

1. a) 同步: 根据  $V_{k+1}(s) = \max_{a \in A} (r_s^a + \gamma \sum_{s' \in S} P_{ss'}^a V_k(s'))$ , 计算得:

$$V_2(a) = -8 + 0.3 \times V_1(b) = -7.4$$

$$V_2(b) = \max \{ 2 + 0.3 V_1(a), -2 + 0.3 V_1(c) \} = 2.2$$

2.6

$$V_2(c) = \max \{ 0.25(4 + 0.3 V_1(a)) + 0.75(0 + 0.3 V_1(c)), 8 + 0.3 V_1(b) \} = 8.6$$

由确定性贪心策略  $\pi_*(s) = \arg \max_{a \in A} (r_s^a + \gamma \sum_{s' \in S} P_{ss'}^a V_*(s'))$ , 得到策略  $\pi_2(a|s)$  为:

$$\pi_2(a=ab|s=A) = 1$$

$$\pi_2(a=ba|s=B) = 1, \quad \pi_2(a=bc|s=B) = 0$$

$$\pi_2(a=ca|s=C) = 0, \quad \pi_2(a=cb|s=C) = 1$$

b) 异步: 根据  $V(s) = \max_{a \in A} (r_s^a + \gamma \sum_{s' \in S} P_{ss'}^a V(s'))$ , 计算得:

$$V(a) = -8 + 0.3 V(b) = -7.4$$

$$V(b) = \max \{ 2 + 0.3 V(a), -2 + 0.3 V(c) \} = -0.22$$

$$V(c) = \max \{ 0.25(4 + 0.3 V(a)) + 0.75(0 + 0.3 V(c)), 8 + 0.3 V(b) \} = 7.934$$

由确定性贪心策略, 得到策略  $\pi'_2(a|s)$  为:

$$\pi'_2(a=ab|s=A) = 1$$

$$\pi'_2(a=ba|s=B) = 1, \quad \pi'_2(a=bc|s=B) = 0$$

$$\pi'_2(a=ca|s=C) = 0, \quad \pi'_2(a=cb|s=C) = 1$$

2. a) 根据状态价值贝尔曼期望方程  $V_\pi(s) = \sum_{a \in A} \pi(a|s) (r_s^a + \gamma \sum_{s' \in S} P_{ss'}^a V_\pi(s'))$

$$\begin{cases} V_\pi(A) = P_{AA}(3 + V_\pi(A)) + P_{AB}(1 - 3 + V_\pi(B)) \\ V_\pi(B) = P_{BA}(3 + V_\pi(A)) + P_{BB} \cdot 0 \end{cases} \quad \text{解得} \begin{cases} V_\pi(A) = -1 \\ V_\pi(B) = 1 \end{cases}$$

$\therefore$  状态价值函数  $V(A) = -1, V(B) = 1$

b) 首次访问:

$$\because G_1(A) = +2 + 3 - 5 + 5 - 2 = 3, \quad G_1(B) = -5 + 5 - 2 = -2$$

$$G_2(A) = +3 - 3 = 0, \quad G_2(B) = -2 + 3 - 3 = -2$$

$$\therefore V(A) = \frac{1}{2} (G_1(A) + G_2(A)) = 1.5, \quad V(B) = \frac{1}{2} (G_1(B) + G_2(B)) = -2$$

每次访问:

$$\because G_{11}(A) = +2 + 3 - 5 + 5 - 2 = 3, \quad G_{12}(A) = +3 - 5 + 5 - 2 = +1, \quad G_{13}(A) = +5 - 2 = 3$$

$$G_{11}(B) = -5 + 5 - 2 = -2, \quad G_{12}(B) = -2$$

$$G_{21}(A) = +3 - 3 = 0, \quad G_{21}(B) = -2 + 3 - 3 = -2, \quad G_{22}(B) = -3$$

$$\therefore V(A) = \frac{1}{4} (G_{11}(A) + G_{12}(A) + G_{13}(A) + G_{21}(A)) = 1.75, \quad V(B) = \frac{1}{4} (G_{11}(B) + G_{12}(B) + G_{21}(B) + G_{22}(B)) = -2.25$$

综上, 使用首次访问, 估计状态价值函数  $V(A) = 1.5, V(B) = -2$ ;

使用每次访问, 估计  $V(A) = 1.75, V(B) = -2.25$