1. 解： $\because i_c = C\dfrac{dU_c}{dt}$，$I = i_c + i_L$

$\therefore \dot{U}_c = \dfrac{dU_c}{dt} = \dfrac{I - i_L}{C}$

$\because U_L = L\dfrac{di_L}{dt}$

$\therefore \dot{i}_L = \dfrac{U_L}{L} = U_c - i_L R$

∴ 有状态方程：

$$\begin{pmatrix}\dot{U}_c\\ \dot{i}_L\end{pmatrix} = \begin{pmatrix}0 & -\frac{1}{C}\\ 1 & -R\end{pmatrix}\begin{pmatrix}U_c\\ i_L\end{pmatrix} + \begin{pmatrix}\frac{1}{C}\\ 0\end{pmatrix} I$$

2. 解： 对水箱I分析可得：

$$\frac{dx_1(t)}{dt} = u - \frac{x_1}{R}$$

对水箱II分析可得：

$$\frac{dx_2(t)}{dt} = \frac{x_1}{R} - \frac{x_1+x_2}{R}$$

∴ 有状态方程

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\end{pmatrix} = \begin{pmatrix}-\frac{1}{R} & 0\\ 0 & -\frac{1}{R}\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} + \begin{pmatrix}1\\ 0\end{pmatrix}u$$

又 $y(t) = \dfrac{x_1+x_2}{R} = \begin{pmatrix}\frac{1}{R} & \frac{1}{R}\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}$

∴ 状态空间表达式为：

$$\begin{cases}\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\end{pmatrix} = \begin{pmatrix}-\frac{1}{R} & 0\\ 0 & -\frac{1}{R}\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} + \begin{pmatrix}1\\ 0\end{pmatrix}u\\ y = \begin{pmatrix}\frac{1}{R} & \frac{1}{R}\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}\end{cases}$$

3. 解： (1) 设车开始运动时弹簧为自然状态。

$$x_2 = v = \frac{dy}{dt} = \frac{dx_1}{dt}$$

$$m\cdot\frac{dx_2}{dt} = u - kx_1 - fx_2$$

∴ 有状态方程 $\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\end{pmatrix} = \begin{pmatrix}0 & 1\\ -\frac{k}{m} & -\frac{f}{m}\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} + \begin{pmatrix}0\\ 1\end{pmatrix}u$

又 $y = x_1 = \begin{pmatrix}1 & 0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}$

∴ 状态空间表达式为

$$\begin{cases}\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\end{pmatrix} = \begin{pmatrix}0 & 1\\ -\frac{k}{m} & -\frac{f}{m}\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} + \begin{pmatrix}0\\ 1\end{pmatrix}u\\ y = \begin{pmatrix}1 & 0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}\end{cases}$$

(2) $m \cdot \frac{d^2 x_1}{dt^2} = u - kx_1 - f\frac{dx_1}{dt}$

即 $u = m\frac{d^2x_1}{dt^2} + f\frac{dx_1}{dt} + kx_1$

$\therefore \frac{Y(s)}{U(s)} = \frac{1}{ms^2+fs+k}$ 即 $G(s) = \frac{1}{ms^2+fs+k}$

4. 解：$\because -\ddot{y} - 5\dot{y} - 6y = u$.

$\therefore \frac{Y(s)}{U(s)} = \frac{1}{s^2+5s+6}$ 即 $G(s) = \frac{1}{s^2+5s+6}$

5. 解：

$$\frac{U(s)}{X(s)} = C(sI-A)^{-1}B + D$$

$$= \begin{pmatrix} 2 & 1 \end{pmatrix} \begin{pmatrix} s-3 & -2 \\ 2 & s+1 \end{pmatrix}^{-1} \begin{pmatrix} 4 \\ 3 \end{pmatrix} + 1$$

$$= \begin{pmatrix} 2 & 1 \end{pmatrix} \begin{pmatrix} \frac{s+1}{(s-1)^2} & \frac{2}{(s-1)^2} \\ \frac{-2}{(s-1)^2} & \frac{s-3}{(s-1)^2} \end{pmatrix} \begin{pmatrix} 4 \\ 3 \end{pmatrix} + 1$$

$$\therefore G(s) = \frac{s^2+9s+4}{(s-1)^2}$$

解：

$$\frac{Y(s)}{X(s)} = C(sI-A)^{-1}B = \begin{pmatrix} 1 & 1 & 1 \\ -2 & -3 & -4 \end{pmatrix} \begin{pmatrix} s+2 & 0 & 0 \\ 0 & s+3 & 0 \\ 0 & 0 & s+4 \end{pmatrix}^{-1} \begin{pmatrix} 1 & -1 \\ -1 & 4 \\ 5 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 & 1 \\ -2 & -3 & -4 \end{pmatrix} \begin{pmatrix} \frac{1}{s+2} & 0 & 0 \\ 0 & \frac{1}{s+3} & 0 \\ 0 & 0 & \frac{1}{s+4} \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -1 & 4 \\ 5 & -3 \end{pmatrix}.$$

$\therefore G(s)$

$$= \begin{pmatrix} \frac{1}{s+2} - \frac{1}{s+3} + \frac{5}{s+4}, & -\frac{1}{s+2} + \frac{4}{s+3} - \frac{3}{s+4} \\ -\frac{2}{s+2} + \frac{3}{s+3} - \frac{20}{s+4}, & \frac{2}{s+2} - \frac{12}{s+3} + \frac{12}{s+4} \end{pmatrix}$$