Proceedings of the 2009 IEEE
International Conference on Robotics and Biomimetics
December 19 -23, 2009, Guilin, China

Analysis of a Biped Powered Walking Model Based on Potential Energy Compensation

Xiaoyue Zhang and Mingguo Zhao

Abstract—**The most important factor in powered walking based on Passive Dynamic Walking is to counter-balance the energy which is lost at heel-strike. In this paper, we discuss a model with only one mass at the hip, which compensates the lost energy with the potential energy through extending and shortening the stance leg instantaneously over one walking cycle. The results show that there exists periodical gait when the model has appropriate length shorten ratio β, inter-leg angle φ_0, extension angle θ_{II}, and shortening angle θ_{IV}, and it has asymptotic stability within a wide range. The energy efficiency can be modulated by adjusting these parameters, and the highest energy efficiency can be 0.0569. Besides, it can realize varying-speed walking under the parameters tunning.**

I. INTRODUCTION

In the early 1990s, McGeer proposed the concept of Passive Dynamic Walking and then he proved it through theoretical analyses and experiments in real robots [1]. After McGeer, Garcia [2], Goswami [3] and Wisse [4] *et al* studied the walking stability and energy efficiency of some other different Passive Dynamic Walking models. Since the results of their studies have confirmed this concept, it is nature that people begin to think about the level ground walking based on Passive Dynamic Walking Theory. The most important problem in level ground walking is the compensation for the lost energy. The lost energy is counter-balanced by means of gravity in slope walking which doesn't need any sources. While McGeer was studying the passive walking, he had proposed several ways to make passive robots walk on level ground with active sources [5]. They have been realized by the followers. Collins *et al* have realized impulse application on the trailing leg as it leaves the ground in Cornell Biped [6], which resulted in highly energy efficient gaits. Wisse *et al* have realized torque application between legs in the Delft pneumatic biped Denise [7]. Hobblen *et al* study the application on the stance leg ankle joint which has been realized in Meta [8]. These applications all compensate for the lost energy with kinetic energy. In addition, Goswami *et al* propose active control algorithm using Passive Dynamic Walking as a reference model [3]. Grizzle *et al* have realized the concept of virtual constraints and the concept of hybrid zero dynamic in robot Rabbit [9]. Asano *et al* introduce "virtual gravity field" toward the horizontal direction which acts as a driving force for level ground walking [10][11]. But all these means depend on the model and force the system to act according to the model, thus losing the feature of "freedom" in passive theory. There are some ways to realize a level walking from the potential energy restoration point of view. Recently, Asano and Luo *et al* propose a way to compensate for the lost energy using the extending and shortening of the swing leg, demonstrating the potential energy compensation can realize a high energy efficiency and speed gait [12]. Besides, the Robots lab in Automation Department in Tsinghua University introduce the "virtual slope walking" which compensates for the lost energy through extending the stance leg and shortening the swing leg [13]. These two ways make use of the rise of some part of the body to increase the system's potential energy and then use this energy to compensate for the lost energy at heel-strike.

In this paper, we propose another model based on potential energy compensation. This model compensates for the lost energy through extending and shortening the stance leg. In order to analyze the existence and stability of the periodical gait, and the influence of the controllable parameters, we take the system's mechanical energy as its state variable. This makes it easy for us to find the fixed point by means of the stride function.

This paper is organized as follows. In section Ⅱ, the model and its parameters are introduced. In section Ⅲ, the stride function of this model is presented. In section Ⅳ, we analyze the existence and stability of the model's periodical gait by means of the fixed point of the stride function. In section Ⅴ, we study the model parameters' influence on the gait. Section Ⅵ is about the conclusion and future work and the last section the acknowledgement.

II. MODEL AND PARAMETERS DESCRIPTION

The model in this paper is a simplified model with only one mass at the hip. In this model, we assume that the legs are massless, and the other parameters are as follows. m is the point mass at the hip. r_s is the length of stance leg before extension. r_e is the length of stance leg after extension, and here we define the length shorten ratio $\beta = r_s / r_e$. θ is the angle between the stance leg and the vertical direction (counter clock wise). φ is the angle between the stance leg and the swing leg. ω is the angle velocity of the stance leg.

Zhang Xiaoyue is with the Department of Automation, Tsinghua University, Beijing, China (phone: (0)13811791481; e-mail: zhangxy08@mails.tsinghua.edu.cn)

Zhao Mingguo is with the Faculty of the Department of Automation, Tsinghua University, Beijing, China (e-mail: mgzhao@mail.tsinghua.edu.cn)

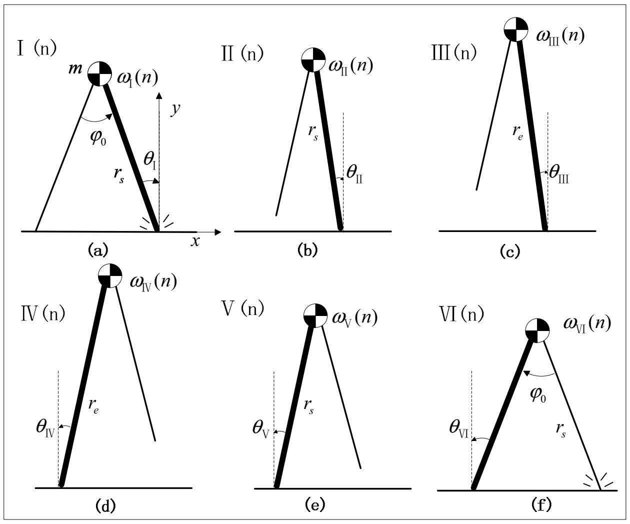


Fig. 1. The n_{th} walking cycle of the model

Fig. 1 shows the n_{th} walking cycle. A walking step starts at the instant Ⅰ when the prior swing leg (the heavier line) has just made contact with the ground and the prior stance leg (the lighter line) is ready to leave the ground, as shown in Fig. 1(a). The new stance leg swings freely to a position with the leg length r_s, as shown in Fig. 1(b). And at the same instant, the stance leg extends to the length of r_e, as shown in Fig. 1(c). Here we have $\theta_{\mathrm{III}} = \theta_{\mathrm{II}}$. After the extension, the stance leg swings freely to a position with the longer leg length r_e, as shown in Fig. 1(d). And then it is shortened back to the length of r_s, as shown in Fig. 1(e). Here we have $\theta_{\mathrm{V}} = \theta_{\mathrm{IV}}$. Finally it swings until it strikes the floor, as shown in Fig. 1(f). At the heel-strike, the length of two legs are equal and the strike is a plastic (no slip, no bounce) collision. After the strike, the prior swing leg sticks to the floor and the prior stance leg is ready to leave the floor. The transformation of the two legs happens within a short instance. As we have assumed that the swing leg is massless, we can control it to swing to the position where the angle between two legs is φ_0 without causing energy.

III. STRIDE FUNCTION

A. Kinetic Equations

We create the coordinates in the model as Fig. 1(a), where the x-axis is along horizontal direction, y-axis is along the vertical direction and the origin is the stance foot contact point. We take the ground as the zero potential energy plane. The kinetic energy and the potential energy of the model are

$$\begin{cases} E_T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) = \frac{1}{2}mr_s^2\dot{\theta}^2 \\ E_V = mgr_s\cos\theta \end{cases} \tag{1}$$

According to Lagrange Equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta}}\right) - \frac{\partial L}{\partial\theta} = 0 \tag{2}$$

where $L = E_T - E_V$ is the Lagrange Function. So the kinetic equations in free swing phase from (n) to Ⅱ(n) is:

$$\ddot{\theta}(t) = \frac{g}{r_s}\sin\theta(t) \tag{3}$$

Here we rescale time by $\tau = \sqrt{g/r_e}\,t$, so (3) can be written as:

$$\ddot{\theta}(\tau) = \frac{1}{\beta}\sin\theta(\tau) \tag{4}$$

From Ⅱ(n) to Ⅲ(n), the stance leg is extended from r_s to r_e instantaneously. According to conservation of angular momentum about the stance foot contact point, we have

$$m\omega_{\mathrm{III}}(n)r_e^2 = m\omega_{\mathrm{II}}(n)r_s^2 \tag{5}$$

Substituting $\beta = r_s/r_e$, we have

$$\omega_{\mathrm{III}}(n) = \beta^2\omega_{\mathrm{II}}(n) \tag{6}$$

Similar to the process Ⅰ(n) to Ⅱ(n), the kinetic equation from Ⅲ(n) to Ⅳ(n) is:

$$\ddot{\theta}(\tau) = \sin\theta(\tau) \tag{7}$$

Similar to the process Ⅱ(n) to Ⅲ(n), we can obtain the connection between Ⅳ(n) and Ⅴ(n)

$$\omega_{\mathrm{V}}(n) = \frac{1}{\beta^2}\omega_{\mathrm{IV}}(n) \tag{8}$$

The process Ⅴ(n) to Ⅵ(n) is a free swing phase with the length r_s until the swing leg strikes the floor, so the kinetic equations is the same as (4).

The process Ⅵ(n) to Ⅰ(n+1) is the transformation between before heel-strike and after heel-strike. The prior swing leg becomes the stance leg, and vice versa. Fig. 2 is the strike transition stage. At the strike, both two feet are on the ground, and we can get the new initial angle according to the geometric collision condition.

$$\theta_{\mathrm{I}}(n+1) = -\theta_{\mathrm{VI}}(n) \tag{9}$$

We assume the strike is plastic and is an instantaneous transition stage, as shown in Fig. 2. According to the conservation of angular momentum about the new stance foot contact point, we have

$$m\omega_{\mathrm{I}}(n+1)r_s^2 = m\omega_{\mathrm{VI}}(n)r_s^2\cos(2\theta_{\mathrm{VI}}(n)) \tag{10}$$

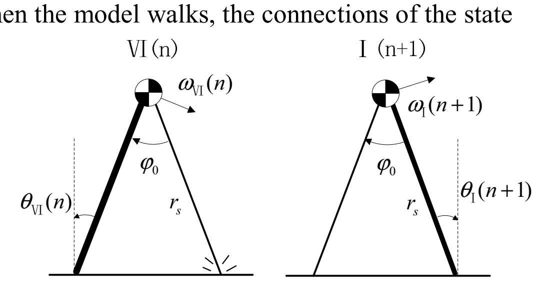
It can be simplified to

$$\omega_{\mathrm{I}}(n+1) = \omega_{\mathrm{VI}}(n)\cos(\varphi_0) \tag{11}$$

So the strike functions are (9) and (11).

B. Stride Function

When the model walks, the connections of the state



(a) Before strike (b) After strike

Fig. 2. Strike transition

variables between two steps can be described as a mapping, which is named Stride Function by McGeer [1]. In this model, we choose the instant when the swing leg strikes the floor as the Poincare section and the mapping of state variables between two successive sections is a Stride Function. The Stride Function about this model is the kinetic equations of every processes and the strike function. We use gait simulation to study the motion trajectory and the limit cycle of state variables. Fig. 3 shows the simulation results. As we can see from the simulation results, the stance leg's motion trajectory is sectionally continuous. These sectionally continuous trajectories correspond to the free swing phase with leg length r_s, the instantaneous extension of stance leg, the free swing phase with leg length r_e, the instantaneous shortening of stance leg, the free swing phase with leg length r_s and the collision respectively.

IV. FIXED POINT

A. *The Analytic Expression of the Fixed Point*

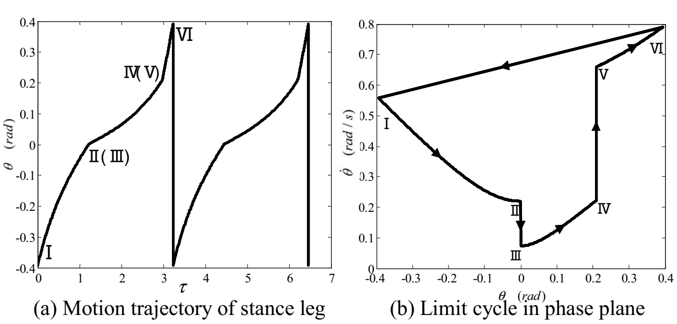
Now we have the stride function of this model, so we can get the solution of this function, or , fixed point, which is the gait limit cycle. From the stride function, we have the connections between $\omega_{\mathrm{II}}(n)$ and $\omega_{\mathrm{III}}(n)$, $\omega_{\mathrm{IV}}(n)$ and $\omega_{\mathrm{V}}(n)$, $\omega_{\mathrm{VI}}(n)$ and $\omega_{\mathrm{I}}(n+1)$, if we can find the connections between $\omega_{\mathrm{I}}(n)$ and $\omega_{\mathrm{II}}(n)$, $\omega_{\mathrm{III}}(n)$ and $\omega_{\mathrm{IV}}(n)$, $\omega_{\mathrm{V}}(n)$ and $\omega_{\mathrm{VI}}(n)$, we will have the walking map between two successive steps. Here we use the conservation of energy in the free swing phases to create the connections between them. The model has the same mechanical energy expression in every instant. It is:

$$E_i(n) = \frac{1}{2}m\dot{\theta}_i(t)^2 r_i^2 + mgr_i\cos\theta_i \qquad i = \mathrm{I},\ \mathrm{II},\ \mathrm{III},\ \mathrm{IV},\ \mathrm{V},\ \mathrm{VI} \tag{12}$$

Substituting $t=\sqrt{r_e/g}\,\tau$, $\beta = r_s/r_e$ and r in every key instant, we can get their energy's expressions.

Then according to the energy conservation during the free swing phases

$$\begin{cases} E_{\mathrm{I}}(n) = E_{\mathrm{II}}(n) \\ E_{\mathrm{III}}(n) = E_{\mathrm{IV}}(n) \\ E_{\mathrm{V}}(n) = E_{\mathrm{VI}}(n) \end{cases} \tag{13}$$



(a) Motion trajectory of stance leg (b) Limit cycle in phase plane

Fig. 3. Gait simulation results of the model, where the parameters are $\beta = 0.58, \varphi_0 = \pi/4, \theta_{\mathrm{II}} = 0, \theta_{\mathrm{IV}} = \pi/15$

we can obtain

$$\omega_{\mathrm{I}}^2(n+1) = \cos^2\varphi_0\,\omega_{\mathrm{I}}^2(n) + \frac{2\cos^2\varphi_0(1-\beta^3)}{\beta^4}[\cos\theta_{\mathrm{II}}(n) - \cos\theta_{\mathrm{IV}}(n)] = f(\omega_{\mathrm{I}}^2(n)) \tag{14}$$

Since θ_{I} is a constant that depends on φ_0 after strike, there leaves only one state variable after strike, that is the angle velocity ω_{I}. This is why we choose the strike instant as the Poincare section, which reduces the variable in state space from two to one. So we consider the kinetic energy $q=\omega_{\mathrm{I}}^2$ as the state variable in Poincare section, and according to the fixed point's definition

$$q = f(q) \tag{15}$$

the analytic expression of fixed point is:

$$q^f = \frac{2\cos^2\varphi_0(1-\beta^3)[\cos\theta_{\mathrm{II}}(n) - \cos\theta_{\mathrm{IV}}(n)]}{\beta^4(1-\cos^2\varphi_0)} \tag{16}$$

The fixed point in the Poincare section corresponds to the periodical gait of the model. From (16), we can say that the periodical gait depends on the model's parameters β, φ_0, θ_{II} and θ_{IV}.

B. *The Existence of the Fixed Point*

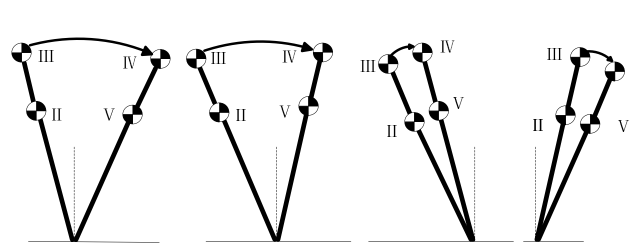
There are four schemes for this model, as shown in Fig. 4. In scheme 1, the stance leg extends at the position before vertical axis and shortens after vertical axis, and the extension position is closer to vertical axis. Compared to scheme 1, the difference in scheme 2 is that the shortening position is closer to vertical axis. In scheme 3, the extension and shortening are both before vertical axis, while in scheme 4, they are both after vertical axis.

According to the definition of q, we can have $|\theta_{\mathrm{II}}(n)| < |\theta_{\mathrm{IV}}(n)|$ from (16). Only scheme 1 and 4 satisfy this condition. Besides this condition, there are some other constraint conditions:

1) The stance leg's velocity must be high enough to make the model pass the vertical axis, or it will fall backward.

In scheme 1, we assume that when the model reaches the vertical axis, the velocity is zero. This is a critical condition of the initial velocity. In order to avoid falling backward, we must make sure the initial state ω_{I} is larger than that critical condition. Thus the fixed point must satisfy

$$\begin{cases} \theta_{\mathrm{IV}}(n) > -\theta_{\mathrm{II}}(n) > 0 \\ \omega_{\mathrm{I}}^2 > 2\left[\dfrac{1-\cos\theta_{\mathrm{II}}(n)}{\beta^4} + \dfrac{\cos\theta_{\mathrm{II}}(n) - \cos\theta_{\mathrm{I}}(n)}{\beta}\right] \end{cases} \tag{17-a}$$



(a) Scheme 1 (b) Scheme 2 (c) Scheme 3 (d) Scheme 4

Fig. 4. The realization schemes of the model

In scheme 4, we make the same assumption as scheme 1, and the fixed point must satisfy

$$\begin{cases} \theta_{\mathrm{IV}}(n) > \theta_{\mathrm{II}}(n) > 0 \\ \omega_{\mathrm{I}}^2 > \dfrac{2[1-\cos\theta_{\mathrm{I}}(n)]}{\beta} \end{cases} \tag{17-b}$$

2) During the swing phase, the stance leg's velocity can't be too high, or the stance foot will leave the floor. It means when component force of gravity in the stance leg's direction isn't sufficient to provide the centripetal force, the model will lose contact with the ground.

3) The stance leg must extend after a step begins and shortens before heel-strike. Thus we have

$$|\theta_{\mathrm{I}}(n)| > |\theta_{\mathrm{IV}}(n)| > |\theta_{\mathrm{II}}(n)| \tag{18}$$

4) The stance leg's velocity after heel-strike must be larger than zero. According to (11), φ_0 must satisfy

$$\varphi_0 < \frac{\pi}{2} \tag{19}$$

In order to make the model walk with a periodical gait, the initial states must satisfy the (17) to (19). So these three equations are the constraint conditions of fixed point's existence.

C. The Stability of the Fixed Point

Once we have obtained the fixed point based on the above constraint conditions, we would like to know whether it is stable or not. The stable fixed point is what we care about. The eigenvalues of the Jacobian matrix of the stride function is a way to study the stability. If all eigenvalues are inside the unit cycle, which means a small disturbance will decay over time, then the fixed point is asymptotical stable, else if there is one outside the unit cycle, it is unstable.

From the fixed point's expression (16), we can calculate the eigenvalue of Jacobian matrix.

$$\lambda = \frac{\partial f}{\partial q} = \cos^2\varphi_0 \tag{20}$$

According to (19), we have $\cos^2\varphi_0 < 1$, so the fixed point is asymptotically stable. This can be confirmed by the simulation results, as shown in Fig. 5. It shows the simulation convergence of a initial state which is not a fixed point. But this initial state can't be too far away from the fixed point. As we can see from the figure, after several cycles it converges to the limit cycle. So if the initial state is proper, it will return back to the fixed point.

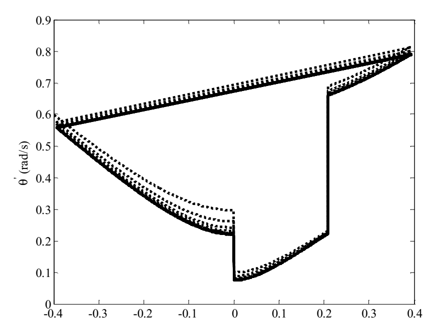


Fig. 5. The phase plane of the process of simulation convergence

However, the eigenvalues of Jacobian matrix can only reflect local stability. Wisse *et al* propose the basin of attraction to analyze the influence of large disturbance [14]. Basin of attraction is an area where the initial states can lead to a stable walking step instead of falling. Fig. 6 shows the basin of attraction when the model has different β. The lower boundary is the minimum velocity for the model to pass vertical axis, while the upper boundary is the maximum velocity for the model to avoid losing contact with the ground. Compared to Passive Dynamic Walking [14], the basin of attraction of this model is larger, which means it can cope with larger disturbance.

V. THE PARAMETERS' INFLUENCE ON GAIT

A. The Parameters' Influence on Energy Efficiency

The energy efficiency of walking model can be described as the dissipation energy when the model has unit mass and walks the unit distance. It is

$$c_t = \frac{E_c}{mg\Delta x} \tag{21}$$

In every walking cycle, the processes when the energy is changing are II(n) to III(n), IV(n) to V(n), and VI(n) to I(n+1). The energy is losing during the collision process VI(n) to I(n+1), so the system must compensate the lost energy during the process II(n) to V(n), namely the compensation energy is

$$E_c = E_{\mathrm{V}}(n) - E_{\mathrm{II}}(n) \tag{22}$$

Substituting the energy's expression as (12) in every key instant, we have

$$E_c = mgr_e[\cos\theta_{\mathrm{II}}(n) - \cos\theta_{\mathrm{IV}}(n)](\frac{1-\beta^3}{\beta^2}) \tag{23}$$

Substituting (21) with (23) and $\Delta x = 2\beta r_e \sin(\frac{\varphi_0}{2})$, we have

$$c_t = \frac{[\cos\theta_{\mathrm{II}}(n) - \cos\theta_{\mathrm{IV}}(n)](1-\beta^3)}{2\beta^3 \sin(\varphi_0/2)} \tag{24}$$

We can see that c_t depends on β, φ_0, absolute value of $\theta_{\mathrm{II}}(n)$ and $\theta_{\mathrm{IV}}(n)$. Since these four parameters have the same meanings in scheme1 and 4, we only analyze their influences on c_t in scheme 1, as shown in Fig. 7. Fig. 7 (a) shows that c_t decreases when β increases, this is because when β is larger, the extension and shortening of stance leg's length is smaller,

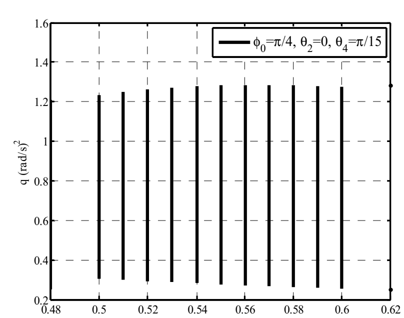


Fig. 6. The basin of attraction of a fixed point

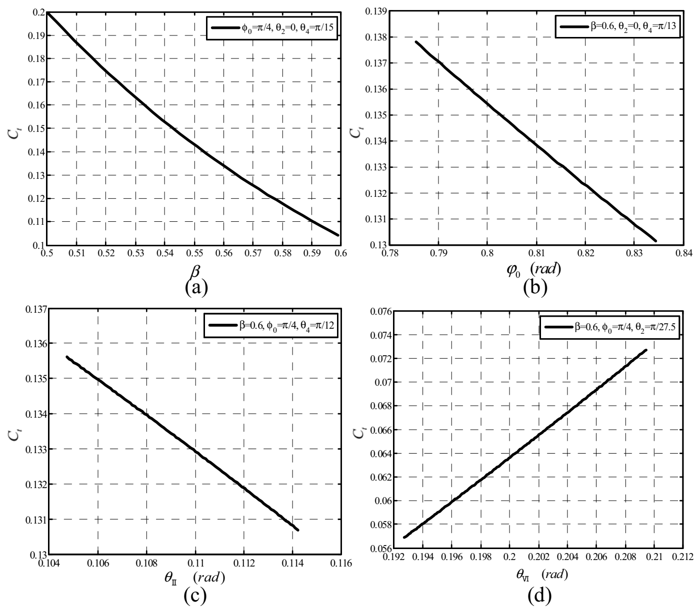


Fig. 7. Influence of model parameters on energy efficiency

thus the model needs less energy. Fig. 7 (b) shows that c_t decreases when φ_0 increases, this is because when φ_0 is larger, the walking distance is larger, thus the energy efficiency is higher. Fig. 7 (c) shows that c_t decreases when the absolute value of $\theta_{II}(n)$ increases, and Fig. 7 (d) shows that c_t increases when the absolute value of $\theta_{IV}(n)$ increases, this is because the compensation potential energy is directly proportional to the extension length's projection to the vertical axis and inversely proportional to the shortening length's projection to the vertical axis, namely when the absolute value of $\theta_{II}(n)$ is larger and the absolute value of $\theta_{IV}(n)$ is smaller, the compensation potential energy is less. We have the highest energy efficiency 0.0569 in Fig. 7(d). Comparing the vertical axes in these four figures, we find that c_t's change range is widest with respect to β, so the most important factor is β.

In table 1 [15], we list the energy efficiency of some other models both passive and active, and also that of human being. We can see that the energy efficiency of Passive Dynamic Walking is almost equal to that of human being. Our model's energy efficiency can be modulated by adjusting the controllable parameters β, φ_0, absolute value of $\theta_{II}(n)$ and $\theta_{IV}(n)$. When the model has appropriate parameters, its energy efficiency will be high.

B. The Parameters' Influence on Walking Velocity

Besides how much energy the model costs, we also care about how fast it can walk. Walking velocity is another index to evaluate the model's performance. Here we refer to the relative average walking velocity during one cycle.

TABLE I
COMPARISON OF ENERGY EFFICIENCY

Type of Dynamic Walkers	Energy Efficiency
Humna	0.05
Honda's Asimo	1.6
Cornell Biped	0.055
Passive Dynamic Walking	$\sin\gamma(\gamma = 0.005-0.09)$
Virtual Passive Dynamic	$\tan\gamma(\gamma = 0.005-0.09)$
Parametric excitation with elasticity	0.031
Virtual Slope Walking	$\frac{1-\cos^2\varphi_0}{1-\cos^2\varphi_0\beta^2}\bullet\frac{(1-\beta^3)\cos\theta_{II}^* -(\beta-\beta^3)\cos\theta_I}{\sqrt{1+\beta^2-2\beta\cos\varphi_0}}$ (0.0146 − 0.281)
Model in this paper	$\frac{[\cos\theta_{II}(n)-\cos\theta_{IV}(n)](1-\beta^3)}{2\beta^3\sin(\varphi_0/2)}$ (0.0569 − 0.3012)

$$\bar{v} = \frac{2\beta\sin(\varphi_0/2)}{t} \tag{25}$$

where t is the periodic time. We must get τ through numerical simulation at first, and then calculate t through $t = \sqrt{r_e/g}\,\tau$.

Similar to the parameters' influence on energy efficiency, we also study their influence on walking velocity, as shown in Fig. 8. Comparing these two figures, we can find that the parameters' influence on energy efficiency and walking velocity are the same. This is because when the energy efficiency is higher, the compensation energy is less and the walking velocity is lower. Among these four parameters, the length shorten ratio β plays a major role in this model.

Since the model's performances depend on these four parameters, we can realize many different kinds of gait through adjusting these parameters.

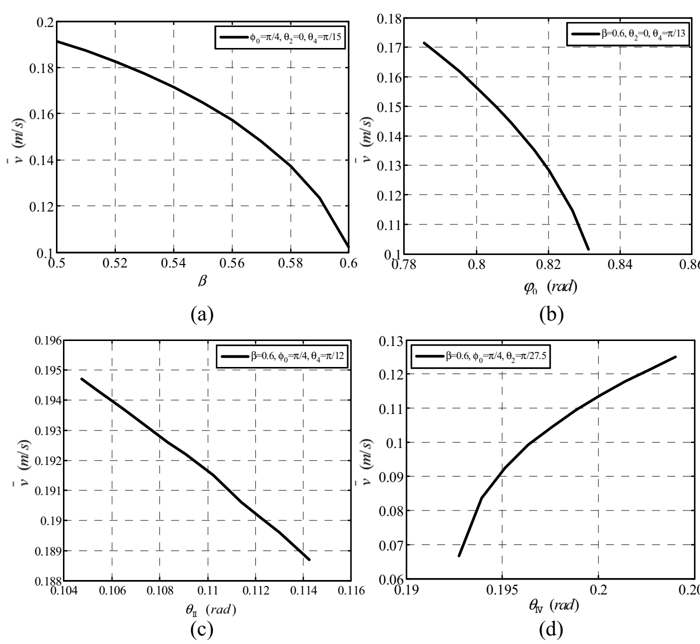


Fig. 8. Influence of model parameters on walking velocity

VI. CONCLUSION AND FUTURE WORK

A. Conclusion

In this paper, we analyze a powered model walking on level ground which is based on the Passive Dynamic Walking. we analyze the characteristic of every key instant and prove that it is practicable to realize compensation for the lost energy by extending and shortening the stance leg. The results show that when the model has appropriate structure parameters and initial conditions, and the stance leg extends and shortens at appropriate angle, it can realize stable periodical gait. Apart from realizing the periodical gait, we also find the constraint conditions of the gait's existence. We analyze the model's stability using eigenvalue of the Jacobian matrix and the basin of attraction. In the end, we study the parameters' influence on the performances of the model.

B. Future Work

The model proposed in this paper has only one mass at the hip and we ignore the coupling between the legs and the hip. We will study more complicated models which consider the masses in the legs and the coupling between the masses in the body. We assume the stance leg extends and shortens instantaneously which makes it easier to analyze. In the future, we will study the effect of extending and shortening over a period of time, and then design the generation algorithm of this gait and realize the gait in real robot.

VII. ACKNOWLEDGMENTS

This work is supported in part by the National Nature Science Foundation of China (No. 60875065) and Open Project Foundation of National Robotics Technology and System Key Lab of China (No. SKLRS200718).

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