

## 数字量和模拟量

- 数字量：在时间上和数量上都是**离散、不连续的**。  
(存在一个最小数量单位 $\Delta$ )
- 模拟量：数字量以外的物理量。

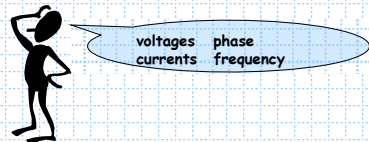
## Continuous versus Discrete

- Which are "continuous"?
  - Color
  - English letters
  - Light
  - Cars
  - Sound
  - Height and weight
  - Dogs
  - Electric current and voltage

**Many natural phenomena are continuous**

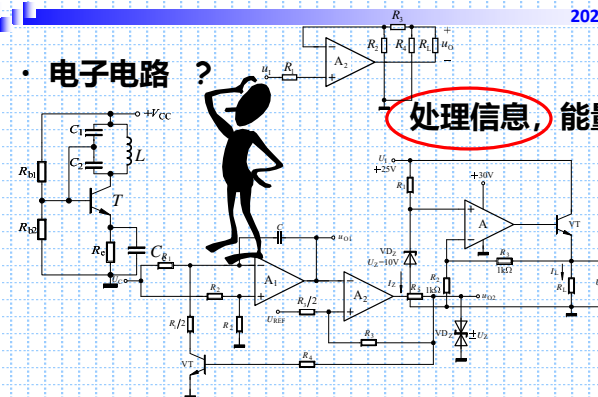
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- 模拟量：数字量以外的物理量。
- 数字**电路**和模拟**电路**：  
**工作信号**、研究的对象、分析/设计方法以及所用的数学工具都有显著的不同。



## • 电子电路 ?

**处理信息, 能量转换**



- 模拟电路：用连续的模拟电压/流值来表示信息
- 数字电路：用**离散的电压序列**来表示信息

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## 电子技术的发展

Electronic Design Automation

- 48年 贝尔实验室制成第一只晶体管
- 58年 集成电路 (4-12-100-1000)
- 69年 大规模集成电路 (10万)
- 75年 超大规模集成电路(15万)
- ...

EDA技术

SSI MSI LSI VLSI ULSI GLSI

第一片集成电路只有4个晶体管，而97年一片集成电路上有40亿个晶体管。预测集成度按10倍/6年的速度-----

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## How do you build systems with >1G components?

Personal Computer: Hardware & Software

Circuit Board:  $\approx 1-8$  / system, 1-2G devices

Integrated Circuit:  $\approx 8-16$  / PCB, 25M-16M devices

Module:  $\approx 8-16$  / IC, 100K devices

MOSFET

Scheme for representing information

Gate:  $\approx 2-16$  / Cell, 8 devices

Cell:  $\approx 1K-10K$  / Module, 16-64 devices

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## What do we see?

- **Structure 结构**
  - hierarchical design
  - limited complexity at each level
  - reusable building blocks
- **Interfaces 接口**
  - Key elements of system engineering
  - Isolate technologies, allow evolution
  - Major abstraction mechanism
- **What makes a good system design?**
  - minimal mechanism, maximal function
  - reliable in a wide range of environments
  - accommodates future technical improvements

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## The Art of Managing Complexity

- **Abstraction**
  - Hiding details when they aren't important
- **Discipline**
  - Intentionally restrict design choices
  - Example: Digital discipline
    - Discrete voltages instead of continuous
- **The Three -y's**
  - **Hierarchy**: A system divided into modules and submodules
  - **Modularity**: Having well-defined functions and interfaces
  - **Regularity**: Encouraging uniformity, so modules can be easily reused

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## Our plan of attack...

- Understand **how things work**, **bottom-up**
- Encapsulate our understanding using **appropriate abstractions**
- Study **organizational principles**: abstractions, interfaces, API (Application Programming Interface)
- Roll up our sleeves and **design at each level of hierarchy**
- Learn **engineering tricks**
  - History
  - Systematic approaches
  - Algorithms
  - Diagnose, fix, and avoid bugs

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面包板搭建**中小规模电路**——调试, 工具的使用  
 可编程器件实现**大规模电路**——EDA工具, 硬件描述语言 (提前)  
 PCB板实现**成熟电路**——EDA工具, 焊接调试

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## 第一章 信息和编码

编码的目的  
二进制编码

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## What is "Information 信息"?

**information, n.**  
 Knowledge, communicated or received concerning a particular fact or circumstance.

中国男足输了  
 Tell me something new...

"Really, 一点儿都不难!"

**Information resolves uncertainty.**  
 Information is simply that which cannot be predicted.  
 The less predictable a message is, the more information it conveys!

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## Quantifying Information


(Claude Shannon, 1948)

Suppose you're faced with  $N$  equally probable choices, and I give you a fact that narrows it down to  $M$  choices. Then I've given you  $\log_2(N/M)$  bits of information

Information is measured in bits (binary digits) = number of 0/1's required to encode choice(s)

Examples:

- information in one coin flip:  $\log_2(2/1) = 1$  bit
- roll of 2 dice:  $\log_2(36/1) = 5.2$  bits



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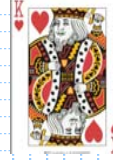

## Quantifying Information

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data	$P_{\text{data}}$	$\log_2(1/P_{\text{data}})$
a heart	13/52	2 bits
not the Ace of spades	51/52	0.028 bits
a face card (J,Q,K)	12/52	2.115 bits
The "suicide king"	1/52	5.7 bits

Get **more information** when the data **resolves more uncertainty** about the randomly selected card,


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## Encoding 编码

- Encoding describes the process of **assigning representations to information**
- Choosing an appropriate and efficient encoding is a real engineering challenge
- Impacts design at many levels
  - Mechanism (devices, # of components used)
  - Efficiency (bits used)
  - Reliability (noise)
  - Security (encryption)



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## 2020011141

- 数制：表示数量的规则
- 码制：表示事物的规则

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- 数制:
  - ① 每一位的构成
  - ② 从低位向高位的进位规则

我们常用到的:  
十进制, 二进制, 八进制, 十六进制

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二进制, 八进制, 十进制, 十六进制

- A binary digit has only 2 possibilities 逢二进一  
0 1
- An octal digit has 8 possibilities 逢八进一  
0 1 2 3 4 5 6 7
- A decimal digit has 10 possibilities 逢十进一  
0 1 2 3 4 5 6 7 8 9
- A hexadecimal (hex) digital has 16 possibilities 逢十六进一  
0 1 2 3 4 5 6 7 8 9 A B C D E F

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Encoding numbers

$$v = \sum_{i=0}^{n-1} 2^i b_i$$

$2^{11} 2^{10} 2^9 2^8 2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0$   
 0 1 1 1 1 1 1 0 1 0 0 0 0 = 2000<sub>10</sub>  
 7 d 0

03720 0x7d0

Octal - base 8 Hexadecimal - base 16

000 - 0	0000 - 0	1000 - 8
001 - 1	0001 - 1	1001 - 9
010 - 2	0010 - 2	1010 - a
011 - 3	0011 - 3	1011 - b
100 - 4	0100 - 4	1100 - c
101 - 5	0101 - 5	1101 - d
110 - 6	0110 - 6	1110 - e
111 - 7	0111 - 7	1111 - f

Often times we will find it convenient to cluster groups of bits together for a more compact notation. Two popular groupings are clusters of 3 bits and 4 bits.

Seems natural to me!

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Bits, Bytes, Nibbles... (MSB, LSB)

- Bits  
10010110  
most significant bit least significant bit
- Bytes & Nibbles  
byte  
10010110  
nibble
- Bytes  
CEBF9AD7  
most significant byte least significant byte

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## Powers of 2

- $2^0 = 1$
- $2^1 = 2$
- $2^2 = 4$
- $2^3 = 8$
- $2^4 = 16$
- $2^5 = 32$
- $2^6 = 64$
- $2^7 = 128$
- $2^8 = 256$
- $2^9 = 512$
- $2^{10} = 1024$
- $2^{11} = 2048$
- $2^{12} = 4096$
- $2^{13} = 8192$
- $2^{14} = 16384$
- $2^{15} = 32768$

- $2^{10} = 1 \text{ kilo} \approx 1000 (1024)$
- $2^{20} = 1 \text{ mega} \approx 1 \text{ million } (1,048,576)$
- $2^{30} = 1 \text{ giga} \approx 1 \text{ billion } (1,073,741,824)$

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## Sign/Magnitude Numbers

- 1 sign bit,  $N-1$  magnitude bits
- Sign bit is the most significant (left-most) bit
  - Positive number: sign bit = 0
  - Negative number: sign bit = 1
- Example, 4-bit sign/mag representations of  $\pm 6$ :
  - $+6 = 0110$
  - $-6 = 1110$
- Range of an  $N$ -bit sign/magnitude number:
  - $[-(2^{N-1}-1), 2^{N-1}-1]$

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## Sign/Magnitude Numbers


- Problems:
  - Addition doesn't work:
 

$$\begin{array}{r} 1110 \\ + 0110 \\ \hline 10100 \end{array}$$

$$\begin{array}{r} 1100 \\ + 0100 \\ \hline 10000 \end{array}$$

?
  - Two representations of 0 ( $\pm 0$ ):
 

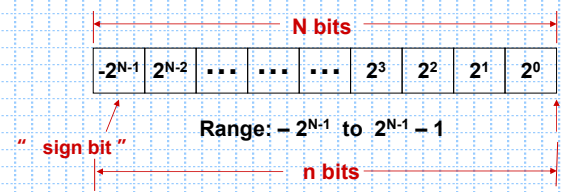
$$\begin{array}{r} 1000 \\ 0000 \end{array}$$



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Range:  $-2^{N-1}$  to  $2^{N-1}-1$

- 最高位为符号位 (0为正, 1为负)
- 正数的补码和它的原码相同
- 负数的补码 = 数值位逐位求反 + 1

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## 二进制数的补码:

- 最高位为符号位 (0为正, 1为负)
- 正数的补码和它的原码相同
- 负数的补码 = 数值位逐位求反 + 1

$$+5 = (0 \ 101)$$

$$-5 = (1 \ 011)$$

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## 两个补码表示的二进制数相加时的符号位讨论

例: 用二进制补码运算求出  
13 + 10、13 - 10、-13 + 10、-13 - 10

注意编码的取值范围

$+13 \ 0 \ 01101$	$+13 \ 0 \ 01101$
$+10 \ 0 \ 01010$	$-10 \ 1 \ 10110$
$+23 \ 0 \ 10111$	$+3 \ 0 \ 00011$

$-13 \ 1 \ 10011$	$-13 \ 1 \ 10011$
$+10 \ 0 \ 01010$	$-10 \ 1 \ 10110$
$-3 \ 1 \ 11101$	$-23 \ 1 \ 01001$

结论: 将两个加数的符号位和来自最高位数字位的进位相加, 结果就是和的符号

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$+13 \ 0 \ 1101$	$+13 \ 0 \ 1101$
$+10 \ 0 \ 1010$	$-10 \ 1 \ 0110$
$+23 \ 1 \ 0111$	$+3 \ 0 \ 0011$

$-13 \ 1 \ 0011$	$-13 \ 1 \ 0011$
$+10 \ 0 \ 1010$	$-10 \ 1 \ 0110$
$-3 \ 1 \ 1101$	$-23 \ 0 \ 1001$

???

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## Increasing Bit Width

- Extend number from NtoM bits ( $M > N$ ):
  - Sign-extension
  - Zero-extension
- Zero-Extension
- Sign-Extension

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## Zero-Extension

- Zeros copied to msb's
- Value changes for negative numbers
- Example 1:
  - 4-bit value =  $0011_2 = 3_{10}$
  - 8-bit zero-extended value:  $00000011 = 3_{10}$
- Example 2:
  - 4-bit value =  $1011 = -5_{10}$
  - 8-bit zero-extended value:  $00001011 = 11_{10}$

## Sign-Extension

- Sign bit copied to msb's
- Number value is same
- Example 1:
  - 4-bit representation of 3 =  $0011$
  - 8-bit sign-extended value:
- Example 2:
  - 4-bit representation of -5 =  $1011$
  - 8-bit sign-extended value:

## Sign-Extension

- Sign bit copied to msb's
- Number value is same
- Example 1:
  - 4-bit representation of 3 =  $0011$
  - 8-bit sign-extended value:  $00000011$
- Example 2:
  - 4-bit representation of -5 =  $1011$
  - 8-bit sign-extended value:  $11111011$

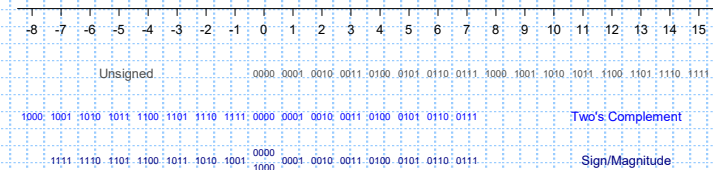
## Number System Comparison

What about fractions?

Number System	Range
Unsigned	$[0, 2^N-1]$
Sign/Magnitude	$[-(2^{N-1}-1), 2^{N-1}-1]$
Two's Complement	$[-2^{N-1}, 2^{N-1}-1]$



For example, 4-bit representation:





## Numbers with Fractions

- Two common notations:

- **Fixed-point:** binary point fixed
- **Floating-point:** binary point floats to the right of the most significant 1

- 例

$$\begin{aligned}(101.11)_B &= 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\ &\quad + 1 \times 2^{-1} + 1 \times 2^{-2} \\ &= (5.75)_D\end{aligned}$$

$$\begin{aligned}(2A.7F)_H &= 2 \times 16^1 + 10 \times 16^0 \\ &\quad + 7 \times 16^{-1} + 15 \times 16^{-2} \\ &= (42.4960937)_D\end{aligned}$$

$$D = \sum K_i N^i \quad \text{任意进制。} \dots$$

## Fixed-Point Numbers

- 6.75 using 4 integer bits and 4 fraction bits:

01101100

0110.1100

$$2^2 + 2^1 + 2^{-1} + 2^{-2} = 6.75$$

- Binary point **is implied**
- The number of integer and fraction bits must be agreed upon **beforehand**
- Represent  $7.5_{10}$  using 4 integer bits and 4 fraction bits.

**0111.1000**

## Signed Fixed-Point Numbers

- Represent  $7.5_{10}$  using 4 integer bits and 4 fraction bits.

**0111.1000**

- Representations:

- Sign/magnitude
- Two's complement

- Example: Represent  $-7.5_{10}$  using 4 integer and 4 fraction bits

- **Sign/magnitude:** **1111.1000**
- **Two's complement**

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### Signed integers: 2's complement (二进制数的补码)

Range:  $-2^{N-1}$  to  $2^{N-1} - 1$

By moving the implicit location of "decimal" point, we can represent fractions too:

1101.0110

$$= -2^3 + 2^2 + 2^0 + 2^{-2} + 2^{-3}$$

$$= -8 + 4 + 1 + 0.25 + 0.125$$

$$= -2.625$$

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### Signed Fixed-Point Numbers

- Represent  $7.5_{10}$  using 4 integer bits and 4 fraction bits.  
**0111.1000**
- Representations:
  - Sign/magnitude
  - Two's complement
- Example: Represent  $-7.5_{10}$  using 4 integer and 4 fraction bits
  - Sign/magnitude: **11111000**
  - Two's complement:
 

1. +7.5:	0111.1000
2. Invert bits:	1000.0111
3. Add 1 to LSB:	$\begin{array}{r} 1000.0111 \\ + 1 \\ \hline 1000.1000 \end{array}$

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### • 码制

用不同数码表示不同事物时遵循的规则  
例如：学号，身份证号，车牌号。。。

- 目前，数字电路中都采用二进制
- 表示数量时称二进制
- 表示事物时称二值逻辑

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