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The Instantaneous Leg Extension Model of Virtual Slope Walking

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Abstract—**In our previous work, we have realized Virtual Slope Walking that a robot can walk on level ground as it walks down a virtual slope by leg length modulation. In this paper, we present the instantaneous leg extension model of Virtual Slope Walking to analyze the essentials of Virtual Slope Walking. It has two straight massless legs and a point mass body at the hip. The stance leg is extended instantaneously while the swing leg is swung and shortened actively. We demonstrate that this model can exhibit stable walking cycles on level ground. We obtain the sufficient conditions for the existence of the fixed point. We then illustrate the effect of the model parameters on the fixed point to show how the fixed point can be determined by adjusting the parameters. Further, we theoretically proved that the fixed point is asymptotically stable, meaning that it is independent on the initial conditions. The validity of the proposed model has been examined by numerical simulations.**

I. INTRODUCTION

PASSIVE Dynamic Walker can walk down a shallow slope without any control or actuation, which is first demonstrated by McGeer though simulations and experiments [1]. Since then, Goswami *et al* [2], Coleman *et al* [3], and Garcia *et al* [4] confirmed McGeer's finding and studied various passive walking in detail.

Since the passive walker performs nature and highly efficient gaits, its concept has been used as a starting point for designing actuated walkers that are able to walk on level ground [5]. The slope in Passive Dynamic Walking is not a fundamental property, but rather just a convenient energy source which can be replaced by a variety of other sources. As McGeer stated in [6], there are several options for adding power and control to the passive walking model for level walking, such as torque application between legs which is realized in the delft pneumatic biped Denise [7], torque on the stance leg ankle joint which is realized in Meta [8], impulse application on the trailing leg as it leaves the ground which is realized in the Cornell biped [9]. On the other hand, Goswami *et al* [10] studied the behavior of mechanical energy in Passive Dynamic Walking and suggested a powered walking on level ground from the mechanical energy restoration point of view, leading to work by Spong [11]. Asano et al [12] also propose a level walking by pumping the swing leg.

From the mechanical energy restoration point of view in Passive Dynamic Walking, we proposed Virtual Slope Walking by introducing the leg length modulation [13]. The swing leg is shortened relative to the stance leg prior to the heelstrike, and then the effect would be like taking a downhill step, which we named Virtual Slope. By actively extending the stance leg and shortening the swing leg, a balance between the complementary energy and the dissipation energy is realized in Virtual Slope Walking.

In this paper, we propose the instantaneous leg extension model as the simplest special case of Virtual Slope Walking. This model allows the possibility to analyze the essentials of Virtual Slope Walking in the analytic way, without caring much about the detail of the control algorithm. The fixed point can be studied from an asymptotic solution. Moreover, it can be theoretically investigated how the fixed point is asymptotically stable in this simplest model.

The remainder of this paper is organized as follows. In Section 2, Virtual Slope Walking is introduced. In Section 3, the instantaneous leg extension model of Virtual Slope Walking is presented. In Section 4, we analyze the walking model in the aspects of criteria of existence, characteristic and stability of the fixed point. Section 5 presents the results with discussion and Section 6 the conclusion and future work.

II. PRINCIPLE OF VIRTUAL SLOPE WALKING

In Passive Dynamic Walking, a robot can descend a gentle slope with no energy input other than gravity, and have no active control, as shown in Fig. 1. The lost gravitational potential energy while the robot walks downhill is transformed to the walking kinetic energy and gets dissipated at heelstrike [1]. If the slope angle is appropriate, the complementary gravitational potential energy E_s is equal to the dissipation energy E_r, a stable gait can be synthesized [1].

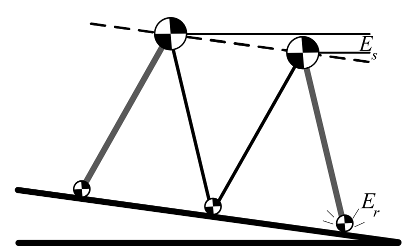

Fig. 1. Passive Dynamic Walking

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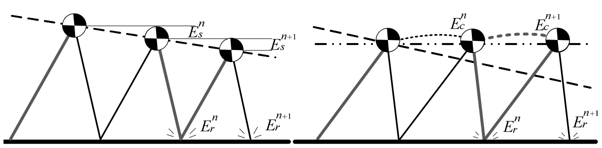
We suppose that the robot leg length can be shortened infinitely and the swing leg can be swung around arbitrarily quickly to the position with constant inter-leg angle for heelstrike. In level walking, the swing leg is shortened by a constant ratio during each step. In this way, the center of gravity experiences a virtual slope as shown in Fig. 2(a). Just as in Passive Dynamic Walking, if the angle of the virtual slope is appropriate, a stable gait can be achieved continuously. However, in practical walking, the leg length of the robot cannot be shortened infinitely. So after heelstrike, it is required that the stance leg shortened in the previous walking step should be actively extended during the following swing phase with an amount of potential energy E_c added into the walking system, as shown in Fig. 2(b). If the complementary energy E_c is equal to the dissipation energy E_r, a stable gait can be synthesized on level ground.

The Virtual Slope Walking is then defined as a combined



(a) Center of Gravity Descending Through the Swing Leg (b) Actively Extending the Stance Leg for Energy Complement

Fig. 2. Virtual Slope Walking

process of actively extending the stance leg and actively swinging & shortening the swing leg.

We have built a planar biped robot named Stepper-2D as the test bed of Virtual Slope Walking, shown in Fig. 3. It is 36.8cm by height and 780g by weight. Both the length of thigh and shank is 12.5cm. The leg length modulation can be realized by bending or unbending the knee joints. We introduce the simple sinusoids to generate the stance leg extension in a period of time. This prototype achieves a relative speed of 4.48 leg/s, which is the fastest relative speed among the known biped robot. We demonstrate the validity of the principle of Virtual Slope Walking in the prototype experiments, as can be found on our website www.au.tsinghua.edu.cn/robotlab/rwg/Robots/Stepper_2D.htm

For the existence of comprehensive function of multiple effects in the prototype, it is hardly to analytically study the essentials of Virtual Slope Walking. We then present the instantaneous leg extension model as the theoretical investigation of Virtual Slope Walking in the following section.

Fig. 3. Stepper-2D
A planar biped with point foot.

III. THE MODEL

A. Model Description

A cartoon of our walking model is shown in Fig. 4. We define a walking step starts when the new stance leg (lighter line) has just made contact with the ground in the upper left picture, namely instant **I**. The stance leg swings to the position in the upper right picture, namely instant **II**, and extends instantaneously in the bottom left picture, namely instant **III**. The swing leg (heavier line) is shortened and swings to the position with constant inter-leg angle before heelstrike in the bottom middle picture, and hits the ground in the bottom right picture, namely instant **IV**.

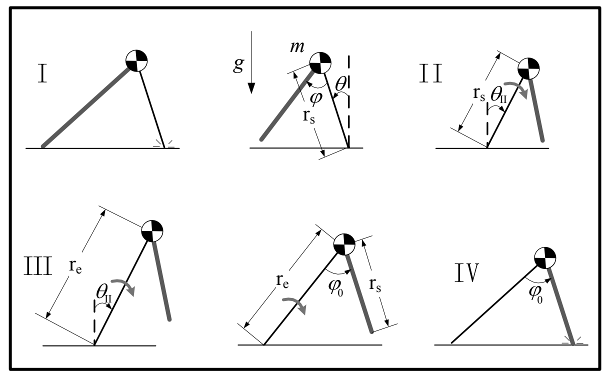


Fig. 4. The instantaneous leg extension model of Virtual Slope Walking. The top-center picture gives a description of the variables and parameters that we use. m: the mass of the body, r_s, r_e: the length of the stance leg before and after leg extension, θ: the angle of the stance leg with respect to the vertical with the negative sign, φ: the inter-leg angle, g: gravitational acceleration.

The details of the model and the underlying assumptions are listed below:

Mass: the model has two straight massless legs and a point mass body at the hip. This model is based on the basic assumption that humans have compact bodies and light legs. And the forces the legs exert on the upper body act through the center of mass, and therefore, applying very little rotational moment on the upper body.

Leg: the two legs are modeled as a telescoping actuator with a point foot. The stance leg length is extended from r_s to r_e, and the swing leg length is shortened from r_e to r_s. The length shorten ratio is defined as $\beta= r_s / r_e$.

Actuation: the stance leg is actuated for instantaneous extension, while the swing phase is an unactuated inverted pendulum. The swing leg is actuated for shortening and swinging to the position with constant inter-leg angle.

Instantaneous Extension Transition: the stance leg extends instantaneously as shown in Fig. 4 from instant **II** to instant **III**, which implies that during the instantaneous transition stage:

--the robot configuration remains unchanged, except the stance leg length,

--the angular momentum of the robot about the point of the support on the ground of its stance leg is conserved. This conservation law leads to a discontinuous change in the velocity of the point mass body.

Heelstrike: the impact of the swing leg with the ground is assumed to be fully inelastic (no slip, no bounce). This implies that during the instantaneous transition stage:

--the robot configuration remains unchanged,

--the angular momentum of the robot about the swing foot contact point is conserved. This conservation law leads to a discontinuous change in the velocity of the point mass body.

--double stance is assumed to occur instantaneously. When the swing leg hits the ground and sticks, the previous stance leg lifts up.

--for analyzing the main effect of stance leg extension, the inter-leg angle φ at heelstrike is always constant, which means that the swing leg can be swung around arbitrarily quickly to the position with constant inter-leg angle before heelstrike. Since the swing leg is modeled as having zero mass, this assumption exists distinctly.

B. Governing Equations

The governing equations of the robot consist of nonlinear differential equations for the swing phase and algebraic equations for the transitions of leg extension and heelstrike.

1) Swing phase from **I** *to* **II***:* Using the *Lagrangian Equations*, the second-order differential equation of motion is given below for the swing phase of the stance leg:

$$\ddot{\theta}(t) = \frac{g}{r_s}\sin\theta(t) \tag{1}$$

Rescaling the time by defining dimensionless time $\tau = \sqrt{g/r_e}\,t$, (1) can be written as:

$$\ddot{\theta}(\tau) = \frac{1}{\beta}\sin\theta(\tau) \tag{2}$$

For the simplicity, we will refer to dimensionless time τ as the time variable, henceforward.

2) Transition from **II** *to* **III***:* From the conservation of angular momentum about the stance foot contact point in the instantaneous leg extension, we obtain the following transition equation:

$$\omega_{\mathrm{III}} = \beta^2\omega_{\mathrm{II}} \tag{3}$$

where ω is the angular velocity of the stance leg and the '**II**' and '**III**' subscripts denote the instant **II** and **III** respectively.

3) Swing phase from **III** *to* **IV***:* Similar to the equation in *1)*, the equation of motion for the stance leg with the length of r_e can be written as:

$$\ddot{\theta}(\tau) = \sin\theta(\tau) \tag{4}$$

4) Transition from **IV** *to* **I** *of the subsequent step:* The heelstrike from step *n* to the subsequent step *n*+1 occurs when the geometric collision condition

$$\begin{cases} \theta_{\mathrm{I}}(n+1) = -(\varphi_0 - \theta_{\mathrm{IV}}(n)) \\ \beta\cos\theta_{\mathrm{I}}(n+1) = \cos\theta_{\mathrm{IV}}(n) \end{cases} \tag{5}$$

is met, where the '**I**' and '**IV**' subscripts denote the instant **I** and **IV** respectively, φ_0 is the constant of the inter-leg angle at heelstrike. Equation (5) also reflects a change of names for the two legs. The swing leg becomes the stance leg, and vice versa.

From the conservation of angular momentum about the swing foot contact point at heelstrike, we obtain the following transition equation:

$$\omega_{\mathrm{I}}(n+1) = \frac{\cos\varphi_0}{\beta}\omega_{\mathrm{IV}}(n) \tag{6}$$

Equations (2)-(6) construct the dynamic equations of hybrid system of the instantaneous leg extension model.

IV. Analysis of The Model

A. Poincaré Section and Walking Map

The general procedure for the study of this model is based on interpreting a step as a Poincaré map, or, as McGeer termed it, a 'stride function' [1]. Gait limit cycles are fixed points of this function.

Our Poincaré section is at the start of a step, namely instant **I** in Fig. 4, just after heelstrike. Given the state of the system at instant **I**, the Poincaré map **f** determines the state just after the next heelstrike. Note that in the geometric collision condition (5), the stance leg angle θ_{I} is constant with inter-leg angle φ_0. So the heelstrike transition reduces this problem in 2D state space $\{\theta_{\mathrm{I}}, \omega_{\mathrm{I}}\}$ to a one dimensional map **f**, only consisting of angular velocity ω_{I}.

So, while the system has only one independent initial condition, we need to specify ω_{I} at the start of walking step *n* to fully determine the subsequent motion at steps *n*+1, *n*+2,… so that $\omega_{\mathrm{I}}(n+1)=$ **f**$(\omega_{\mathrm{I}}(n))$. Applying energy conservation in the two swing stages (from instant **I** to instant **II** and from instant **III** to instant **IV**) and transition equations in leg extension and heelstrike, the walking map can be written as:

$$\begin{aligned} \omega_{\mathbf{I}}^2(n+1) &= \beta^2\cos^2\varphi_0\,\omega_{\mathbf{I}}^2(n) \\ &\quad + 2\cos^2\varphi_0[\cos\theta_{\mathbf{II}}(\frac{1}{\beta^2}-\beta) - \cos\theta_{\mathbf{I}}(\frac{1}{\beta}-\beta)] \end{aligned} \tag{7}$$

where θ_{II} is the stance leg angle at leg extension.

To simplify the definition of the map **f**, a new variable *q* is taken to be the square of the angular velocity

$$q = \omega_{\mathrm{I}}^2 \tag{8}$$

Since the stance leg angle θ_{I} is constant, variable *q* indicating the kinetic energy can represent the total mechanical energy of the system. This state variable is different from that in Passive Dynamic Walking. With the change in variables, the Poincaré map **f** is linear in *q*

$$\begin{aligned} \mathbf{f}(q) &= \beta^2\cos^2\varphi_0\,q \\ &\quad + 2\cos^2\varphi_0[\cos\theta_{\mathbf{II}}(\frac{1}{\beta^2}-\beta) - \cos\theta_{\mathbf{I}}(\frac{1}{\beta}-\beta)] \end{aligned} \tag{9}$$

We will refer to *q* as the state variable, henceforward.

B. Existence of Fixed Points

A point q^f is a fixed point of **f** if **f**$(q^f) = q^f$. So the fixed point of **f** is:

$$q^f = \frac{2\cos^2\varphi_0[\cos\theta_{\mathrm{II}}(1-\beta^3) - \cos\theta_{\mathrm{I}}(\beta-\beta^3)]}{\beta^2(1-\cos^2\varphi_0\beta^2)} \tag{10}$$

The stance leg angle θ_{I} at Poincaré section can be obtained from (5) as follows:

$$\theta_{\mathrm{I}} = -\arctan\frac{\beta - \cos\varphi_0}{\sin\varphi_0} \tag{11}$$

Three sufficient conditions have to be satisfied for the existence of fixed point:

1) Referring the definition of q in (8), the sign of q is always positive, this requirement is

$$q^f > 0 \tag{12}$$

Combing (10) and (12), this inequality can be rewritten as a criterion relating the length shorten ratio β, the leg extension angle θ_{II}, and the inter-leg angle φ_0

$$\frac{\cos\theta_{\mathrm{II}}}{\cos\theta_{\mathrm{I}}} > \frac{\beta - \beta^3}{1-\beta^3} \tag{13}$$

where θ_{I} is the function of β and φ_0 in (11).

2) For a fixed point to exist, the robot must be able to pass the apex at mid-stance repeatedly after each heelstrike, i.e., the robot will not fall backward. This requirement can be described as a criterion relating the initial condition of each step

$$q > \begin{cases} \dfrac{2(1-\cos\theta_{\mathrm{I}})}{\beta} & \theta_{\mathrm{II}} \ge 0 \\ 2\left[\dfrac{1-\cos\theta_{\mathrm{II}}}{\beta^4} + \dfrac{\cos\theta_{\mathrm{II}} - \cos\theta_{\mathrm{I}}}{\beta}\right] & \theta_{\mathrm{II}} < 0 \end{cases} \tag{14}$$

3) Geometric restrictions are listed below:

--the sign of the stance leg angle θ_{I} should be negative, this requires

$$\beta > \cos\varphi_0 \tag{15}$$

--the stance leg angular velocity just after heelstrike should be greater than zero, this requires

$$\varphi_0 < \frac{\pi}{2} \tag{16}$$

--the stance leg should extend before heelstrike, this requires

$$\theta_{\mathrm{II}} < \frac{\varphi_0}{2} \tag{17}$$

C. Effect of Model Parameters on the Fixed Point

Referring the analytical expression of fixed point q^f in (10) and (11), q^f is determined by three parameters: the length shorten ratio β, the leg extension angle θ_{II}, and the inter-leg angle φ_0. The influence of each parameter on q^f is presented individually as follows.

1) q^f is shown as a function of the length shorten ratio β in Fig. 5.

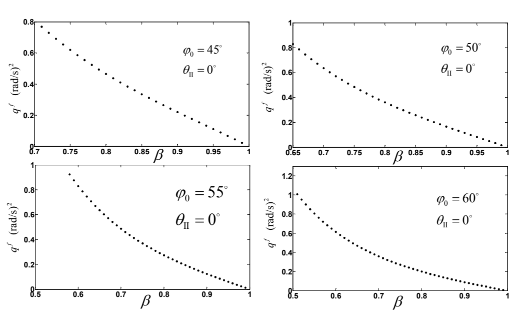


Fig. 5. The variation of q^f with respect to the length shorten ratio β for $\theta_{\mathrm{II}}=0^\circ$ and $\varphi_0=45^\circ$, 50°, 55°, and 60°.

It is indicated from Fig. 5 that q^f decreases with an increase in the length shorten ratio β. An increase in β causes a net decrease in the extended leg length, resulting in a net decrease in the complementary energy E_c. On the other hand, the dissipation energy E_r at heelstrike can be represented as the function of β and φ_0

$$E_r = \frac{1}{2} m g r_e q \tan^2\varphi_0 \beta^2 \tag{18}$$

E_r increases as β increases. Consequently, the total mechanical energy E is lowered, and q^f decreases. The main conclusion from this graph is that a decrease in β leads to a greater fixed point q^f. However, there is a lower limitation of β shown in (15). And β is also restricted by the physical parameters of the real robot.

2) q^f is shown as a function of the leg extension angle θ_{II} in Fig. 6.

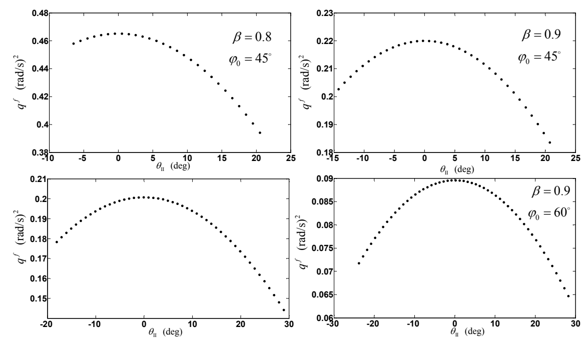


Fig. 6. The variation of q^f with respect to the leg extension angle θ_{II} for β=0.8 and 0.9, $\varphi_0=45^\circ$ and 60°.

Fig. 6 shows a second order relationship between q^f and the leg extension angle θ_{II}. As θ_{II} approaching zero from both side, q^f increases and reaches a maximal value at $\theta_{\mathrm{II}}=0^\circ$. As θ_{II} approaching zero, the vertical projection of leg length extension increases, and more potential energy is complemented. As a consequence, the total energy E increases, and q^f increases. The vertical projection of leg length extension reaches its maximum at $\theta_{\mathrm{II}}=0^\circ$. It can be concluded from this graph that extending the stance leg more close to mid-stance will result in a greater fixed point q^f.

3) q^f is shown as a function of the inter-leg angle φ_0 in Fig. 7.

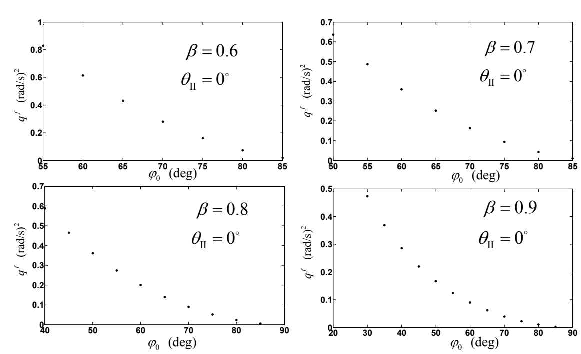


Fig. 7. The variation of q^f with respect to the inter-leg angle φ_0 for $\theta_{\mathrm{II}}=0^\circ$ and β=0.6, 0.7, 0.8, and 0.9.

As shown in Fig. 7, q^f decreases with an increase in the inter-leg angle φ_0. The dissipation energy E_r increases as φ_0 increases in (18). As a consequence, the total energy E decreases, and q^f decreases. So we can conclude that a larger φ_0 results in a smaller q^f, but does not mean a slower walking for the step length is larger simultaneously.

We can conclude that q^f can be parameterized by β, θ_{II}, and φ_0. The desired fixed point can be easily produced by adjusting these three controllable parameters, which is a significant improvement for Passive Dynamic Walking.

D. Stability Analysis

The eigenvalues of the jacobian **J** of the Poincaré map **f** govern the stability of the fixed point. If all eigenvalues are within the unit circle, rendering a small perturbation will decay with time, then the gait cycle is asymptotically stable.

Considering the one dimensional map **f** in (9), the eigenvalues of the jacobian **J** can be represented as the first derivative of the map **f** at the fixed point

$$\lambda = \left.\frac{d\mathbf{f}}{dq}\right|_{q=q^f} = \beta^2 \cos^2 \varphi_0 \tag{19}$$

Since the length shorten ratio β and $\cos\varphi_0$ are always less than one, the eigenvalue of the jacobian **J** is within the unit circle. The fixed point is asymptotically stable.

E. Energy Analysis

This walking model is not conservative holonomic system, since energy is gained in stance leg extension and lost at heelstrike, which is different from Passive Dynamic Walking. As discussed indirectly in [14], the dissipative collision allows the possibility of asymptotic stability. How asymptotic stability is approached can be seen by considering the energy of the system. The energy gained by the stance leg extension and the energy lost at heelstrike are linear functions of the state variable q.

The complementary energy E_c is a net change of the total mechanical energy in stance leg extension transition, which can be represented below

$$\begin{aligned} E_c &= -\frac{1}{2} mgr_e(\beta^2 - \beta^4)q \\ &\quad + mgr_e[(1-\beta^3)\cos\theta_{II} - (\beta - \beta^3)\cos\theta_I] \end{aligned} \tag{20}$$

Once given the model parameters m, r_e, β, θ_{II}, and φ_0, E_c is proportional to the state variable q.

The dissipation energy E_r is a net change of the total mechanical energy at heelstrike, which can be represented below

$$\begin{aligned} E_r &= \frac{1}{2} mgr_e\beta^4(1-\cos^2\varphi_0)q \\ &\quad + mgr_e(1-\cos^2\varphi_0)[(1-\beta^3)\cos\theta_{II} - (\beta-\beta^3)\cos\theta_I] \end{aligned} \tag{21}$$

Once given the model parameters m, r_e, β, θ_{II}, and φ_0, E_r is also proportional to the state variable q.

We define E_Δ as the net change of the total mechanical energy in one step. So E_Δ can be represented as follows

$$E_\Delta = E_c - E_r \tag{22}$$

Substituting (20) and (21) into (22), we obtain

$$E_\Delta = -\frac{1}{2} mgr_e\beta^2(1-\beta^2\cos^2\varphi_0)q + C \tag{23}$$

$$C = mgr_e\cos^2\varphi_0[(1-\beta^3)\cos\theta_{II} - (\beta-\beta^3)\cos\theta_I]$$

Once given the model parameters m, r_e, β, θ_{II}, and φ_0, E_Δ has a linear relationship with the state variable q.

From the conservation of energy, E_Δ should be zero at fixed point. Once there is a perturbation on q, the state variable will approach the fixed point q^f at a rate of $\beta^2\cos^2\varphi_0$, which can be obtained in (19) by the linearization of Poincaré map **f**. As a consequence, E_Δ will approach zero with the same rate, meaning that the complementary energy E_c and The dissipation energy E_r will be balanced asymptotically.

V. Results and Discussion

A. Typical Gait Cycles

Stable gait cycles can be found if the existence criteria of the fixed point are satisfied. Using numerical simulation, a typical plot of the cyclic walking motion is shown in Fig. 8.

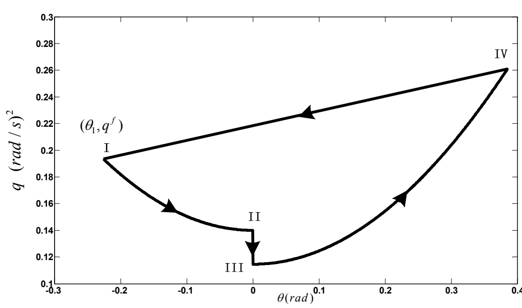


Fig. 8. A typical gait cycles in phase space. The transition instants match the plot of Fig. 4. At a gait cycle, heelstrike returns the system to its initial conditions. $\theta_{II}=0^o$, β=0.8, and $\varphi_0=45^o$.

As shown in Fig. 8, a walking step starts at instant **I**, and the stance leg swings to instant **II** as an unactuated inverted pendulum, following the arrowheads in the phase trajectory. The phase trajectory from Instant **II** to instant **III** corresponds to the stance leg instantaneous extension, with a discontinue change of the stance leg angular velocity and unchanged stance leg angle. From instant **III** to instant **IV**, the stance leg also swings like an inverted pendulum. Heelstrike happens at the phase trajectory from instant **IV** to instant **I**, resulting in a discontinue change of the stance leg angular velocity and the exchange of the two legs.

The dynamic behavior in Fig. 8 is the same as that in Passive Dynamic Walking, except the state jump in the stance leg extension. It is indicated that the effect of the stance leg extension is the same as the slope in Passive Dynamic Walking for energy complement. Moreover, there is more freedom to affect the fixed point by stance leg extension than that in Passive Dynamic Walking.

B. Basin of Attraction

The basin of attraction for each θ_{II} is shown in Fig. 9 from numerical simulations. All initial conditions leading to the fixed point are contained inside the basin of attraction.

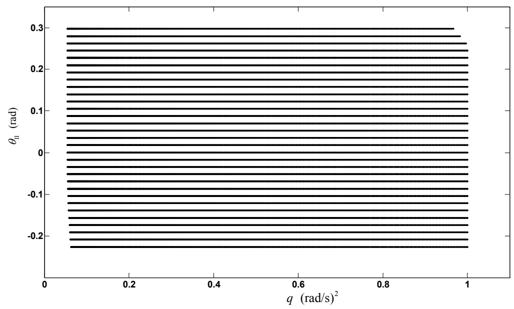


Fig. 9. The basin of attraction for the fixed point with the variation of θ_{II}. β=0.8 and φ_0=45°.

The left boundary in Fig. 9 is defined as the critical angular velocities that make the robot pass the apex at mid-stance, as shown in (14). Below this boundary, the robot falls backward. The right boundary is defined as the critical angular velocities that make the stance foot lose contact with the ground. Note that this is more or less equal to the commonly used boundary of Froude number $(\omega^2 r)/g>1$ [15]. The upper limitation of leg extension angle θ_{II} is determined by (13) and (17). As shown in Fig. 9, any initial condition is attracted to the limit cycle motion, as long as the existence criteria are satisfied. This conclusion is identical with the stability analysis in section IV.

Comparing with the small region of BoA in passive Dynamic Walking [16], the BoA in this model is enlarged for the whole possible state space, which can be explained as follows. Firstly, the complementary energy by stance leg extension helps the robot approach to the fixed point once disturbed. Then, the effect of swing leg is neglected, and the inter-leg angle at heelstrike is constant, resulting in an asymptotically stable fixed point [17].

VI. Conclusion and Future Work

In this paper, we presented the instantaneous leg extension model of Virtual Slope Walking. The results of this study are summarized as follows:

1) We demonstrate that the instantaneous leg extension model of Virtual Slope Walking can exhibit stable gait cycles. And three sufficient mathematical conditions for the existence of the fixed point are obtained in the analytical way.
2) Based on the analytical expression of the fixed point, the effect of the model parameters on the fixed point can be concluded as follows. A decrease in the length shorten ratio β has a positive effect on increasing the fixed point q^f. And the optimal leg extension angle θ_{II}=0° yields a maximum fixed point q^f. An increase in the inter-leg angle φ_0 has a negative effect on increasing the fixed point q^f. Finally, the fixed point can be controlled flexibly by adjusting these three parameters.
3) We theoretically proved that the fixed point is asymptotically stable, meaning that the fixed point is independent on the initial conditions. The BoA gives a validation of the theoretical result.

The model we presented in this paper partly demonstrates the effect of stance leg extension in Virtual Slope Walking. We will introduce a compass-like biped model to investigate the effect of swing leg on mechanical energy restoration in Virtual Slope Walking for the future work.

We find that the exact trajectories of the telescopic leg motion are of little concern in Stepper-2D's experiment results. As long as the energy balance condition is satisfied, the robot can generate stable walking with various shapes of smooth trajectories. Our long term goal of this research is to prove that Virtual Slope Walking can generate a stable gait in a simple way without the elaborate trajectory control algorithm.

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