1. 解：

$$x(s)=(sI-A)^{-1}[x(0)+Bu(s)]$$

$$=\begin{pmatrix} s+1 & 0 \\ 0 & s+2 \end{pmatrix}^{-1}\begin{pmatrix} 2 \\ 2 \end{pmatrix}=\begin{pmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{s+2} \end{pmatrix}\begin{pmatrix} 2 \\ 2 \end{pmatrix}=\begin{pmatrix} \frac{2}{s+1} \\ \frac{2}{s+2} \end{pmatrix}$$

作反拉氏变换有 $x(t)=\begin{pmatrix} 2e^{-t} \\ 2e^{-2t} \end{pmatrix}$

2. 解：$X(s)=(sI-A)^{-1}[x(0)+Bu(s)]$

$$=\begin{pmatrix} s & -1 \\ 2 & s+3 \end{pmatrix}^{-1}\begin{pmatrix} -1 \\ 1 \end{pmatrix}=\begin{pmatrix} \frac{s+3}{(s+1)(s+2)} & \frac{1}{(s+1)(s+2)} \\ \frac{-2}{(s+1)(s+2)} & \frac{s}{(s+1)(s+2)} \end{pmatrix}\begin{pmatrix} -1 \\ 1 \end{pmatrix}=\begin{pmatrix} \cancel{\frac{-(s+2)}{(s+1)(s+2)}}\ -\frac{1}{s+1} \\ \frac{1}{s+1} \end{pmatrix}$$

作反拉氏变换有 $x(t)=\begin{pmatrix} -e^{-t} \\ e^{-t} \end{pmatrix}$

3. 解：

$$(sI-A)^{-1}=\begin{pmatrix} s & 0 & 0 \\ 0 & s & \theta \\ 0 & -\theta & s \end{pmatrix}^{-1}=\begin{pmatrix} \frac{1}{s} & 0 & 0 \\ 0 & \frac{s}{s^2+\theta^2} & \frac{-\theta}{s^2+\theta^2} \\ 0 & \frac{\theta}{s^2+\theta^2} & \frac{s}{s^2+\theta^2} \end{pmatrix}$$

$$\therefore e^{At}=L^{-1}\left[(sI-A)^{-1}\right]$$

$$=L^{-1}\begin{pmatrix} \frac{1}{s} & 0 & 0 \\ 0 & \frac{\frac{1}{2}}{s+\theta i}+\frac{\frac{1}{2}}{s-\theta i} & \frac{-\frac{i}{2}}{s+\theta i}+\frac{\frac{i}{2}}{s-\theta i} \\ 0 & \frac{\frac{i}{2}}{s+\theta i}+\frac{-\frac{i}{2}}{s-\theta i} & \frac{\frac{1}{2}}{s+\theta i}+\frac{\frac{1}{2}}{s-\theta i} \end{pmatrix}$$

$$=\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2}(e^{i\theta}+e^{-i\theta}) & \frac{i}{2}(e^{i\theta}-e^{-i\theta}) \\ 0 & -\frac{i}{2}(e^{i\theta}-e^{-i\theta}) & \frac{1}{2}(e^{i\theta}+e^{-i\theta}) \end{pmatrix}$$

$$=\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & i\cos\theta \\ 0 & -i\cos\theta & \cos\theta \end{pmatrix}$$

4. 解：

$$e^{At} = L^{-1}\left[(sI-A)^{-1}\right] \qquad (sI-A)^{-1} = L^{-1}(e^{At})$$

$$= \begin{pmatrix} \frac{1}{s+1} & 0 & 0 \\ 0 & \frac{1}{s+2} - \frac{2}{(s+2)^2} & \frac{4}{(s+2)^2} \\ 0 & -\frac{1}{(s+2)^2} & \frac{1}{s+2} + \frac{2}{(s+2)^2} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{s+1} & 0 & 0 \\ 0 & \frac{s}{(s+2)^2} & \frac{4}{(s+2)^2} \\ 0 & -\frac{1}{(s+2)^2} & \frac{s+4}{(s+2)^2} \end{pmatrix}$$

$\therefore sI-A$

$$= \begin{pmatrix} s+1 & 0 & 0 \\ 0 & s+4 & -4 \\ 0 & 1 & s \end{pmatrix}$$

$$\therefore A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -4 & 4 \\ 0 & -1 & 0 \end{pmatrix}$$

5. 解：

$$x(t) = L^{-1}\left[(sI-A)^{-1}\cdot x(0)\right] \qquad \therefore L^{-1}[x(t)] = (sI-A)^{-1}\cdot x(0)$$

$$\therefore \begin{pmatrix} \frac{1}{s+2} \\ \frac{-1}{s+2} \end{pmatrix} = (sI-A)^{-1}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \qquad \begin{pmatrix} \frac{2}{s+1} \\ \frac{-1}{s+1} \end{pmatrix} = (sI-A)^{-1}\begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

即 $$(sI-A)^{-1} = \begin{pmatrix} \frac{1}{s+2} & \frac{2}{s+1} \\ \frac{-1}{s+2} & \frac{-1}{s+1} \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -1 & -1 \end{pmatrix}^{-1} = \begin{pmatrix} \frac{1}{s+2} & \frac{2}{s+1} \\ \frac{-1}{s+2} & \frac{-1}{s+1} \end{pmatrix}\begin{pmatrix} -1 & -2 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} \frac{s+3}{(s+1)(s+2)} & \frac{2}{(s+1)(s+2)} \\ \frac{-1}{(s+1)(s+2)} & \frac{s}{(s+1)(s+2)} \end{pmatrix}$$

$$\therefore sI-A = \begin{pmatrix} s & -2 \\ 1 & s+3 \end{pmatrix} \qquad \therefore A = \begin{pmatrix} 0 & 2 \\ -1 & -3 \end{pmatrix}$$

$$\therefore \phi(t) = e^{At} = L^{-1}\left[(sI-A)^{-1}\right] = L^{-1}\begin{pmatrix} \frac{-1}{s+2} + \frac{2}{s+1} & \frac{-2}{s+2} + \frac{2}{s+1} \\ \frac{1}{s+2} + \frac{-1}{s+1} & \frac{2}{s+2} - \frac{1}{s+1} \end{pmatrix}$$

$$= \begin{pmatrix} -e^{-2t} + 2e^{-t} & -2e^{-2t} + 2e^{-t} \\ e^{-2t} - e^{-t} & 2e^{-2t} - e^{-t} \end{pmatrix}$$