Expert Systems with Applications 37 (2010) 4368–4378

Contents lists available at ScienceDirect

Expert Systems with Applications

journal homepage: www.elsevier.com/locate/eswa

Fuzzy logic control with genetic membership function parameters optimization for the output regulation of a servomechanism with nonlinear backlash

Nohe R. Cazarez-Castro [a,*], Luis T. Aguilar [b], Oscar Castillo [c]

[a] Facultad de Ciencias Quimicas e Ingenieria, Universidad Autonoma de Baja California, Calzada Universidad No. 14418, Mesa de Otay, Tijuana BC 22390, Mexico
[b] Centro de Investigacion y Desarrollo de Tecnologia Digital, Instituto Politecnico Nacional, Av. del Parque No. 1310, Mesa de Otay, Tijuana BC 22510, Mexico
[c] Division de Estudios de Posgrado e Investigacion, Insituto Tecnologico de Tijuana, Calzada Tecnologico S/N, Tijuana, BC 22414, Mexico

ARTICLE INFO

Keywords:
Type-2 fuzzy systems
Fuzzy control
Genetic algorithm
Backlash

ABSTRACT

The paper presents a hybrid architecture, which combines Type-1 or Type-2 fuzzy logic system (FLS) and genetic algorithms (GAs) for the optimization of the membership function (MF) parameters of FLS, in order to solve to the output regulation problem of a servomechanism with nonlinear backlash. In this approach, the fuzzy rule base is predesigned by experts of this problem. The proposed method is far from trivial because of nonminimum phase properties of the system. The simulation results illustrate the effectiveness of the optimized closed-loop system.

1. Introduction

The design of FLSs is a heavy task that FLSs practitioners face every time that they try to use fuzzy logic (FL) as a solution to some problem, the design of FLSs implies at least two stages: design of rules and design of MFs.

There has been some publications in the design of Type-1 FLS (T1FLS) using GA, Grefenstette (1986) presents GAs as optimization method for control parameters, they optimize parameters of the closed-loop system but not of the T1FLS. In Lee and Takagi (1993), GAs are used to optimize all the parameters of a T1FLS. In Melin and Castillo (2001), a hybridization of Neural Networks and GAs are presented to optimize a T1FLSs. A Hierarchical GA (HGA) is proposed by Castillo, Lozano, and Melin (2004) to optimize rules and MFs parameters of a T1FLS.

Type-2 FLSs (T2FLSs) (Mendel, 2001) are a generalization of T1FLSs (Zadeh, 1975a, 1975b, 1975c), which allow us to deal with the uncertainty induced into a mechanical system by noise, frictions, backlash, etc. In the few last years, a growing interest in the research of theories and applications of T2FLSs can be seen from the academic and industry sectors. In Hagras (2007) is presented an extended review of T2FLS in control applications, and in Sepulveda, Castillo, Melin, Rodriguez-Diaz, and Montiel (2007) is presented a comparison between Type-1 and T2FLSs, giving conditions for the use of each one.

As T2FLSs are relatively new and this type of FLSs has more parameters to be optimized than T1FLSs, some options have emerged to optimize some of its parameters. In Al-Jaafreh and Al-Jumaily (2007) was proposed a particle swarm optimization (PSO) method to optimize parameters of the primary MFs of T2FLS. The human evolutionary model (HEM) is proposed by Sepulveda, Castillo, Melin, Montiel, and Aguilar (2007) for the optimization of interval Type-2 MFs, HGAs are proposed by Castillo, Huesca, and Valdez (2005) to optimize gaussian Type-2 MFs.

T2FLSs allow us to deal with uncertainty, but this uncertainty must to be modeled in form of T2MFs, which can carry a new problem in the designing of FLSs. In Castillo, Aguilar, Cazarez, and Cardenas (2008), it is shown that making a uniform modification to the MF's parameters to a certain limit, the closed-loop system keeps some properties like stability but will lose or gain in some others like performance.

Taking in account the main advantage described earlier, the output regulation problem will be solved for an electrical actuator consisting of a motor part driven by DC motor and a reducer part (load) operating under uncertainty conditions in the presence of nonlinear backlash effects. In the regulation problem, the objective is to drive the load to a desired position while providing the boundedness of the system motion and attenuating external disturbances. Due to practical requirements (Lagerberg & Egardt, 1999), the motor angular position is assumed to be the only information available for feedback.

This problem was first solved byAguilar, Orlov, Cadiou, and Merzouki (2007), using nonlinear H_∞ control, but the result reported do not provide robustness evidence. In Cazarez-Castro, Aguilar, Castillo, and Crdenas (2008), authors report a solution to the regulation problem using a T1FLS. In Cazarez-Castro, Aguilar, and Castillo (2008a, 2008b), authors report solutions using T2FLS, and making a genetic optimization of the MF's parameters, but do not specify the criteria used in the optimization process and GA design. In

* Corresponding author. Tel./fax: +52 6646827681.
E-mail address: nohe@ieee.org (N.R. Cazarez-Castro).

0957-4174/$ - see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.eswa.2009.11.091

Cazarez-Castro, Aguilar, Castillo, and Rodriguez (2008) is reported a comparison of the use of GA to optimize T1FLS and T2FLS, but a method to achieve this optimization is not provided.

The solution that we propose in this paper is to optimize the MFs parameters of both Type-1 and Type-2 FLSs with a genetic algorithm, this optimization is in order to obtain the closed-loop system in which the load of a driver is regulated to a desired position. To make the optimization, we propose a systematical methodology, and the hybrid architecture obtained with this method is called a fuzzy genetic architecture (FGA).

The contribution of this paper is as follows:

- We propose a systematic methodology to optimize the parameters of FLSs.
- With the proposed methodology, we obtain a hybrid architecture: FGA.
- We solve the output regulation problem for an electrical actuator operating under uncertainty conditions in the presence of nonlinear backlash effects.
- We present via simulations that the resulting FLSs are so robust to deal with additional uncertainties (noise).

The paper is organized as follows. The dynamic model of the servomechanism is presented in Section 2. The problem statement is in Section 3. Section 4 addresses fuzzy sets and systems theory. GAs are described in Section 5. The methodology to design FGA is presented in Section 6. Section 7 presents the case of design for T1FLSs and Section 8 makes for T2FLSs. The numerical simulations for the FLSs found by the GA are presented in Section 9. Conclusions are presented in Section 10.

2. Dynamic model

The dynamic model of the angular position $q_i(t)$ of the DC motor and that $q_o(t)$ of the load are given according to

$$J_0 N^{-1}\ddot{q}_0 + f_0 N^{-1}\dot{q}_0 = T + w_0, \\ J_i\ddot{q}_i + f_i\dot{q}_i + T = \tau_m + w_i, \tag{1}$$

hereafter, $J_0,\ f_0,\ \ddot{q}_0$ and $\dot{q}_0$ are respectively the inertia of the load and the reducer, the viscous output friction, the output acceleration, and the output velocity. The inertia of the motor, the viscous motor friction, the motor acceleration, and the motor velocity denoted by $J_i,\ f_i,\ \ddot{q}_i$ and $\dot{q}_i$, respectively. The input torque τ_m serves as a control action, and T stands for the transmitted torque. The external disturbances, $w_i(t),\ w_0(t)$, have been introduced into the driver equation (1) to account for destabilizing model discrepancies due to hard-to-model nonlinear phenomena, such as friction and backlash.

The transmitted torque T through a backlash with an amplitude j is typically modeled by a dead-zone characteristic (Nordin, Bodin, & Gutman, 2001, p. 7):

$$T(\Delta q) = \begin{cases} 0 & |\Delta q| \leqslant j, \\ K\Delta q - Kj\mathrm{sign}(\Delta q) & \text{otherwise} \end{cases} \tag{2}$$

with

$$\Delta q = q_i - Nq_0, \tag{3}$$

where K is the stiffness, and N is the reducer ratio. Such a model is depicted in Fig. 1. Provided the servomotor position, $q_i(t)$ is the only available measurement on the system, the above model (1)–(3) appears to be nonminimum phase because along with the origin the unforced system possesses a multivalued set of equilibria (q_i, q_0) with $q_i = 0$ and $q_0 \in [-j, j]$.

To avoid dealing with nonminimum phase system, we replace the backlash model (2) with its monotonic approximation:

$$T = K\Delta q - K\eta(\Delta q), \tag{4}$$

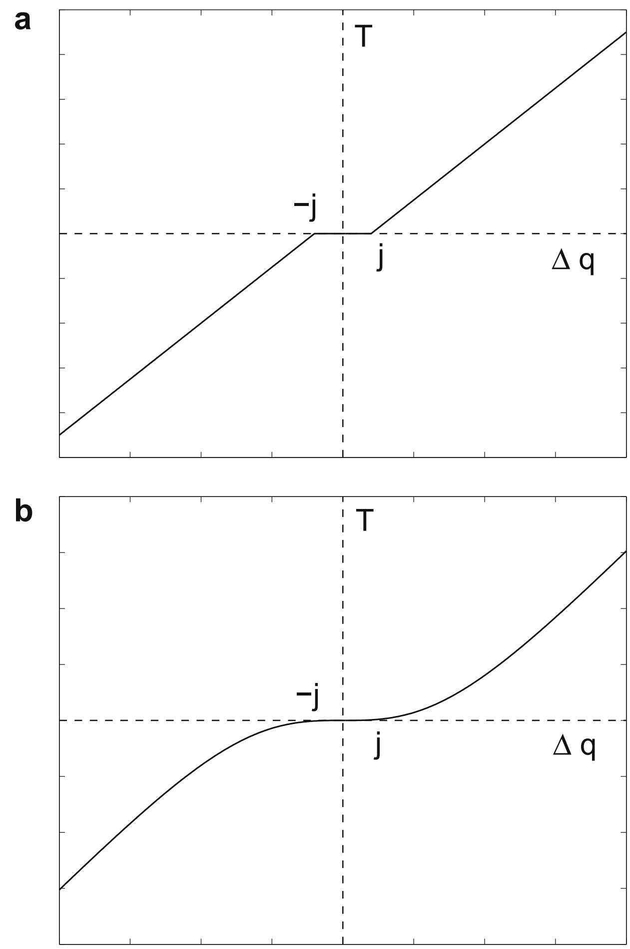

Fig. 1. (a) The dead-zone model of backlash and (b) the monotonic approximation of the dead-zone model.

where

$$\eta = -2j\frac{1 - \exp\left\{-\frac{\Delta q}{j}\right\}}{1 + \exp\left\{-\frac{\Delta q}{j}\right\}}. \tag{5}$$

The present backlash approximation is inspired from Merzouki, Cadiou, and M'Sirdi (2004). Coupled to the drive system (1) subject to motor position measurements, it is subsequently shown to continue a minimum phase approximation of the underlying servomotor, operating under uncertainties $w_i(t),\ w_0(t)$ to be attenuated. As a matter of fact, these uncertainties involve discrepancies between the physical backlash model (2) and its approximation (4) and (5).

3. Problem statement

To formally state the problem, let us introduce the state deviation vector $x = [x_1, x_2, x_3, x_4,]^T$ with

$$x_1 = q_0 - q_d, \\ x_2 = \dot{q}_0, \\ x_3 = q_i - Nq_d, \\ x_4 = \dot{q}_i,$$

where x_1 is the load position error, x_2 is the load velocity, x_3 is the motor position deviation from its nominal value, and x_4 is the motor velocity. The nominal motor position Nq_d has been pre-specified in such a way to guarantee that $\Delta q = \Delta x$, where

$$\Delta x = x_3 - Nx_1.$$

Then, system (1)–(5), represented in terms of the deviation vector x, takes the form

$$\begin{aligned}
\dot{x}_1 &= x_2,\\
\dot{x}_2 &= J_0^{-1}[KNx_3 - KN^2x_1 - f_0x_2 + KN\eta(\Delta q) + w_o],\\
\dot{x}_3 &= x_4,\\
\dot{x}_4 &= J_i^{-1}[\tau_m + KNx_1 - Kx_3 - f_ix_4 + K\eta(\Delta q) + w_i].
\end{aligned} \tag{6}$$

The zero dynamics

$$\begin{aligned}
\dot{x}_1 &= x_2,\\
\dot{x}_2 &= J_0^{-1}[-KN^2x_1 - f_0x_2 + KN\eta(-Nx_1)],
\end{aligned} \tag{7}$$

of the undisturbed version of system (6) with respect to the output

$$y = x_3 \tag{8}$$

is formally obtained (see Isidori (1995) for details) by specifying the control law that maintains the output identically zero.

The objective of the fuzzy control output regulation of the nonlinear driver system (1) with backlash (4) and (5) is thus to design a fuzzy controller so as to obtain the closed-loop system in which all these trajectories are bounded, and the output $q_0(t)$ asymptotically decays to a desired position q_d as $t \to \infty$ while also attenuating the influence of the external disturbances $w_i(t)$ and $w_0(t)$.

4. Fuzzy sets and systems

4.1. Type-1 fuzzy sets and systems

A Type-1 fuzzy set (T1FS), denoted A, is characterized by a Type-1 membership function (T1MF) $\mu_A(z)$ (Castillo & Melin, 2008), where $z \in Z$, been Z the domain of definition of a variable, i.e,

$$A = \{(z, \mu(z))|\forall z \in Z\}, \tag{9}$$

where $\mu(z)$ is called Type-1 *membership function* (T1MF) of the T1FS A. The T1MF maps each element of Z to a membership grade (or membership value) between 0 and 1.

Type-1 fuzzy logic systems (T1FLS) – also called Type-1 fuzzy inference systems (T1FIS) – are both intuitive and numerical systems that maps crisp inputs into a crisp output. Every T1FIS is associated with a set of rules with meaningful linguistic interpretations, such as:

$$R^l : \text{IF } y \text{ is } A_1^l \text{ AND } \dot{y} \text{ is } A_2^l \text{ THEN } u \text{ is } B_1^l, \tag{10}$$

which can be obtained either from numerical data or from experts familiar with the problem at hand. In particular (10) is in the form of Mamdami fuzzy rule (Mamdani & Assilian, 1975, 1976). Based on this kind of statements, actions are combined with rules in an antecedent/concecuent format and then aggregated according to approximate reasoning theory to produce a nonlinear mapping from input space $U = U_1 \times U_2 \times \cdots U_n$ to output space W, where $A_k^l \subset U_k,\ k = 1, 2, \ldots, n$, and the output linguistic variable is denoted by τ_m.

A T1FIS consist of four basic elements (see Fig. 2): the ***Type-1 fuzzifier***, the ***Type-1 fuzzy rule-base***, the ***Type-1 inference engine***, and the ***Type-1 defuzzifier***. The ***Type-1 fuzzy rule-base*** is a collection of rules in the form of (10), which are combined in the ***Type-1 inference engine***, to produce a fuzzy output. The ***Type-1 fuzzifier*** maps the crisp input into T1FS, which are subsequently used as inputs to the ***Type-1 inference engine***, whereas the ***Type-1 defuzzifier*** maps the T1FSs produced by the ***Type-1 inference engine*** into crisp numbers.

In this paper, to get the crisp output of Fig. 2, we compute a centroid of area (COA) (Castillo & Melin, 2008) as *Type-1 defuzzifier*. The COA is defined as follows:

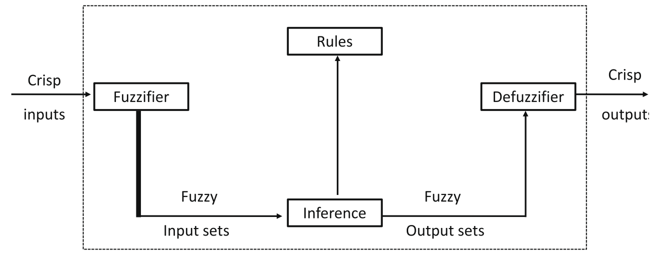

Fig. 2. Type-1 fuzzy inference system (Mendel et al., 2006).

$$\tau_m = u_{COA} = \frac{\int_u \mu_A(u)u\,du}{\int_u \mu_A(u)\,du} \tag{11}$$

where $\mu_A(u)$ is the aggregated output T1MF. This is the most widely adopted defuzzification strategy, which is reminiscent of the calculation of expected values of probability distributions.

4.2. Type-2 fuzzy sets and systems

As the T1FS, the concept of Type-2 fuzzy set (T2FS) was introduced by Zadeh (1975a, 1975b, 1975c) as an extension of the concept of an ordinary fuzzy set (T1FS).

A T2FS, denoted $\widetilde{A}$, is characterized by a Type-2 membership function (T2MF) $\mu_{\widetilde{A}}(z, \mu(z))$ (Mendel & John, 2002), where $z \in Z$ and $\mu \in J_z \subseteq [0, 1]$, i.e,

$$\widetilde{A} = \{((z, \mu(z)), \mu_{\widetilde{A}}(z, \mu(z)))|\forall z \in Z, \forall \mu(z) \in J_z \subseteq [0, 1]\} \tag{12}$$

in which $0 \leqslant \mu_{\widetilde{A}}(z, \mu(z)) \leqslant 1$. $\widetilde{A}$ can also be expressed as follows (Mendel & John, 2002):

$$\widetilde{A} = \int_{z \in Z} \int_{\mu(z) \in J} \mu_{\widetilde{A}}(z, \mu(z))/(z, \mu(z)), \tag{13}$$

where $J_z \subseteq [0, 1]$ and $\int \int$ denotes union over all admissible z and $\mu(z)$ (Mendel & John, 2002).

J_z is called primary membership of z, where $J_z \subseteq [0, 1]\ \forall z \in Z$ (Mendel & John, 2002). The uncertainty in the primary memberships of a T2FS $\widetilde{A}$ consists of a bounded region that is called the *footprint of uncertainty* (FOU) (Mendel & John, 2002). It is the union of all primary memberships (Mendel & John, 2002).

An interval Type-2 fuzzy set (IT2FS) $\widetilde{A}$ is to date the most widely used kind of T2FS and is the only kind of T2FSs that is considered in this paper. It is described as

$$\widetilde{A} = \int_{z \in Z} \int_{\mu(z) \in J_z} 1/(z, \mu(z)) = \int_{z \in Z} \left[\int_{\mu(z) \in J_z} 1/\mu(z)\right] \Big/ z, \tag{14}$$

where z is the *primary variable*, $J_z \subseteq [0, 1]$ is the *primary membership* of z, $\mu(z)$ is the *secondary variable*, and $\int_{\mu(z) \in J_z} 1/\mu(z)$ is the *secondary membership function* at z. Note that (14) means $\widetilde{A} : Z \to \{[a, b] : 0 \leqslant a \leqslant b \leqslant 1\}$. Uncertainty about $\widetilde{A}$ is conveyed by the union of all of the primary memberships, called the FOU of $\widetilde{A}[\mathrm{FOU}(\widetilde{A})]$, i.e.,

$$\mathrm{FOU}(\widetilde{A}) = \bigcup_{z \in X} J_z = \{(z, \mu(z)) : \mu(z) \in J_z = [\underline{A}(z), \overline{A}(z)] \subseteq [0, 1]\}. \tag{15}$$

Note that an IT2FS can also be represented as

$$\widetilde{A} = 1/\mathrm{FOU}(\widetilde{A}) \tag{16}$$

with the understanding that this means putting a secondary grade of one at all points of $\mathrm{FOU}(\widetilde{A})$.

A FLS described using at least one T2FS is called a Type-2 fuzzy logic systems (T2FLS) – also called Type-2 fuzzy inference systems (T2FIS). T1FIS are unable to directly handle rule uncertainties, this is because they use T1FSs that are certain. On the other hand, T2FISs are very useful in circumstances where it is difficult to

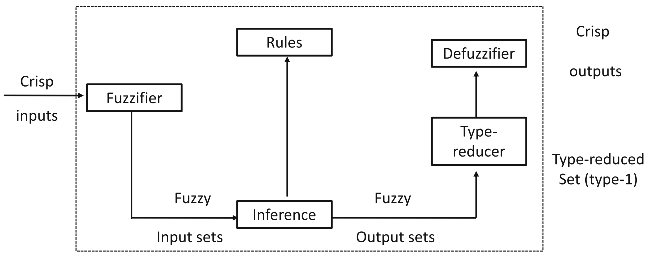

Fig. 3. Type-2 fuzzy inference system (Mendel et al., 2006).

Table 1
Fuzzy IF-THEN rules.

No.	Error	Change of error	Control
1	n	n	p
2	n	z	p
3	n	p	z
4	z	n	z
5	z	z	z
6	z	p	z
7	p	n	z
8	p	z	n
9	p	p	n

determine an exact and measurement uncertainties (Mendel, 2001).

It is known that T2FSs let us to model and to minimize the effects of uncertainties in rule-based FLS. Unfortunately, T2FSs are more difficult to use and understand than T1FSs; hence, their use is not widespread yet.

Similar to a T1FIS, a T2FIS includes ***Type-2 fuzzifier***, ***Type-2 rule-base***, ***Type-2 inference engine***, and substitutes the ***Type-1 defuzzifier*** by the ***output processor***. The ***output processor*** includes a ***type-reducer*** (Mendel, 2001) and a ***Type-2 defuzzifier***; it generates a T1FS output (from the *type-reducer*), or a crisp number (from the ***Type-2 defuzzifier***). A T2FIS is again characterized by IF-THEN rules, but its antecedent and consequents sets are now of the Type-2, see (17). T2FISs can be used when the circumstances are too uncertain to determine exact membership grades. A model of a T2FIS is shown in Fig. 3

$$R^l : \text{IF } y \text{ is } \widetilde{A}_1^l \text{ AND } \dot{y} \text{ is } \widetilde{A}_2^l \text{ THEN } u \text{is } \widetilde{B}_1^l, \tag{17}$$

Note that for both, T1FIS and T2FIS in the ***rule-base***, we are describing the IF-THEN rules following the Mamdani (Mamdani & Assilian, 1975; Mamdani, 1976) type of fuzzy rules.

In this paper, we are computing a centroid type-reducer (CTR) (Mendel, 2001) as *Type-2 defuzzifier*, the CTR is defined as follows:

$$\tau_m = \int_{\theta_j \in J_N} \cdots \int_{\theta_N \in J_{u_N}} [f_{u_1}(\theta_1) \bigstar \cdots \bigstar f_{u_N}(\theta_N)] \Bigg/ \frac{\int u_i \theta_i d\theta}{\int \theta_i d\theta}. \tag{18}$$

A detailed definition of (18) can be found in Mendel (2001).

5. Genetic algorithms

GAs are derivative-free optimization methods based on the concepts of natural selection and evolutionary process (Castillo & Melin, 2005). They were first proposed and investigated by Holland (1975). As a general-purpose optimization tool, GAs are moving out of academic sectors and finding significant applications in many areas. Their popularity can be attributed to their freedom from dependence on functional derivatives and their incorporation of other characteristics reported by Castillo and Melin (2005).

The main idea of a GA is to maintain a ***population*** of solutions of a problem that evolves over a time through a process of competition and controlled variation. Each individual in the population represents a candidate solution to the specific problem, and each individual has associated a *fitness* to determine which individuals are used to form (by sexual reproduction and mutation) new ones in the process of competition.

The sexual reproduction of GAs consists basically in a **Selection Process** (Holland, 1975), where a set of individuals are selected to be passed through a **Crossover Operation** (Holland, 1975), which consist in to take a pair of individuals and interchanging its gens from one (or more) random selected cross point to the end of the chromosome. **Mutation** (Holland, 1975) consists in to change one or more randomly selected gens of the chromosome in some of the selected individuals.

The ***objective function*** (Holland, 1975) of a GA is the value (*fitness*) that the method must maximize or minimize.

6. Methodology to built FGAs to evolve FLCs

In order to evolve MF parameters of your FLC to achieve the control objective, we propose the following methodology:

1. Design the structure of the FLS: selecting input and output variables, its granulation and the type of MF to be used, and the fuzzy rules for the FIS.
2. Identify the necessary parameters to represent the MF that was selected.
3. Built a chromosome by sorting the parameters of the MF of each variable.
4. Design and objective function.
5. Select the type of GA operations and its parameters.
6. Implement the necessary restrictions to built valid individuals.
7. Execute the GA to evolve the FLC.

7. FGA to evolve T1FLSs

7.1. Design the structure of the T1FLS

To solve the fuzzy control output regulation problem, we implement a two-input and one-output fuzzy system.

The particular choice of each $\mu_{B^l}(z)$ will depend on the heuristic knowledge of the experts over the plant. We select triangular T1MFs for each input (error and change of error) and output (control) variables, each one of these variables was granulated in three T1FSs *negative* (**n**), *zero* (**z**), and *positive* (**p**).

These input and output variables are combined in fuzzy rules in the form of (10), we select the nine fuzzy rules shown in Table 1.

For the inference process was used the Mamdani (Mamdani & Assilian, 1975; Mamdani, 1976) type of fuzzy inference, with minimum as disjunction operator, maximum as conjunction operator, minimum as implication operator, maximum as aggregation operator, and COA as our defuzzification method.

7.2. Identify the necessary parameters to represent the T1MF that was selected

Triangular T1MF are defined as

$$\mu(z) = \begin{cases} 0, & z \leqslant a, \\ \frac{z-a}{b-a}, & a \leqslant z \leqslant b, \\ \frac{c-z}{c-b}, & b \leqslant z \leqslant c, \\ 0, & c \leqslant z, \end{cases} \tag{19}$$

where z is the crisp value to be fuzzified, and we can see that a triangular T1MF needs a domain of definition z and three parameters a, b, and c to be defined, where a, b, and c are constant so that $a \leqslant b \leqslant c$ (Castillo & Melin, 2005).

7.3. Built a chromosome by sorting the parameters of the T1MF of each variable

In this work, we use a GA to optimize the parameters of the T1MFs of the FLS, considering triangular shapes for each T1MF, we encode each individual (solution – FLS) in a 27 real gens chromosome, where are represented the three variables, with each one of its three membership function; in this chromosome, we are representing three parameters needed to represent the *negative*, three parameters for the *zero*, and the three parameters needed to represent the *positive* triangular T1MF, this is called a genotype (Goldberg, 1989) of the population.

The chromosome of the genotype that we use is shown Fig. 4.

7.4. Design and objective function

In our optimization process, we want to minimize the objective function

$$\text{fitness}_i = min(mean|error|). \tag{20}$$

7.5. Select the type of GA operations and its parameters

The set of parameters of the GA are shown in Table 2.

7.6. Implement the necessary restrictions to built valid individuals

In out particular case, we implement the following restrictions:

1. The central parameter of the triangular T1MF *negative* must to be minor that the central parameter or the triangular T1MF *zero*.
2. The central parameter of the triangular T1MF *positive* must to be major that the central parameter or the triangular T1MF *zero*.
3. At each triangular T1MF, the three parameters a, b, and c must satisfy the restriction imposed to (19).

7.7. Execute the GA to evolve the T1FLS

Briefly in Cazarez-Castro et al. (2008), authors solved the regulation problem by using T1FLS, but they use seven rules that report that was obtained by experts on the problem; moreover, authors use a different configuration of the T1FLS, they use minimum as disjunction operator, maximum as conjunction operator, minimum as implication operator, maximum as aggregation operator, and mean of maximums (Castillo & Melin, 2005) as our defuzzification method. Then, to improve the results given in Cazarez-Castro et al. (2008), we perform simulations of the T1FLS regulator using motor position feedback, these issues were tested on numerical simulations using Matlab® and Simulink®. The parameters of the dynamical model (1) are given in Table 3 while $N = 3$, $j = 0.2$ (rad), and $K = 5$ (N m/rad). These parameters are taken from of the experimental testbed (see Fig. 5) installed in the Robotics & Control Laboratory of CITEDI-IPN, which involves a DC motor linked to a mechanical load through an imperfect contact gear train (Aguilar et al., 2007). The input–output motion graph of Fig. 6 reveals the gear backlash effect of the system.

Table 2
Parameters of the genetic algorithm.

Parameter	Value
Representation	Real
Population size	10
Generations	40
Selection method	Roulette
Cross method	Scattered
Rate of cross	0.8
Mutation method	Gaussian
Rate of mutation	0.1
Elitism	2

Table 3
Nominal parameters.

Description	Notation	Value	Units
Motor inertia	J_i	2.8×10^{-6}	kg m^2
Load inertia	J_o	1.07	kg m^2
Motor viscous friction	f_i	7.6×10^{-7}	N m s/rad
Load viscous friction	f_o	1.73	N m s/rad

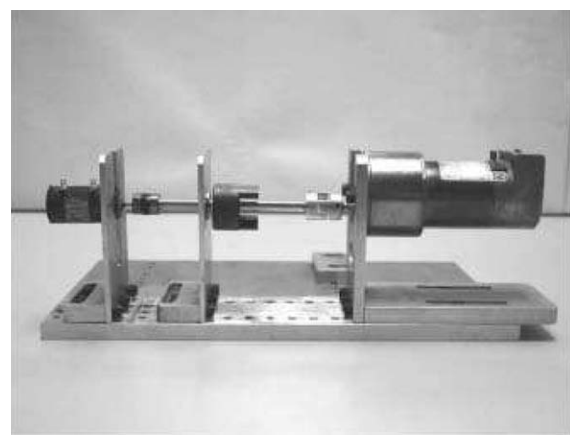

Fig. 5. Experimental testbed of Robotics & Control Laboratory of CITEDI-IPN.

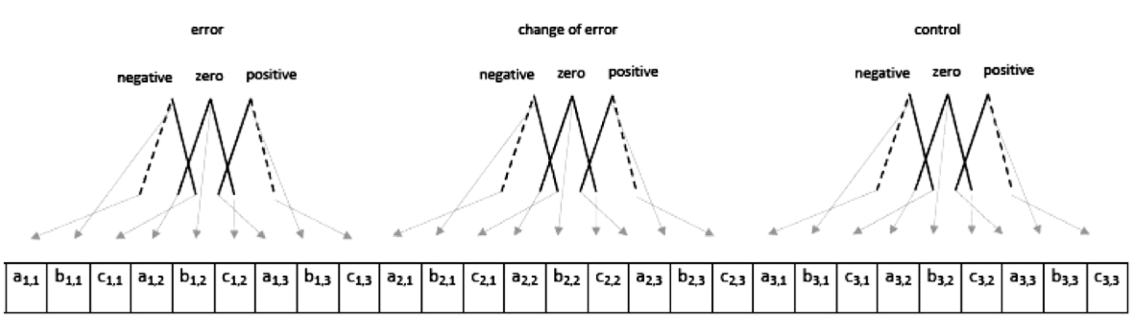

Fig. 4. Genotype for T1FIS.

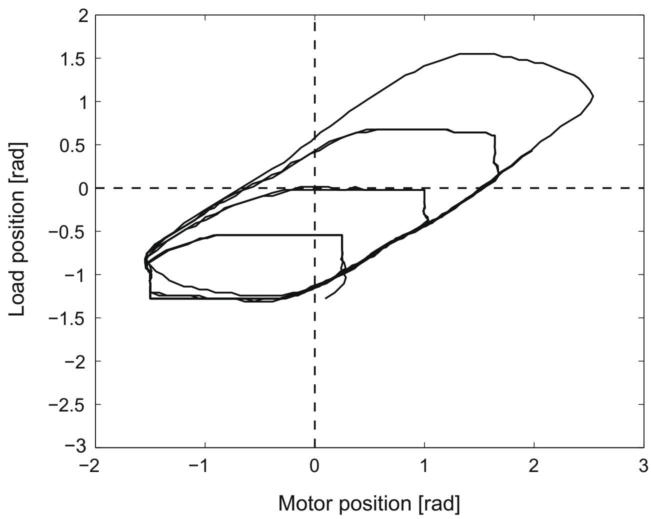

Fig. 6. Backlash hysteresis before compensation.

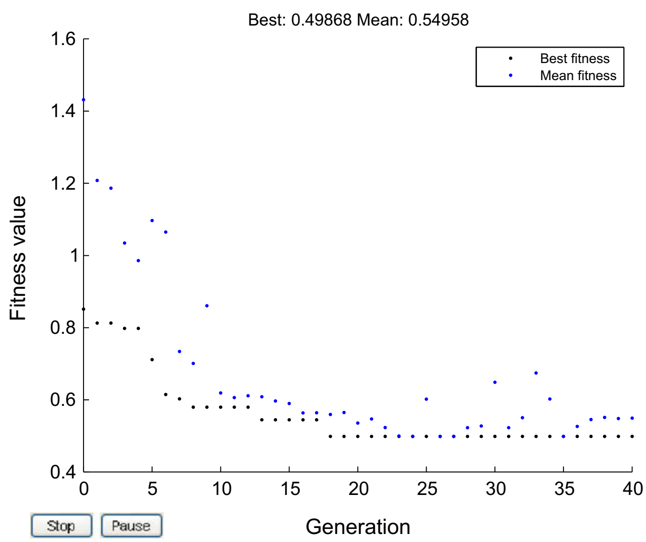

Fig. 7. Evolution of GA to evolve T1FLSs.

The GA was implemented using the Matlab Genetic Algorithm and Direct Search Toolbox®. In the simulations, the load was required to move from the initial static position $q_0(0) = 0$ (rad) to the desired position $q_d = \pi/2$ (rad). In order to illustrate the size of the attraction domain, the initial load position was chosen reasonably far from the desired position.

The GA was executed in a PC with Intel Pentium processor of 2.4 GHz and 512 Mb of RAM. The execution conclude satisfactory in about 200 h, the evolution of the GA is depicted in Fig. 7, and the solution is concentrated in Table 4, where the best solution (minimal fitness value) is the same in individuals number 1, 2, 4, 6, 7, 8 and 9, the chromosome of these individuals is in Table 5. In Fig. 8 is depicted the phenotype (Castillo & Melin, 2005) of the T1FLS with its nine optimized T1MFs.

8. FGA to evolve T2FLSs

As many of the specifications of this section are the same as that in Section 7, in this section we will specify only the changes to extend the explication to the Type-2 case.

Table 4
Results given by the GA to evolve T1FLS.

Individual	Fitness (rad)
1	0.4987
2	0.4987
3	0.7048
4	0.4987
5	0.6399
6	0.4987
7	0.4987
8	0.4987
9	0.4987
10	0.6604

8.1. Design the structure of the T2FLS

We select interval type-2 triangular MFs (IT2TMFs) for each variable. The input and output variables are combined in fuzzy rules in the form of (17).

For the inference process, we use the Mamdani (Mamdani & Assilian, 1975; Mamdani, 1976) type of fuzzy inference for T2FLSs (Mendel, 2001) and CRT as our defuzzification method.

8.2. Identify the necessary parameters to represent the T2MF that was selected

In this paper, we are using the Type-2 fuzzy logic toolbox reported in Castro, Castillo and Melin (2007) and Castro, Castillo and Martinez (2007), and we need six parameters to parameterize each IT2TMF, that is, each IT2TMF is parameterized by two triangular T1MFs, so, the restrictions of Section 7.2 now apply to each pair of parameters a, b, and c.

8.3. Built a chromosome by sorting the parameters of the T2MF of each variable

In this work, we use a GA to optimize the parameters of the T2MFs of the FLS, considering triangular shapes for each IT2TMF, we encode each individual (solution – FLS) in a 54 real gens chromosome, where are represented the six variables, with each one of its three membership function; in this chromosome, we are representing six parameters needed to represent the *negative*, six parameters for the *zero*, and the six parameters needed to represent the *positive* triangular IT2TMF.

The chromosome of the genotype that we use is shown in Fig. 9.

8.4. Design and objective function

In our optimization process, we want to minimize the same objective function (20) of Section 7.4.

8.5. Select the type of GA operations and its parameters

The set of parameters of the GA are the same as shown in Table 2.

8.6. Implement the necessary restrictions to built valid individuals

In out particular case, we implement the following restrictions:

1. The central parameter of the triangular IT2TMF *negative* must to be minor that the central parameter or the triangular IT2TMF *zero*.
2. The central parameter of the triangular T1MF *positive* must to be major that the central parameter or the triangular IT2TMF *zero*.

Table 5
Data of the *best* individuals (T1FLSs) found by the FGA.

Variable	Membership function	*a*	*b*	*c*
Error	Negative	−1.5000	−1.0000	−0.5377
	Zero	−0.1173	−0.1173	2.3416
	Positive	−0.5380	1.0000	1.5000
Change of error	Negative	−1.5000	−1.0000	0.0258
	Zero	−0.4957	0.6905	0.6905
	Positive	−0.5658	1.0000	1.5000
Control	Negative	−1.5000	−1.0000	−0.6166
	Zero	−0.6619	0.7550	0.7550
	Positive	0.0800	1.0000	1.5000

3. At each triangular IT2TMF, the three parameters a, b, and c must satisfy the restriction imposed to Castro, Castillo and Melin (2007) and Castro, Castillo and Martinez (2007).

8.7. Execute the GA to evolve your T2FLC

Again, to improve (Cazarez-Castro et al., 2008) results, we perform simulations of the T2FLS regulator using motor position feedback, these issues were tested on numerical simulations using

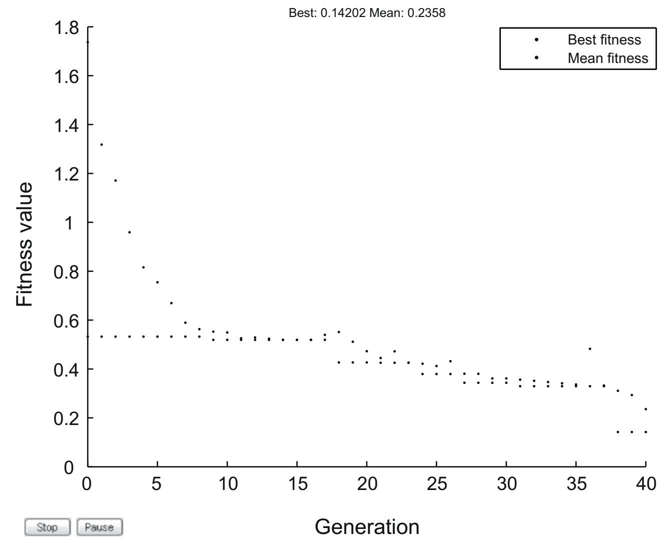

Fig. 10. Evolution of GA to evolve T2FLSs.

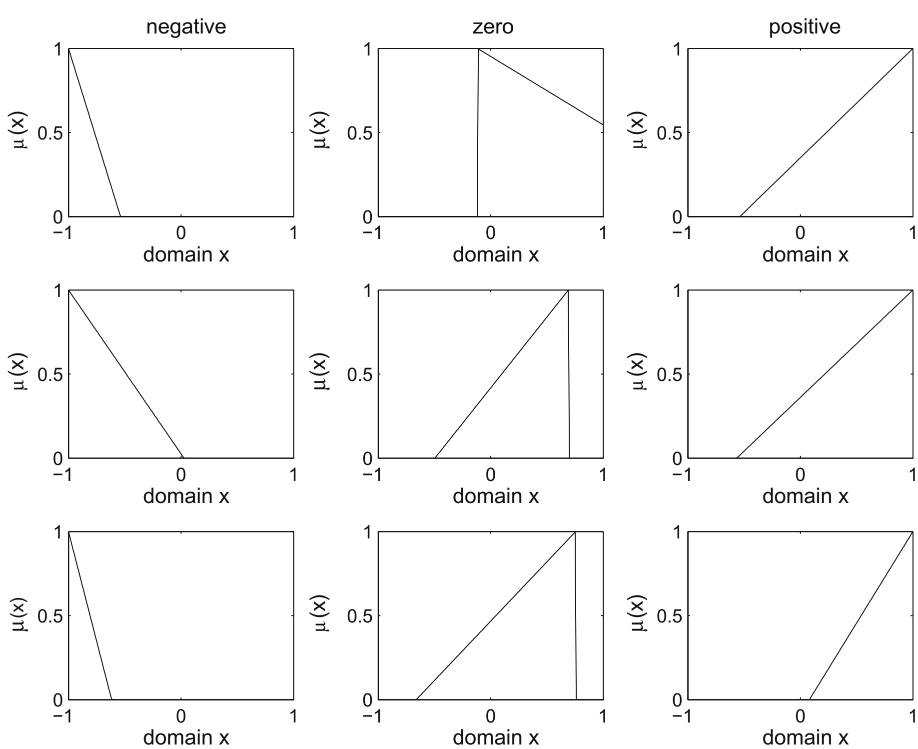

Fig. 8. Phenotype of the *best* T1FLS found by the FGA. Up-down rows: *error*, *change of error* and *control*.

Fig. 9. Genotype for T2FIS.

Table 6
Results given by the GA to evolve T2FLS.

Individual	Fitness (rad)
1	0.3296
2	0.3296
3	0.3296
4	0.3296
5	0.1420
6	0.1420
7	0.1420
8	0.1420
9	0.1420
10	0.3296

Matlab® and Simulink®. All the parameters for the simulation are the same of Section 7.7.

The GA was executed in a PC with Intel Pentium processor of 2.4 GHz and 512 Mb of RAM. The execution conclude satisfactory in about 350 h, the evolution of the GA is depicted in Fig. 10, and the solution is concentrated in Table 6, where the best solution (minimal fitness value) is the same in individuals number 5, 6, 7, 8 and 9, the chromosome of these individuals is in Table 7. In

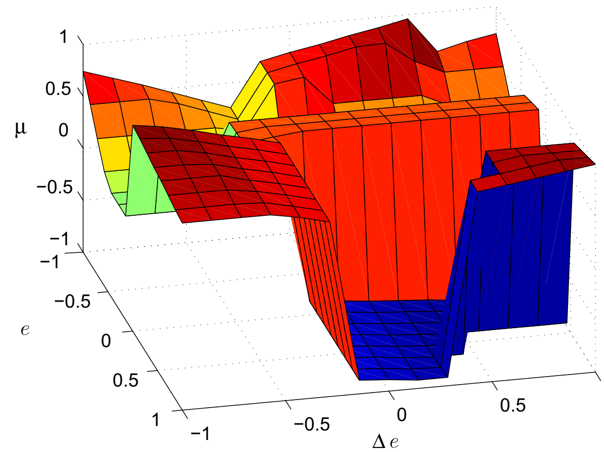

Fig. 12. Surface of control of the T1FLS found by the GA.

Table 7
Data of the *best* individuals (T2FLSs) found by the FGA.

Variable	*μ*	*a1*	*b1*	*c1*	*a2*	*b2*	*c2*
Error	Negative	−1.0000	0.2176	0.2176	0.3012	0.4293	0.4293
	Zero	−0.1982	−0.1982	−0.1912	−0.4260	0.8378	0.8378
	Positive	0.2669	0.2669	1.0000	−0.5479	−0.5479	1.0000
Change of error	Negative	−1.0000	0.3504	0.6897	0.0926	0.9659	0.9659
	Zero	−0.6963	−0.6963	−0.1490	−0.4170	0.0000	0.4005
	Positive	0.2611	0.4405	1.0000	−0.9561	−0.9561	1.0000
Control	Negative	−1.0000	0.4586	0.4586	−0.6673	0.7247	0.7247
	Zero	−0.4571	−0.4571	0.7067	0.0446	0.0446	0.0446
	Positive	0.6220	0.9432	1.0000	−0.0605	−0.0605	1.0000

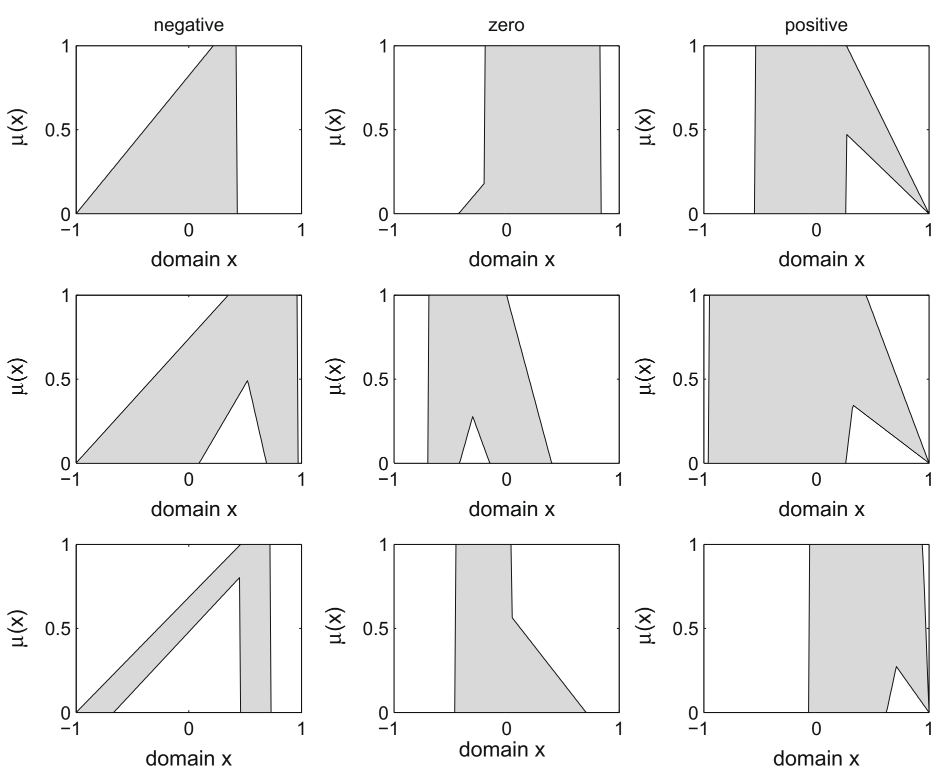

Fig. 11. Phenotype of the *best* T2FLS found by the FGA. Up-down rows: *error*, *change of error* and *control*.

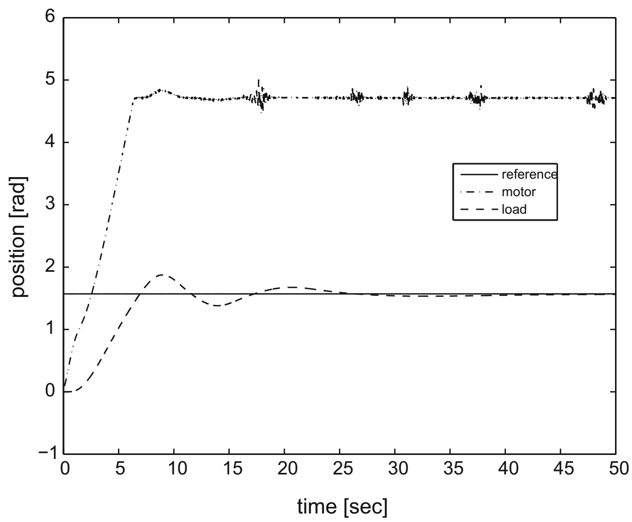

Fig. 13. Simulation result of the T1FLS found by the GA.

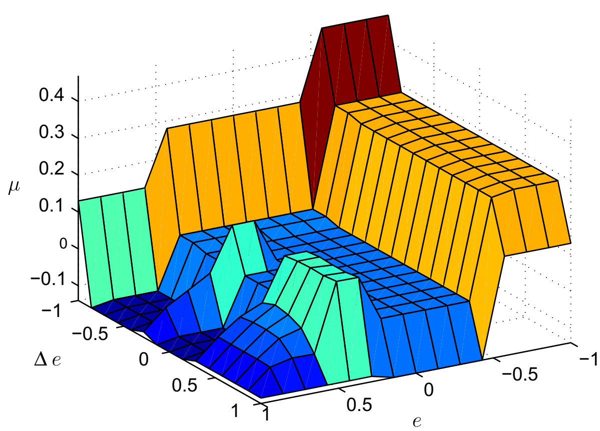

Fig. 14. Surface of control of the T2FLS found by the GA.

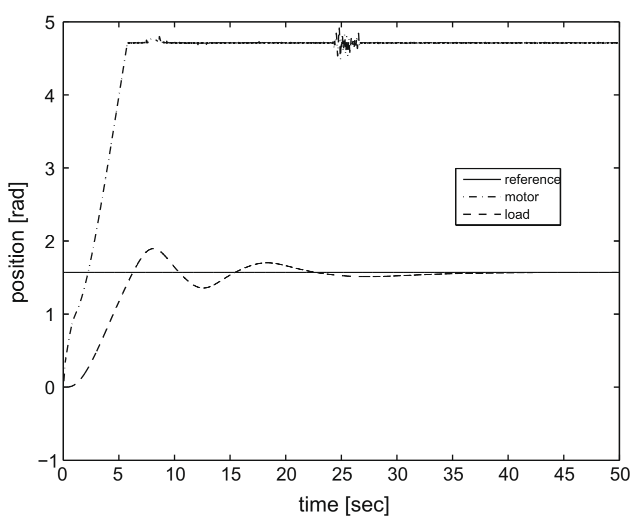

Fig. 15. Simulation result of the T2FLS found by the GA.

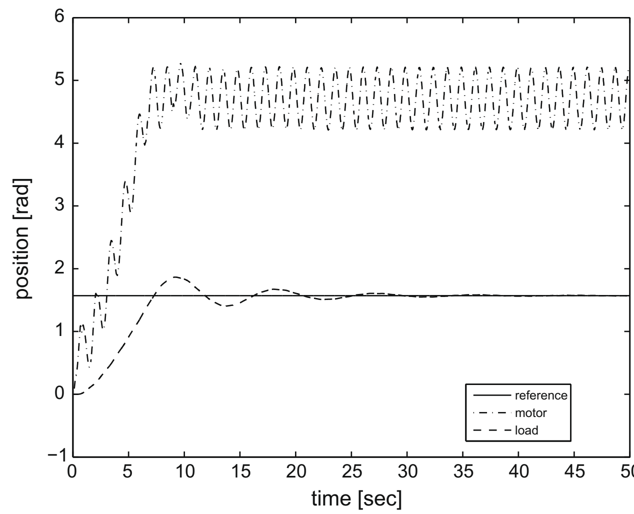

Fig. 16. Simulation result of the T1FLS found by the GA and a noisy closed-loop.

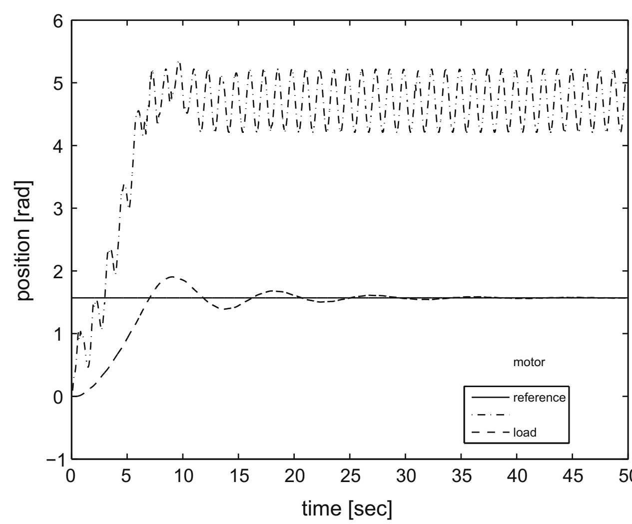

Fig. 17. Simulation result of the T2FLS found by the GA and a noisy closed-loop.

Fig. 11 is depicted the phenotype (Castillo & Melin, 2005) of the T2FLS with its nine optimized IT2TMFs.

9. Simulation results

In this section, we present the simulation results for the closed-loop systems for both, the T1FLS and the T2FLS.

9.1. Simulation results for T1FLS found by the GA

Performing a simulation of the closed-loop system (1)–(4) with the T1FLS found by the GA in Section 7.7, we have the following results: the surface of control of Fig. 12 and the output of Fig. 13. It can be seen that load trajectories reach the desired position as was predicted.

Table 8
Comparison between results of Cazarez-Castro et al. (2008), T1FLS and T2FLS found by the GA.

Controller	Overshoot (rad)	Settling time (s)	ise	iae	itae	itse
In Cazarez-Castro et al. (2008)	1.5919	19.8768	166970.0000	1.1940e+5	2.8218e+5	3.7132e+5
T1FLC	1.7059	14.5254	650.0983	2.4438e+3	4.1093e+4	4.4868e+3
T2FLC	1.6514	9.4689	551.7236	1.0085e+3	2.9564e+4	2.7546e+3

Table 9
Comparison between results of the T1FLS and T2FLS found by the GA applying noise.

Controller	Overshoot (rad)	Settling time (s)	ise	iae	itae	itse
T1FLC	1.9452	13.6684	633.1284	3.0568e+3	3.9586e+4	3.9258e+3
T2FLC	1.7758	10.0016	593.8564	1.4258e+3	3.1258e+4	2.7367e+3

9.2. Simulation results for T2FLS found by the GA

Performing a simulation of the closed-loop system (1)–(4) with the T2FLS found by the GA in Section 8.7, we have the following results: the surface of control of Fig. 14 and the output of Fig. 15. It can be seen that load trajectories reach the desired position as was predicted.

9.3. Comparison

The method implemented and reported in this paper to evolve T1FLS and T2FLS has performed in satisfactory fashion, each one with particular characteristics. Table 8 summarizes the results obtained in this paper and that reported in Cazarez-Castro et al. (2008). In Cazarez-Castro et al. (2008), the MF parameters was selected arbitrary.

9.4. Simulation results adding noise

To evaluate the robustness of the T1 and T2FLSs found by the FGAs, we implement a simulation adding noise in the closed-loop system, that is, we add a noise signal to the output equation (8), and the new output system equation is redefined as

$$y = x_3 + w, \tag{21}$$

where

$$w = sin(t). \tag{22}$$

Fig. 16 shows a simulation of the closed-loop system with noise, and using the T1FLS found in Section 7 and Fig. 17 shows a simulation of the closed-loop system using the T2FLS found in Section 8. It can be seen that load trajectories reach the desired position as was predicted, but the motor shows a perturbed behavior due to the noise (21) and (22) added to the closed-loop system. It is important to see that the load trajectories reach the desired position because the FLSs are processing the uncertainty added to the system with the noise, and Table 9 allow us to conclude that the T2FLS have best results than the T1FLS, this is because in all the design parameters measured, T2FLS have best performance than the T1FLS.

10. Conclusion

The main goal of this paper was to propose a systematic methodology to optimize the parameters of T1 and T2FLSs to solve the output regulation problem of a servomechanism with nonlinear backlash.

With the proposed methodology, we build hybrid architecture, this hybrid architecture combines FLSs and GAs, this architectures is what we call a FGA.

These FGAs were used to optimize the MF's parameters of the T1 and T2FLSs, this optimization was performed in order to achieve the regulation problem.

The regulation problem was solved as was predicted, this affirmation is supported with simulation results where both, the T1FLS and the T2FLS, found by following the proposed methodology achieve the regulation problem. We do not improve the results reported in Aguilar et al. (2007), but our methodology found FLCs that result to be robust under external perturbations, under these conditions T2FLS again result to be best than the T1FLS.

The combination of a GA and T1 and T2FLSs results to be a good method to solve the proposed problem, but the time necessary to run each generation of the GA was to log.

References

Aguilar, L. T., Orlov, Y., Cadiou, J. C., & Merzouki, R. (2007). Nonlinear H_∞-output regulation of a nonminimum phase servomechanism with backlash. *Journal of Dynamic Systems, Measurement, and Control, 129*(4), 544–549. doi:10.1115/1.2719774<http://www.link.aip.org/link/?JDS/129/544/1>.

Al-Jaafreh, M., & Al-Jumaily, A. (2007). Training type-2 fuzzy system by particle swarm optimization. In *IEEE congress on evolutionary computation, 2007 – CEC 2007* (pp. 3442–3446). doi:10.1109/CEC.2007.4424917.

Castillo, O., Aguilar, L. T., Cazarez, N., & Cardenas, S. (2008). Systematic design of a stable type-2 fuzzy logic controller. *Journal of Applied Soft Computing, 8*(3), 1274–1279.

Castillo, O., Lozano, A., & Melin, P. (2004). Hierarchical genetic algorithms for fuzzy system optimization in intelligent control. In *IEEE annual meeting of the fuzzy information processing NAFIPS'04* (Vol. 1, pp. 292–297). doi:10.1109/NAFIPS.2004.1336294.

Castillo, O., Huesca, G., & Valdez, F. (2005). Evolutionary computing for optimizing type-2 fuzzy systems in intelligent control of non-linear dynamic plants. In *Annual meeting of the North American fuzzy information processing society, 2005 – NAFIPS 2005* (pp. 247–251). doi:10.1109/NAFIPS.2005.1548542.

Castillo, O., & Melin, P. (2005). *Hybrid intelligent systems for pattern recognition using soft computing: An evolutionary approach for neural networks and fuzzy systems*. Berlin: Springer-Verlag.

Castillo, O., & Melin, P. (2008) (1st ed.). *Type-2 fuzzy logic: Theory and applications* (Vol. 223). 978-3-540-76283-6. Heidelberg, Germany: Springer-Verlag.

Castro, J. R., Castillo, O., & Martinez, L. G. (2007). Interval type-2 fuzzy logic toolbox. *Journal of Engineering Letters, 15*(1), 89–98.

Castro, J., Castillo, O., & Melin, P. (2007). An interval type-2 fuzzy logic toolbox for control applications. In *IEEE international fuzzy systems conference, 2007 – FUZZ-IEEE 2007* (pp. 1–6). doi:10.1109/FUZZY.2007.4295341.

Cazarez-Castro, N., Aguilar, L., & Castillo, O. (2008a). Hybrid genetic-fuzzy optimization of a type-2 fuzzy logic controller. In *Eighth international conference on hybrid intelligent systems, 2008 – HIS'08* (pp. 216–221). doi:10.1109/HIS.2008.170.

Cazarez-Castro, N., Aguilar, L., & Castillo, O. (2008b). Genetic optimization of a type-2 fuzzy controller for output regulation of a servomechanism with backlash. In *5th international conference on electrical engineering, computing science and automatic control, 2008 – CCE 2008* (pp. 268–273). doi:10.1109/ICEEE.2008.4723381.

Cazarez-Castro, N. R., Aguilar, L. T., Castillo, O., Crdenas, S. (2008a). *Soft computing for hybrid intelligent systems. Studies in computational intelligence. Ch. Fuzzy Control for Output Regulation of a Servomechanism with Backlash* (1st ed., Vol. 154, pp. 19–28). Berlin: Springer.

Cazarez-Castro, N., Aguilar, L., Castillo, O., & Rodriguez, A. (2008). Optimizing type-1 and type-2 fuzzy logic systems with genetic algorithms. *Research in Computing Science, 39*, 131–153.

Goldberg, D. E. (1989). *Genetic Algorithms in Search. Optimization and Machine Learning*. Boston, MA, USA: Addison-Wesley, Longman Publishing Co., Inc..

Grefenstette, J. (1986). Optimization of control parameters for genetic algorithms. *IEEE Transactions on Systems, Man and Cybernetics, 16*(1), 122–128. doi:10.1109/TSMC.1986.289288.

Hagras, H. (2007). Type-2 flcs: A new generation of fuzzy controllers. *Computational Intelligence Magazine, IEEE, 2*(1), 30–43. doi:10.1109/MCI.2007.357192.

Holland, J. M. (1975). *Adaptation in natural and artificial systems*. Ann Arbor, MI: University of Michigan Press.

Isidori, A. (1995). *Nonlinear Control Systems*. New York, Inc./Secaucus, NJ, USA: Springer-Verlag.

Lagerberg, A., & Egardt, B. (1999). Estimation of backlash with application to automotive powertrains. In *Proceedings of the 42th conference on decision and control* (pp. 135–147).

Lee, M., & Takagi, H. (1993). Integrating design stage of fuzzy systems using genetic algorithms. In *Second IEEE international conference on fuzzy systems* (Vol. 1, pp. 612–617). doi:10.1109/FUZZY.1993.327418.

Mamdani, E. (1976). Advances in the linguistic synthesis of fuzzy controllers. *Journal of Man–Machine Studies, 8*, 669–678.

Mamdani, E., & Assilian, S. (1975). An experiment in linguistic synthesis with fuzzy logic controller. *Journal of Man–Machine Studies, 7*(1), 1–13.

Melin, P., & Castillo, O. (2001). Intelligent control of complex electrochemical systems with a neuro-fuzzy-genetic approach. *IEEE Transactions on Industrial Electronics, 48*(5), 951–955. doi:10.1109/41.954559.

Mendel, J. (2001). *Uncertain rule-based fuzzy logic systems: Introduction and new directions*. Upper Saddle River, NJ: Prentice-Hall.

Mendel, J., & John, R. (2002). Type-2 fuzzy sets made simple. *IEEE Transactions on Fuzzy Systems, 10*(2), 117–127. doi:10.1109/91.995115.

Mendel, J. M., John, R. I., & Liu, F. (2006). Interval type-2 fuzzy logic systems made simple. *IEEE Transactions on Fuzzy Systems, 14*(6), 808–821. doi:10.1109/TFUZZ.2006.879986.

Merzouki, R., Cadiou, J., & M'Sirdi, N. (2004). Compensation of friction and backlash effects in an electrical actuator. *Journal of System and Control Engineering, 218*(12), 75–84.

Nordin, M., Bodin, P., & Gutman, P. (2001). *Adaptive control of nonsmooth dynamic systems. Ch. New models and identification methods for backlash and gear play* (pp. 1–30). Berlin: Springer.

Sepulveda, R., Castillo, O., Melin, P., Rodriguez-Diaz, A., & Montiel, O. (2007). Experimental study of intelligent controllers under uncertainty using type-1 and type-2 fuzzy logic. *Information Sciences, 177*(10), 2023–2048.

Sepulveda, R., Castillo, O., Melin, P., Montiel, O., & Aguilar, L. (2007). Evolutionary optimization of interval type-2 membership functions using the human evolutionary model. In *IEEE international on fuzzy systems conference – FUZZ-IEEE 2007* (pp. 1–6). doi:10.1109/FUZZY.2007.4295401.

Zadeh, L. A. (1975a). The concept of a linguistic variable and its application to approximate reasoning – i. *Information Sciences, 8*(3), 199–249.

Zadeh, L. A. (1975b). The concept of a linguistic variable and its application to approximate reasoning – ii. *Information Sciences, 8*(4), 301–357.

Zadeh, L. A. (1975c). The concept of a linguistic variable and its application to approximate reasoning – ii. *Information Sciences, 9*(1), 43–80.