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Extended Virtual Passive Dynamic Walking and Virtual Passivity-mimicking Control Laws

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Abstract

In our previous works, we have proposed "virtual passive dynamic walking" utilizing modified gravity condition with virtual gravity field. The virtual passive walking motion critically depends on the physical parameters and the steady walking pattern is not easily obtained without suitable physical parameters. Based on the observation, in this paper we propose more generalized method of virtual passive walk and a virtual passivity mimicking control law. By the effect of the control laws, we can generate the steady walking pattern even if the physical parameters were not suitable. We call the walking pattern generated by the control methods as "extended virtual passive dynamic walking". The validity of the proposed method is examined by numerical simulations and tested by a prototype experimental machine.

1 Introduction

In this paper we propose a safety and energy-effective control law based on passive dynamic walking originally studied by McGeer[3]. We have proposed "virtual passive dynamic walking" with virtual gravity field in the previous work[4, 5] and investigated its validity using an experimental machine. The advantage of the virtual passive walk is its *autonomy.* The control input is determined by only the information of robot's angular positions since the controlled system is an autonomous system as mentioned later. As a result the walking motion is generated by interactions with the environment.

A problem is that since the walking motion strongly depend on the parameter choice and the virtual slope angle, we must identify the dynamic parameters accurately. And the possibility of realization of steady walk is also restricted to mass balance, the positions of center of gravity, and so on. Based on the observation, in this paper we propose a generalized virtual passive walk; *Extended virtual passive walking* in which the variation of physical parameters are admitted. Further we propose a more generalized method which mimics the behavior of total mechanical energy of a virtual passive walk. The validity of control law is examined by numerical simulations and an experiment.

2 Virtual Passive Dynamic Walking

2.1 The compass gait biped model

Fig. 1 shows the model of a compass-gait biped. The model is compass-like biped robot which has actuators at ankle and hip and the motion is assumed to be constrained in the sagittal plane. It is well known that this robot with suitable parameter choice can walk down a gentle slope without any external energy sources except the gravity effect and the gait pattern converges 1-periodic stable limit cycle[3, 1, 6, 7]. On the level ground, however, the robot cannot walk without control forces because there does not exist any driving force toward the walking direction. If small virtual gravity field toward the horizontal direction is exerted, the robot can exhibit "virtual passive" walking and the walking motion also converges 1-periodic stable limit cycle[4]. The condition is seemed to be on a gentle slope ϕ in the nominal gravity field $g/\cos\phi$. Since the value of ϕ is very small, this condition is seemed to be very closed to real condition, so the gait pattern can be considered to be *natural.*

The dynamic equation during a single support phase of the robot is given by

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + g(\theta) = \tau, \tag{1}$$

where $\theta = \begin{bmatrix} \theta_1 & \theta_2 \end{bmatrix}^T$ is the vector of joint angles, $M(\theta) = [2 \times 2]$ is the inertia matrix, $C(\theta, \dot{\theta}) = [2 \times 2]$ is the colioris matrix and $g(\theta) = [2 \times 1]$ is the gravity matrix. τ is the control input in a stational coordinate system and which has a relation with u as follows:

$$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = Su = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

where u_1 and u_2 are torques at the joints. Thus, the ankle torque and the hip torque are given by $u_1 = \tau_1 + \tau_2$ and $u_2 = -\tau_2$ respectively. The heel-strike collision is assumed to be inelastic and without sliding. The stance and swing legs switch during the transition. In the simulations, we set the physical parameters in Fig. 1 as $m = 5.0$, $m_H = 10.0$ [kg], $a = 0.50$, $b = 0.50$, $l = a + b = 1.0$ [m]. Please see [5] for the detail.

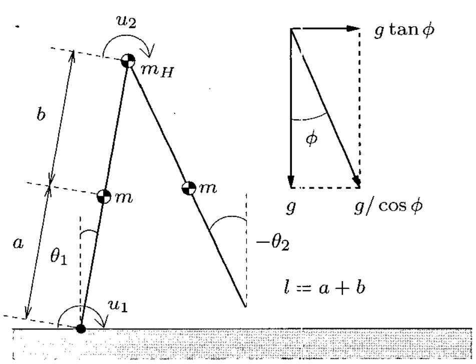


Figure 1: The model of a compass-gait biped and virtual gravity field

2.2 Torque transformation and active walking

We can transform the virtual gravity effect to the actuator's torque as

$$\tau(\theta, \phi) = \begin{bmatrix} (m_H l + ma + ml)\cos\theta_1 \\ -mb\cos\theta_2 \end{bmatrix} g \tan\phi \tag{2}$$

On the other hand, the structure of the gravity term is described as

$$\begin{aligned} g(\theta) &= \begin{bmatrix} -(m_H l + ma + ml)g\sin\theta_1 \\ mbg\sin\theta_2 \end{bmatrix} \\ &= \begin{bmatrix} \sin\theta_1 & \\ & \sin\theta_2 \end{bmatrix} \begin{bmatrix} -(m_H l + ma + ml)g \\ mbg \end{bmatrix}. \end{aligned}$$

and we write this in the following form:

$$g(\theta) = R_s(\theta)\bar{g}. \tag{3}$$

Then the control input by the virtual passive dynamic walking can be rewritten in the following form:

$$\begin{aligned} \tau(\theta, \phi) &= \begin{bmatrix} \cos\theta_1 & \\ & \cos\theta_2 \end{bmatrix} \\ &\quad \times \begin{bmatrix} (m_H l + ma + ml)g \\ -mbg \end{bmatrix} \tan\phi \\ &= -R_c(\theta)\bar{g}\tan\phi. \end{aligned} \tag{4}$$

2.3 Characteristics of virtual passive walk

The characteristics of virtual passive walk can be enumerated as follows:

1. Since the control input can be written as $\tau = \tau(\theta, \phi)$, the closed system is an autonomous system. So the control input is determined by only the information of angular positions.
2. $\|\tau\|_\infty = |\tau_1|_\infty = (m_H l + ml + ma)g\tan\phi$. Thus we can know the upper bound of the control input of τ in advance. This is important characteristics for biped robots from the ZMP point of view as mentioned later.

Furthermore we have defined "virtual energy" which is the total energy of the robot defined in the modified gravity field and it can be regarded its constant value as an indicator of passive system. Let

$$E_V(\theta, \dot{\theta}) := \frac{1}{2}\dot{\theta}^T M(\theta)\dot{\theta} + P_V(\theta, \phi)$$

where $P_V(\theta, \phi)$ is the virtual potential energy defined as:

$$\begin{aligned} P_V(\theta, \phi) &= (m_H l + ml + ma)\cos(\theta_1 + \phi)\frac{g}{\cos\phi} \\ &\quad -mbg\cos(\theta_2 + \phi)\frac{g}{\cos\phi}. \end{aligned}$$

Then E_V is kept constant, that is, the following equation

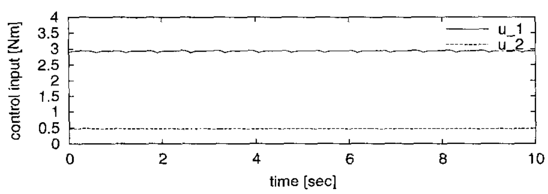
$$\frac{d}{dt}E_V(\theta, \dot{\theta}) = 0$$

holds during the single support phase in the case without any external forces.

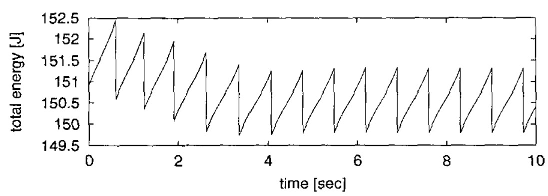
2.4 Numerical simulations of virtual passive walk

Fig. 2 shows the simulated results of actuator torque vs. time where $\phi = 0.02$ [rad]. The initial conditions are set as $\theta(0) = \begin{bmatrix} -0.20 & 0.20 \end{bmatrix}^T$, $\dot{\theta}(0) =$

$\begin{bmatrix} 1.00 & 0.80 \end{bmatrix}^T$. The control is assumed to be implemented as a digital controller, i.e., Z.O.H. is introduced in front of the actuator, where the control interval is 1.0 [msec]. From (a) and (b) we can see that the walking pattern converges 1-periodic motion and its torque are almost constant. The constant-like torque is special feature of virtual passive walk and it is also able to be an indicator of natural motion. The small range of $\boldsymbol{u}$ can realize that of ZMP(Zero-Moment Point) and it is very important feature for biped robots[2]. From (c) we can see that the *real* energy increases during each single support phase constantly and monotonously. This is because the virtual gravity acts as a driving force for the robot toward the walking direction continuously.



(a) control inputs



(b) total energy

Figure 2: Simulated results of a virtual passive walk

3 Extended Virtual Passive Dynamic Walking

We consider generalization of the control input Eq. (4) as $\boldsymbol{R}_c(\boldsymbol{\theta})\boldsymbol{p}$. The vector $\boldsymbol{p}$ is defined as

$$\boldsymbol{p} := \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}, \tag{5}$$

so $\mathbf{p} = -\bar{g}\tan\phi$, virtual passive walk, is the special case of extended virtual passive walk. We call the walking motion which is realized by the control input in the form $\boldsymbol{\tau}(\boldsymbol{\theta}) = \boldsymbol{R}_c(\boldsymbol{\theta})\boldsymbol{p}$ as "*Extended Virtual Passive Dynamic Walking*". The following characteristics from virtual passive walk can be satisfied by the control scheme:

1. $p_1 > 0,\ p_2 < 0$
2. $|p_1| > |p_2|$
3. $\|\tau_i\|_\infty = |p_i|$

The statement 3 is an important feature for a biped from the viewpoint of ZMP and it is the most important property of extended virtual passive walk. The ZMP is calculated by an equation

$$ZMP = \frac{u_1}{R}$$

where R is the vertical reaction force of the foot from the floor. Please see [2] for the detail. Thus the small range of u_1 implies that of ZMP, and the fact that we can obtain approximate value of $|u_1|_\infty$ analytically is important fact for the robot design. The generalized control can extend the acceptable domain of parameter choice.

4 Energy Constrained Control

In this section, we propose a simple control law which mimics the energy behavior of a virtual passive walk.

4.1 The control law

The virtual passive walker is driven toward the walking direction during the single support phase, so its total energy increases almost in a constant speed. The total *real* energy E of the robot can be expressed as $E = \dot{\boldsymbol{\theta}}^T\boldsymbol{M}(\boldsymbol{\theta})\dot{\boldsymbol{\theta}}/2 + P$ where P is the *real* potential energy of the robot. And the power input to the system is the time rate of change of total energy,

$$\dot{E} = \dot{\boldsymbol{\theta}}^T\boldsymbol{\tau}. \tag{6}$$

From Fig. 2, we can see that the total energy increases monotonously and we introduce a control law which mimics this phenomenon, i.e., the following equation is satisfied:

$$\dot{E} = \lambda. \tag{7}$$

The time rate of change of total energy is controlled. $\lambda > 0$ is a tuning parameter. Large λ implies that of virtual slope ϕ. Although there exist many control input $\boldsymbol{\tau}$ which realize Eq.(7), we consider the same form of extended virtual passive walk and determine the control input as

$$\boldsymbol{\tau}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}, \lambda) = \frac{\lambda\boldsymbol{R}_c(\boldsymbol{\theta})\boldsymbol{p}}{\dot{\boldsymbol{\theta}}^T\boldsymbol{R}_c(\boldsymbol{\theta})\boldsymbol{p}}. \tag{8}$$

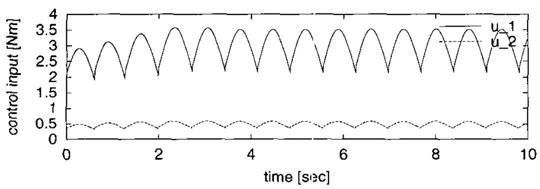
where $\boldsymbol{R}_c(\boldsymbol{\theta})$ introduces the freedom of the control input. It is easily checked that Eq.(7) is satisfied from Eq.(6) and (8). Furthermore, without loss of generality, we can reform Eq.(8) as:

$$\boldsymbol{\tau}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}, \lambda) = \frac{\lambda\boldsymbol{R}_c(\boldsymbol{\theta})\bar{\boldsymbol{p}}}{\dot{\boldsymbol{\theta}}^T\boldsymbol{R}_c(\boldsymbol{\theta})\bar{\boldsymbol{p}}} \tag{9}$$

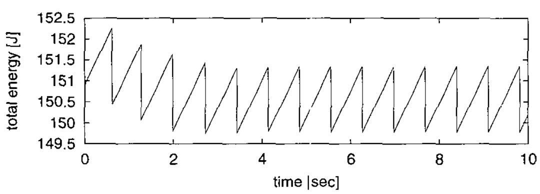
where $\bar{\boldsymbol{p}} = \begin{bmatrix} p & -1 \end{bmatrix}$. Thus, in this case the tuning parameter is only p and λ. So we do not have to measure the dynamic parameters of the robot exactly.

4.2 Numerical simulations

We analyzed the walking pattern by numerical simulations. All simulated conditions are identical with previous section. Fig. 3 shows the simulated results of energy constrained control. The tuning parameters are set as $p = 7.0$ and $\lambda = 2.2$ and the robot starts from the same initial condition. In this case the walking pattern also converges 1-periodic stable limit cycle. The walking motion and energy consumption $\bar{E}_k$ is almost the same with that of virtual passive walk, however, the range of the control input is larger. The only problem of energy constrained control is this phenomenon. From the ZMP point of view, the ankle torque is better for biped robots.



(a) control inputs



(b) total energy

Figure 3: Simulated results of a virtual passive walk

[Remark] The passive walking pattern is generated by only mass ratio $k_m = m_H/m$ and length ratio $k_l = b/a$. Thus, the individual value of mass and length are not required to determine the steady walking pattern. Only the mechanical energy is changed by the actual values. Fig. 4 shows the simulated results of $\dot{E}$ vs. time w.r.t. M where $M = m_H + 2m$ is a total mass of the robot. We can see that $\dot{E}$ behaves flat in the case of light robot. This implies that if the robot is light, its walking pattern of energy constrained control is very close to that of virtual passive walk.

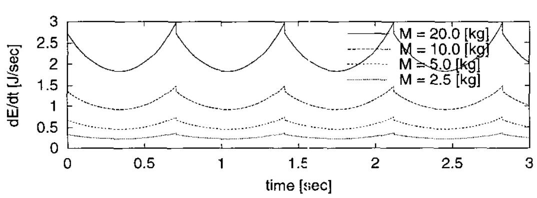


Figure 4: Simulated results of a virtual passive walk

5 Hybrid Control

Each control laws achieves almost same energy consumptions. Which is better control for biped robots ? From the ZMP point of view, the answer is virtual passive walk. Thus we should transfer the control law to virtual passive walk according to the degree of the stability of the gait. A hybrid control law which realizes the adaptability will be explained in the following.

5.1 Some definitions

Here we should define the following terminologies to explain hybrid control as mentioned later.

1. k-th step
   The time duration of a single support phase from k-th heel-strike to just before $k+1$-th one from the start of walking. Thus, the duration until first collision is 0-th step.
2. Step period T_k
   The time duration of k-th step.
3. Average energy consumption $\bar{E}_k$
   In this paper we define the average energy consumptions of 1 step by l_1 norm in the following form because of the constant sign of the control input

$$\bar{E}_k = T_k^{-1} \int_{k \text{ th step}} \|\boldsymbol{\tau}\|_1 dt \tag{10}$$

In the case without external forces, $\bar{E}_k$ converges a limit value $\bar{E}_\infty$ and we should note that $\bar{E}_\infty$ of virtual passive walk converges almost the same value of energy constrained control in the case of same walking speed.

5.2 Combined control law

If the robot is started from energy constrained mode, we must tune only the parameters λ and p. And then, the walking pattern should change to extended virtual passive walk as mentioned before. In order to realize the variable walking pattern, the average energy consumptions of energy constrained control should be defined as follows:

$$\begin{aligned}\bar{E}_k^\lambda &:= T_k^{-1} \int_0^{T_k} \|\boldsymbol{\tau}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}, \lambda)\|_1 dt \\ &\rightarrow T_\infty^{-1} \int_0^{T_\infty} \|\boldsymbol{\tau}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}, \lambda)\|_1 dt =: \bar{E}_\infty^\lambda.\end{aligned}$$

Then we can construct a hybrid control law as follows.

The virtual passive walk is combined with energy constrained control as the following form:

$$\boldsymbol{\tau} = \left[\frac{\lambda s_k}{\dot{\boldsymbol{\theta}}^T \boldsymbol{R}_c(\boldsymbol{\theta})\bar{\boldsymbol{p}}} + \frac{\bar{E}_k^\lambda}{\|\bar{\boldsymbol{p}}\|_1}(1 - s_k) \right] \boldsymbol{R}_c(\boldsymbol{\theta})\bar{\boldsymbol{p}}. \quad (11)$$

In order to realize smooth change of the walking motion, we consider s_k using a sigmoid function as:

$$s_k = \left[1 + \exp\left\{ -\zeta \left(\left| \frac{\bar{E}_{k-1}^\lambda - \bar{E}_{k-2}^\lambda}{\bar{E}_{k-2}^\lambda} \right| - 0.1 \right) \right\} \right]^{-1} \quad (k \geq 2)$$

where $\zeta > 0$ is some constant. In the case without any external energy sources $\bar{E}_k^\lambda$ converges $\bar{E}_\infty^\lambda$ and $s_k \to 0$. Then

$$\boldsymbol{\tau}(\boldsymbol{\theta}) \to \frac{\bar{E}_\infty^\lambda}{\|\bar{\boldsymbol{p}}\|_1} \boldsymbol{R}_c(\boldsymbol{\theta})\bar{\boldsymbol{p}} \quad (12)$$

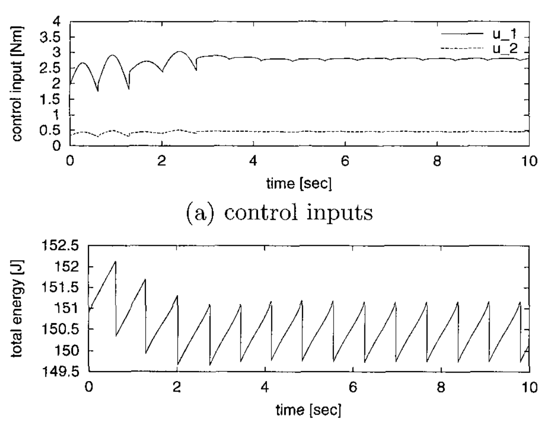
and the vector $\boldsymbol{p}$ is defined as: $\boldsymbol{p} := \bar{E}_\infty^\lambda \|\bar{\boldsymbol{p}}\|_1^{-1} \bar{\boldsymbol{p}}$. Thus, the walking motion changes to extended virtual passive walk. In this case, the energy consumption is calculated as follows:

$$\begin{aligned} \bar{E}_k &\to T_\infty^{-1} \int_0^{T_\infty} \|\boldsymbol{\tau}\|_1 dt \\ &\approx T_\infty^{-1} \int_0^{T_\infty} \frac{\bar{E}_\infty^\lambda \|\bar{\boldsymbol{p}}\|_1}{\|\bar{\boldsymbol{p}}\|_1} dt = \bar{E}_\infty^\lambda. \end{aligned}$$

Thus, the energy consumption is not changed even if the control law is changed.

5.3 Numerical simulations

We analyzed the validity of the control law by numerical simulations. All simulated conditions and tuning parameters are identical with previous sections. Fig. 5 shows the simulated results of the hybrid control where



(a) control inputs

(b) total energy

Figure 5: Simulated results of hybrid control

$\zeta = 50$. The walking motion smoothly changes from energy constrained control to extended virtual passive walk without change of energy consumptions w.r.t. stabilization of walking pattern. And then the velocity feedback was disappeared and the control was changed to positional feedback only.

6 Experimental Results

In order to check the validity of the proposed method, we constructed an experimental system. In Fig. 6 we show the experimental setup(left) and the mechanism of ankle joint and foot(right). The robot has straight legs of the same length and three DC motors with encoders at the hip position. The ankle joints are driven by tinning belts. Because of the symmetric structure the motion is constrained to sagittal plane and the robot can be regarded as a two-legged robot as shown in Fig. 1. The dynamic parameters of the robot were identified off-line as: $m = 0.40$, $m_H = 3.0$ [kg], $a = 0.215$, $b = 0.465$ and $l = 0.68$ [m]. The ankle joint of the swing leg is controlled by PID controller in order to keep the foot posture horizontal.

Since the proposed methods are so called model matching control, the controls are not robust for uncertainty. In this research, we use model following control of the motion generated by VIM(Virtual Internal Model) which is a reference model driven by sensory information. By the use of VIM, the uncertainties of identification which is crucial factor in the case of model matching control can be compensated for. Please see [4] for the detail.

Form Fig. 6 we can see that if the stance leg moves forward, the ZMP is at behind the ankle joint due to the reaction torque. Since our method provides *one-directional* ankle torque, the ZMP is always at behind. So at the transition, since the robot is strongly affected

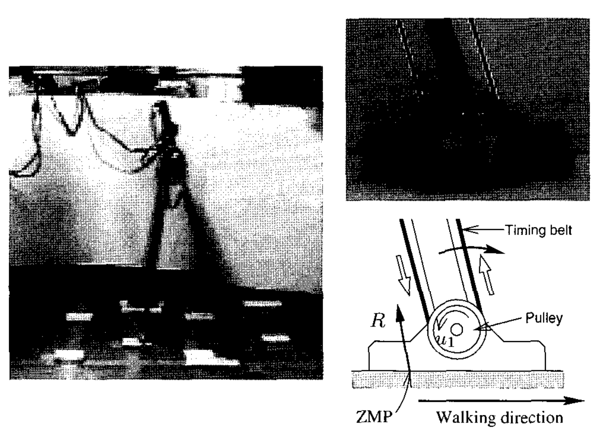


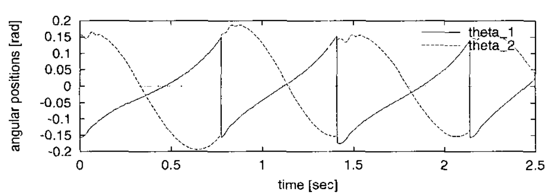
Figure 6: Experimental setup and foot mechanism

by the reaction moment from the floor, the ZMP will be shifted to the heel and walking cannot be continued. Thus the ZMP should be controlled and disposed in the forward area just after the transition in order to cancel the reaction force from the floor. Based on the observation, we propose an easy control law for ZMP via switching λ. The ZMP can be moved to forward area of the sole via change of ankle torque's direction. In our method, this is identical with switching the signal of λ. And we propose λ switching scheme as

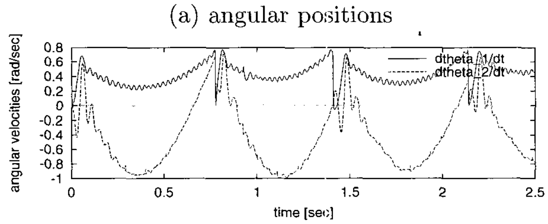
$$\lambda = \begin{cases} \lambda_1 < 0 & (T_s \leq t < T) \\ \lambda_2 > 0 & (T \leq t < T_e). \end{cases}$$

According to the switching control, the ZMP can be controlled forward till $t = T$ [sec] after transition where T_s and T_e are instances when the stance leg touches and detaches the ground. And then the effect of the reaction moment will disappear.

Fig. 7 shows the experimental results. In the experiment the control period was set to 2.0 [msec] and the angular velocity was measured through a filter whose transfer function is $70/(s+70)$ for each actuator. To implement the control law, we used RTMATX[9]. The control parameters are set as $\lambda_1 = -0.02$, $\lambda_2 = 0.15$, $p = 12.8925$ and $T = T_s + 0.05$ [sec]. The results are almost identical with that of virtual passive walk. This is because the total mass of the robot is light as mentioned in the previous remark. So in the case of light robots we can obtain the virtual passive walk-like efficiency by energy constrained control and realize the walking easily only by tuning λ. In the previous works[4, 5], we reported that the experiment cannot be realized because the heel-strike is not inelastic in the real world, however, by the effect of λ control we could solve the transition probrem.



(a) angular positions



(b) angular velocities

Figure 7: Experimental results

7 Conclusions

In this paper we proposed simple control laws based on passive dynamic walking and investigated the validity of the control laws by numerical simulations and experiments. The proposed ZMP control is very simple and effective for the transition which is not inelastic perfectly. The experimental walking is not robust to be repeated and cannot be realized in the case the step is large. So in the future we should consider more robust control laws.

Acknowledgement

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