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1、

(a) 考虑 $\begin{cases} \dot{x}_1 = x_2 = \dot{x} \\ \dot{x}_2 = \ddot{x} \end{cases}$ 则系统方程可写作 $\dot{x}_2 + \sin x_1 = 0$

考虑 $V(x) = 1 - \cos x_1 + \frac{1}{2}x_2^2 \geqslant \frac{1}{2}x_2^2 \geqslant 0$

$V'(x) = \sin x_1 (\dot{x}) + x_2(\ddot{x})$

$V'(x) = (\sin x_1) x_2 + x_2 (\ddot{x})$

$V'(x) = x_2 (\sin x + \ddot{x}) = 0$

功率函数为0, 且 原点为平衡点, 故原点稳定

且对于任意点V(x)都为0

(b)

$\ddot{x} + 5x^4\dot{x} + x^3 = 0 \overset{\substack{x_1 = x \\ x_2 = \dot{x}}}{\Longrightarrow} \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -5x_1^4 x_2 - x_1^3 \end{cases}$

考虑 $V(x) = x_1^4 + 2x_2^2 > 0$

$V'(x) = 4x_1^3 x_2 + 4x_2(-5x_1^4 x_2 - x_1^3)$

$= -20x_1^4 x_2^2 \leq 0$

故 V(x) 正定, V'(x)半负定, 同时$\|x\| \to \infty$, $V(x) \to \infty$ 故原点为全局渐近稳定

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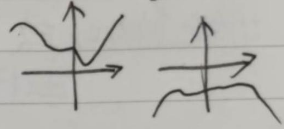
$\ddot{x}+\dot{x}+g(x) \xRightarrow[x_2=\dot{x}]{x_1=x} \begin{cases}\dot{x}_1 = x_2 \\ \dot{x}_2 = -g(x)-x_2\end{cases} \Rightarrow f(0)=0$ 原点为平衡点

考虑 $V(x)=\int_0^{x_1} g(x_1)\,dx_1 + \frac{1}{2}x_2^2$，因为g为连续函数 $\begin{cases} g(0)=0\\ g(x)\neq 0 \quad \forall x\neq 0\end{cases}$

$\Downarrow$

g(x) 只有 x=0 这一0点

$\Downarrow$



又知 $g(x_1)$ 与 x_1 同号，同时 $x_1>0$ 故 $g(x_1)>0$，$\int_0^{x_1} g(x_1)\,dx>0$

故 $V(x)>0$

$$V'(x) = g(x_1)x_2 + x_2(-g(x)-x_2) = -x_2^2 < 0$$

故 $V(x)>0$，$V'(x)<0$，同时这 x^2 发散 故 $\|x\|\to\infty$ 时 $V(x)\to\infty$

~~则原点发散~~ 则原点稳定

3 使用克拉索夫斯基方法判断稳定

$\boxed{d_{ii}=3x_i^2}$

$\dot{x}=Ax-Dx \Rightarrow f(0)=0 \Rightarrow x=0$ 为平衡点　$F(T)=\frac{\partial f(x)}{\partial x^T} = \begin{bmatrix} a_{11}-d_{11} & \cdots & a_{1n}\\ a_{21} & a_{22}-d_{22} & \vdots \\ \vdots & & \vdots \\ a_{n1} & & a_{nn}-d_{nn}\end{bmatrix} = (A-D)$

令 $V(x)=\|f(x)\|^2 = [(A-D)x]^T\cdot[(A-D)x] > 0$

$$V'(x) = [(-A-D)x][(A-D)^T+(A-D)][(A-D)x]$$

$$V'(x) = [(-A-D)x][-A-D+A-D][(A-D)x]$$

$$V'(x) = [(-A-D)x][(A-D)x](-2D)$$

由 $V(x)>0$ 可知 $[(-A-D)x][(A-D)x]>0$，同时D正定，则 $-2D$ 负定

故 $V'(x)$ 负定

因为 $V(x)$ 正定，$V'(x)$ 负定，当 $\|x\|\to\infty$，$V(x)\to\infty$ 故为全局渐近稳定

4 $\begin{cases} \dot{x}_1 = ax_1 + x_2 \\ \dot{x}_2 = x_1 - x_2 + bx_2^5 \end{cases} \Rightarrow F(x) = \begin{bmatrix} a & 1 \\ 1 & -1+5bx_2^4 \end{bmatrix}$

$$F^T(x) + F(x) = \begin{bmatrix} 2a & 2 \\ 2 & -2+10bx_2^4 \end{bmatrix}$$

$\pi_1 = 2a \qquad \pi_2 = -4a + 20abx_2^4 - 4$

① $\pi_1 > 0 \quad \pi_2 < 0$

$\begin{cases} 1 > a > 0 \\ b < 0 \end{cases}$

② $\pi_1 < 0 \quad \pi_2 > 0$

$\Rightarrow \begin{cases} a < 0 \\ a < -1 \text{ 且 } b < 0 \end{cases} \Rightarrow \begin{cases} a < -1 \\ b < 0 \end{cases}$

当 $\|x\| \to \infty$ 时 $V(x) \to \infty$ 故全局渐近稳定

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5、 $\begin{cases} \dot{x}_1 = -2x_1 - 2x_1x_2^4 \\ \dot{x}_2 = -x_2 \end{cases} \Rightarrow f(0)=0 \Rightarrow$ 原点为平衡点

设 $\mathrm{grad}V(x) = \begin{bmatrix} \nabla_1 \\ \nabla_2 \end{bmatrix} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 \\ a_{21}x_1 + a_{22}x_2 \end{bmatrix}$

$$\dot{V}(x) = [a_{11}x_1 + a_{12}x_2,\ a_{21}x_1 + a_{22}x_2] \begin{bmatrix} -2x_1 - 2x_1x_2^4 \\ -x_2 \end{bmatrix}$$

$$V'(x) = (a_{11}x_1 + a_{12}x_2)(-2x_1 - 2x_1x_2^4) - x_2(a_{21}x_1 + a_{22}x_2)$$

$$V'(x) = -2a_{11}x_1^2 - 2a_{11}x_1^2x_2^4 - 2a_{12}x_1x_2 - 2a_{12}x_1x_2^5 - a_{21}x_1x_2 - a_{22}x_2^2$$

取 $a = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow V'(x) = -2x_1^2 - 2x_1^2x_2^4 - x_2^2 < 0$

$$V(x) = \int_0^{x_1} \nabla_1\, dx_1 + \int_0^{x_2} \nabla_2\, dx_2$$

$$= \int_0^{x_1} x_1\, dx_1 + \int_0^{x_2} x_2\, dx_2 \Rightarrow \frac{1}{2}(x_1^2 + x_2^2) \geq 0$$

~~故原点全局稳定~~

$V(x)$正定 $\dot{V}(x)$ 半负定，在 0 以外的点不恒为0，故原点 逐渐稳定