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Parametric Excitation Mechanisms for Dynamic Bipedal Walking

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Abstract— **It is already clarified throughout studies of passive dynamic walking mechanisms that the common necessary condition for dynamic gait generation comes from the requirement on mechanical energy restoration. Until now we have treated only rotational joints of the robot, whereas in this paper we consider a novel dynamic gait generation method based on mechanical energy restoration by parametric excitation using telescopic leg actuation. We first introduce a simple walking model and a control law for the telescopic leg motion, and show the typical walking pattern by numerical simulations. We then analyze the gait performance by adjusting some control and physical parameters. In addition, some extensions of the mechanism and control applications are investigated.**

Index Terms— **Biped walking, legged locomotion, mechanical energy, parametric excitation, gait generation.**

I. Introduction

A. From ZMP robots to passive dynamic walking

There are two basic studies on biped locomotion. One is the study by Vukobratović *et al.*in 1969 [1] which described *why biped locomotion is difficult* throughout the investigation of Zero Moment Point (ZMP). After that, it has been recognized as the central problem of biped gait generation that how to realize the desired motion under the ZMP constraint or ankle torque limitation. The other is the study by McGeer in 1990 [2] which appealed that biped locomotion is *not difficult* and can be realized easily without using any pre-designed walking patterns but utilizing its own physical dynamics effectively. His work brought an important insight to legged locomotion research, and thereafter many studies taking into account passive walking mechanisms have been reported. Nowadays, its concept is familiar to legged locomotion researchers and is as important as ZMP.

The study by Goswami *et al.*[3] is recognized as the first which suggested the importance of mechanical energy restoration in dynamic gait generation. This leads to works by Spong [4] and Asano *et al.*[5] These approaches can easily generate a dynamic gait whose specific resistance is quite small [6]. In order to reproduce passive walking mechanism on a level floor, however, we must introduce foot or torso to exert control torques w.r.t. the legs. Several problems arise along with it. In the case of using foot, ZMP constraint (tilt limitation problem) arises [1][5]. In the case of using torso, a difficulty will arise on how to drive the legs while balancing the torso stably [7]. These are because a rotational actuator needs another link in order to exert the control torque on one link. We must add links to the original passive walker.

One of the most important and remarkable point which McGeer's original passive dynamic walking suggests is realization of dynamic gait generation *by leg only* without using any other link such as torso and feet. If we could succeed this property, that is, if *the leg itself* had a mechanism to restore the mechanical energy, the above problems would ought to be released.

Based on the observations, this paper proposes a novel dynamic gait generation method based on the effectiveness of parametric excitation. The energy restoration in this case is caused by up-and-down motion of the leg mass as explained later. Although several studies on passive walking with telescopic leg motion have been reported [8][9], they did not treat the telescopic leg part's dynamics. This is what the studies on passive walking have overlooked, and is very important factor for gait generation. The fascination of this mechanism is not only out of use of feet nor torso for energy restoration but also foot lifting for obstacle avoidance, that is, achievement of both physical and functional meanings as a legged locomotion.

B. Parametric excitation mechanism

Minakata and Tadakuma experimentally studied and showed dynamic gait generation on a level by telescopic leg actuation using an actual machine named "Pseudo Passive Walking Robot (PPWR)"[10]. They mentioned that the leg length control works as the ankle joint control in their original paper [10], however, its dynamics principle has not been theoretically clarified yet. The authors have investigated it from mechanical energy restoration point of view and found that it can be explained by parametric excitation effect.

Fig. 1 shows the optimal control to increase the mechanical energy. The point mass starts from A, and moves up when $\theta = 0$ from B to C instantaneously. Finally it moves down from D to E. Throughout this control, the

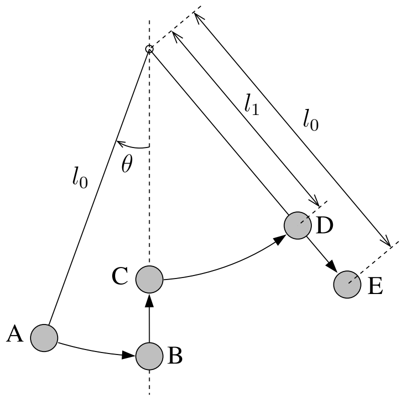


Fig. 1. Optimal control to increase mechanical energy.

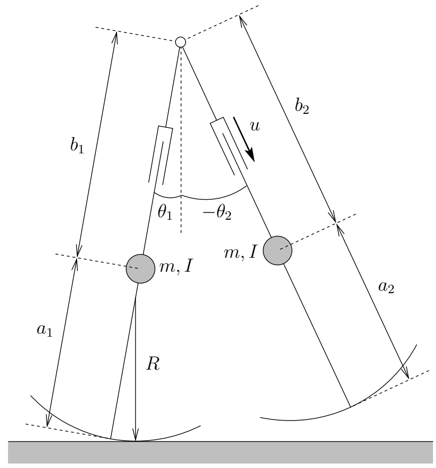


Fig. 2. Model of a telescopic legged robot.

total mechanical energy has been restored effectively. The readers should refer to [11] for further detail. Parametric excitation is a familiar phenomenon to human and can be observed in a *swing* and a crane. This is also known to as "variable length pendulum" in the research area of control theory. The optimal control is the desirable one from the control theory point of view, however, it requires instantaneous movement of the mass and therefore is impossible to realize. This paper then proposes an intuitive control law of the telescopic motion of the swing leg mass, and investigates its effectiveness through numerical simulations.

II. Model of a Telescopic Legged Robot

This paper deals with a planar biped model with telescopic legs shown in Fig. 2. We assume that the robot does not have actuators at the hip and ankle joints, only the telescopic actuators on the leg are available. By moving the swing leg's mass to the leg direction following a proposed method, the robot system can increase the mechanical energy by the effectiveness of parametric excitation. We assume that the stance leg's actuator is mechanically locked during the swing phase keeping the length $b_1 = b$ [m] where b is a positive constant. The lower parts' length a_1 and a_2 are equal to constant a [m]. The swing leg length b_2 is also settled to this values before heel-strike impact. Then the robot can be modeled as a 3-dof system whose generalized coordinate vector is $\boldsymbol{q} = \begin{bmatrix} \theta_1 & \theta_2 & b_2 \end{bmatrix}^{\mathrm{T}}$ as shown in Fig. 2.

The swing phase dynamic equation is given by

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \boldsymbol{S}u, \tag{1}$$

where $\boldsymbol{M}(\boldsymbol{q}) \in R^{3\times 3}$ is the inertia matrix and $\boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) \in R^3$ is the vector of Coriolis, centrifugal and gravity forces, respectively. u is the control input of the telescopic actuator of the swing leg and $\boldsymbol{S} := \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^{\mathrm{T}}$. The total mechanical energy E is defined as

$$E(\boldsymbol{q}, \dot{\boldsymbol{q}}) := \frac{1}{2}\dot{\boldsymbol{q}}^T \boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}} + P(\boldsymbol{q}), \tag{2}$$

where P is the potential energy, and its time derivative satisfies the following relation

$$\dot{E} = \dot{\boldsymbol{q}}^{\mathrm{T}}\boldsymbol{S}u = \dot{b}_2 u. \tag{3}$$

The heel-strike collision is assumed to be perfectly inelastic without slipping, and thus the stance leg and the swing leg are instantaneously switched, that is, double support phase does not exist.

In general, ZMP robots need flat sole to exert the ankle joint torque for the specified control task whereas this robot can use semicircular feet as McGeer's original passive walker because of the free ankle joint property. This feet mechanism is of great advantage to increase the walking speed as described later.

III. Parametrically Excited Dynamic Bipedal Walking

This section proposes a simple control law for the telescopic leg actuation and investigates the typical motion of generated dynamic gait.

A. The control law

The dynamic gait generation can be realized by a simple control of the swing leg length. We choose the telescopic leg length $b_2 = \boldsymbol{S}^{\mathrm{T}}\boldsymbol{q}$ as the output for control. Its second order derivative yields

$$\ddot{b}_2 = \boldsymbol{S}^{\mathrm{T}}\boldsymbol{M}(\boldsymbol{q})^{-1}\boldsymbol{S}u - \boldsymbol{S}^{\mathrm{T}}\boldsymbol{M}(\boldsymbol{q})^{-1}\boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}). \tag{4}$$

Then the control input for trajectory tracking control to $b_{2\mathrm{d}}(t)$ can be considered as

$$u = \left(\boldsymbol{S}^{\mathrm{T}}\boldsymbol{M}^{-1}\boldsymbol{S}\right)^{-1}\left(\bar{u} + \boldsymbol{S}^{\mathrm{T}}\boldsymbol{M}^{-1}\boldsymbol{h}\right), \tag{5}$$

$$\bar{u} = \ddot{b}_{2\mathrm{d}} + k_{\mathrm{d}}(\dot{b}_{2\mathrm{d}} - \dot{b}_2) + k_{\mathrm{p}}(b_{2\mathrm{d}} - b_2), \tag{6}$$

where $k_{\mathrm{d}} > 0$ and $k_{\mathrm{p}} > 0$ are the derivative and proportional gains, respectively. In this paper, we intuitively introduce the following desired time trajectory $b_{2\mathrm{d}}(t)$ for telescopic leg motion.

$$b_{2\mathrm{d}}(t) = \begin{cases} b - A\sin^3\left(\frac{\pi}{T_{\mathrm{set}}}t\right) & (t \le T_{\mathrm{set}}) \\ b & (t > T_{\mathrm{set}}) \end{cases} \tag{7}$$

T_{set} [s] is the desired settling time and we assume that the heel-strike collision occurs after the settling time. In other words, the condition $T \ge T_{\mathrm{set}}$ is assumed to be always satisfied in the case of steady walking where T [s] is the steady step period. A [m] is the amplitude of the telescopic leg motion. In this paper, considering realization of smooth telescopic leg motion, we designed $b_{2\mathrm{d}}$ so that $\ddot{b}_{2\mathrm{d}}$ becomes always continuous during the swing phase.

B. Numerical simulations

Fig. 3 shows the simulation results of parametrically excited dynamic bipedal walking where $A = 0.08$ [m] and $T_{\mathrm{set}} = 0.55$ [s]. The control torques are generated as a digital control system whose period is 1.0 [ms]. Physical parameters are chosen as in Table I. From the results, we can see that a stable limit cycle is generated by the effect of the proposed method. From Figs. 3(b) and (c), we can confirm that the leg length is successfully controlled and settled to the same length b [m] before every transition instant whereas the mechanical energy is restored by the effectiveness of parametric excitation. Since this robot does not require the ankle joint torque, we need not consider the ZMP condition. The ZMP in this case moves forward monotonically from the heel to the tiptoe under the assumption that the condition $\dot{\theta}_1 \ge 0$ holds. This property looks like humans. The mechanical energy behaviour in (c) is greatly different from that of virtual gravity approaches, sawtooth waveform [13], and this means inefficient. We have showed that monotonic mechanical energy restoration is a necessary condition for energy-effective walking in [5], and how to improve this feature will be investigated later. Before it, in the next section, we will investigate the dynamic gait according to several robot parameters in more detail.

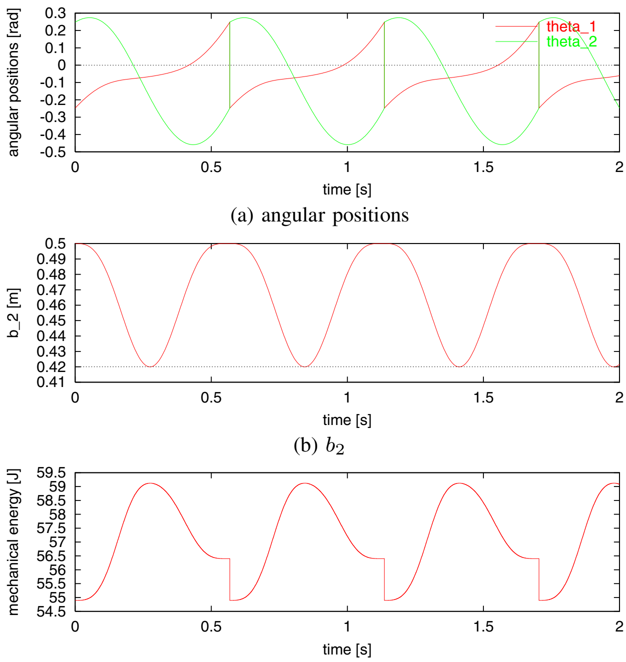
TABLE I

PHYSICAL PARAMETERS OF THE ROBOT

$a\ (= a_1)$	0.5	m
$b\ (= b_1)$	0.5	m
R	0.5	m
m	5.0	kg
I	0.1	kg·m^2

IV. Influence of Robot Parameters on Steady Gaits

This section analyzes changes of the gait according to the control parameters T_{set}, A and the foot radius R.



(a) angular positions

(b) b_2

(c) mechanical energy

Fig. 3. Simulation results of parametrically excited dynamic walking where $A = 0.08$ [m] and $T_{\mathrm{set}} = 0.55$ [s]. (a) angular positions, (b) b_2 and (c) mechanical energy.

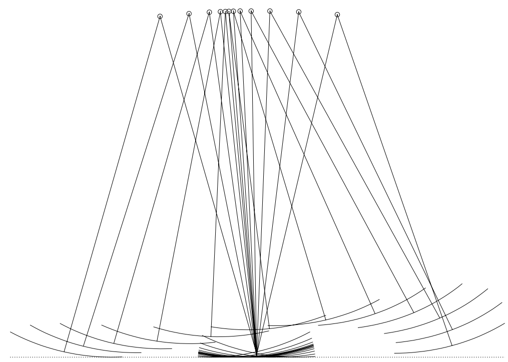


Fig. 4. Stick diagram of parametrically excited dynamic bipedal walking where $T_{\mathrm{set}} = 0.55$ [s] and $A = 0.08$ [m].

A. Effect of settling time T_{set}

We first examine the effect of T_{set}. Fig. 5 shows the result of (a) step period T [s], (b) half-interleg angle α [rad], (c) walking speed v [m/s], (d) restored mechanical energy ΔE [J] for $a = 0.05$, 0.06 and 0.07 [m], respectively. α is determined by using the steady positional information at impact as

$$\alpha := \frac{\theta_1^- - \theta_2^-}{2} = \frac{\theta_2^+ - \theta_1^+}{2} > 0. \tag{8}$$

The walking speed v is determined as a discrete information and is explained in the next section. The restored mechanical energy is determined as $\Delta E := E(T^-) - E(0^+)$. As mentioned, the condition $T_{\mathrm{set}} \le T$ must be satisfied,

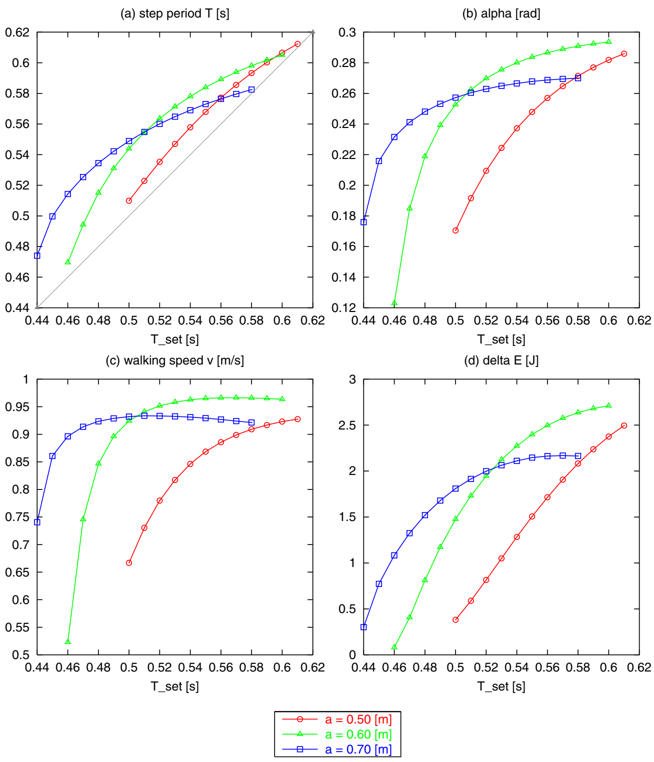


Fig. 5. Simulation results of the discrete events w.r.t. T_{set} [s] for $a = 0.50$, 0.60 and 0.70 [m] where $A = 0.08$ [m].

and it is confirmed from (a). The $45°$ line indicates the limitation of settling time $T_{\text{set}} = T$. From (c), we can see that there exists the maximum walking speed except the case of $a = 0.50$ [m]. In all cases, α and ΔE monotonically increase w.r.t. T_{set}.

B. Effect of amplitude A

It is well known that passive dynamic walking often exhibits period-doubling bifurcation and chaotic behaviour. Parametrically excited dynamic walking also exhibits bifurcation of gait. Fig. 6 shows the evolutions of the step period where $T_{\text{set}} = 0.55$ [s] for $A = 0.08$, 0.09, 0.10, 0.11 and 0.12 [m]. The robot starts walking from the same initial condition. In general, large A leads increasing of v and ΔE, and such an excessive power input causes period-doubling bifurcation. The generated limit cycle then becomes unstable.

C. Effect of foot radius R

It is known that adjustment of the foot radius R can increase the walking speed [14]. With suitable choice of R, the robot can get the maximum walking speed without changing the control law. Fig. 7 shows the results of (a) step period T [s], (b) walking speed v [m/s] for four values of A where $T_{\text{set}} = 0.55$ [s], respectively. We can see that, in each case, T monotonically decreases with increase of R and there exists the optimal R which gives the maximum walking speed. The obtained walking speeds

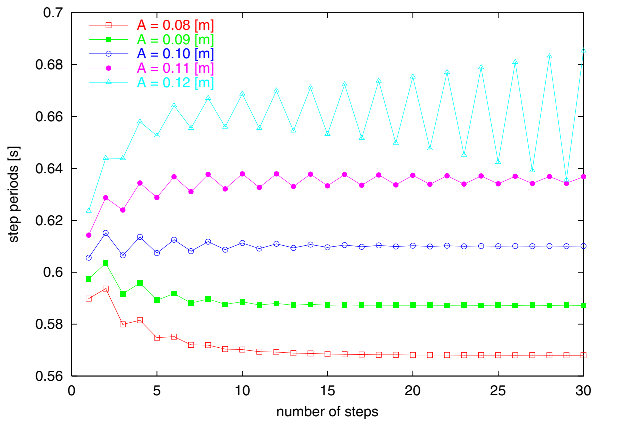


Fig. 6. Evolutions of the step period for $A = 0.08$, 0.09, 0.10, 0.11 and 0.12 [m] where $T_{\text{set}} = 0.55$ [s].

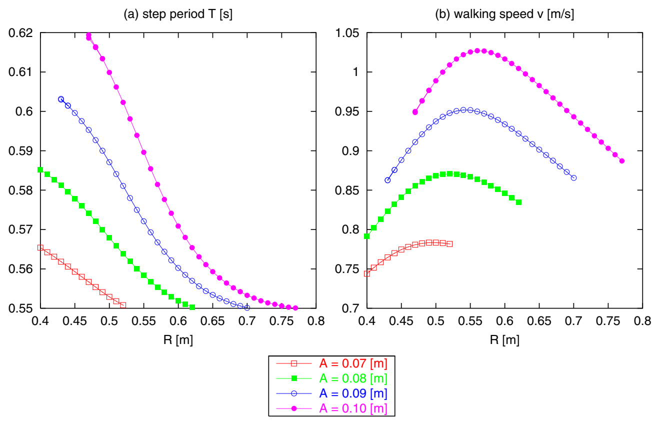


Fig. 7. Simulation results of the discrete events w.r.t. the foot radius R [m] for $A = 0.07$, 0.08, 0.09 and 0.10 [m].

are remarkably faster than that of virtual gravity approaches with ankle actuation using flat sole [3][5].

Investigation of circular sole mechanism is an interesting research theme. Circular sole can increase not only the walking speed but also the robust stability [15]. By using the ankle torque together, realization of more high performance dynamic walking can be expected. In addition, influence of the central position of feet w.r.t. the leg should be analyzed.

V. Improvement of Energy-Efficiency Using Elastic Elements

The telescopic leg actuation requires very large torque to lift-up the whole leg mass and this causes inefficient dynamic walking. As a candidate to solve this problem, utilization of elastic elements can be considered. This section then introduces a model with elastic elements and analyzes its effectiveness by numerical simulations.

A. Model with elastic elements

Fig. 8 shows the biped model with elastic elements where k is the elastic coefficient and b_0 is its nominal

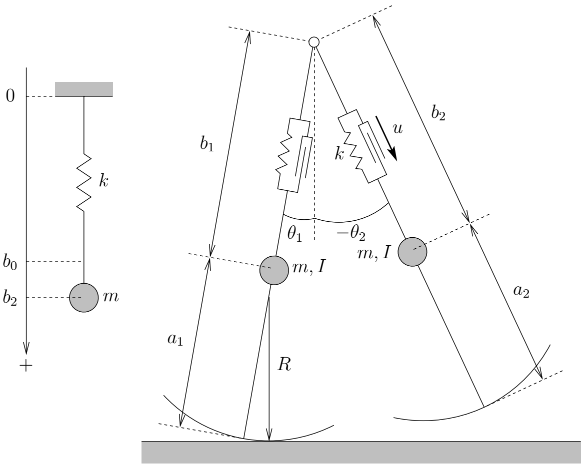


Fig. 8. Model of a telescopic legged robot with elastic elements.

length. Its dynamic equation during the swing phase is given by

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \boldsymbol{S}u - \frac{\partial Q}{\partial \boldsymbol{q}^{\mathrm{T}}}, \tag{9}$$

where Q is the elastic energy defined as

$$Q := \frac{1}{2}k\left(b_2 - b_0\right)^2. \tag{10}$$

The terms except elastic effect in Eq. (9) is the same with those of Eq. (1).

B. Performance analysis

The input torque u for model matching control $b_2 = b_{2\mathrm{d}}$ considering the elastic effect is given by

$$u = \left(\boldsymbol{S}^{\mathrm{T}}\boldsymbol{M}^{-1}\boldsymbol{S}\right)^{-1}\left(\ddot{b}_{2\mathrm{d}} + \boldsymbol{S}^{\mathrm{T}}\boldsymbol{M}^{-1}\left(\boldsymbol{h} + \frac{\partial Q}{\partial \boldsymbol{q}^{\mathrm{T}}}\right)\right), \tag{11}$$

and in this section we give this control input as a continuous-time signal to the walking system in order to examine the exact energy-efficiency.

We introduce some criterion functions. Let p [J/s] the average input power which is defined as

$$p := \frac{1}{T}\int_{0^+}^{T^-} \left|\dot{b}_2 u\right| dt. \tag{12}$$

Average walking speed v [m/s] is also defined as

$$v := \frac{1}{T}\int_{0^+}^{T^-} \dot{x}_{\mathrm{G}}\ dt = \frac{\Delta x_{\mathrm{G}}}{T}, \tag{13}$$

where $\Delta x_{\mathrm{G}} := x_{\mathrm{G}}(T^-) - x_{\mathrm{G}}(0^+)$ [m] is the steady value of the change of horizontal CoM position x_{G} and is equal to the steady step. Based on them, energy-efficiency is defined as v/p [m/J] [12][14]. This means the movable distance in the same energy supply, and is a spatial criterion. Note that in this method of Eq. (9) the walking speed as well as the motion does not change if the control parameters are fixed, only actuator's burden is changed. The maximum efficiency problem then yields to find the condition minimizing p. On the definite integral of absolute function for calculation of p, the following relation holds:

$$p \geq \frac{1}{T}\int_{0^+}^{T^-} \dot{b}_2 u\ dt. \tag{14}$$

Therefore, following Eqs. (13) and (14), we can obtain the relation

$$\frac{v}{p} \leq \frac{\Delta x_{\mathrm{G}}}{\int_{0^+}^{T^-} \dot{b}_2 u\ dt}. \tag{15}$$

Here note that the equality holds in Eq. (14) iff $\dot{b}_2 u \geq 0$. This means that monotonic energy restoration by input torques is a necessary condition for maximum efficiency.

Fig. 9 shows the efficiency w.r.t. the changes of (a) k and (b) b_0. The line of 0.071 [m/J] denotes the efficiency where $k = 0$. From (a), we can see that all cases except $b_0 = 0.50$ have the optimal k. In the case of $b_0 = 0.50$, the efficiency becomes worse regardless of the setting of k. Fig. 10 shows the time evolution of the input power integral for $k = 0$, 75, 150, 225, 300 and 375 [N/m] where $b_0 = 0.30$ [m]. In this case, as shown in Fig. 9(a), $k = 225$ gives the maximum efficiency. Fig. 10 also shows that monotonic mechanical energy restoration, i.e. maximum efficiency, is achieved with $k = 225$. This efficiency is match for that of virtual gravity approaches.

From Fig. 9(b), we can see that each case has the optimal b_0 and the case of $b_0 = 0.46$ gives the same efficiency regardless of k. We consider this reason in the following. Eq. (11) can be expressed as

$$u = u_0 + k(b_2 - b_0), \tag{16}$$

where u_0 is the sum of terms in Eq. (11) except elastic effect. u becomes always negative when $b_0 = b - \frac{A}{2}$, so the sign of $\dot{b}_2 u$ is equivalent to that of $-\dot{b}_2$. Then the absolute input power integral can be divided as follows.

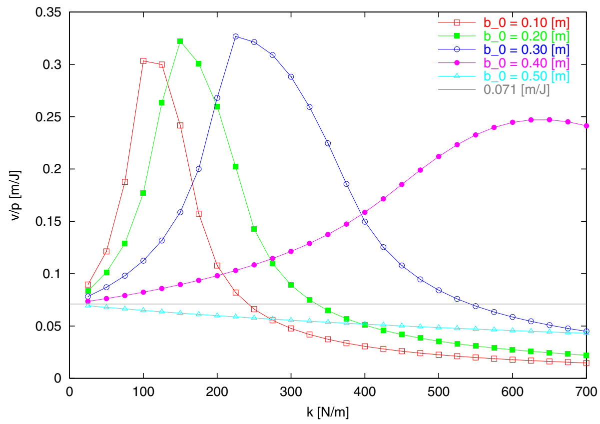
$$\begin{aligned}\int_{0^+}^{T^-} \left|\dot{b}_2 u\right| dt &= \int_{0^+}^{T_{\mathrm{set}}/2} \dot{b}_2 u\ dt - \int_{T_{\mathrm{set}}/2}^{T_{\mathrm{set}}} \dot{b}_2 u\ dt \\ &= \int_{0^+}^{T_{\mathrm{set}}/2} \dot{b}_2\left(u_0 + k(b_2 - b_0)\right) dt \\ &\quad - \int_{T_{\mathrm{set}}/2}^{T_{\mathrm{set}}} \dot{b}_2\left(u_0 + k(b_2 - b_0)\right) dt \end{aligned} \tag{17}$$

Here the following relations hold.

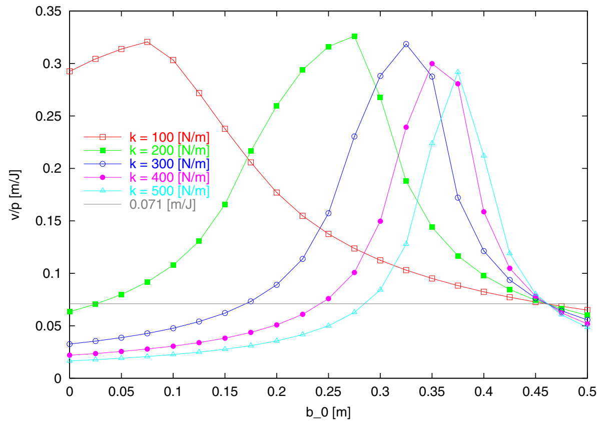
$$\int_{0^+}^{T_{\mathrm{set}}/2} k\dot{b}_2(b_2 - b_0)dt = \left[\frac{1}{2}k(b_2 - b_0)^2\right]_{b_2 = b_0 + \frac{A}{2}}^{b_2 = b_0 - \frac{A}{2}} = 0$$

$$\int_{T_{\mathrm{set}}/2}^{T_{\mathrm{set}}} k\dot{b}_2(b_2 - b_0)dt = \left[\frac{1}{2}k(b_2 - b_0)^2\right]_{b_2 = b_0 - \frac{A}{2}}^{b_2 = b_0 + \frac{A}{2}} = 0$$

This shows that in this case the term of elastic effect does not influence the energy-efficiency at all. Therefore we should choose b_0 less than $b - \frac{A}{2}$ to some extent.



(a) k versus v/p



(b) b_0 versus v/p

Fig. 9. Simulation results of the efficiency w.r.t. the mechanical impedances.

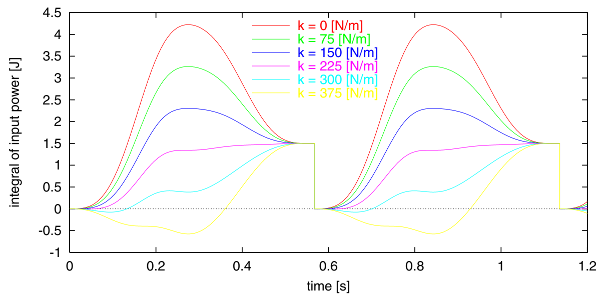


Fig. 10. Time evolution of input power integral.

VI. Some Extensions

This section considers an extension of drive using a rotational actuator together for improvement of the control performances. Feasibility of running is also discussed.

A. Hybrid actuation

The swing phase dynamic equation of the robot in Fig. 2 with hip actuation is given by

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \boldsymbol{S}u + \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} u_H, \tag{18}$$

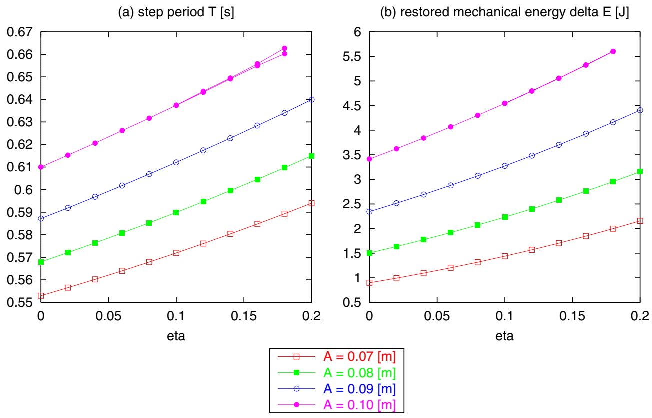


Fig. 11. Step period and restored mechanical energy w.r.t. η for $A =$ 0.07, 0.08, 0.09 and 0.10 [m] where $T_{\mathrm{set}} = 0.55$ [s].

where u_H is the hip joint torque. Although it is possible to use the ankle joint torque together in the case of round sole [15], this paper does not treat it.

As a candidate of hip joint torque formula to promote the mechanical energy restoration effectively, we introduce $u_H = \eta\left(\dot{\theta}_1 - \dot{\theta}_2\right)$ where $\eta > 0$ is the feedback gain. This can always increase the mechanical energy. Fig. 11 shows the changes of (a) T [s] and (b) ΔE [J] w.r.t. η for $A =$ 0.07, 0.08, 0.09 and 0.10 [m], respectively. We can see that the hip actuation promotes mechanical energy restoration, and the settling time margin (time interval between T_{set} and T) is also increased. This control is particularly effective in the case of small A which cannot restore the mechanical energy enough.

B. Go up the stairs

Next we consider an adaptation to going up the stairs utilizing the hybrid actuation effect. In general, limit cycles generated by passive walking approach is fragile against disturbances, especially the shape at the transition instant seriously influences the stability of next step. Therefore, its adaptation to uneven terrain has hardly studied so far.

In the case of $A = 0.08$, the robot without hip actuation can go up the stairs no more than with a level difference of 0.1 [cm]. We then adjust as $A = 0.15$ and the robot can go up a level difference of 2.1 [cm]. Further, by applying hip actuation with $\eta = 0.3$, it becomes possible to go up a level difference of 3.0 [cm].

Fig. 12 shows the stick diagram of dynamic walking going up the stairs with a level difference of 2.0 [cm]. In this case the robot requires enough mechanical energy restoration, so the amplitude A must be chosen large. The motion becomes exaggerated flinging up the swing leg wastefully. More reasonable control approach is expected to create smarter walking pattern.

C. Toward running

The leg retraction by spring is very useful for effective energy restoration for running robots. We assume that the

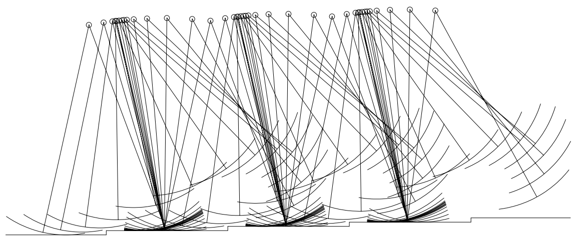


Fig. 12. Stick diagram of dynamic walking going up the stairs with a level difference of 2.0 [cm] where $A = 0.15$ [m] and $T_{\rm set} = 0.55$ [s].

telescopic leg has not only a "compressive" spring attached between the leg mass and the foot but also a "tensile" spring attached between them. With additional lock mechanism, we can utilize both springs as follows: In stance phase, we can add energy by extending the supporting leg as in [16], but also by retracting the swinging leg as described in this paper. In flight phase, retracting the leg makes it possible to swing the leg faster because the leg inertia is decreased. In [17], due to *the energy-preserving condition*, only one-periodic running gaits were found and orbitally stabilized on a level surface. This was also true for a semi-passive biped robot having a torso [18], but if we can restore the energy loss at touchdown (heel-strike) via retraction of swinging leg, it is expected that several passive running gaits on a level surface can be found and stabilized. Since extension and retraction of the legs are realized only by springs, we can expect much effective passive running than ever before.

VII. Conclusions and Future Work

This paper has proposed a novel dynamic gait generation method based on parametric excitation. Throughout modeling and numerical simulations, the effectiveness of the proposed method has been confirmed. Further, improvement of energy-efficiency by introducing an elastic element has been proposed and investigated.

Although in this paper we intuitively introduced a simple desired-time trajectory of Eq. (7), more investigation of the energy restoration mechanism will improve the control performances such as energy-efficiency, walking speed, and adaptation to stairs and irregular terrain. Further study is necessary. Now we are developing an experimental walking machine, and the result will be reported in a future paper.

Since in parametrically excited dynamic bipedal walking the mechanical energy is restored by only swing leg actuation, that is, joint torques are not required, extension to use of a torso will be easily realized. In this mechanism, balancing the torso and driving the leg can be performed independently, so the difficulty on how to balance the torso under gait generation can be released. This is also an interesting subject to be investigated in the future.

McGeer suggested an important philosophy for biped locomotion through his work in 1990, but it had a restriction that biped locomotion is not difficult *if on a downhill*. In order to solve the problem he left, thereafter many researchers including the authors have analyzed its dynamics and mechanisms, proposed control methods, and reported the effectiveness. These previous works, however, could not overcome the essential difficulty of biped locomotion that Vukobratović suggested in 1969. This is because all walking robots in them use *rotational actuators* at the joints, and the essential problem lies in execution of mechanical energy restoration by them. The authors hope that our approach provides an insight that *biped locomotion is not difficult even if on a level ground*.

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