A
SHORT NOTES
ON

INVERTED PENDULUM

(Model Based Control Design for Swing-up & Balance the Inverted Pendulum)

Energy Based Collocated Partial Feedback Linearization Control for Swing up
&
LQR Control for Balance

Submitted by

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Objective: The objective is to design and implement a control system that will Swing up & Balance the Pendulum in the vertical upward position.

Apparatus/software required:

1. MATLAB-2017b.
2. Arduino Support package.
3. Rensselaer Arduino Support package.
4. MATLAB Support for MinGW-w64 C/C++ Compiler.
5. Inverted pendulum experimental setup made out of defective Inkjet Printer.

1. Introduction:

The task is to balance a simple pendulum around its unstable equilibrium, using only horizontal forces on the cart

1.1 Description:

The Inverted Pendulum setup, shown in Fig.1, consists of a Linearguide that is taken out from defective Inkjet Printer. This linear guide further modified by adding two Encoders for the measurement of both Linear & angular position. The Microcontroller we use is Arduino Mega2560.Cytron 10Amp DC motor Driver is used to drive a motor.

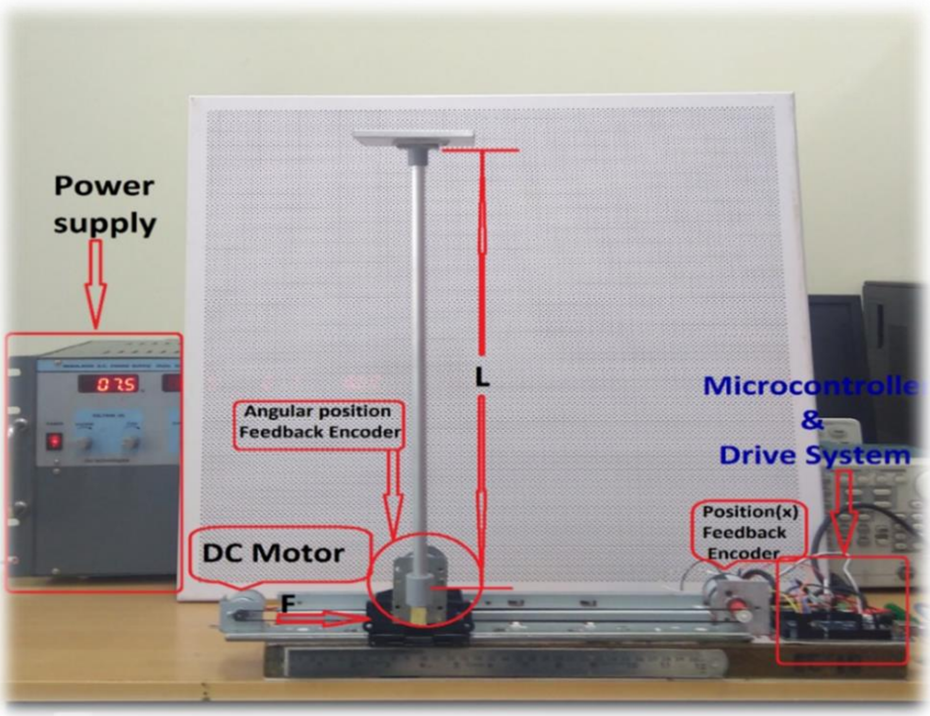


Fig.1 Linear Inverted Pendulum System

1.2 Inverted Pendulum Components:

The component of the Inverted Pendulum module are listed in Table 1 and labelled in Fig.1.

Table 1: Inverted Pendulum components

Sr.No.	Component
1.	Linear Guide taken out from Inkjet Printer.
2.	Pendulum Aluminium Rod.
3.	Encoders 600PPR.
4.	USB A/B Printer Cable 3.0/2.0 (High speed).
5.	Arduino Mega2560.
6.	Cytron 10Amp DC motor drive (MD10C R3).

1.3 Nomenclature:

Table 2: Inverted Pendulum system specifications.

Symbol	Description	Matlab Variable	Value with Units
M	Mass of the pendulum rod.	M	0.1 kg
M_C	Mass of the Cart	M_c	0.135 Kg
L	Pendulum length from pivot to centre of gravity	L	0.2 m
J_m	Motor rotor moment of inertia	Jm	$3.26\text{x}10^{-08}$ kg.m^2
R_m	Motor armature resistance	Rm	12.5 Ω
k_b	Motor back emf constant	Kb	0.031 V/rad/sec
k_t	Motor torque constant	Kt	0.031 N.m/A
R	Motor pinion radius	R	0.006 m
B	Viscous damping at pivot of pendulum	B	0.000078 N.m/rad/sec
C	Viscous friction coefficient for cart displacement	C	0.63 N/m/sec
I	Mass moment of inertia of pendulum rod	I	0.00072 kg.m^2
M	Total cart weight mass including motor inertia	M	0.136 kg
G	Gravitational constant	G	9.81 m/sec^2

Nomenclature

Where:

$$M = M_c + \frac{J_m}{r^2}$$

1. T = Total Kinetic energy of the system.
2. U = Total potential energy of the system.
3. x_p = absolute x – coordinate of the pendulum centre of gravity.
4. y_p = absolute y – coordinate of the pendulum centre of gravity.

The following topics will be covered in this experiment:

I. Modeling the dynamics of the Inverted Pendulum using Euler-Lagrange equations.

II. Obtaining a Linear state-space representation of the system.

III. Design a **Energy based nonlinear swing up**strategy for the Inverted Pendulum.

IV. Designing a state-feedback control system that balances the pendulum at its vertical upward position using **LQR.**

V. Implementing the Digital LQR controller on Arduino Mega2560 Controller via Matlab Simulink.

2. Mathematical Modeling of IP system(EOM):

The Equation of Motion is obtained by fig.2:

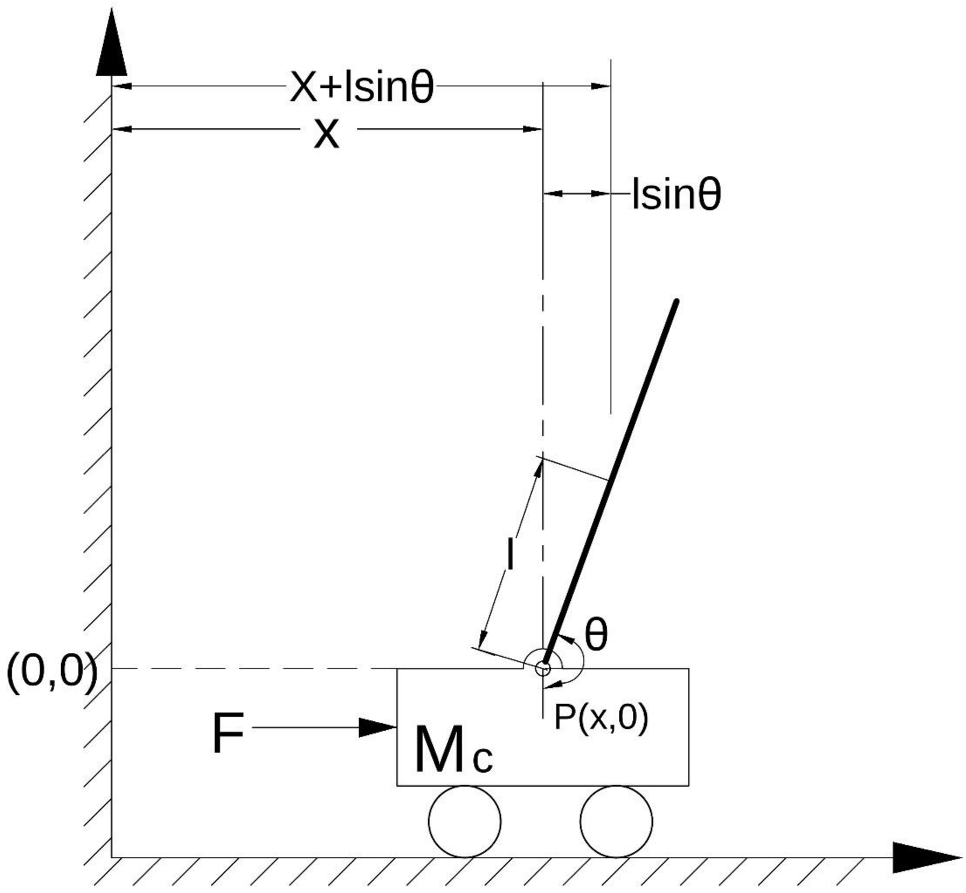


Fig.2 Free Body Diagram of IP system

Kinematics of the system

$$\begin{bmatrix} x_p \\ y_p \end{bmatrix} = \begin{bmatrix} x + l\sin\theta \\ -l\cos\theta \end{bmatrix} \tag{1}$$

The non-relativistic Lagrangian for a system of particles can be defined by

$$\mathcal{L} = T - U \tag{2}$$

Total Potential energy of the system:

$$U = -mgl\cos\theta \tag{3}$$

Total Kinetic Energy of the system is the sum of 'Translational (T_t)' & 'Rotational (T_r)' Kinetic Energy:

$$T = T_t + T_r \tag{4}$$

The Total Translational kinetic energy (T_t) is given by

$$T_t = T_{t_c} + T_{t_p} \tag{5}$$

The translational kinetic energy (T_{t_c}) of cart is given by:

$$T_{t_c} = \frac{1}{2} M_c \dot{x}^2 \tag{6}$$

The translational kinetic energy (T_{t_p}) of the pendulum is the given by:

$$T_{t_p} = \frac{1}{2} m \left[\left(\frac{dx_p}{dt} \right)^2 + \left(\frac{dy_p}{dt} \right)^2 \right]$$

From Eq^n(1), we get:

$$= \frac{1}{2} m \left[\left(\frac{d}{dt}(x + l \sin\theta) \right)^2 + \left(\frac{d}{dt}(-l\cos\theta) \right)^2 \right]$$

Solving the above equation:

$$T_{t_p} = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} ml^2\dot{\theta}^2 + ml\dot{x}\dot{\theta}\cos\theta \tag{7}$$

Form Eq^n(5),(6) & (7), the Total translational Kinetic Energy is :

$$T_t = \frac{1}{2} M_c\dot{x}^2 + \frac{1}{2} m\dot{x}^2 + \frac{1}{2} ml^2\dot{\theta}^2 + ml\dot{x}\dot{\theta}\cos\theta \tag{8}$$

The Rotational Kinetic Energy (T_r) is given by

$$T_r = \frac{1}{2} I\dot{\theta}^2 \tag{9}$$

Then Total Kinetic Energy of system is, from Eq^n(4),(8) & (9)

$$T = \frac{1}{2} M_c\dot{x}^2 + \frac{1}{2} m\dot{x}^2 + \frac{1}{2} ml^2\dot{\theta}^2 + ml\dot{x}\dot{\theta}\cos\theta + \frac{1}{2} I\dot{\theta}^2 \tag{10}$$

Form Eq^n(2),(3) & (10), the Lagrangian is:

$$\mathcal{L} = \frac{1}{2} M_c\dot{x}^2 + \frac{1}{2} m\dot{x}^2 + \frac{1}{2} ml^2\dot{\theta}^2 + ml\dot{x}\dot{\theta}\cos\theta + \frac{1}{2} I\dot{\theta}^2 + mgl\cos\theta \tag{11}$$

Since system has 2DOF, so there are **two Lagrangian equations** of motion given below:-

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}} - \frac{\partial \mathcal{L}}{\partial x} = F - c\dot{x} \tag{12}$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\theta}} - \frac{\partial \mathcal{L}}{\partial \theta} = -b\dot{\theta} \tag{13}$$

Form Eq^n(11),(12) & (13), the Lagrangian yields the Equation of motion:

$$(M+m)\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = F - c\dot{x} \tag{14}$$

$$(I+ml^2)\ddot{\theta} + ml\ddot{x}\cos\theta + mgl\sin\theta = -b\dot{\theta} \tag{15}$$

3. Simulation of Nonlinear dynamics of Inverted Pendulum:

Simulation of above Eqs^n(14) & (15), we get

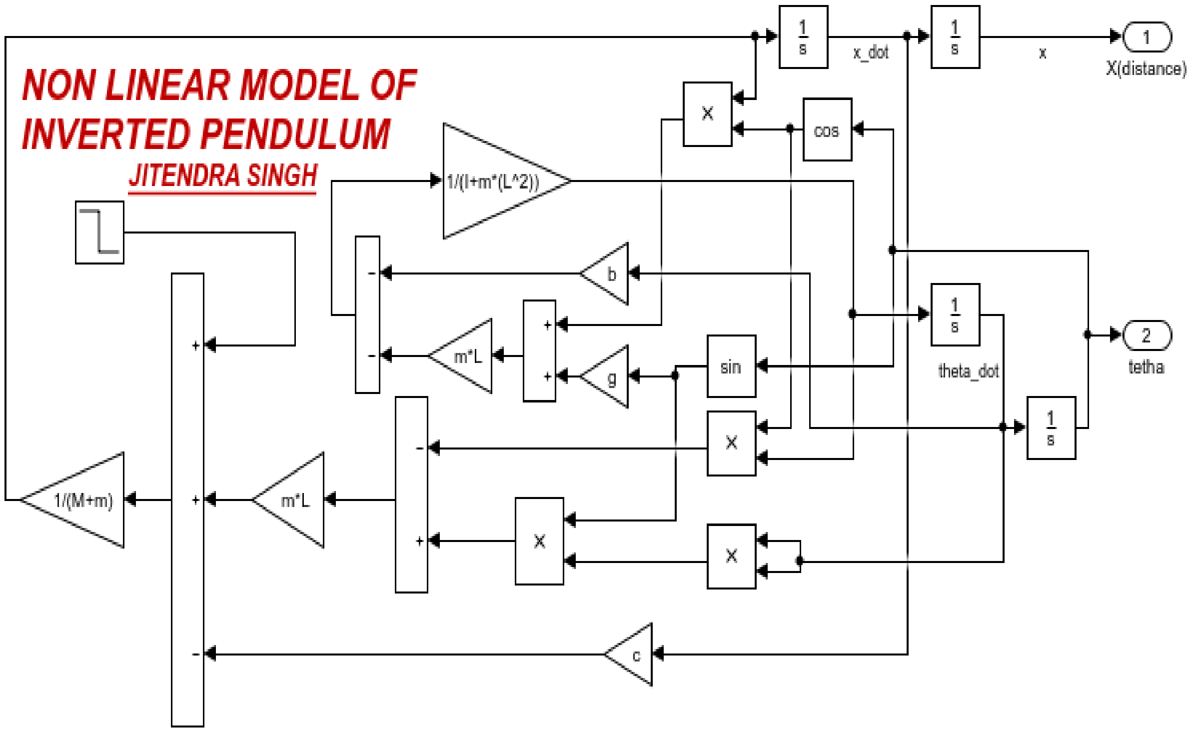


Fig. 3 Simulation of Nonlinear model

Results:

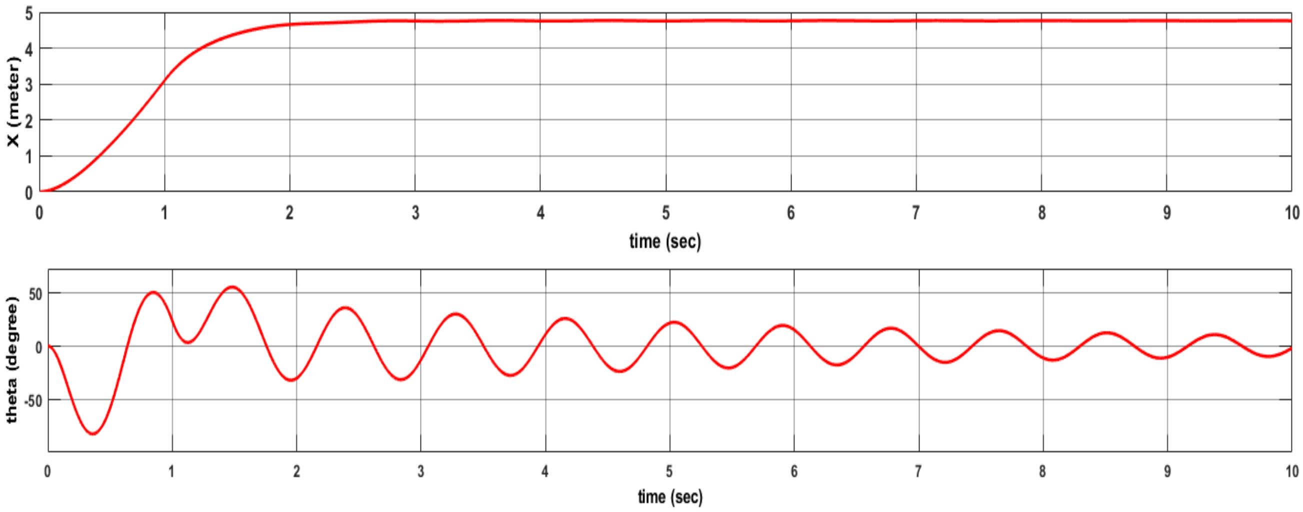


Fig. 4 Time response of states

4. Linear state- space Representation of IP system:

We have non-linear model as:

$$(M+m)\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = F - c\dot{x} \tag{14}$$

$$(I+ml^2)\ddot{\theta} + ml\ddot{x}\cos\theta + mgl\sin\theta = -b\dot{\theta} \tag{15}$$

Take the $\ddot{x}$ from eq^n(15) ,we have

$$\ddot{x} = \frac{-b\dot{\theta} - mgl\sin\theta - (I+ml^2)\ddot{\theta}}{ml\cos\theta}$$

Put the value $\ddot{x}$ in the eq^n(14), we get

$$\ddot{\theta} = \frac{-\left(Fmlcos\theta - cml\dot{x}cos\theta + m^2l^2\dot{\theta}^2 sin\theta cos\theta + (M+m)(b\dot{\theta} + mgl\sin\theta)\right)}{m^2l^2\sin^2\theta + Mml^2 + (M+m)I}. \tag{16}$$

Similarly take $\ddot{\theta}$ from eq^n (14) ,we have

$$\ddot{\theta} = \frac{F - c\dot{x} - (M+m)\ddot{x} + ml\dot{\theta}^2\sin\theta}{ml\cos\theta}$$

Put the value of $\ddot{\theta}$ in the eq^n (15), we get

$$\ddot{x} = \frac{bml\dot{\theta}cos\theta + m^2l^2 gsin\theta cos\theta + (I+ml^2)(F - c\dot{x} + ml\dot{\theta}^2\sin\theta)}{m^2l^2\sin^2\theta + Mml^2 + (M+m)I}. \tag{17}$$

Now to put these equations into state form, we make the following substitutions

$$x_1 = x \ , \ x_2 = \theta, \ x_3 = \dot{x} \ , \ x_4 = \dot{\theta} \tag{18}$$

So we can write $\ddot{x}$ and $\ddot{\theta}$from eq^n (18) as:

$$\dot{x_3} = \frac{(bmlx_4cosx_2 + m^2l^2gsinx_2cosx_2 + (I + ml^2)(F - cx_3 + mlx_4^2 \sin x_2)}{Mml^2 + (M + m)I + m^2l^2sin^2x_2}.$$

$$\dot{x_4} = \frac{-(Fmlcosx_2 - cmlx_3cosx_2 + m^2l^2x_4^2sinx_2cosx_2 + (M + m)(bx_4 + mgl \sin x_2)}{Mml^2 + (M + m)I + m^2l^2sin^2x_2}$$

Thus we have final Non Linear state space equations of inverted pendulum as:

$$\frac{d}{dt} X = \frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \frac{d}{dt}\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix}$$

$$= \begin{bmatrix} x_3 \\ x_4 \\ \dfrac{(bmlx_4cosx_2 + m^2l^2gsinx_2cosx_2 + (I + ml^2)(F - cx_3 + mlx_4^2 \sin x_2)}{Mml^2 + (M + m)I + m^2l^2sin^2x_2} \\ \dfrac{-(Fmlcosx_2 - cmlx_3cosx_2 + m^2l^2x_4^2sinx_2cosx_2 + (M + m)(bx_4 + mgl \sin x_2)}{Mml^2 + (M + m)I + m^2l^2sin^2x_2} \end{bmatrix} \tag{19}$$

$$Y = CX = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix}$$

If we desire a linearized system around the upright stationary point, we simply linearize the non-linear system.

$$\boldsymbol{f}(\boldsymbol{X}, \boldsymbol{U}) = \begin{bmatrix} x_3 \\ x_4 \\ \dfrac{(bmlx_4cosx_2 + m^2l^2gsinx_2cosx_2 + (I + ml^2)(F - cx_3 + mlx_4^2 \sin x_2)}{Mml^2 + (M + m)I + m^2l^2sin^2x_2} \\ \dfrac{-(Fmlcosx_2 - cmlx_3cosx_2 + m^2l^2x_4^2sinx_2cosx_2 + (M + m)(bx_4 + mgl \sin x_2)}{Mml^2 + (M + m)I + m^2l^2sin^2x_2} \end{bmatrix}$$

Linearization Using Taylor Series expansion &Jacobian Matrix:

We have a non-linear model

$$\dot{X} = f(X, U) ,$$

We want to find a “local”, linear model around an operating point

$$(X_0 , U_0) \rightarrow \ (X = X_0 + \delta X , \quad U = U_0 + \delta U)$$

$$\delta\dot{X} = f(X_0 + \delta X\ ,\ U_0 + \delta U\ )$$

Using Taylor Series expansion:

$$\dot{\delta X} = f(X_0, U_0) + \frac{\partial f}{\partial X}(X_0, U_0)\delta X\ +\ \frac{\partial f}{\partial U}(X_0, U_0)\delta U + H.O.T$$

Assumptions: **Operating Points**

$$f(X_0, U_0) = 0$$

Reference Point:

$$(\boldsymbol{X_0}\ , \boldsymbol{U_0}) = \ \ ([\mathbf{0}\ \ \boldsymbol{\pi}\ \ \mathbf{0}\ \ \mathbf{0}]\ , \mathbf{0})$$

And neglecting H.O.T

We have:

$$\dot{\delta X} = A\delta X + B\delta U$$

$$Y = C\delta X$$

Re-define: $\delta X \cong X\ , \delta U \cong U$

This leads to:

$$\dot{X}\ = AX + BU$$

$$Y = CX$$

Where $A = \left.\frac{\partial f}{\partial X}\right|_{(X_0, U_0)}\ ,\ B = \left.\frac{\partial f}{\partial U}\right|_{(X_0, U_0)}$

So we can also linearize our non-linear model (**Eq^{n}(19)**) around a reference point:

$$A\ =\ \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} & \frac{\partial f_1}{\partial x_4} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} & \frac{\partial f_2}{\partial x_4} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} & \frac{\partial f_3}{\partial x_4} \\ \frac{\partial f_4}{\partial x_1} & \frac{\partial f_4}{\partial x_2} & \frac{\partial f_4}{\partial x_3} & \frac{\partial f_4}{\partial x_4} \end{bmatrix}_{(\boldsymbol{X_0}\ , \boldsymbol{U_0})} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{m^2 l^2 g}{\alpha} & \frac{-(I + ml^2)c}{\alpha} & \frac{-bml}{\alpha} \\ 0 & \frac{mgl(M+m)}{\alpha} & \frac{-mlc}{\alpha} & \frac{-b(M+m)}{\alpha} \end{bmatrix}$$

Where: F (force) as Input U

$$U = F$$

$$\alpha = I(M + m) + Mml^2$$

$$B = \begin{bmatrix} \frac{\partial f_1}{\partial F} \\ \frac{\partial f_2}{\partial F} \\ \frac{\partial f_3}{\partial F} \\ \frac{\partial f_4}{\partial F} \end{bmatrix}_{(\boldsymbol{X_0}, \boldsymbol{U_0})} = \begin{bmatrix} 0 \\ 0 \\ \frac{(I + ml^2)}{\alpha} \\ \frac{ml}{\alpha} \end{bmatrix}$$

Linear State Space Model, F (force) as Input U

$$\dot{X} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{m^2 l^2 g}{\alpha} & \frac{-(I + ml^2)c}{\alpha} & \frac{-bml}{\alpha} \\ 0 & \frac{mgl(M+m)}{\alpha} & \frac{-mlc}{\alpha} & \frac{-b(M+m)}{\alpha} \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ \frac{(I + ml^2)}{\alpha} \\ \frac{ml}{\alpha} \end{bmatrix} F \qquad (20)$$

The force applied on the cart, F, is generated by the PMDC motor, the Relation between F &applied voltage, V_m, is given by the equation

$$F = \frac{k_t V_m r - k_t k_b \dot{x}}{R_m r^2} \qquad (21)$$

$$= \frac{k_t V_m r - k_t k_b x_3}{R_m r^2}$$

<u>Final Linear State Space Model, V_m (Voltage) as Input U</u> (from Eq^n(20) &Eq^n(21)):

$$\dot{X} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{m^2 l^2 g}{\alpha} & \frac{-(I + ml^2)}{\alpha}\left(c + \frac{k_t k_b}{R_m r^2}\right) & \frac{-bml}{\alpha} \\ 0 & \frac{mgl(M+m)}{\alpha} & \frac{-ml}{\alpha}\left(c + \frac{k_t k_b}{R_m r^2}\right) & \frac{-b(M+m)}{\alpha} \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ \frac{(I + ml^2)k_t}{\alpha R_m r} \\ \frac{mlk_t}{\alpha R_m r} \end{bmatrix} V_m$$

$$\dot{X} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 5.51 & -18.29 & -0.002 \\ 0 & 64.9 & -77.53 & -0.026 \end{bmatrix} \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 2.73 \\ 11.59 \end{bmatrix} V_m,$$

$$Y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix}$$

5. Inverted Pendulum Swing Up controller design:

In this section a control scheme is developed for swinging up the pendulum from its vertical downward position. The controller is an energy based controller.

Energy based controller simply control the total amount of the energy in the system such as adding enough energy, the pendulum is swing up from the hanging position to its unstable equilibrium point.

Many different control algorithms can be used to perform the swing up control such as, trajectory tracking, rectangular reference input swing up type. The energy shaping controller tend to be the most natural to derive and perhaps the most well–known.

(a). <u>Homoclinic Orbit</u>

For a simple pendulum phase portrait, the orbits of this phase plot are defined by countours of constant energy. One very special orbit, known as *Homoclinic Orbit*, is the orbit which passes through the unstable fixed point. In below figure Red orbit is Homoclinic orbit.

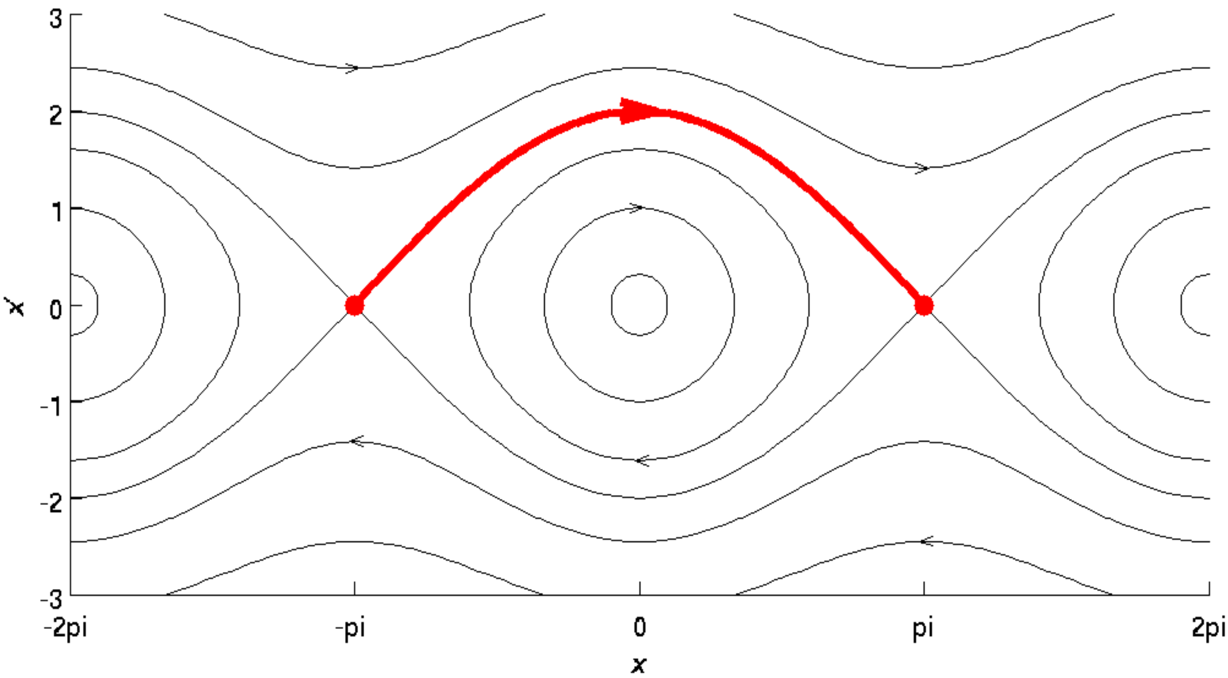


Fig.5 Phase portrait of simple pendulum motion with undamped & unactuated system

(b). <u>Nonlinear Control Law Formulation by using Collocated Partial Feedback Linearization</u>

We seek to design a nonlinear feedback control policy which drives the simple pendulum from any initial condition to the unstable fixed point, a very reasonable strategy would be to use actuation to regulate the energy of the pendulum to place it on this homoclinc orbit. The basic idea:

1. Use collocated Partial Feedback Linearization to simplify the dynamics.
2. Use Energy shaping to regulate the pendulum to it'shomoclinc orbit.
3. Then to add a few term to make sure the cart stays near the origin.

The Collocated PFL:

From Eq^n(15), simplify for $\ddot{\theta}$

$$\ddot{\theta} = \frac{-b\dot{\theta} - ml\ddot{x}^d \cos\theta - mgl\sin\theta}{I + ml^2} \tag{22}$$

Put $\ddot{\theta}$ from Eq^n(22), into Eq^n(14), we get Nonlinear Eq^n for Control Force:

$$F = (M+m)\ddot{x}^d + c\dot{x} - ml\dot{\theta}^2 \sin\theta - ml\left(\frac{b\dot{\theta} + ml\ddot{x}^d \cos\theta + mgl\sin\theta}{I + ml^2}\right)\cos\theta \tag{23}$$

For appropriate control law:(Energy shaping control)

$$\ddot{x}^d = u$$

Now put ($\ddot{x}^d = u$), in Eq^n(22), we get,

$$\ddot{\theta} = \frac{-b\dot{\theta} - mlu\cos\theta - mgl\sin\theta}{I + ml^2} \tag{24}$$

The Total Energy of the Simple pendulum is given by:

$$E = \frac{1}{2}(I + ml^2)\dot{\theta}^2 + mgl(1 - \cos\theta) \tag{25}$$

The desired energy, equivalent to the energy at the desired fixed-point, is

$$E_r = mgl(1 - \cos\pi) = 2mgl \tag{26}$$

If we define Error dynamics $\tilde{E} = E - E_r$, Differentiating with respect to time we obtain:

$$\dot{\tilde{E}} = \dot{E} = (I + ml^2)\dot{\theta}\ddot{\theta} + mgl\sin\theta\dot{\theta}$$

Put the value of $\ddot{\theta}$ from Eq^n(24), into above Eq^n:

We get,

$$\dot{\tilde{E}} = -b\dot{\theta}^2 - mlu\dot{\theta}\cos\theta$$

Therefore, if we design a controller of the form

$$u = k\dot{\theta}\cos\theta\,\tilde{E}, \qquad k > 0$$

Then the resulting error dynamics are:

$$\dot{\tilde{E}} = -b\dot{\theta}^2 - mlk\dot{\theta}^2\cos^2\theta\,\tilde{E},$$

The damping term b is very small, then we can neglect this,

These error dynamics imply an exponential convergence:

$$\tilde{E} \to 0,$$

Except for states where $\dot{\theta}$=0. The essential property is that when $E > E_r$, we should remove energy from the system (damping) and when $E < E_r$, we should add energy (negative damping).

Even if the control action are bounded, the convergence is easily preserved.

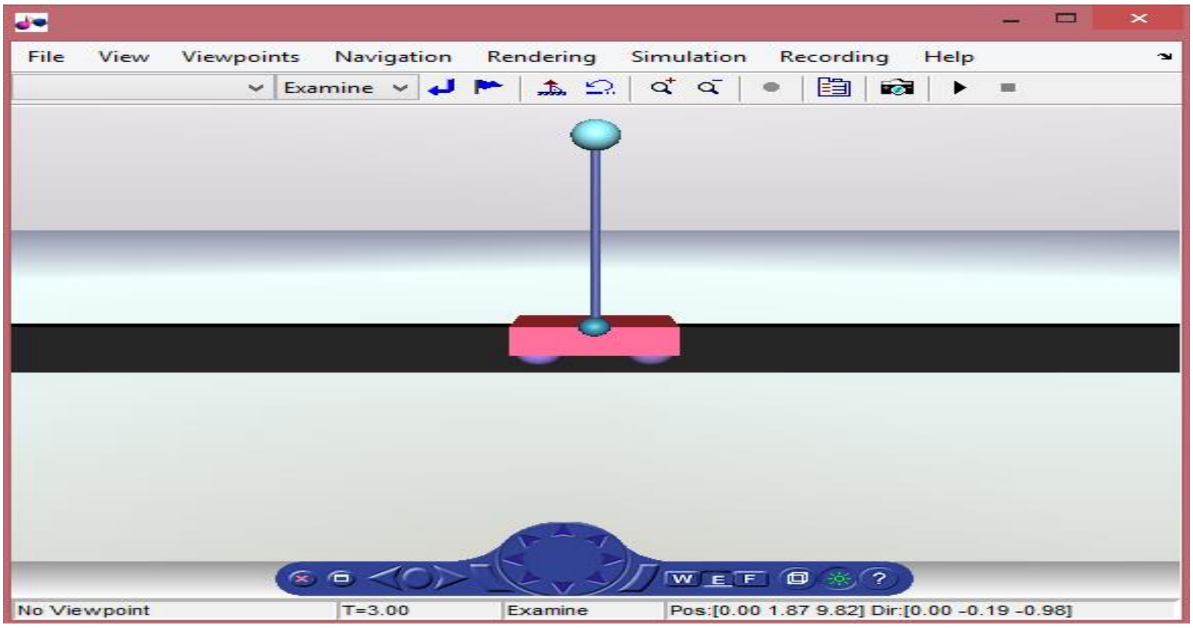
However for energy to change quickly the magnitude of the control signal must be fairly large. As a result a tunable gain k is multiplied by the above and the controller is saturated at the maximum acceleration deliverable by the motor u_{max} :

$$u = sat_{u_{max}}\big(k(E - E_r)Sign(\dot{\theta}\cos\theta)\big) \tag{27}$$

From Eq^n(27) &Eq^n(23), **Collocated Partial Feedback Linearization**control law, that is the appropriate Nonlinear Control Law in term of Force.

& from Eq^n(21),We get the relation between Control Voltage & applied Force.

3D VRML Model of Inverted Pendulum:



6. LQR balanced Control Design:

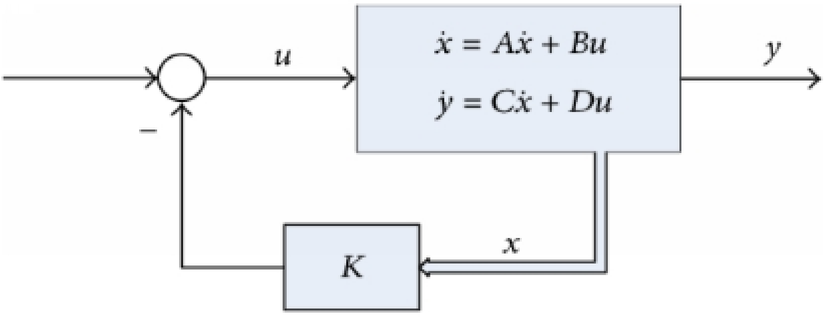


Fig.6 Full state Feedback control

Stability Test:

To check the stability means to analyze whether the open-loop system (without any feedback) is stable or not. The eigenvalues of the system state matrix, 'A', can determine the stability. That is equivalent to finding the poles of the transfer function of the system.

$$AI = \lambda I.$$

Where λ is the eigenvalue and I is the eigenvector.The eigenvalues tell us how the matrix A "acts" in different directions (eigenvectors).

The eigenvalues of the A matrix are the values of s where $\det(sI - A) = 0$.A system is stable if real part of all its eigenvalues must be lied in the left- half of the s-plane such as negative number.

>> Poles = $eig(A)$

ans =

0

-19.6327

6.9178

-5.6006

Controllability Test:

Complete state controllability (or simply controllability if no other context is given) describes the ability of an external input to move the internal state of a system from any initial state to any other final state in a finite time interval.

Controllability Matrix

For LTI (linear time-invariant) systems, a system is reachable if and only if its **controllability matrix**, ζ, has a full row rank of *p*, where *p* is the dimension of the matrix A, and $p \times q$ is the dimension of matrix B.

$$\zeta = [B\ AB\ A^2B \ldots\ldots A^{p-1}B] \in R^{p\ x\ pq}$$

A system is controllable when the rank of the system matrix A is *p*, and the rank of the controllability matrix is equal to:

$$Rank(\zeta) = Rank(A^{-1}\zeta) = p$$

MATLAB allows one to easily create the controllability matrix with the **ctrb** command. To create the controllability matrix ζ simply type:

$\gg \zeta = \mathrm{ctrb}(\mathrm{A}, \mathrm{B});$

$\gg \mathrm{Rank} = \mathrm{rank}(\zeta);$

Optimal Full State Feedback Control Law:$V_m = -KX$

$$K = R^{-1}B^{T}P$$

$$V_m = -R^{-1}B^{T}PX$$

$P$is the solution of **Algebraic Riccati Equation:**

$$PA + A^{T}P + Q - PBR^{-1}B^{T}P = 0$$

$$Q = \begin{bmatrix} 500 & 0 & 0 & 0 \\ 0 & 1000 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R = 0.008$$

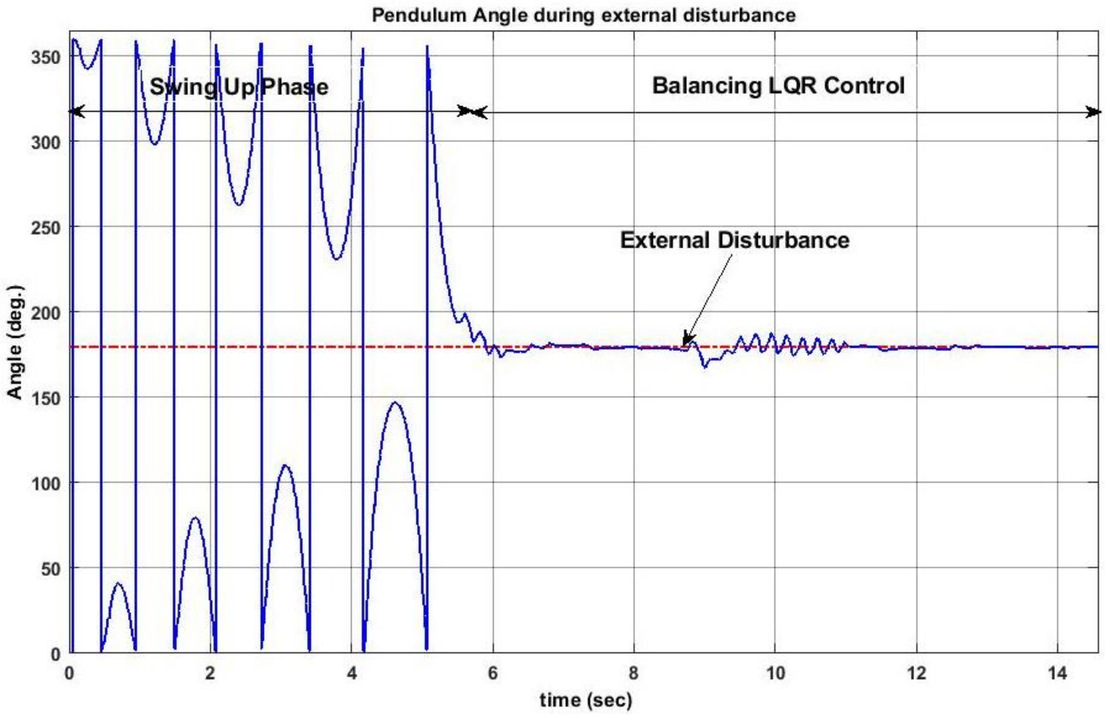
MATLAB Command for find the LQR gain **K**

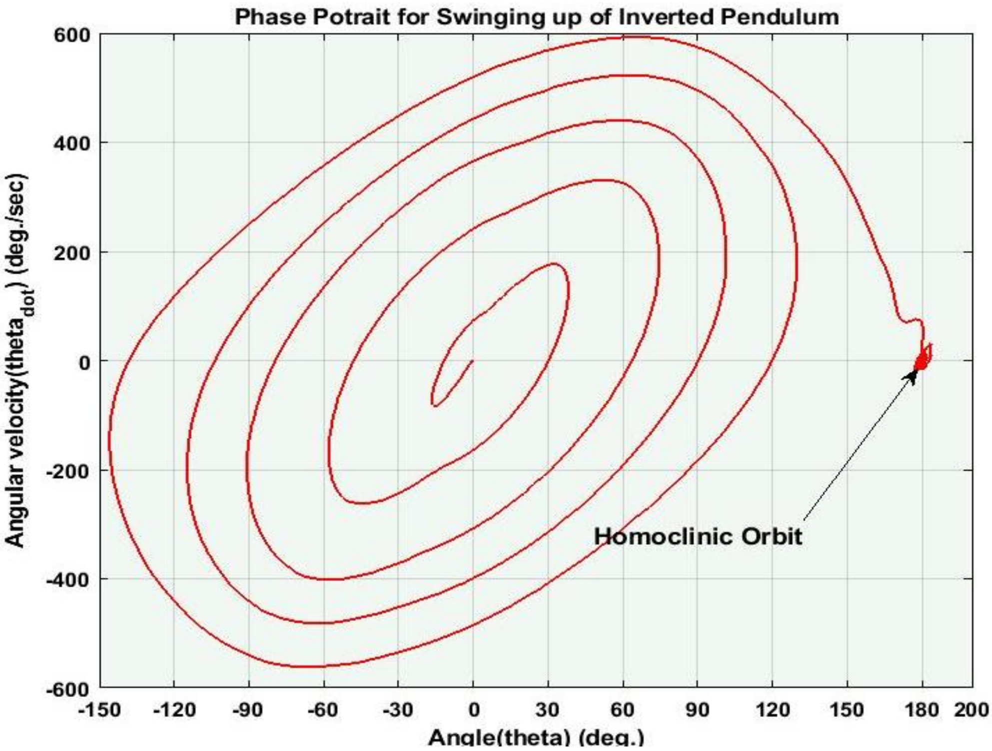
$$K = lqr(A, B, Q, R)$$

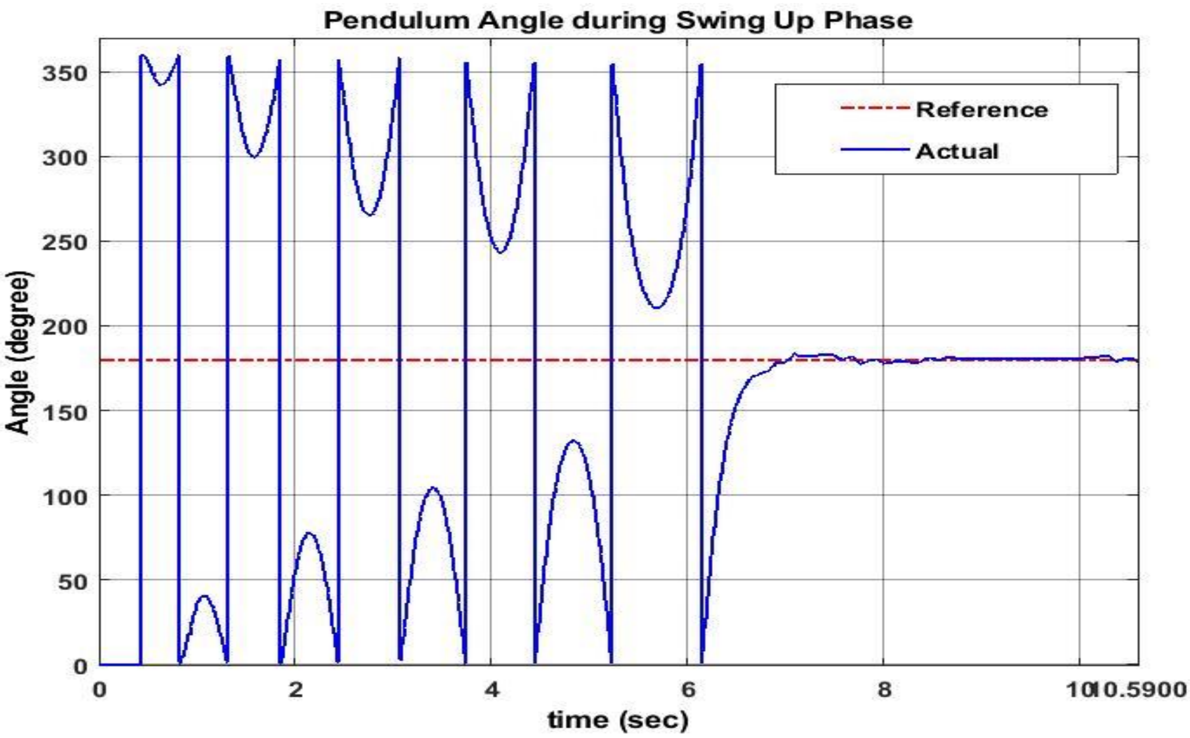
7. Result Discussion :

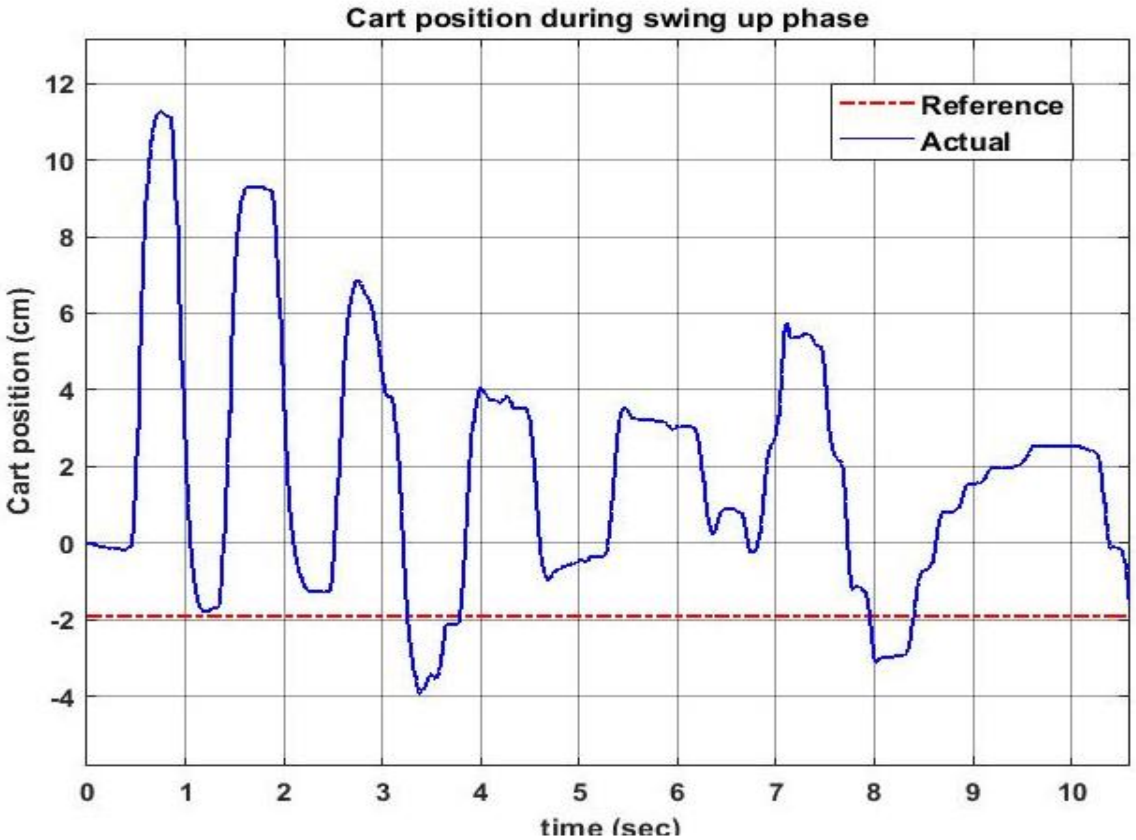
The Experimental results are showing in graphs to illustrate that the **Swing Up**& **LQR**controller are working well with desire control specifications.

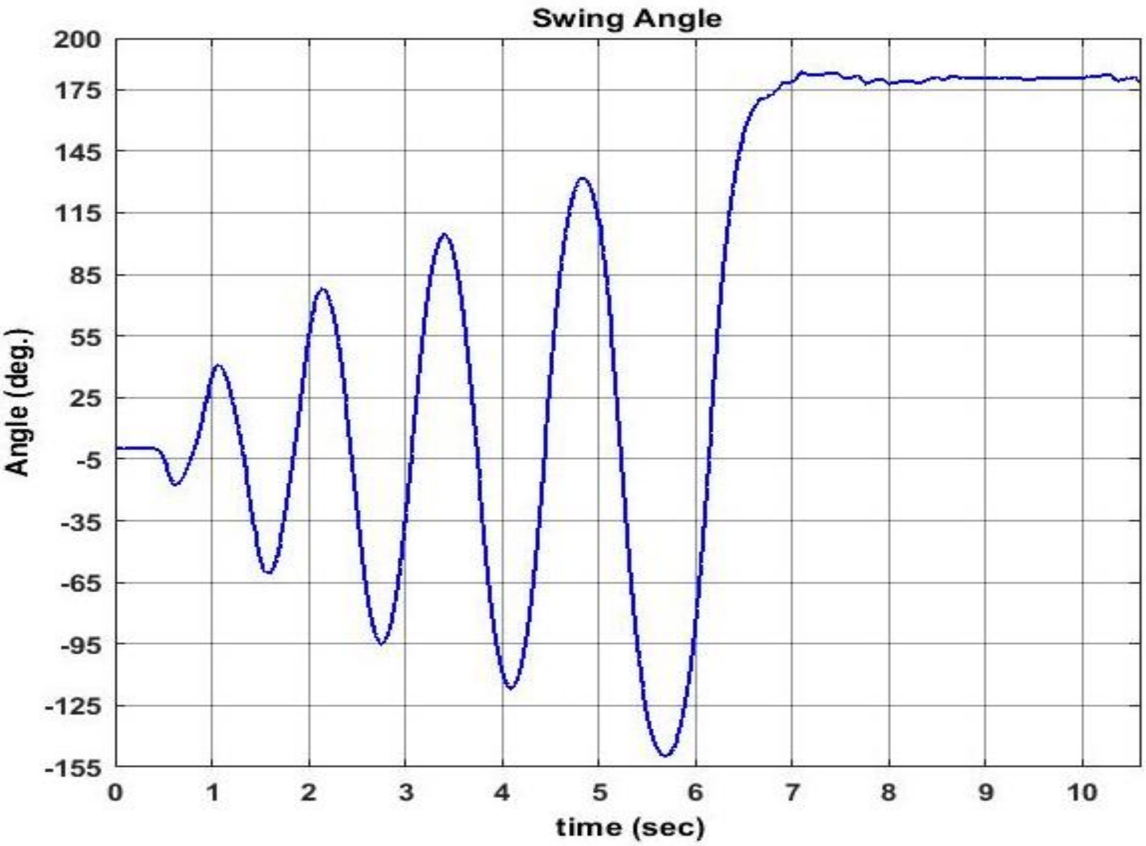
Results are listed below:

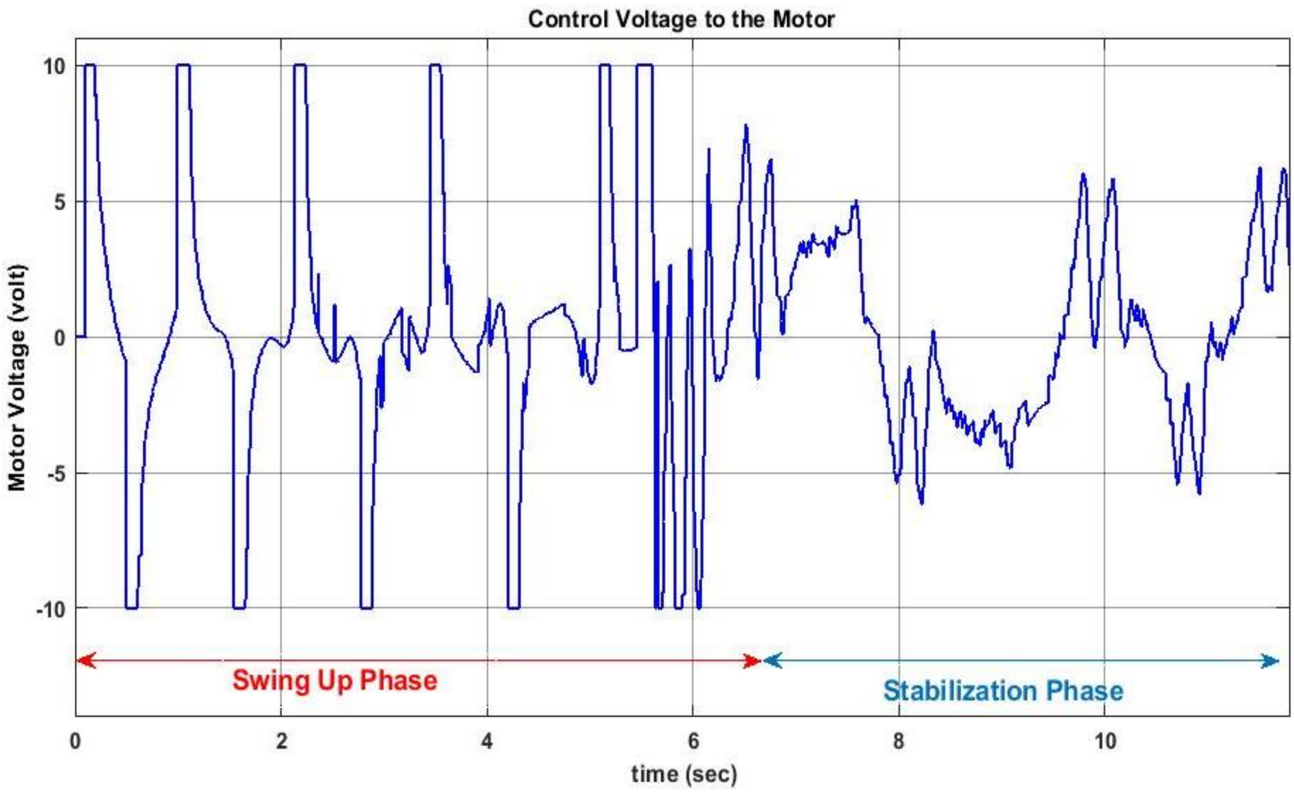


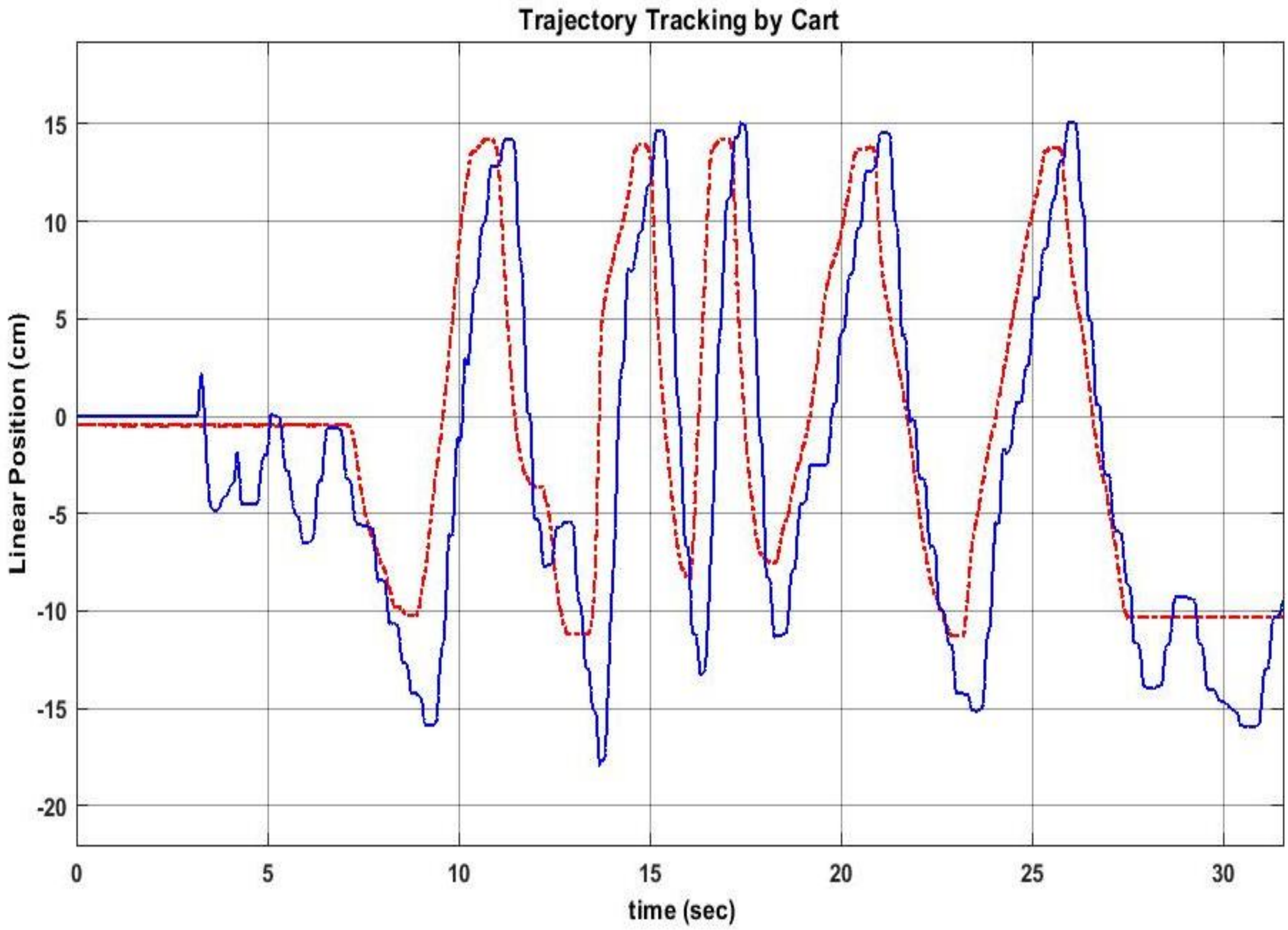

Phase Potrait for Swinging up of Inverted Pendulum
Angular velocity(theta$_{dot}$) (deg./sec)
Angle(theta) (deg.)
Homoclinic Orbit


Pendulum Angle during Swing Up Phase
Reference
Actual
Angle (degree)
time (sec)


Cart position during swing up phase
Reference
Actual
Cart position (cm)
12
10
8
6
4
2
0
-2
-4
0
1
2
3
4
5
6
7
8
9
10
time (sec)


Swing Angle
Angle (deg.)
200
175
145
115
85
55
25
-5
-35
-65
-95
-125
-155
0
1
2
3
4
5
6
7
8
9
10
time (sec)


Control Voltage to the Motor
Motor Voltage (volt)
time (sec)
Swing Up Phase
Stabilization Phase
10
5
0
-5
-10
0
2
4
6
8
10


Trajectory Tracking by Cart
Linear Position (cm)
time (sec)
15
10
5
0
-5
-10
-15
-20
0
5
10
15
20
25
30

8. In-Lab Procedures:

The LQRArd Simulink diagram shown in Fig.5 is used to control the IP system using the Arduino support package. The INVERTED_PENDULUM_SYSTEM subsystem contains Arduino block that interface with the DC motor and sensors (encoders) of the IP system.

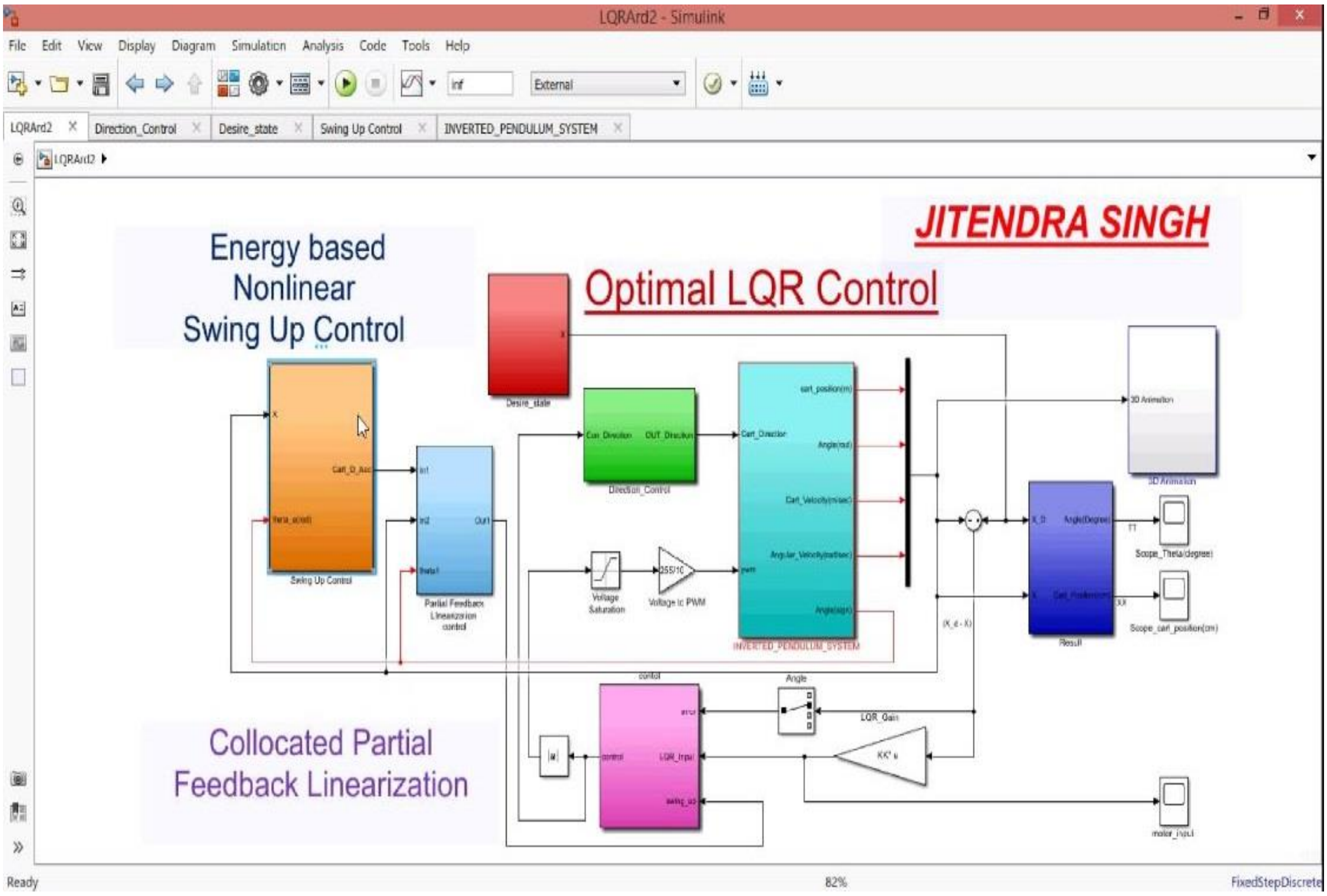


Fig.5: Simulink model used to control the IP system

Follow these steps to implement a complete control system on the actual IP plant:

1. Load the MATLAB software.
2. Browse through the current Directory window in MATLAB and find the folder than contains the controller files.
3. Double-click on the LQRArd.slx file to open the Simulink diagram shown in Fig.5.
4. Double-click on the setup_LQRArd.m to open the setup script for this same.
5. Double-click on the startup.m to load Rensselaer Arduino Support package in Simulink library.