$i_R(\infty) = \frac{6 + 9}{3} = 5\text{mA}$$

$$i_{R\text{ZSR}} = 5 + (4.2 - 5)e^{-\frac{t}{\tau}} = 5 - 0.8e^{-\frac{t}{\tau}}\text{mA} \quad t \geq 0^+$$

$$i_R = i_{R\text{ZSR}} + i_{R\text{ZIR}}$$

$$= 5 - 0.8e^{-\frac{t}{\tau}} - 1.2e^{-\frac{t}{\tau}} = 5 - 2e^{-\frac{t}{\tau}}\text{mA} \quad t \geq 0^+$$

$$u_C = A + Be^{-\frac{t}{\tau}} \quad t > 0$$



$$u'_C = A(1 - e^{-\frac{t}{\tau}}) \quad t > 0$$

零状态响应

$$u''_C = (A + B)e^{-\frac{t}{\tau}} \quad t > 0$$

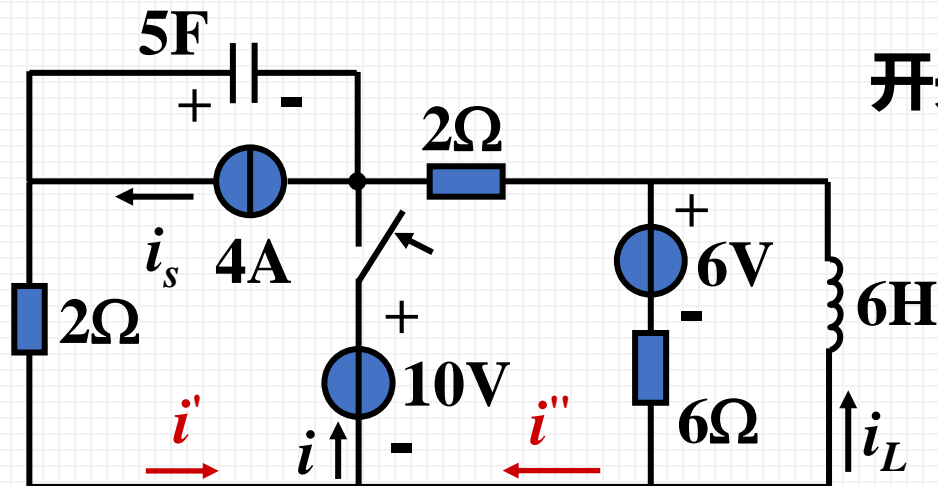
零输入响应

只有 i_L 和 u_C 在已知全响应的情况下，可以通过上述方法得到零状态响应和零输入响应。其他变量不能这么拆分。

如：

$$i_R = 5 - 2e^{-\frac{t}{\tau}} \text{mA} \begin{cases} i_{R\text{零输入}}(t) = -1.2e^{-\frac{t}{\tau}} \text{mA} & t > 0 \\ i_{R\text{零状态}}(t) = 5 - 0.8e^{-\frac{t}{\tau}} \text{mA} & t > 0 \end{cases}$$

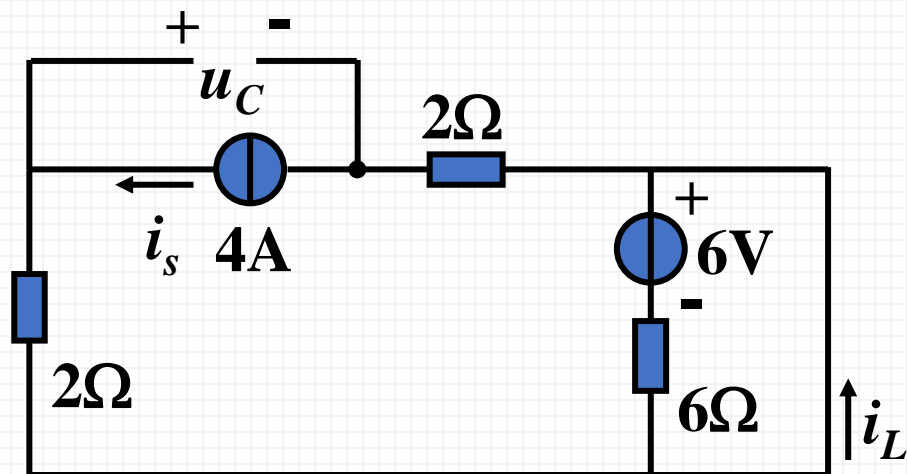
4.



开关在 $t = 0$ 时刻闭合. 求 $i(t)$ 。

$$i = i' + i''$$

0-时刻电路:

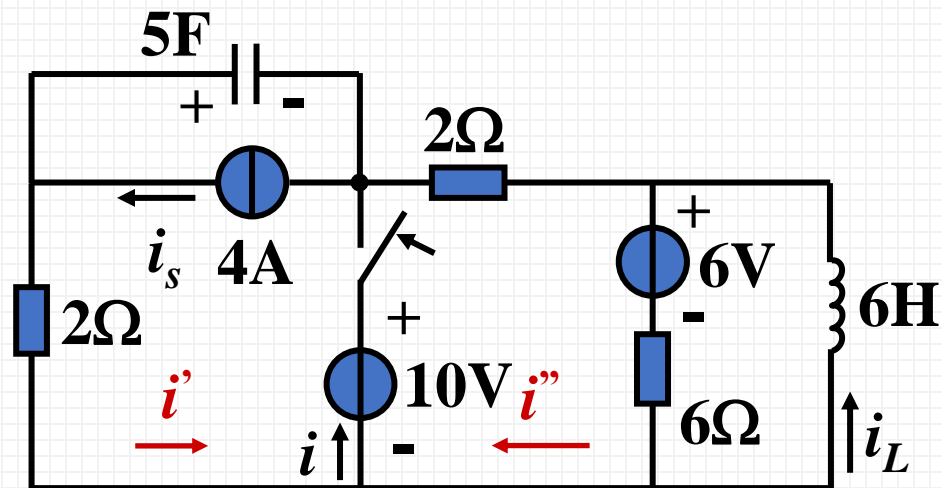


$$u_C(0^-) = 16V$$

$$i_L(0^-) = 3A$$

$$u_C(0^+) = 16V$$

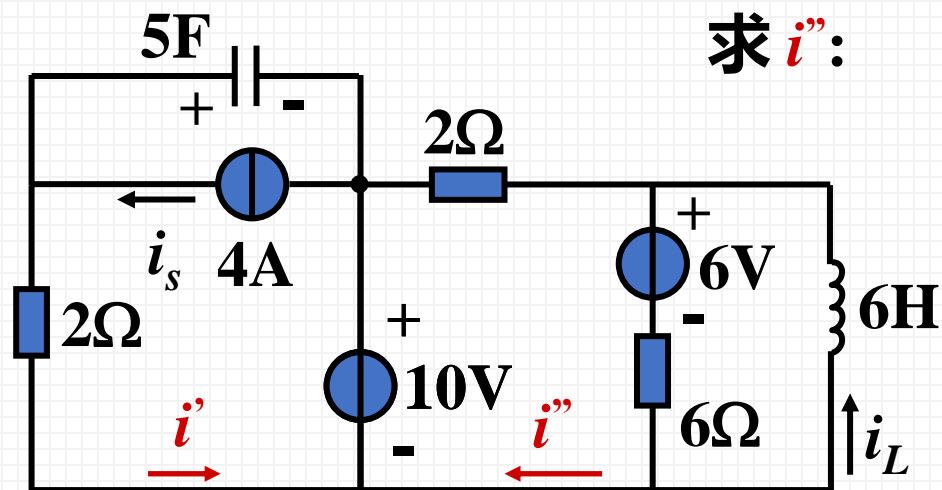
$$i_L(0^+) = 3A$$



一阶还是二阶？

并联的恒压源

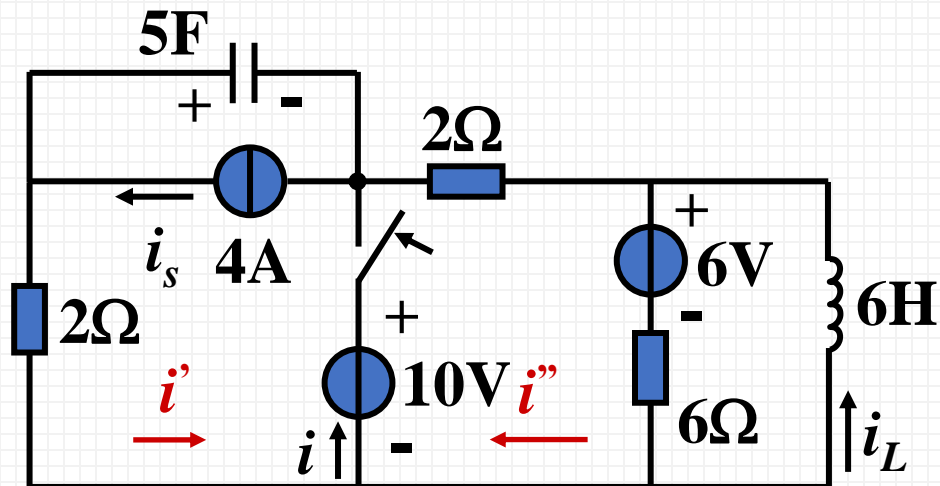
求 i' :



求 i'' :

$$u_C(0^+) = 16V$$

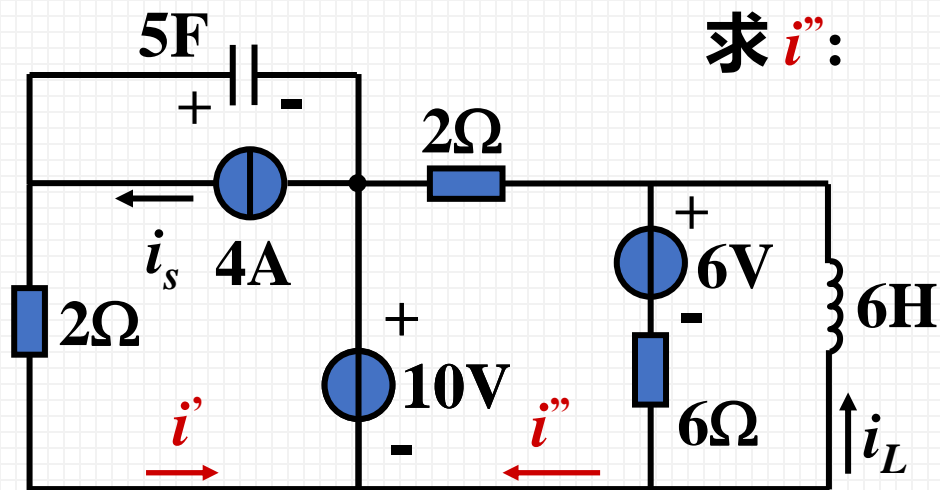
$$i_L(0^+) = 3A$$



一阶还是二阶？

并联的恒压源

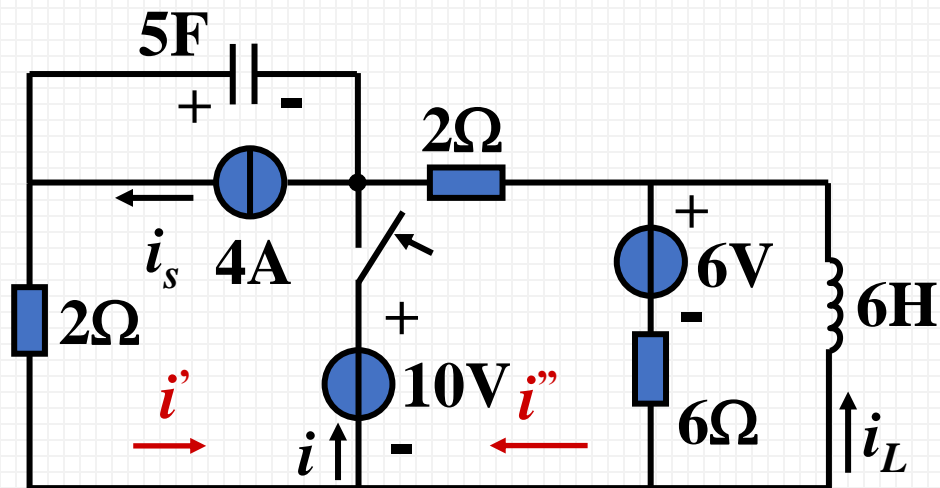
求 i' :



求 i'' :

$$u_C(0^+) = 16V$$

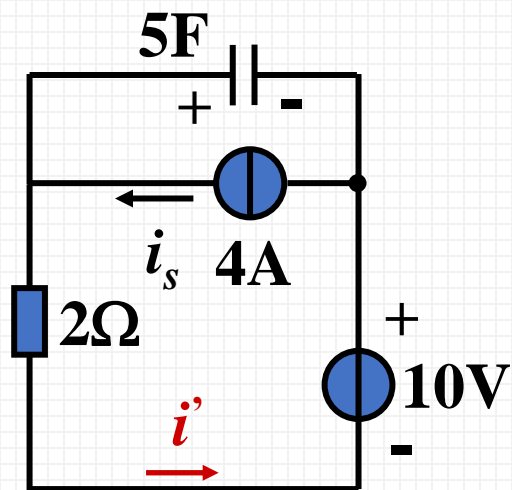
$$i_L(0^+) = 3A$$



一阶还是二阶?

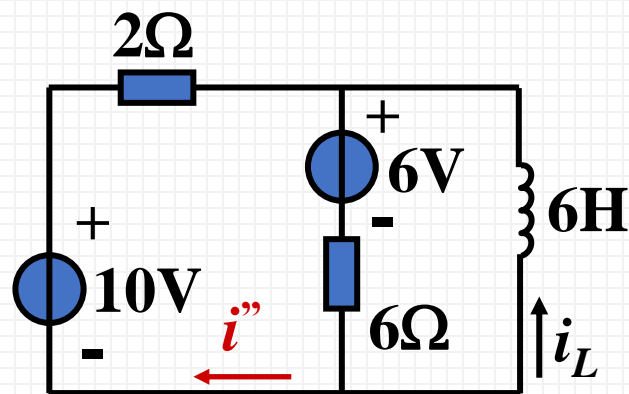
并联的**恒压源**

求 i' :

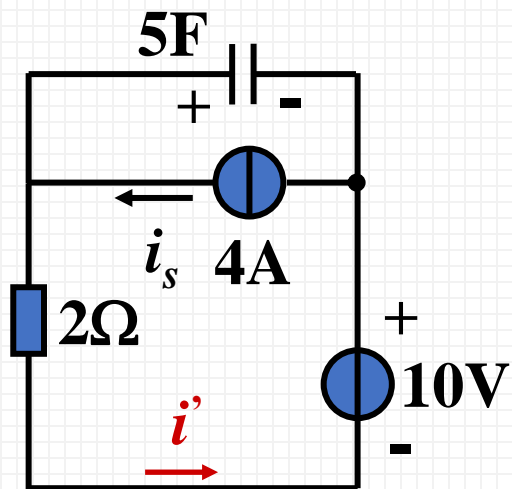


$$u_C(0^+) = 16V$$

求 i'' :



$$i_L(0^+) = 3A$$



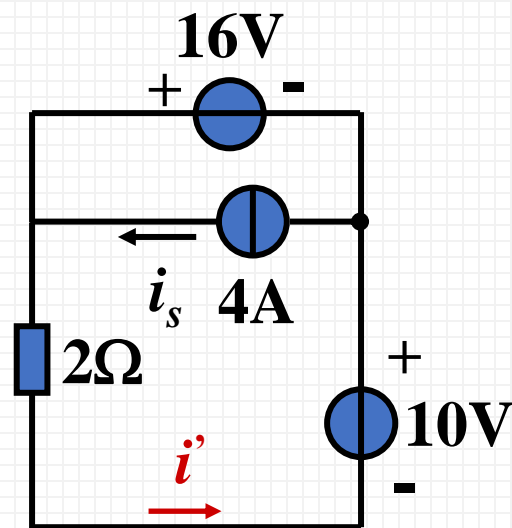
$$u_C(0^+) = 16\text{V}$$

$$i'(0^+) = 13\text{A}$$

$$i'(\infty) = 4\text{A}$$

$$\tau_1 = 10\text{s}$$

$$i' = 4 + 9e^{-0.1t} \text{ A}$$



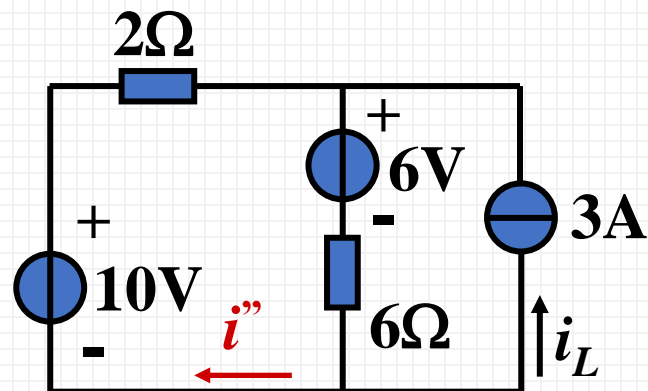
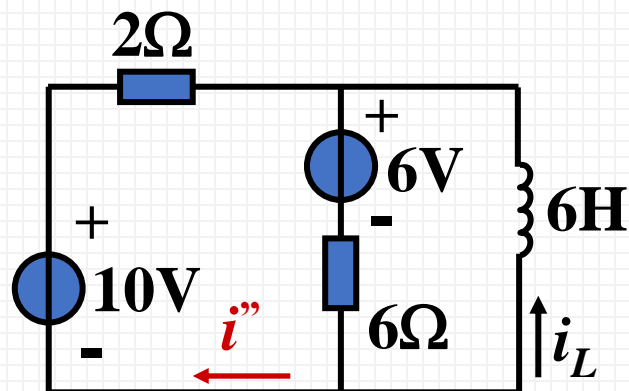
$$i_L(0^+) = 3\text{A}$$

$$i''(0^+) = \frac{10 - 6}{2 + 6} - 3 \times \frac{6}{2 + 6} = -1.75\text{A}$$

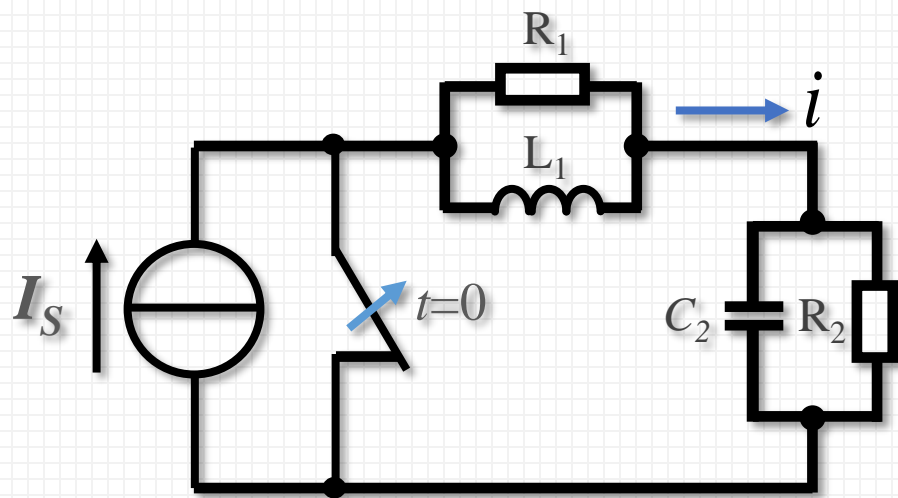
$$i''(\infty) = 5\text{A}$$

$$\tau_2 = 4\text{s}$$

$$i'' = 5 - 6.75e^{-0.25t} \text{ A}$$

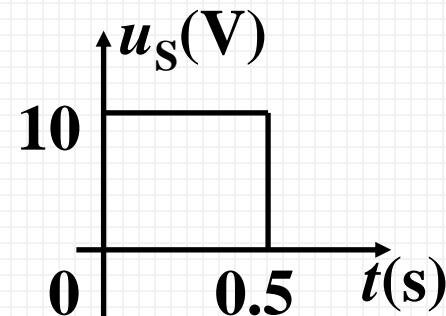
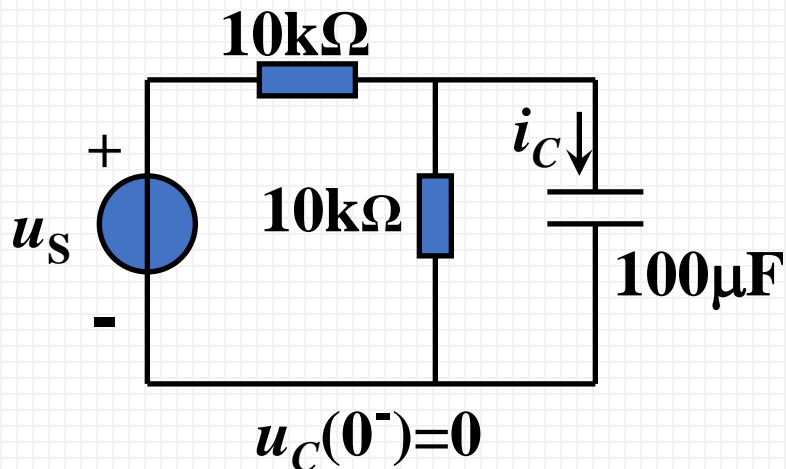


$$i = i' + i'' = 9 + 9e^{-0.1t} - 6.75e^{-0.25t} \text{ A} \quad t \geq 0^+$$



另外一种含LC的一阶
串联的恒流源

5. 求 $i_C(t)$.



解: $0 \leq t \leq 0.5$

$$u_C(0^+) = u_C(0^-) = 0$$

$$u_C(\infty) = 5V$$

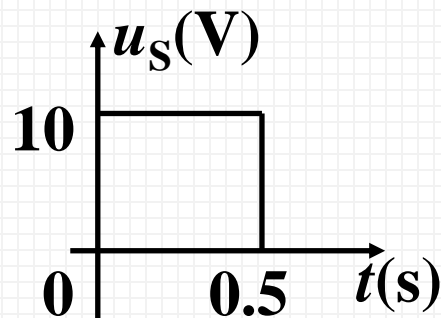
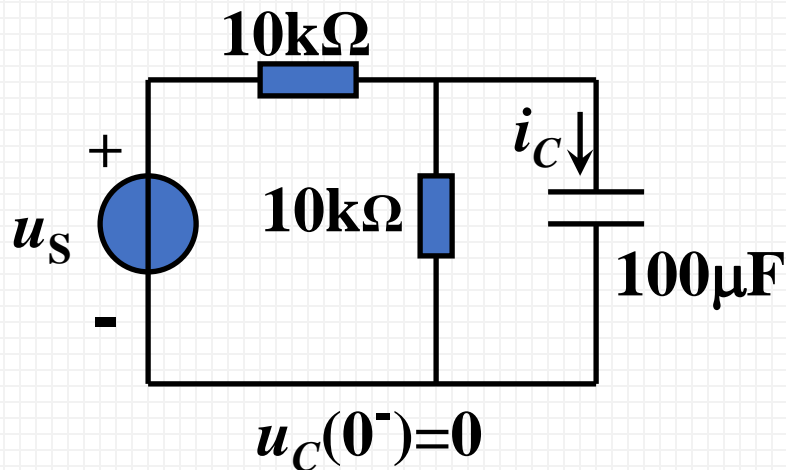
$$\tau = R_{eq}C = 0.5s$$

$$u_C(t) = 5(1 - e^{-2t})V$$

$$i_C(t) = C \frac{du_C(t)}{dt} = e^{-2t}mA$$

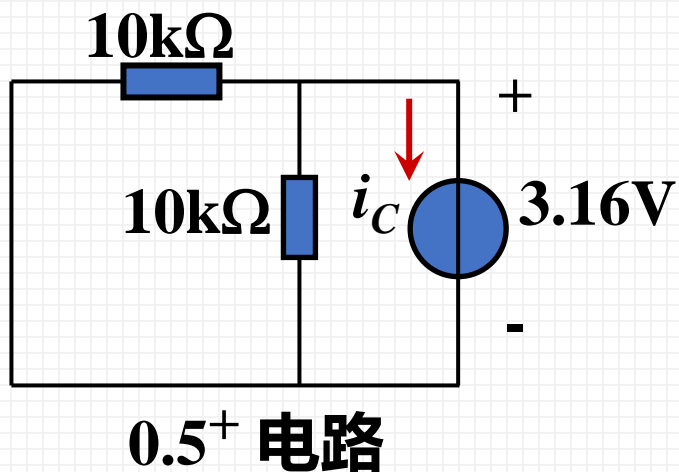
$$\begin{aligned} i_C(0^+) &= 1mA \\ i_C(\infty) &= 0 \\ \tau &= R_{eq}C = 0.5s \end{aligned}$$





$$0 \leq t \leq 0.5 \quad u_C(t) = 5(1 - e^{-2t}) \text{ V}$$

$$t \geq 0.5 \quad u_C(0.5^+) = u_C(0.5^-) = 3.16 \text{ V}$$



$$i_C(0.5^+) = -0.632 \text{ mA}$$

$$i_C(\infty) = 0$$

$$\tau = R_{\text{eq}} C = 0.5 \text{ s}$$

$$i_C(t) = -0.632 e^{-2(t-0.5)} \text{ mA} \quad t > 0.5^+$$

换路在 t_0 时刻发生:

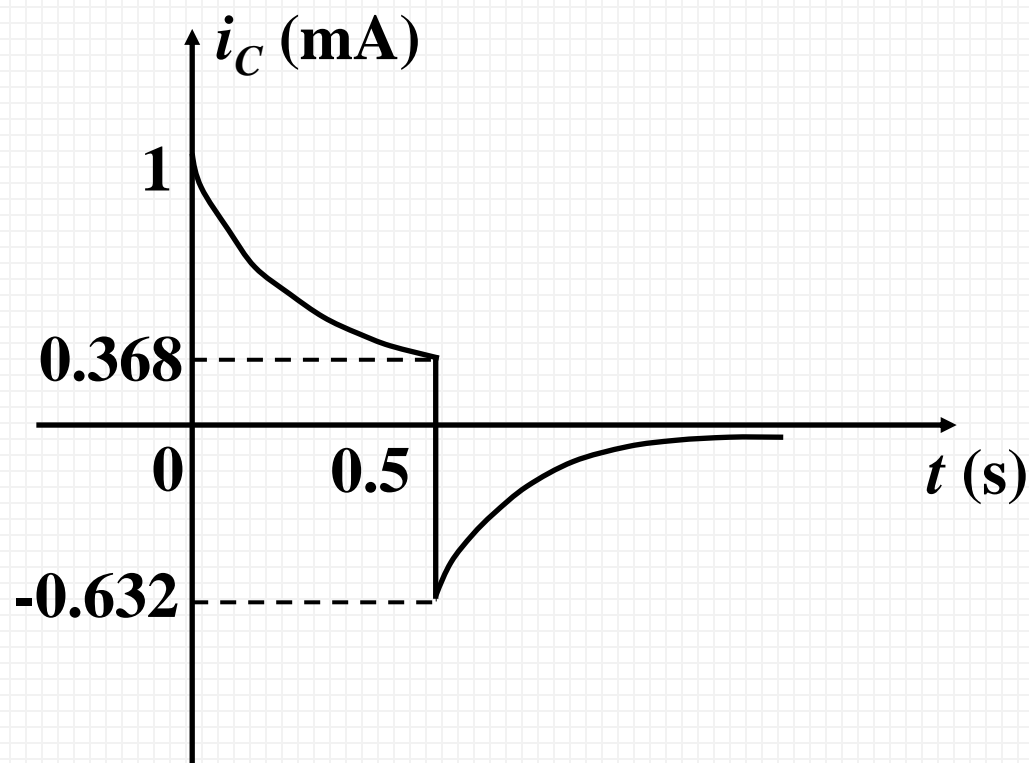
$$f(t) = f(\infty) + [f(t_0^+) - f(\infty)] e^{-\frac{t-t_0}{\tau}} \quad t > t_0$$

$$0 \leq t \leq 0.5$$

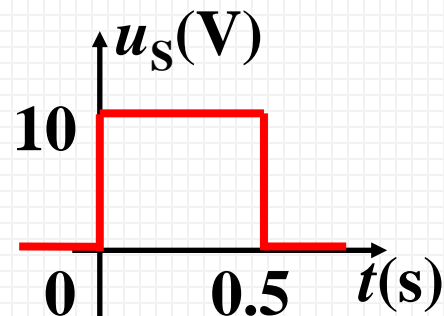
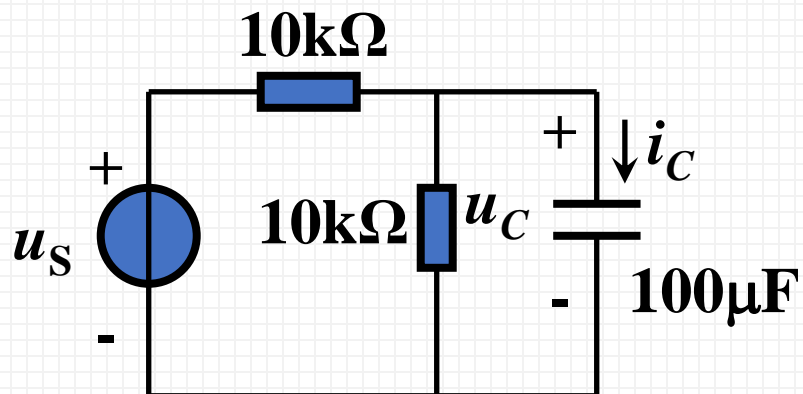
$$i_C(t) = e^{-2t} \text{ mA}$$

$$t \geq 0.5$$

$$i_C(t) = -0.632e^{-2(t-0.5)} \text{ mA}$$



求图示电路中电流 $i_C(t)$.



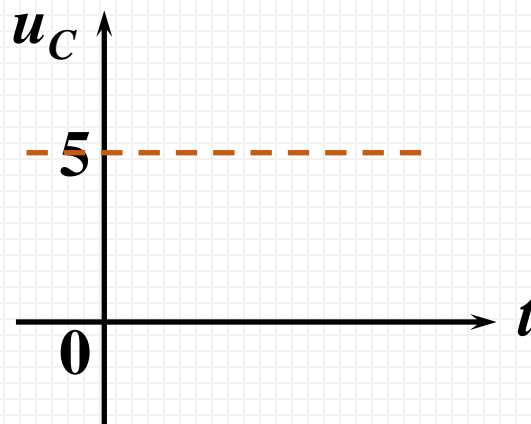
若 $u_C(0^-) = 5\text{V}$ 怎么求 i_C ?

$$u_C(0^+) = u_C(0^-) = 5\text{V}$$

$$u_C(\infty) = 5\text{V}$$

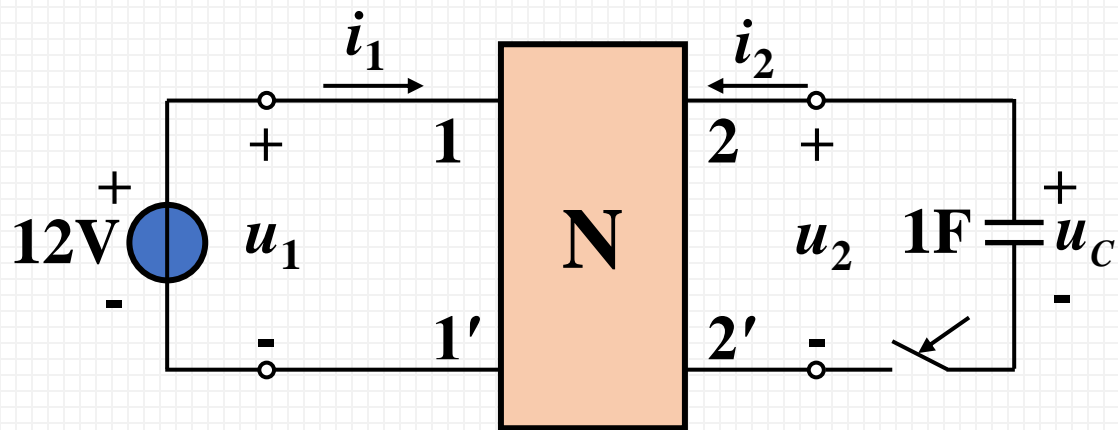
不存在过渡过程直接进入稳态

$$i_C(t) = 0$$



6. 二端口N的传输参数矩阵为 $T = \begin{bmatrix} 2 & 8\Omega \\ 0.5\text{S} & 2.5 \end{bmatrix}$ 。

$t=0$ 时刻闭合开关, 已知 $u_C(0^-)=1\text{V}$, 求 $u_C(t)$ 。



解法1: $u_C(0^+) = u_C(0^-) = 1\text{V}$

$$\begin{cases} u_1 = 2u_2 - 8i_2 \\ i_1 = 0.5u_2 - 2.5i_2 \end{cases}$$

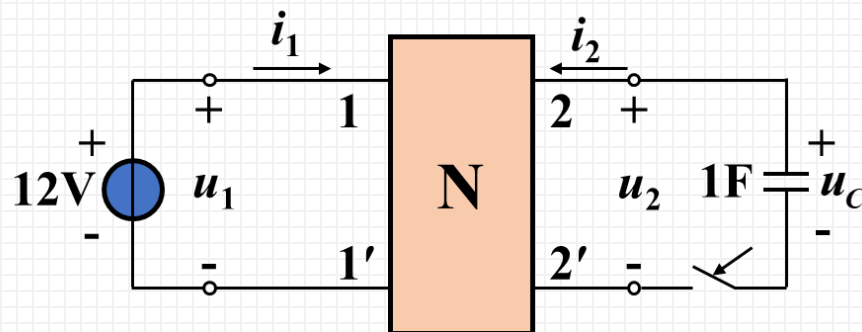
$$u_1 = 12\text{V} \quad i_2 = 0$$

$$\longrightarrow u_C(\infty) = 6\text{V}$$

$$u_C(0^+) = u_C(0^-) = 1\text{V}$$

$$u_C(\infty) = 6\text{V}$$

$$\begin{cases} u_1 = 2u_2 - 8i_2 \\ i_1 = 0.5u_2 - 2.5i_2 \end{cases}$$

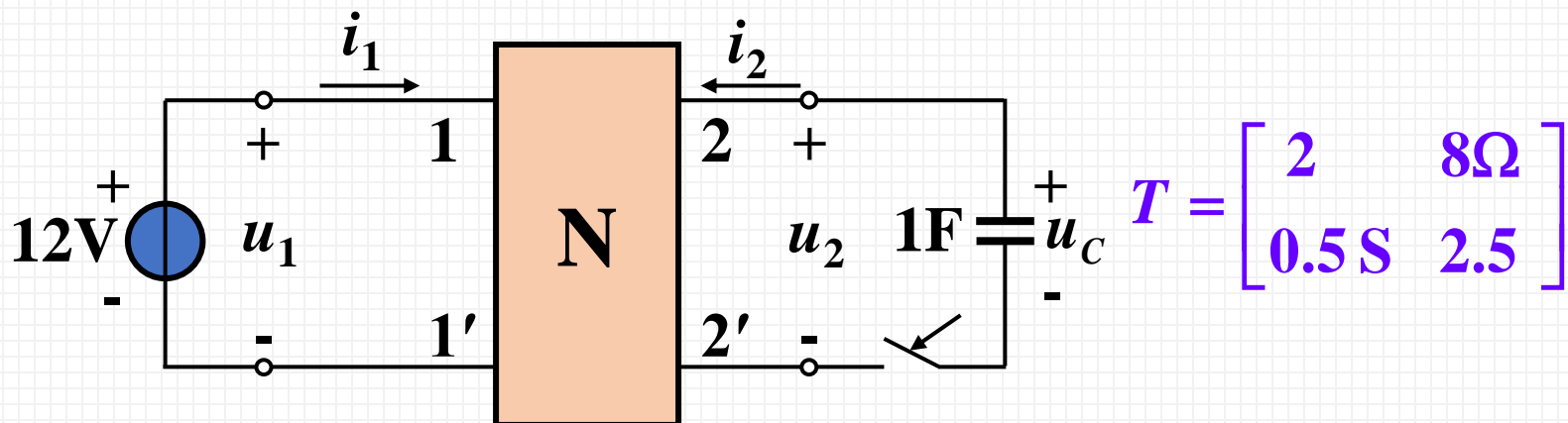


$$u_1 = 0 \Rightarrow R_{\text{eq}} = \frac{u_2}{i_2} = 4\Omega$$

$$\tau = R_{\text{eq}}C = 4\text{s}$$

$$u_C(t) = 6 - 5e^{-0.25t} \quad \text{V} \quad t \geq 0$$

解法2



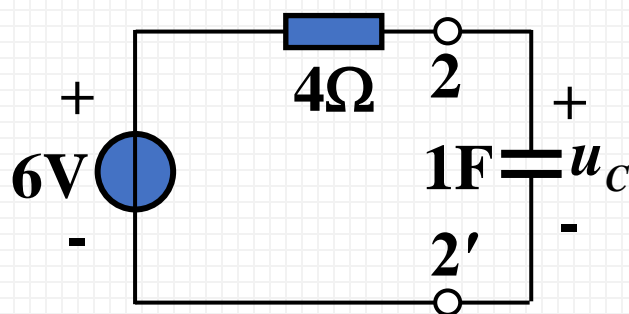
求2 - 2'左端的戴维南等效电路

$$\begin{cases} u_1 = 2u_2 - 8i_2 \\ i_1 = 0.5u_2 - 2.5i_2 \end{cases}$$

$$u_2|_{i_2=0} = u_1 / 2 = 6V$$

$$i_2|_{u_2=0} = -u_1 / 8 = -1.5A$$

$$R_{eq} = \frac{u_2|_{i_2=0}}{-i_2|_{u_2=0}} = 4\Omega$$



$$u_c(0^+) = u_c(0^-) = 1V$$

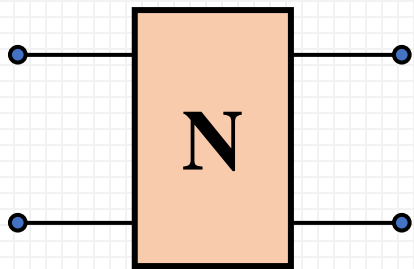
$$u_c(\infty) = 6V$$

$$\tau = R_{eq}C = 4s$$

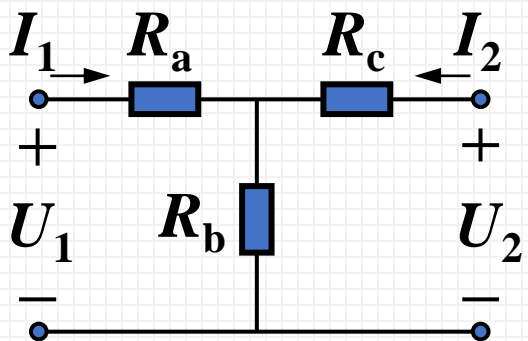
$$u_c(t) = 6 - 5e^{-0.25t}V \quad t \geq 0$$

解法3 T 参数满足 $T_{11}T_{22}-T_{12}T_{21}=1$, 因此, 是互易二端口, 不含受控源。

求等效电路:



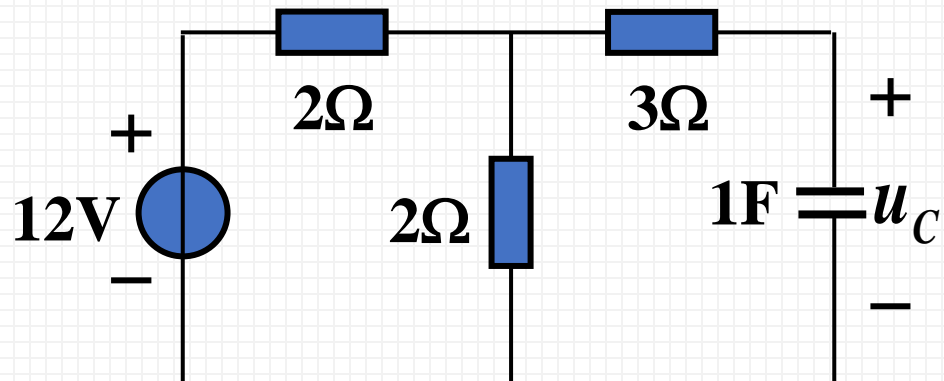
$$T = \begin{bmatrix} 2 & 8\Omega \\ 0.5\text{S} & 2.5 \end{bmatrix} \quad \left\{ \begin{array}{l} U_1 = T_{11}U_2 - T_{12}I_2 \\ I_1 = T_{21}U_2 - T_{22}I_2 \end{array} \right.$$



$$\left\{ \begin{array}{l} T_{11} = \frac{U_1}{U_2} \Big|_{I_2=0} = \frac{R_a + R_b}{R_b} = 2 \\ T_{21} = \frac{I_1}{U_2} \Big|_{I_2=0} = \frac{1}{R_b} = 0.5 \\ T_{22} = \frac{I_1}{-I_2} \Big|_{U_2=0} = \frac{R_c + R_b}{R_b} = 2.5 \end{array} \right.$$

解之得:

$$\left\{ \begin{array}{l} R_a = 2\Omega \\ R_b = 2\Omega \\ R_c = 3\Omega \end{array} \right.$$



$$u_C(0^+) = u_C(0^-) = 1\text{V}$$

$$u_C(\infty) = 6\text{V}$$

$$\tau = R_{\text{eq}}C = 4\text{s}$$

$$u_C(t) = 6 - 5e^{-0.25t}\text{V} \quad t \geq 0$$

7. 列状态方程

方法1

1) 电容节点列KCL, 电感回路列KVL

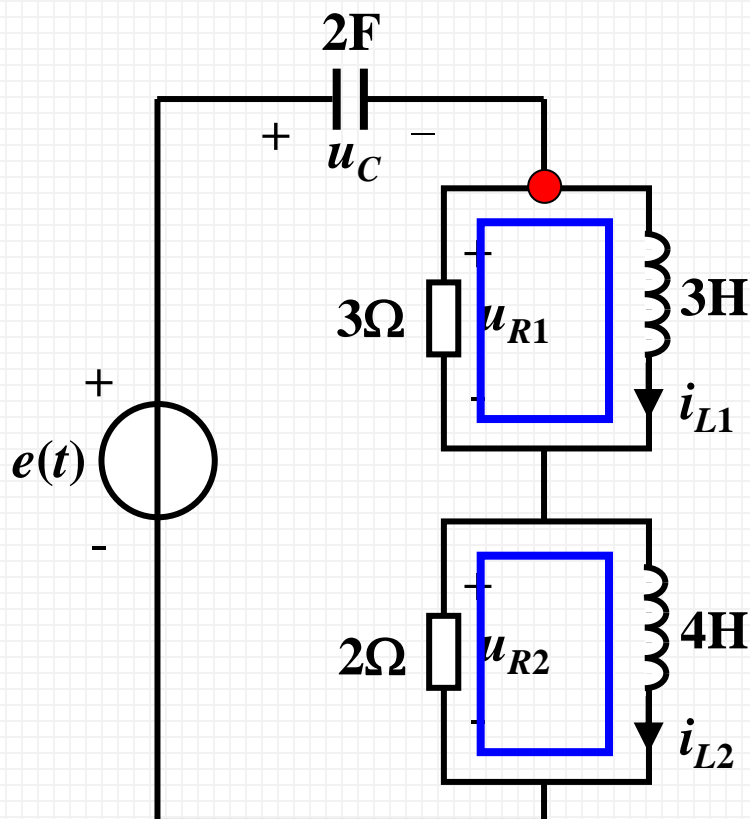
$$2 \frac{du_C}{dt} = i_{L1} + \frac{u_{R1}}{3} \quad 3 \frac{di_{L1}}{dt} = u_{R1}$$

$$4 \frac{di_{L2}}{dt} = u_{R2}$$

2) 消去非状态量 u_{R1} , u_{R2}

$$\left. \begin{aligned} u_C + u_{R1} + u_{R2} &= e(t) \\ \frac{u_{R1}}{3} - \frac{u_{R2}}{2} &= i_{L2} - i_{L1} \end{aligned} \right\}$$

含非状态量的两个
独立方程有时较难
一下子找到



$$u_{R1} = -0.6 u_C - 1.2 i_{L1} + 1.2 i_{L2} + 0.6 e(t)$$

$$u_{R2} = -0.4 u_C + 1.2 i_{L1} - 1.2 i_{L2} + 0.4 e(t)$$

$$\begin{aligned} 2\dot{u}_C &= i_{L1} - 0.2 u_C - 0.4 i_{L1} + 0.4 i_{L2} + 0.2 e(t) \\ &= -0.2 u_C + 0.6 i_{L1} + 0.4 i_{L2} + 0.2 e(t) \end{aligned}$$

$$3\dot{i}_{L1} = -0.6 u_C - 1.2 i_{L1} + 1.2 i_{L2} + 0.6 e(t)$$

$$4\dot{i}_{L2} = -0.4 u_C + 1.2 i_{L1} - 1.2 i_{L2} + 0.4 e(t)$$

$$\begin{bmatrix} \dot{u}_C \\ \dot{i}_{L1} \\ \dot{i}_{L2} \end{bmatrix} = \begin{bmatrix} -0.1 & 0.3 & 0.2 \\ -0.2 & -0.4 & 0.4 \\ -0.1 & 0.3 & -0.3 \end{bmatrix} \begin{bmatrix} u_C \\ i_{L1} \\ i_{L2} \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.2 \\ 0.1 \end{bmatrix} e(t)$$

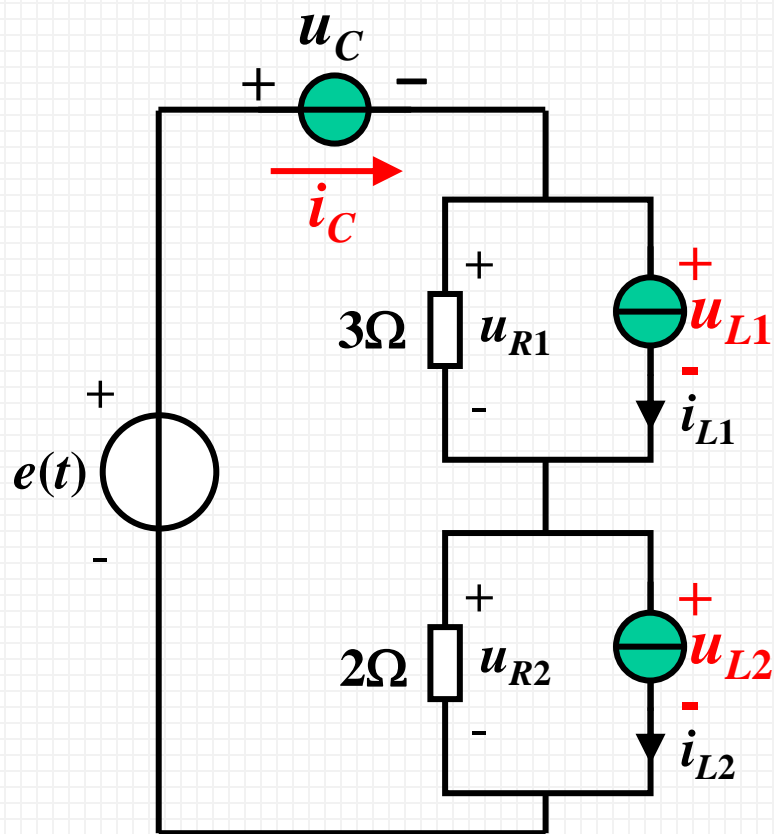
$$2 \frac{du_C}{dt} = i_{L1} + \frac{u_{R1}}{3}$$

$$3 \frac{di_{L1}}{dt} = u_{R1}$$

$$4 \frac{di_{L2}}{dt} = u_{R2}$$

2、列状态方程

方法2



$$\dot{i}_C = \dot{i}_{L1} + \frac{u_{L1}}{3} = \dot{i}_{L2} + \frac{u_{L2}}{2}$$

$$u_C + u_{L1} + u_{L2} = e(t)$$

$$u_C + 3\dot{i}_C - 3\dot{i}_{L1} + 2\dot{i}_C - 2\dot{i}_{L2} = e(t)$$

$$\Rightarrow 5\dot{i}_C = -u_C + 3\dot{i}_{L1} + 2\dot{i}_{L2} + e(t)$$

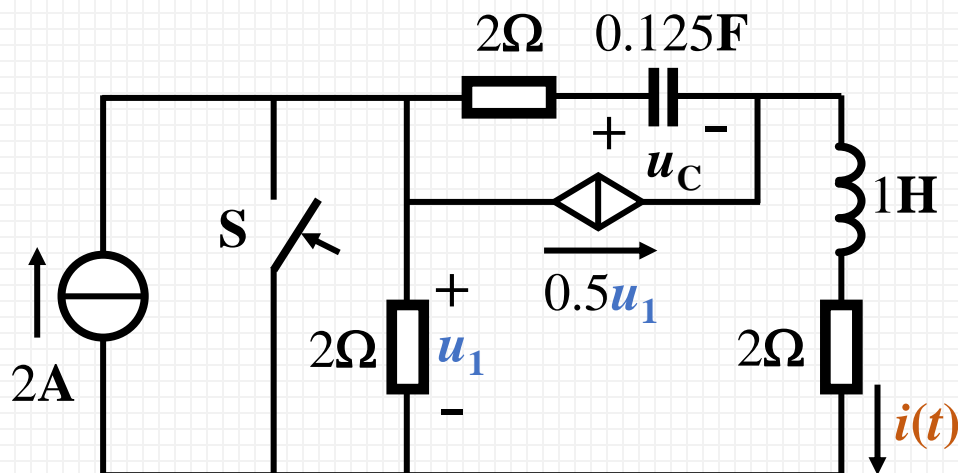
$$\dot{i}_C = C \frac{du_C}{dt}$$

$$\Rightarrow i_C = \frac{1}{C} \bar{i}_C$$

同理, u_{L1}, u_{L2} .

8. 换路前电路已达稳态，在 $t=0$ 时闭合开关S。求换路后的 $i(t)$ 。

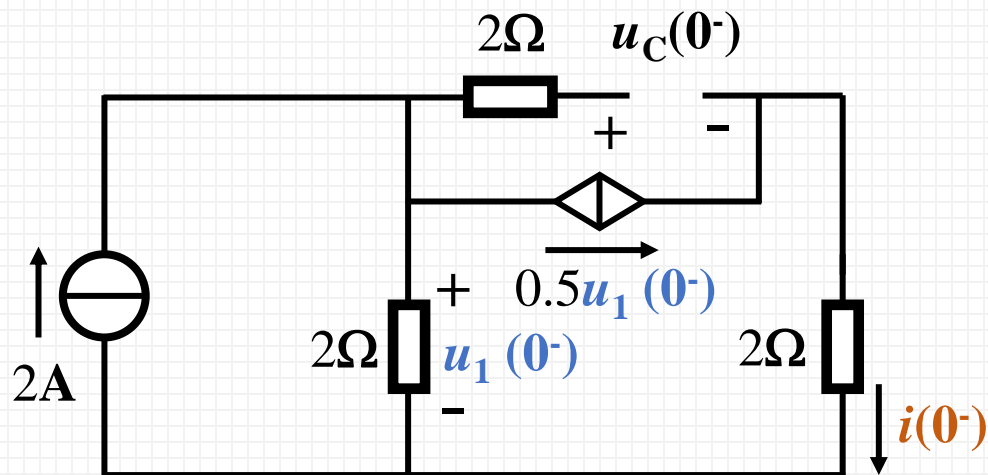
用状态方程法求解。

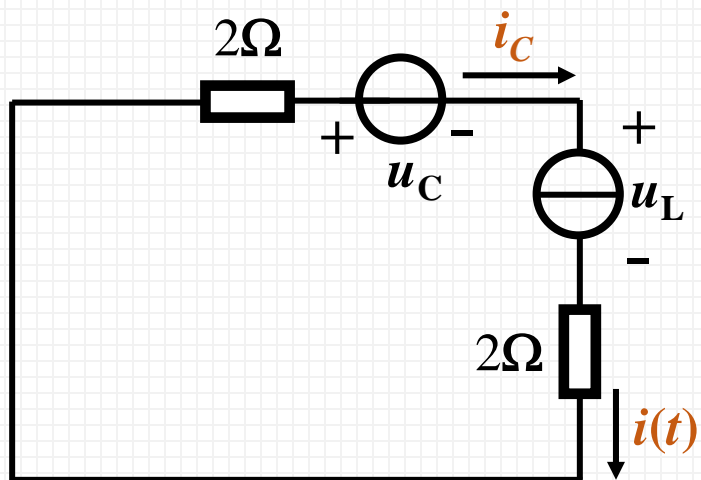
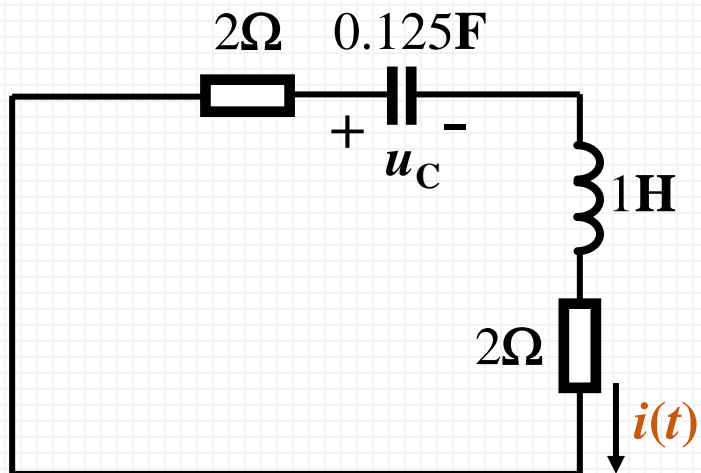


解：换路定理

$$i(0^+) = i(0^-) = 1 \text{ A}$$

$$u_C(0^+) = u_C(0^-) = 0$$





列状态方程和输出方程

$$\dot{u}_C = \dot{u}_L$$

$$2\dot{u}_C + u_C + u_L + 2\dot{u}_L = 0$$

$$u_L = -u_C - 4\dot{u}_L$$

$$\begin{pmatrix} \dot{u}_C \\ \dot{u}_L \end{pmatrix} = \begin{pmatrix} 0 & 8 \\ -1 & -4 \end{pmatrix} \begin{pmatrix} u_C \\ u_L \end{pmatrix}$$

$$\dot{u}_C = \dot{u}_L$$

求特征根

$$\begin{pmatrix} u_C' \\ \dot{i}_L' \end{pmatrix} = \begin{pmatrix} 0 & 8 \\ -1 & -4 \end{pmatrix} \begin{pmatrix} u_C \\ \dot{i}_L \end{pmatrix}$$

$$\begin{vmatrix} p & -8 \\ 1 & p+4 \end{vmatrix} = p^2 + 4p + 8 = 0$$

$$p_{1,2} = -2 \pm j2$$

$$\begin{cases} \alpha = 2 \\ \omega_d = 2 \end{cases}$$

$$\dot{i}_L(t) = K e^{-2t} \sin(2t + \theta)$$

$$\frac{d^2 u_C}{dt^2} + 2\alpha \frac{du_C}{dt} + \omega_0^2 u_C = 0$$

$$\omega_0^2 = \omega_d^2 + \alpha^2$$

$$u_C = K e^{-\alpha t} \sin(\omega_d t + \theta)$$

$$i(t) = K e^{-2t} \sin(2t + \theta)$$

$$i(0^+) = 1, \quad \left. \frac{di}{dt} \right|_{0^+} = -4$$

$$i(0^+) = i(0^-) = 1 \text{ A}$$

$$u_C(0^+) = u_C(0^-) = 0$$

$$\begin{pmatrix} u_C' \\ i_L' \end{pmatrix} = \begin{pmatrix} 0 & 8 \\ -1 & -4 \end{pmatrix} \begin{pmatrix} u_C \\ i_L \end{pmatrix}$$

$$\begin{cases} 1 = K \sin \theta \\ -4 = -2K \sin \theta + 2K \cos \theta \end{cases}$$

$$K = -\sqrt{2}$$

$$\theta = -45^\circ$$

$$i(t) = -\sqrt{2} e^{-2t} \sin(2t - 45^\circ) \text{ A}$$