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Pseudo Virtual Passive Dynamic Walking and Effect of Upper Body as Counterweight

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***Abstract*— This paper investigates the effect of an upper body on efficient dynamic bipedal walking utilizing its natural dynamics. We introduce an upper body as a one-link torso and add it to a simple biped model by means of a bisecting hip mechanism (BHM). We first mathematically analyze the effect of the upper body with the BHM as a counterweight and discuss how it affects natural swinging motion of the swing leg. Second, we propose a simple method for generating efficient dynamic bipedal gait imitating the property of virtual passive dynamic walking, and numerically analyze the gait efficiency. Simulation results show that the walking system exhibits period-doubling bifurcation, and we discuss how the efficiency changes in the multiple-period gait.**

I. INTRODUCTION

Passive-dynamic walkers generally consists of leg links only, and many efficient dynamic walkers inspired by passive dynamic walking (PDW) [1] do not utilize the upper body dynamics. It is clear that an upper body strongly affects human bipedal walking and plays an important role in the gait generation. The effect in robotic bipedal locomotion, especially PDW or efficient walking utilizing the natural dynamics, has not been deeply understood. This paper then mathematically investigates what effect an upper body had on dynamic bipedal walking, especially the effect as a counterweight.

We added an upper body to a planar biped model by means of a bisecting hip mechanism (BHM) [2] so as not to destroy the robot natural dynamics. By using the BHM, the upper body can be passively stabilized to remain upright and efficient level dynamic walking can be achieved [3]. In this paper, we investigate the effect of the upper body incorporating the BHM in more detail, and mathematically clarify that it helps or obstructs the natural swinging motion of the swing leg in accordance with the parameter settings.

On the other hand, the authors theoretically investigated the effect of semicircular feet on dynamic bipedal walking through analysis of underactuated virtual passive dynamic walking (UVPDW) [4][5], and clarified that such feet provides virtual ankle-joint torque and reduce the mechanical energy loss caused by heel strike. In UVPDW, however, the specified control input has a singularity which disrupts smoothness of the generated gait [3]. This paper then proposes a novel gait generation method imitating the property of VPDW based on observations. The control input defined following the proposed method does not have any singularities, and the robot is driven by the constant-like torque smoothly.

Further, it is known that passive or semi-passive gaits often exhibit period-doubling bifurcation and chaotic behaviour [6][7][8][9][10]. However, it has not been investigated how the bifurcation changes the gait efficiency. Our simulation results show that period-doubling bifurcation also occurs in accordance with the method this paper proposes. We then discuss whether the gait efficiency improves or worsen with respect to the bifurcation.

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II. MODELING AND ANALYSIS OF BIPED ROBOT WITH BISECTING HIP MECHANISM

This section describes the basic definitions and analyzes the effect of the upper body incorporating the BHM as a counterweight.

A. Adding upper body by means of bisecting hip mechanism

This paper deals with a planar underactuated biped model with semicircular feet and a torso as shown in Fig. 1. A bisecting hip mechanism (BHM) [2], which is a mechanism to bisect the relative hip-joint angle with respect to the torso passively, is used to connect the torso with the biped so as not to destroy the robot's natural dynamics. By the synergistic effect of BHM, we can generate a dynamic bipedal gait efficiently without having to maintain the torso's posture

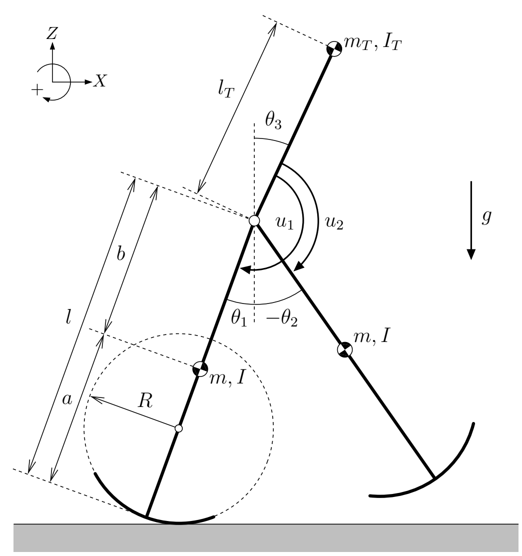


Fig. 1. Model of planar underactuated biped robot with semicircular feet and torso

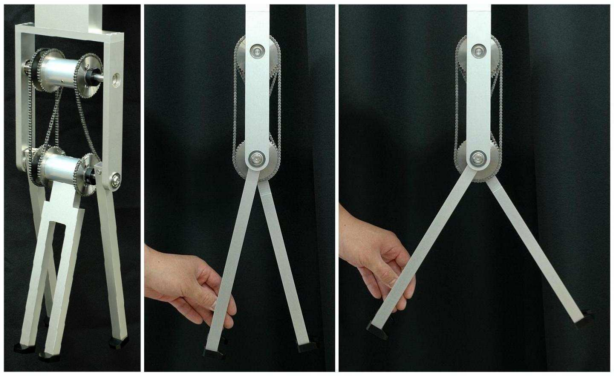


Fig. 2. Bisecting hip mechanism

actively [3]. Semicircular feet are also very effective for generating an efficient dynamic gait [4][5]. Two joint torques between each leg and torso, u_1 and u_2, can be exerted. Let $\boldsymbol{\theta} = \begin{bmatrix} \theta_1 & \theta_2 & \theta_3 \end{bmatrix}^{\mathrm{T}}$ be the generalized coordinate vector, then dynamic equation of the biped model becomes

$$\boldsymbol{M}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \boldsymbol{h}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = \boldsymbol{S}\boldsymbol{u} + \boldsymbol{J}_H^{\mathrm{T}}\lambda_H, \tag{1}$$

where

$$\boldsymbol{S}\boldsymbol{u} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \tag{2}$$

and $\boldsymbol{J}_H^{\mathrm{T}}\lambda_H \in \mathbb{R}^3$ stands for the constraint force of the BHM. The geometric relation between the torso and legs according to the BHM is given by

$$\theta_3 = \frac{\theta_1 + \theta_2}{2} + \psi, \tag{3}$$

where ψ [rad] is the offset angle of the torso [3]. This paper chooses it as zero and does not investigate its effect. Time-derivative of Eq. (3) then becomes

$$\dot{\theta}_3 = \frac{\dot{\theta}_1 + \dot{\theta}_2}{2}. \tag{4}$$

This can be simply rearranged as

$$\boldsymbol{J}_H\dot{\boldsymbol{\theta}} = 0, \ \boldsymbol{J}_H = \begin{bmatrix} 1 & 1 & -2 \end{bmatrix}. \tag{5}$$

This leads to $\boldsymbol{J}_H\ddot{\boldsymbol{\theta}} = 0$, and by using this we can solve Eq. (1) for λ_H as

$$\lambda_H = -X_H(\boldsymbol{\theta})^{-1}\boldsymbol{J}_H\boldsymbol{M}(\boldsymbol{\theta})^{-1}\left(\boldsymbol{S}\boldsymbol{u} - \boldsymbol{h}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})\right), \tag{6}$$

$$X_H(\boldsymbol{\theta}) = \boldsymbol{J}_H\boldsymbol{M}(\boldsymbol{\theta})^{-1}\boldsymbol{J}_H^{\mathrm{T}}. \tag{7}$$

By substituting this into Eq. (1), we further simplify the robot's dynamic equation as

$$\boldsymbol{M}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} = \boldsymbol{Y}_H(\boldsymbol{\theta})\left(\boldsymbol{S}\boldsymbol{u} - \boldsymbol{h}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})\right), \tag{8}$$

where

$$\boldsymbol{Y}_H(\boldsymbol{\theta}) = \boldsymbol{I}_3 - X_H(\boldsymbol{\theta})^{-1}\boldsymbol{J}_H^{\mathrm{T}}\boldsymbol{J}_H\boldsymbol{M}(\boldsymbol{\theta})^{-1}. \tag{9}$$

The heel-strike is modeled as an inelastic collision, but this paper omits the detail. Please see [3] for further details of the model.

By using this mechanism, we can passively keep the upper body upright under the influence of gravity. It is a great advantage that we can add an upper body without having to consider PD control for stabilizing it [11][12][13]. If the hip joint does not have such passive constraints or synergistic effect, the upper body falls down and the control synthesis becomes complicated.

B. Mechanical energy

Let E [J] be the robot's total mechanical energy, which is defined as the sum of kinetic and potential energies:

$$E(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = \frac{1}{2}\dot{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{M}(\boldsymbol{\theta})\dot{\boldsymbol{\theta}} + P(\boldsymbol{\theta}), \tag{10}$$

and its time derivative satisfies the following relation:

$$\dot{E} = \left(\dot{\theta}_1 - \dot{\theta}_3\right)u_1 + \left(\dot{\theta}_2 - \dot{\theta}_3\right)u_2. \tag{11}$$

By using Eq. (4), this can be rearranged as

$$\dot{E} = \frac{\dot{\theta}_H u_1}{2} - \frac{\dot{\theta}_H u_2}{2}, \tag{12}$$

where $\theta_H := \theta_1 - \theta_2$ [rad] is the relative hip-joint angle. This implies that each joint torque actuates the hip-joint alternately or these two control inputs are redundant actuations for one joint.

C. Effect of upper body as counterweight

Here, we analyze the effect of the upper body incorporating the BHM as a counterweight. We consider two cases: (a) the torso is put to the ceiling, and (b) the stance leg is put to the level floor, as shown in Fig. 3.

1) Case (a): The dynamic equation of the legs is

$$\begin{bmatrix} I + mb^2 & 0 \\ 0 & I + mb^2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} mbg\sin\theta_1 \\ mbg\sin\theta_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}\lambda, \tag{13}$$

where $\lambda \in \mathbb{R}$ is the constraint force of the BHM, and it can be solved as

$$\lambda = \frac{mbg\left(\sin\theta_1 + \sin\theta_2\right)}{2}. \tag{14}$$

Considering $\theta_1 = -\theta_2$, we can find $\lambda = 0$. Thus, we can conclude that the BHM does not destroy the natural swinging motion of the legs at all.

2) Case (b): The dynamic equation in this case is

$$\begin{bmatrix} I + mb^2 & 0 \\ 0 & I_T + m_T l_T^2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{bmatrix} + \begin{bmatrix} mbg\sin\theta_2 \\ -m_T l_T g\sin\theta_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}\lambda. \tag{15}$$

By linearizing this around the equilibrium point, $\theta_2 = \theta_3 = 0$, and using the relation $\theta_2 = 2\theta_3$, we get

$$\ddot{\theta}_2 = \frac{(m_T l_T - 4mb)g\theta_2}{I_T + 4I + m_T l_T^2 + 4mb^2}, \tag{16}$$

$$\ddot{\theta}_3 = \frac{(m_T l_T - 4mb)g\theta_2}{2(I_T + 4I + m_T l_T^2 + 4mb^2)}. \tag{17}$$

Further, by considering the relation $\ddot{\theta}_2 = 2\ddot{\theta}_3$, we can find that these equations are equivalent. Thus, we can conclude that the whole motion of this system can be described

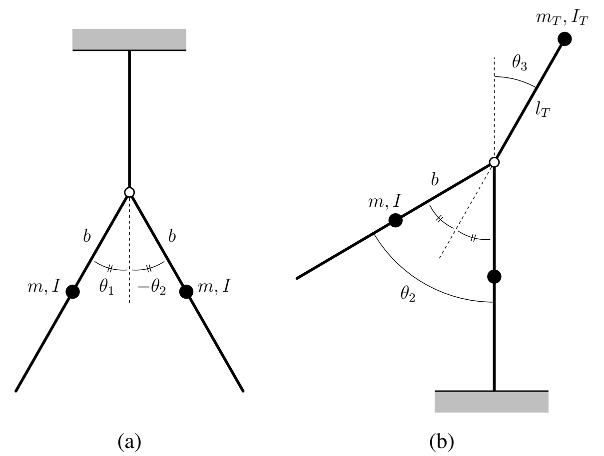


Fig. 3. Two effects of bisecting hip mechanism on leg-swinging motion

only by Eq. (16). This system exhibits simple harmonic oscillation if $m_T l_T < 4mb$ around the stable equilibrium point, whereas it becomes unstable if $m_T l_T > 4mb$, i.e., the robot cannot keep the standing posture and falls down. If $m_T l_T = 4mb$, the rotational moment force around the hip joint vanishes and the leg's swinging motion disappears. In other words, the upper body obstructs the swing-leg motion naturally swinging forward during the first half cycle when $m_T l_T \geq 4mb$. The standing posture is essentially unstable in this case. In contrast, in $m_T l_T < 4mb$, the standing posture becomes stable and the BHM does not obstruct the natural swinging motion.

For these reasons, it is expected that the walking speed would decrease as l_T increases because it is necessary to effectively drive the robot CoM forward during the first half cycle to overcome the potential barrier. In Section IV, we confirm this through numerical simulations.

III. PSEUDO VIRTUAL PASSIVE DYNAMIC WALKING

This section proposes a simple method for generating an efficient gait based on the concept of VPDW [15]. In VPDW, we introduce a small virtual gravity in the walking direction whose magnitude is $g \tan\phi$ [m/s^2]. The control input is then synthesized in accordance with the specified relation between the input power and the horizontal velocity of CoM, and the restored mechanical energy per step, ΔE [J], satisfies the following relation:

$$\Delta E = Mg\tan\phi \Delta X_g, \tag{18}$$

where ϕ [rad] is the angle of virtual slope and ΔX_g [m] is the travel distance of CoM which is equal to the step length [15]. The robot exhibits efficient dynamic walking on a virtual slope, i.e., on level ground, in a virtual gravity field whose magnitude is $g/\cos\phi$ [m/s^2].

For simplicity, we assume $u_2 = 0$ and Eq. (12) then becomes

$$\dot{E} = \frac{\dot{\theta}_H u_1}{2}. \tag{19}$$

Considering this, let us define u_1 as

$$u_1 = Mg\tan\phi\left(R + (l-R)\cos\frac{\theta_H}{2}\right). \tag{20}$$

The restored mechanical energy then satisfies the following relation:

$$\begin{aligned}\frac{\Delta E}{Mg\tan\phi} &= \int_{0^+}^{T^-} \frac{\dot{\theta}_H}{2}\left(R + (l-R)\cos\frac{\theta_H}{2}\right)\mathrm{d}t \\ &= \left[\frac{R\theta_H}{2} + (l-R)\sin\frac{\theta_H}{2}\right]_{\theta_H=-2\alpha}^{\theta_H=2\alpha} \\ &= 2\left(R\alpha + (l-R)\sin\alpha\right) = \Delta X_g. \end{aligned}\tag{21}$$

Therefore, this control input is found to satisfy the condition of Eq. (19): the restored mechanical energy is kept being proportional to the step length. In the sense, this approach should be termed as the pseudo virtual passive dynamic walking (PVPDW). As an aside, also in this case, the target condition for the mechanical energy during stance phases can be written in the same form as VPDW by introducing the concept of pseudo CoM which is defined as

$$\hat{X}_g = \frac{R\theta_H}{2} + (l-R)\sin\frac{\theta_H}{2}. \tag{22}$$

Following Eqs. (19) and (20), its time-derivative can be arranged as

$$\begin{aligned}\dot{\hat{X}}_g &= \frac{\dot{\theta}_H}{2}\left(R + (l-R)\cos\frac{\theta_H}{2}\right) \\ &= \frac{\dot{\theta}_H}{2}\frac{u_1}{Mg\tan\phi} = \frac{\dot{E}}{Mg\tan\phi},\end{aligned}\tag{23}$$

and this leads to

$$\frac{\partial E}{\partial \hat{X}_g} = Mg\tan\phi. \tag{24}$$

Fig. 4 shows the simulation results of PVPDW where $\phi = 0.01$ [rad]. Here, (a) is the angular positions, (b) the angular velocities, (c) the control input, (d) the mechanical energy, and (e) the input power. Fig. 5 plots the stick diagram for the steady gait. The system parameters were chosen as in Table I. As in VPDW, the mechanical energy is monotonically restored during the stance phases and dissipates by heel strikes. Note that the following relation holds.

$$\text{Specific resistance} := \frac{p}{Mgv} \geq \frac{\Delta E}{Mg\Delta X_g} = \tan\phi, \tag{25}$$

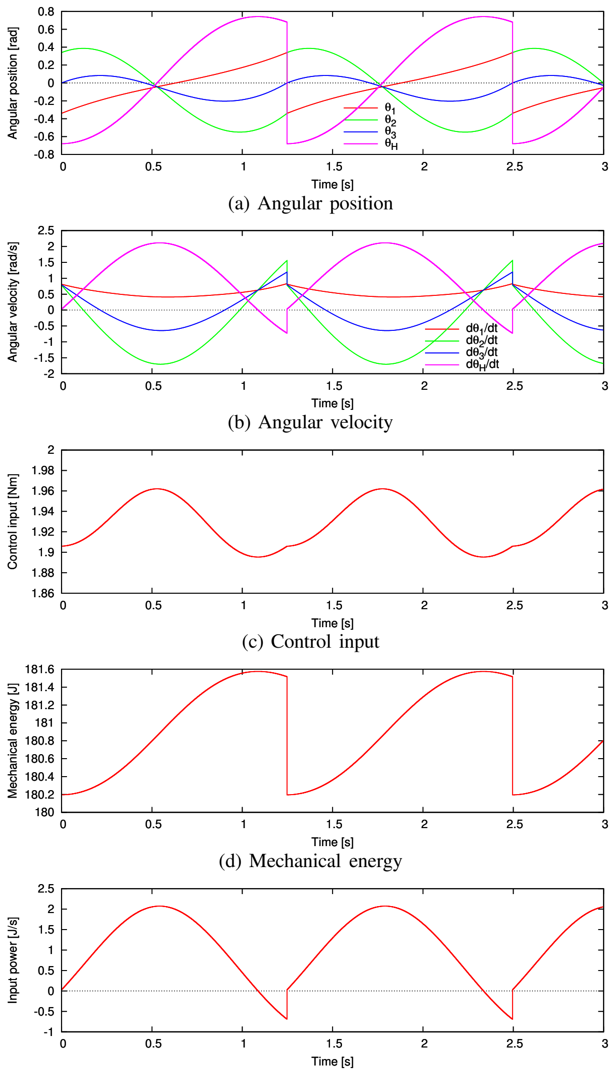
where p [J/s] is the average input power and v [m/s] is the average walking speed, respectively defined as

$$p := \frac{1}{T}\int_{0^+}^{T^-}\left|\left(\dot{\theta}_1 - \dot{\theta}_3\right)u_1\right|\mathrm{d}t = \frac{1}{2T}\int_{0^+}^{T^-}\left|\dot{\theta}_H u_1\right|\mathrm{d}t, \tag{26}$$

$$v := \frac{\Delta X_g}{T}. \tag{27}$$

If negative power does not occur during the actuation, the equality in Eq. (25) holds and the specific resistance then yields minimum. In this result, however, it yielded 0.0109 [-] and was not minimum. This is because negative input power occurred due to swing-leg retraction prior to impact

[14]. Since the control input u_1 is positive and constant-like, $\dot{\theta}_H < 0$ due to swing-leg retraction causes negative input power. Nevertheless, the efficiency was sufficient.



(a) Angular position

(b) Angular velocity

(c) Control input

(d) Mechanical energy

(e) Input power

Fig. 4. Simulation results for pseudo virtual passive dynamic walking with upper body

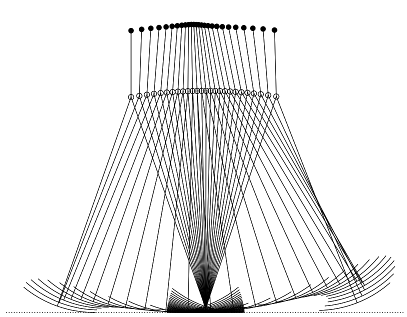


Fig. 5. Stick diagram for steady gait

TABLE I

PARAMETER SETTINGS FOR BIPED ROBOT

m		5.0	kg
m_T		10.0	kg
a		0.50	m
b		0.50	m
l	$(= a + b)$	1.00	m
l_T		0.30	m
I		0.001	kg·m^2
I_T		0.001	kg·m^2
R		0.30	m
ψ		0.00	rad

In this approach, on the other hand, robust stability of the generated gait is not strong so much because the control input is given without having any stabilizing properties. Some robust control laws should be considered in the future. Following control to a reference trajectory of the mechanical energy [15] is a strong candidate for it.

IV. EFFICIENCY ANALYSIS

This section numerically analyzes efficiency of the pseudo virtual passive dynamic gait with respect to the changes of upper body's parameters.

A. Analysis results

Fig. 6 shows the gait descriptors of PVPDW with respect to l_T for two values of R. Here, (a) is the step period, (b) the walking speed, (c) the specific resistance, (d) the restored mechanical energy, (e) the half inter-leg angle at impact, and (f) the step length. We can see that the efficiency, the walking speed and specific resistance, grows worth in the increase of l_T. It should be pointed out that, from (b), the mean walking speed decreases more rapidly after the first bifurcation point. In addition, from (c), we can also see that the mean specific resistance increases more rapidly after the same point. We can conclude that the gait efficiency worsen more rapidly due to the bifurcation. Note that, from (d), the step length does not bifurcate if the gait changes to 2-period gait. This is understood through the calculation of ΔX_g. Let α_1 and α_2 be the steady values of half inter-leg angle at impact in a 2-period gait, then we get

$$\Delta X_g = R\,(\alpha_1 + \alpha_2) + (l - R)\,(\sin\alpha_1 + \sin\alpha_2)\,, \quad (28)$$

and this does not change even if α_1 and α_2 are exchanged. This is strongly supported by Fig. 7, it's visually obvious. The transition posture, however, must be symmetrical and its horizontal CoM position also must be the same as the hip-joint's one. The step length thus bifurcates from 4-period gait. The restored mechanical energy, ΔE, also bifurcates from 4-period gait in accordance with the method of PVPDW. This feature is common to the original PDW [1], VPDW [15], and UVPDW [3][4]. It is in common with these methods that the restored mechanical energy supplied by gravity or control input is proportional to the step length. It is also remarkable that the step length monotonically increases as l_T increases but it changes to decrease after the first bifurcation point. The reason is, however, unclear at present.

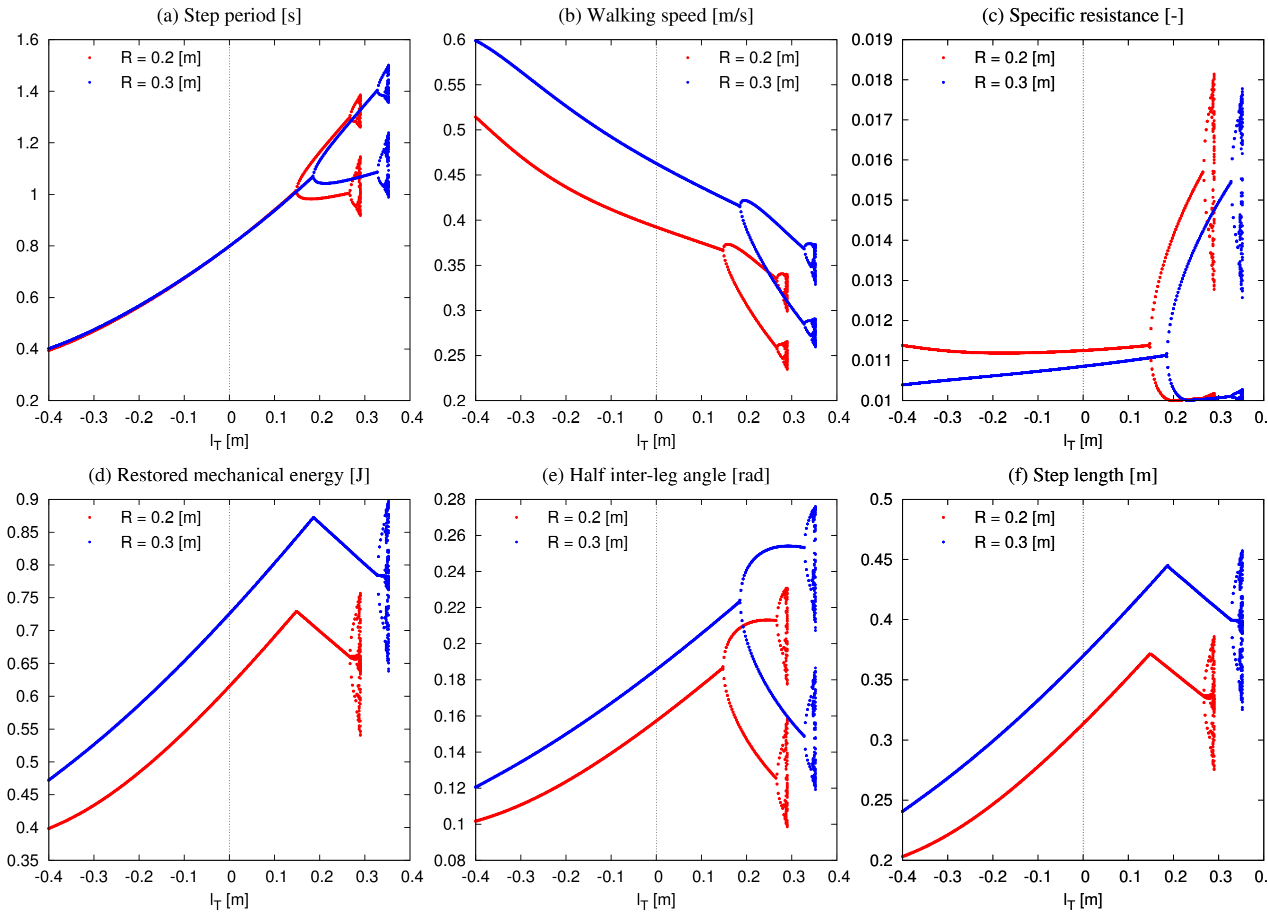


Fig. 6. Gait descriptors of pseudo virtual passive dynamic walking with respect to l_T for two values of R

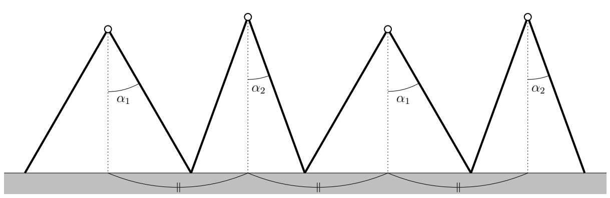


Fig. 7. Steady 2-period gait and its constant step length

In this case, the balanced condition $m_T l_T = 4mb$ is satisfies when $l_T = 4mb/m_T = 1.0$ [m]. The simulation results, however, show that the limit of l_T for generating a stable gait is less than 0.4 [m], much shorter than the balanced condition. This is because overcoming the potential barrier at mid-stance is also difficult for the underactuated biped without ankle-joint actuation. If the foot radius, R, was set larger, the stable domain for gait generation would be increased.

In $l_T < 0$, the upper body hangs under the hip-joint, and as expected from Eq. (16), it propels swinging motion of the swing leg during the first half cycle. Fig. 6 (b) shows that the walking speed monotonically increases as l_T decreases, whereas (a) shows that the step period shortens. These results implies that the upper body effectively propels the swinging motion but the natural frequency becomes larger as l_T decreases.

B. Efficiency of 2^n-period gait

Period-doubling bifurcation is an attractive phenomenon in passive-dynamic gait, and many researchers have investigated how it appears and what effect it has on the efficiency. Since the discovery by Goswami *et al.* [6], however, no concrete investigations have been reported. It is still unclear how the bifurcation changes the gait efficiency. We then analyze the change of the efficiency in accordance with bifurcations based on observations.

Fig. 8 plots an enlargement of the walking speed in PVPDW where $R = 0.3$ [m]. The mean value is plotted with blue points. As previously mentioned, the walking speed monotonically decreases as l_T increases due to the effect of upper body as a counterweight. After the first bifurcation point, however, we can see that its average decreases more rapidly than that in 1-period gait. In other words, deterioration speed is accelerated by the first period-doubling.

Fig. 9 plots an enlargement of the specific resistance where $R = 0.3$ [m]. We can confirm that the specific resistance

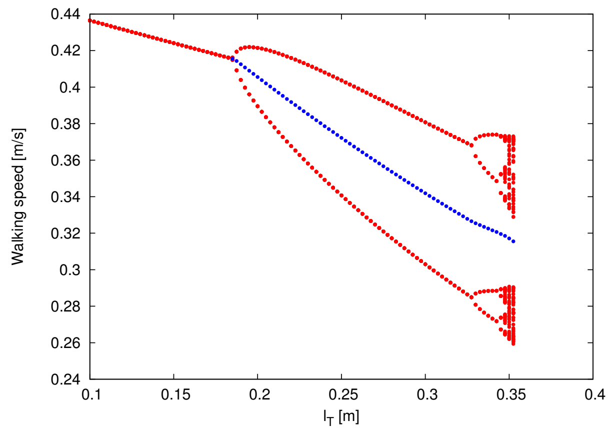


Fig. 8. Walking speed and its mean value in 2^n-period gait

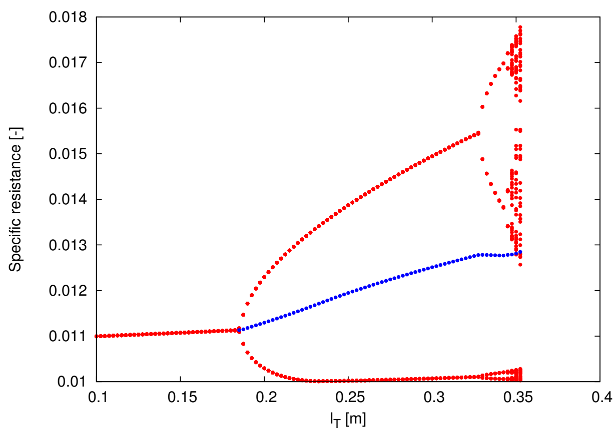


Fig. 9. Specific resistance and its mean value in 2^n-period gait

is more than $\tan\phi \approx 0.01$ [-]. In 2-period gait, the stable limit cycle consists of two walking patterns; the influence of swing-leg retraction is weak and strong. In the latter walking pattern, swing-leg retraction occurs prior to impact, and the input power then becomes negative because the control input of Eq. (20) is always positive. This leads to inefficiency. In addition, we can see that the average specific resistance (blue points) increases more rapidly after the first bifurcation point, in other words, the energy-efficiency worsen more rapidly after the point. This result comes from the fact that the walking speed decreases more rapidly after the first bifurcation point but the consumed energy does not change relatively so much.

V. CONCLUSION AND FUTURE WORK

In this paper, we proposed PVPDW for an underactuated biped with an upper body incorporating the BHM, and numerically analyzed the gait efficiency. We theoretically discussed that the upper body affects the gait generation as a counterweight and obstructs the natural swinging motion of the swing leg. It has been validated through numerical simulations.

In the future, we will investigate in more detail why and how the efficiency worsen more rapidly after the first period-doubling bifurcation. Controlling chaos in passive-dynamic gait has been actively investigated [16][17], but its positive applications have not been proposed until now. Our results showed that, however, multiple-period gait implies the deterioration of gait efficiency. In this sense, we should control it to a stable 1-period gait without leaving as it is. Anyway, the mechanism of period-doubling should be theoretically clarified, and after that, we should challenge to actively utilize and apply the nature of multiple-period gait.

VI. ACKNOWLEDGMENTS

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