American Journal of Physics

PAPERS | APRIL 01 1998

Resource Letter ICQM-1: The Interplay between Classical and Quantum Mechanics

Martin C. Gutzwiller

Am. J. Phys. 66, 304–324 (1998)
https://doi.org/10.1119/1.19065

RESOURCE LETTER

Roger H. Stuewer, *Editor*
School of Physics and Astronomy, 116 Church Street
University of Minnesota, Minneapolis, Minnesota 55455

This is one of a series of Resource Letters on different topics intended to guide college physicists, astronomers, and other scientists to some of the literature and other teaching aids that may help improve course content in specified fields. [The letter E after an item indicates elementary level or material of general interest to persons becoming informed in the field. The letter I, for intermediate level, indicates material of somewhat more specialized nature; and the letter A, indicates rather specialized or advanced material.] No Resource letter is meant to be exhaustive and complete; in time there may be more than one letter on some of the main subjects of interest. Comments on these materials as well as suggestions for future topics will be welcomed. Please send such communications to Professor Roger H. Stuewer, Editor, AAPT Resource Letters, School of Physics and Astronomy, 116 Church Street SE, University of Minnesota, Minneapolis, MN 55455.

Resource Letter ICQM-1: The Interplay between Classical and Quantum Mechanics

Martin C. Gutzwiller
IBM Research Center, Yorktown Heights, New York 10598

(Received 8 March 1997; accepted 9 November 1997)

This Resource Letter provides a guide to the literature that is concerned with the connections between classical and quantum mechanics. After the three centuries of classical mechanics, this fundamental problem in physics became one of the important issues at the beginning of the twentieth century, in the transitional period from 1900 to 1925. It then lost some of its urgency during the modern period from the discovery of Schrödinger's equation to the middle of the 1960s, when the validity of quantum mechanics was tested in all the areas of application. The most recent 30 years are called the Post-Modern Period because both the classical and quantal aspects of nature are seen in their full complexity, and the advantages of viewing one in the light of the other have brought a better understanding of many special fields. The literature is divided into these four periods, and the list of references in each of them is subdivided according to the main areas of interest. Most of the bibliographic descriptions are accompanied by a short comment that is supposed to complete the information in the title.

I. INTRODUCTION

Newton created classical mechanics almost single-handedly in less than three years, starting in the late summer of 1684 and completing his *Mathematical Principles of Natural Philosophy* (usually called the *Principia* from their Latin title) in the early summer of 1687. The creation of quantum mechanics took a little longer, starting with de Broglie's wave nature of the electron in 1924 and ending with the general quantization of wave fields by Heisenberg and Pauli in 1929. The intervening five years present a unique flowering of new ideas that form the conceptual base for all of modern physics.

Quantum mechanics describes physics on the atomic scale. The word atom was chosen to apply to an object that cannot be cut into smaller pieces. Therefore, its mathematical description requires concepts that contradict to some extent the foundations of classical mechanics. Since our intuition is based on our experience with objects on the macroscopic scale, however, we have to deal with the contrast and take advantage of any connections between the apparent opposites when we try to solve particular problems. The alternative would be a purely logical application of abstract rules, such as one could expect a computing machine to handle all by itself. The results would still have to be interpreted just as any data from a survey or a controlled experiment. Thus, in order to understand modern physics, we have to be aware of the interplay between classical and quantum mechanics.

The same situation occurs in the relations between geometric (ray) optics and wave optics, although the contrast between the macroscopic and the microscopic scale is not as extreme and the special applications may be quite different. The time interval for working out the conceptual difficulties, however, was much longer since the diffraction of light is first described in the work of Grimaldi, which was published in 1665. The resulting controversy was only settled by Young and Fresnel 150 years later. Another century passed before the electromagnetic theory of light got its final triumph when the diffraction of x rays by crystals was discovered by Laue in 1912. This happened after Einstein had revalidated the corpuscular nature of light in his work on the light quanta.

Wave phenomena could be observed during all this time most easily on the surface of water. The mathematical theory of hydrodynamics and acoustics was developed without much controversy. In the *Principia*, Newton discusses the motion of a fluid through the opening of a screen, complete with the standard picture of a wave that is diffracted as it passes a narrow opening in a wall. The interplay between classical and quantum mechanics has a close analog both in the electromagnetic theory of light, and its wave phenomena themselves can be visualized most directly in the propagation of sound and water waves.

These analogies played a crucial role in the historical development of physics on the atomic scale, and they are an important part of the introductory teaching in modern physics. But they are usually hidden in the textbooks and monographs, and they are rarely acknowledged in the articles of the main scientific journals. Apparently, most authors nowadays feel completely comfortable with applying quantum mechanics to their special area of research. They are concerned exclusively with proving that their results are in total agreement with the mathematical predictions of quantum mechanics. Consequently, they tend to hide any approximate and intuitive idea that could be used (and probably was used) as a simple preliminary check, and provide some motivation to the reader. Many theoretical physicists seem to imitate the mathematicians in eliminating any crude approximation from their publications because it could be misleading to the readers.

This Resource Letter is meant to provide some relevant information for three distinct, but related purposes: (i) to cover the areas of physics where the interplay between classical and quantum mechanics may help in explaining the basic ingredients to the novice; (ii) to list the most easily accessible sources, primary as well as secondary, that deal with the historic transition from classical to quantum mechanics; and (iii) to provide a rough idea of the recent applications where the semiclassical approach to quantum mechanics has been used with some success.

II. THE HISTORICAL APPROACH

The main purpose of this Resource Letter might be best served if the transition from classical to quantum mechanics is studied in the chronological order of events. Such a program is hard to carry out, however, because many crucial problems were solved quite unexpectedly and completely in a very short time. It was as if a mighty fortress had been beleaguered for decades, and then was taken by storm after the troops had been able to penetrate through a small opening in the walls, and overwhelm all further resistance. The exact process of conquest is of less interest than the stubborn resistance that preceded the capture.

The development from classical to quantum mechanics will be divided into four periods. Each period is defined as a time interval between some definite years; these limits were chosen to some extent in order to have simple dates that are related to our year of 1996.

The Classical Period—from 1596 to 1895. This period starts with the year of Kepler's first publication, *Mysterium Cosmographicum* (*The Secret of the Universe*), which is in rough coincidence with Galileo's first works, *De Motu* (*On Motion*) of 1590 and *Le Meccaniche* (*On Mechanics*) of 1600. Preceding both is a marvelous book by Simon Stevin of Brugge in Flanders of 1586, *De Beginselen der Weeghconst* (*The Beginnings of the Art of Weighing*).

These three centuries cover several basic developments in classical physics: the progress of mechanics from its first formulation by Isaac Newton to the sophisticated formalism of Lagrange, Hamilton, and Jacobi; the advance of optics from the geometry of light rays to the wave phenomena of diffraction and interference; and the development of field theories to cope with hydrodynamics, acoustics, elastic vibrations, and particularly, electricity and magnetism.

The nineteenth century also produced the laws of thermodynamics and the beginning of statistical mechanics; the same century brought us spectroscopy and its many applications in gases, liquids, and solids; finally, there is the spectacular development of chemistry that led to the structure of many molecules and the periodic table of elements. All of these areas had a profound effect on the birth of quantum mechanics, and were in turn deeply affected by it.

The Transition Period—from 1896 to 1925. Some of the world-shaking discoveries that mark its beginning include: x rays by Wilhelm Conrad Roentgen in 1895, radioactivity by Antoine-Henri Becquerel in 1896, and the electron by Joseph John Thomson in 1897. This period covers the early work by the great pioneers Max Planck, Albert Einstein, and Niels Bohr, who found the first explanations for the simplest problems in the interaction between isolated atoms like hydrogen and helium, and the electromagnetic field. They were the masters in stretching the validity of classical mechanics way beyond its natural domain with the help of purely empirical rules.

The Modern Period—from 1926 to 1965. In the first six months of this period the four monumental papers on wave mechanics by Erwin Schrödinger were produced. Physics once more looked like an open book where the answers to all questions can be reduced to very few elementary principles that are expressed in precise mathematical language. There no longer seemed any need to fall back on the magical inventions of the transition period, because the explanation for every concrete experiment seemed reducible to a tractable, mathematical problem. No discrepancy was ever found in the comparison between the results of the laboratory and of the quantum-mechanical theory.

The Post-Modern Period—from 1966 until now. This period is characterized by the realization, on one hand, that the basic issues concerning nuclear and elementary-particle physics are still far from resolved, and on the other hand, that most of the important phenomena on the atomic scale and above cannot be readily deduced from the accepted foundations in these fields.

In spite of the tremendous increase of scientific activity in these last 30 years, the simple-minded optimism of the Modern Period for an ultimate understanding of the whole universe is no longer justified. Physics has acquired many of the attributes of engineering where the known fundamental principles are used to think of inventions and novel applications, and complete explanations are a luxury to be left for later generations.

Since the problems to be tackled have become much harder, there is more emphasis on intuitive insight, and approximate methods are appreciated because they provide a better understanding. Therefore, classical pictures and models are again called upon, as they were in the period of transition. But they are now used in the full knowledge either of what an exact treatment would demand in problems from the atomic scale on up, or of the large territory still to be discovered in the nuclear and elementary-particle domain. The date of 1966 was chosen by the author because it marks the beginning of his work on the connection between classical and quantum mechanics, which was the first to take into account the possibility of chaotic behavior in classical mechanics.

Each of these four periods has contributed in its own way to the interplay between classical and quantum mechanics. It seemed, therefore, most natural to divide up the vast number of individual connections according to their origin in one of these four time intervals. The bridges from one to the other will be pointed out if they appear to be particularly significant. But within each period the various items will be grouped together according to their technical areas.

Each of the four consecutive periods will be covered in this Resource Letter. The text dealing with each period will be subdivided into sections that are devoted to the main topics of investigation during that period, as far as they are related to the basic issue of this Resource Letter. Inside each section, we will try to adhere to the usual format with subsections for each of the various modes of publication.

III. SOME REMARKS CONCERNING THE CHOICE OF THE REFERENCES

Sections V–VII contain mostly the titles of textbooks and monographs, whereas Sec. VIII contains mostly the titles of conference proceedings and collections, plus a large selection of articles in contemporary scientific journals. Many institutional libraries have recently adopted a policy where books are kept in the accessible stacks, but bound journals older than ten years are relegated to some warehouse where they can be recovered on special request; browsing in old journals is essentially discouraged.

Most journals are actively trying to limit the number and length of the articles that are published. Unfortunately, the authors do not seem to cooperate, and the result is an exponential growth at a rapid rate of the number of pages published in almost every journal. At the same time, either the area covered in any one article gets smaller in order to ensure rapid dissemination of new information, or the same information gets spread out with minor variations and in different combinations, particularly in conference proceedings.

Nevertheless, there is a lot of very good work coming out all the time. But the monthly tables of content demand more patience and concentration if a potential reader wants to find the items that are most interesting in any special field. General classifications are hard to establish because any article is likely to touch more than one area. Section VIII is subdivided into many more categories than Secs. V–VII to help the reader through the available literature. These sections are not meant to propose a systematic and exhaustive scheme, but only a practical way to distinguish among the existing publications.

The original intention of the author was to put together a list of references that could make some claims to completeness, at least for the purpose of an introduction concerning the interplay between classical and quantum mechanics. This topic, however, not only covers all of the twentieth century and pervades a large part of the active branches in physics, but it has also become ever more active in the last 30 years, with many applications coming into view all the time. Therefore, the present list is hardly more than a personal collection coming from somebody who has been an active participant in the recent revival. Some helpful suggestions by Doug Stone and Turgay Uzer are gratefully acknowledged.

IV. JOURNALS

American Journal of Physics
Annals of Physics (New York)
CHAOS
Chemical Physics Letters
International Journal of Bifurcation and Chaos
Journal of Chemical Physics
Journal of Mathematical Physics
Journal of Physics A and B (occasionally G)
Nature
Nonlinearity
Nuclear Physics A
Physica D (occasionally A)
Physical Review (before being split into different parts)
Physical Review A and E (occasionally B and D)
Physical Review Letters
Physics Reports
Physics Today
Physics World
Proceedings of the Royal Society of London, Series A
Progress of Theoretical Physics
Reports of Progress in Physics
Reviews of Modern Physics
Science
Science News
Scientific American
SIAM (Society for Industrial and Applied Mathematics) Review
Zeitschrift für Physik B and D

V. THE CLASSICAL PERIOD

Although some of the original texts from this period may be hard to obtain, the author has found many of them languishing in university libraries, completely unappreciated for their critical role in the development of physics. The author would like to encourage the reader to take at least a casual look at them, even if they are written in an unfamiliar language, nothing worse than Latin in any case, and often available in English translation. The density of new information is usually orders of magnitude above the standard publication in our time.

A. Some historical documents of classical mechanics

1. **The Art of Weighing**, Simon Stevin (De Beghinselen der Weeghconst beschreven duer Simon Stevin van Brugghe, tot Leyden Inde Druckeye van Christoffel Plantijn, By Francoys van Raphelinghen, 1586). Reprinted with English translation in *Principal Works of Simon Stevin*, edited by E. J. Dijksterhuis (C. V. Swets & Zeitlinger, Amsterdam, 1955), Vol. 1, General Introduction, Mechanics. A marvelous exposition of the statics, including hydrostatics, shortly before the beginning of the seventeenth century when dynamics became the focus of scientific activity. This text is full of illustrations and practical examples, and its title page is decorated with a sketch of the experimental evidence for equilibrium on an inclined plane. (E)
2. **On Motion and On Mechanics**, Galileo Galilei. Comprising De Motu (~1590) translated with Introduction and Notes by I. E. Drabkin and Le Meccaniche (~1600), translated with Introduction and Notes by Stillman Drake (The University of Wisconsin Press, Madison, 1960). In contrast to Stevin's work these texts are relatively short and much more philosophical; they are also accompanied by extended historical comments. Obviously, Galileo is still bound to the scientific discussions of antiquity and middle ages, and his skill in problems of statics hardly matches Stevin's. Yet, we are less than 100 years away from Newton's breakthrough in the *Principia*. (I)
3. **Dialogues Concerning Two Sciences**, Galileo Galilei, translated by Henry Crew and Alfonso de Salvio (MacMillan, London, 1914; reprinted by Dover, New York, 1954). [Also translated, with a Foreword by Albert Einstein as well as Introduction and Notes by Stillman Drake (University of California Press, Los Angeles, 1953 and 1967).] New Translation with Introduction and Notes by Stillman Drake (University of Wisconsin Press, Madison, 1974). Galileo's claim to be the

father of modern mechanics is based on this work, written after his condemnation by the Church and published in 1638. The two new sciences cover problems in statics, and in dynamics, particularly the motion on an inclined plane, the pendulum, and parabolic trajectories of projectiles. Although written as a conversation between three people, it proceeds through numerous simple theorems in geometry as well as some experiments where the modern reader is left wondering whether they were real. (E)

4. **Sir Isaac Newton's Mathematical Principles of Natural Philosophical and His System of the World**, translated into English by Andrew Motte in 1729, revised by Florian Cajori (California U.P., Berkeley, 1934), reprinted in **Great Books of the Western World** (Encyclopedia Britannica, Chicago, 1952), Vol. 34 as well as in several other recent translations, e.g., by I. B. Cohen and Anne Whitman (to appear in the University of California Press, Los Angeles, 1997). For any person interested in physics, there is nothing at all comparable to reading this fundamental text, which is the foundation of everything that has happened since its first publication in 1687. The technical requirements are no more than high-school geometry, and an appreciation for the great leap forward that occurred with this book. (A)
5. **The Key to Newton's Dynamics—The Kepler Problem and the Principia**, J. Bruce Brackenridge with English translation from the Latin by May Ann Rossi (University of California Press, Berkeley, 1995). A mostly technical presentation of the general reasoning and the detailed arguments of Newton to prove Kepler's laws from the assumption of the gravitational force between two masses. The text follows the original draft *On Motion* and the relevant sections of the *Principia*; a high-school student should be able to understand everything with the help of all the figures. Learned historical comments are not always well separated, and may discourage some people's interest. (I)
6. **Newton's Principia for the Common Reader**, S. Chandrasekhar (Oxford U.P., New York, 1995). Covering most of the *Principia* without many historical comments, giving Newton's arguments in modern form with the help of new diagrams and rather elaborate algebra. (I)

B. The history of (classical) mechanics

7. **The Mechanization of the World Picture (Pythagoras to Newton)**, E. J. Dijksterhuis, translated by C. Dikshoorn (Oxford U.P., London, 1961; reprinted by Princeton U.P., Princeton, NJ, 1986). One of the best and most complete accounts of the historical process that led from the Greek views of science through late antiquity, the Middle Ages, and the Renaissance to Newton's synthesis: The world runs like a mechanical contraption on the basis of some well-defined mathematical relations that allow precise predictions. (I)
8. **The Great Physicists from Galileo to Einstein,** George Gamow, originally published as **Biography of Physicists** (Harper Brothers, New York, 1961; reprinted by Dover, New York, 1988). Most enjoyable discussion of physics in its historical development, requiring some mathematical literacy, but lightened with many of the author's drawings. Special effort is devoted to the discussion of wave phenomena in various contexts. (E)

C. Ray and wave optics

9. **Traité de la Lumière,** Christiaan Huygens, Leiden 1690 (original written in French, translated by S. P. Thompson in volume 34 of Great Books of the Western World, Encyclopedia Britannica, Chicago 1952, pages 545–619). Generally recognized as the first discussion of wave optics, and of Huygens's principle where each disturbance spreads in a spherical wave, and every point in turn serves as the origin of a secondary wave. This idea leads directly to Feynman's path integral in quantum mechanics. It is important to emphasize, however, that Huygen's principle deals with a single pulse traveling at the speed of light, and not with a sinusoidal wave as in most optical experiments (such as Grimaldi's). (E)
10. **Opticks or a Treatise of the Reflexions, Refractions, Inflexions and Colours of Light,** Isaac Newton (London, 1704). Published many times thereafter; available with a foreword by A. Einstein in Dover Publications, New York, 1979, as well as in **Great Books of the Western World**, Encyclopedia Britannica, Chicago 1952, pp. 373–544), Vol. 34. Although Newton's many discoveries in optics were made 30 years earlier, this work is his definitive explanation. While the bulk deals with ray optics and particularly with the refraction of white light by a prism, the last part deals with ''inflexion,'' Newton's word for diffraction that has not survived, and for a good reason. He recognizes Grimaldi's work in the first sentence, but he has an explanation in terms of particle trajectories that are bent in the neighborhood of a sharp edge. (I)
11. **The Wave Theory of Light—Memoirs of Huygens, Young, and Fresnel,** edited by Henry Crew (American, New York, 1900). A short history of the most important developments in the nineteenth century precedes the crucial papers by Huygens, Young, and Fresnel, each of which is followed by a short biographical sketch. A very useful collection. (E)
12. **The Nature of Light—A Historical Survey,** V. Ronchi, translated by V. Barocas (Harvard U.P., Cambridge, MA, 1970). A readable account of the long history that starts in antiquity and the middle ages, and leads to the great debate between ray and wave optics at the beginning of the nineteenth century. This story is important in our context because the relations between classical and quantum mechanics are clearly foreshadowed in the optical phenomena. They have directly inspired a lot of the crucial experiments that established quantum mechanics as the ultimate theory on the atomic scale. (E)
13. **The Mathematical Principles of Huygens' Principle,** Bevan B. Baker and E. T. Copson (Clarendon, Oxford, 1950). A concise and very readable treatise on the mathematical theory (in traditional terms) of the wave motion in acoustics, electromagnetism, and optics. Particular attention is given to the theories of the diffraction on a screen by Kirchhoff and Sommerfeld. Highly recommended to any student approaching this subject. (A)
14. **The Classical Theory of Fields,** L. D. Landau and E. M. Lifshitz, translated by Morton Hamermesh (Pergamon, New York, 1951, 1962, 1971). Unique textbook, both understandable and extremely varied, contains a good chapter on light propagation, as well as the motion of charges, and the analogous phenomena in the relativistic gravitational field. (I)
15. **The Feynman Lectures on Physics, Volume I: Mainly Mechanics, Radiation, and Heat,** Richard P. Feynman, Robert B. Leighton, and Matthew Sands (Addison–Wesley, Reading, MA, 1963). Among the many elementary textbooks on classical physics, this one is mentioned because of its five chapters, 26–30, which cover various parts of optics, from Fermat's principle to diffraction, with a minimum of mathematical formalism and a maximum of intuition. A person with only a beginner's training in mathematics should be able to learn about the most important ideas. (E)
16. **Almost All About Waves,** John R. Pierce (MIT, Cambridge, MA, 1974). A very general introduction to vibration and wave phenomena requiring only relatively simple mathematics. (E)
17. ''Optical Caustics in Natural Phenomena,'' J. A. Lock and J. H. Andrews, Am. J. Phys. **60**, 397–407 (1992). Caustics are explained in simple terms as a wave phenomenon that is related to geometric optics. (E)

D. Classical field theories

18. **A History of the Theories of Aether and Electricity, Volume I: The Classical Theories,** Sir Edmund Whittaker (First published by Nelson and Sons, London, 1910, revised and enlarged in 1951; reprinted by Humanities, New York, 1973). A masterpiece of complete reporting and explaining that starts in the seventeenth century and goes through the nineteenth century achievements of Maxwell, J. J. Thomson, and Lorentz. (I)
19. **Principles of Optics: Electromagnetic Theory of Propagation, Interference and Diffraction of Light,** Max Born and Emil Wolf (Pergamon, New York, 1959). Completely new edition of a textbook by Born, entitled *Optik, Ein Lehrbuch der elektromagnetischen Lichttheorie* (Springer, Berlin, 1933). After a historical introduction with a careful exposition of geometrical optics, the emphasis is on starting systematically with Maxwell's equations including their phenomenological equivalent in solids, liquids, and gases. Coherence and interference are treated. A complete picture of a classical field theory emerges with many of its practical applications. (I)

E. Thermodynamics and statistical mechanics

20. **Elements of Classical Thermodynamics for Advanced Students of Physics,** A. B. Pippard (Cambridge U.P., Cambridge, 1957). In spite of its title the demands on mathematical sophistication are quite limited, but the discussions are simple and clear. (E)

21. **Elementary Statistical Physics,** C. Kittel (Wiley, New York, 1958). Covering not only fundamental principles, but also fluctuations, noise, and transport phenomena requiring a minimum of mathematical skills. (I)

VI. TRANSITION

Since the Transition Period came to an end 70 years ago, it is receding into the distant past. Its eminent representatives died a generation ago, and their work is no longer studied for its own merit, but only as a stepping stone to a more complete and definitive theory. Nevertheless, their struggles with preliminary results and ad hoc explanations help us appreciate the inherent difficulties of quantum mechanics, which offers only an abstract approach to reality instead of the concrete pictures in classical mechanics. The preliminary concepts from the transition period very often were shown much later to provide a valid approximation to the mathematical results of quantum mechanics.

A. Historical surveys

1. Textbooks and monographs

22. **Planck's Original Papers in Quantum Physics,** German and English edition, annotated by Hans Kangro, translated by D. ter Haar and Stephen Brush (Wiley, New York, 1972). This small paperback contains the two short papers that Planck read at the meetings of October 19 and December 14, 1900, to interpret the recent measurements of the blackbody radiation. The arguments in the second paper are quite simple, and the constant *h* is defined, with a numerical value too small by only 1%. (E)
23. **Thirty Years that Shook Physics—The Story of the Quantum Theory,** George Gamow (Doubleday, New York, 1966; reprinted by Dover, New York, 1985). A highly entertaining introduction for the layperson who is able to read simple mathematical formulas, and enjoy the author's amusing drawings as well as the photographs of all the participants. The Appendix contains the English translation (from the original German) of the famous Copenhagen ''Faust,'' a physicist's spoof of Goethe's drama. (E)
24. **Sources of Quantum Mechanics,** edited with historical introduction by B. L. van der Waerden (North-Holland, Amsterdam, 1967; reprinted by Dover, New York, 1968). A selection of the essential papers including some letters from the short, but crucial period when modern quantum mechanics was developed first by Bohr and his school with the help of the correspondence principle, and then by Heisenberg, Born, Dirac, and Pauli. All in English translation with illuminating comments. (I)
25. **Paul Ehrenfest—Volume 1: The Making of a Theoretical Physicist,** M. J. Klein (North-Holland, Amsterdam, 1970). Chapters 10 and 11 contain valuable discussions of the quantum hypothesis and the adiabatic principle. (I)
26. **The Genesis of Quantum Theory (1899–1913),** A. Hermann (MIT, Cambridge, 1971). A slender volume centered around the main figures of Planck, Lorentz, Einstein, Stark, Haas, Sommerfeld, Nernst, and Bohr. (E)
27. **The History of Quantum Theory,** Friedrich Hund, translated by Gordon Reece (Harper & Row, New York, 1974). A rather detailed historical account in fairly simple language, but requiring a good knowledge of the underlying physics. Mathematical formulas are used freely, and many references to the original works are provided. A very useful introduction into a more thorough study of the whole development. (E)
28. **Niels Bohr's Times (in Physics, Philosophy, and Polity),** Abraham Pais (Clarendon, Oxford, 1991). Perhaps the best biography of Niels Bohr as the great innovator, skillful organizer, and fearless leader in the transition from classical to quantum mechanics. It is written for the scientist who is already familiar with the most important developments in the first half of this century, and contains a great many details about the individual events and their main actors, but without giving any of the formal mathematical arguments. (I)
29. **Order, Chaos, Order—Transition from Classical to Quantum Physics,** Philip Stehle (Oxford U.P., New York, 1994). A well-researched history of the 30 years from 1895 to 1925, which is written for the beginning student in physics. Some of the crucial experiments are described, and a few theoretical developments are explained in separate boxes. The reader will gain appreciation for the difficulties and occasional detours that led to the ultimately satisfying outcome. (E)

2. Articles in scientific journals

30. ''On the Quantization Condition of Sommerfeld and Epstein,'' Albert Einstein, translation by Charles Jaffe of the article ''Zum Quantensatz von Sommerfeld und Epstein'' von A. Einstein in Verhandlungen der Deutschen Physikalischen Gesellschaft **19**, 82–92 (1917), Joint Institute for Laboratory Astrophysics (JILA) Report No. 116 (1980). The quantization condition is rewritten in a form that does not refer to any special coordinate system. But Einstein (with a reference to Poincaré) is the first to realize that even for this form to be applied, the system cannot be allowed to be chaotic (to use the modern term). The seminal importance of this deceptively simple paper was not recognized for another 40 years, and not fully understood for another 20 years. (I)
31. ''Elementary Examples of Adiabatic Invariance,'' F. S. Crawford, Am. J. Phys. **58**, 337–344 (1990). The adiabatic invariance is shown explicitly for a number of simple systems, including its relation to the simple rules of quantization. (I)

B. Photons

1. Textbooks and monographs

32. **Black-Body Theory and the Quantum Discontinuity 1894–1912**, T. S. Kuhn, second edition with a New Afterword (The University of Chicago Press, Chicago, 1987; first Edition, Oxford U.P., New York, 1978). A historical account of the thermodynamics, and then statistical mechanics, concerned with the blackbody radiation. The central role of Max Planck in this development is studied with great care, with the help of many details and numerous notes at the end as well as an extensive bibliography. There have been some vocal objections, however, both to Kuhn's interpretation of science history in terms of revolutions, and to his view of the developments in Planck's views; see R. Jost (1995) Ref. 34. (I)
33. **A History of Theories of Aether and Electricity, Volume II: The Modern Theories 1900–1926**, Sir Edmund Whittaker (first published by Nelson and Sons, London, 1933; reprinted by Humanities, New York, 1973). This second volume covers the transition period again very thoroughly (cf. the mention of the first volume in the preceding chapter), including special relativity, the old quantum theory, gravitation, and the discovery of the new quantum mechanics. It has acquired a special reputation because it assigns most of the credit for the discovery of special relativity to Poincaré and Lorentz. (I)
34. **Das Märchen vom Elfenbeinernen Turm—Reden und Aufsätze** (The Tale of the Ivory Tower—Lectures and Essays), Res Jost, edited by K. Hepp, W. Hunziker, and W. Kohn (Springer-Verlag, Berlin, 1995). A very worthwhile collection of essays of interest to physicists, including ''The critique of Planck by T. Kuhn'' on pp. 67–78. (A)

C. Atoms

1. Textbooks and monographs

35. **Atomic Structure and Spectral Lines**, Arnold Sommerfeld, translated from the third edition of *Atombau und Spektrallinien* by H. L. Brose (Methuen, London, 1923). The whole generation of physicists who started the modern period was raised on the strength of this basic text. It still is the best introduction to the atomic physics of the transition period, an ideal combination of simple qualitative arguments with a minimum of elementary mathematics, covering the whole area including experiments. (E)
36. **The Mechanics of the Atom**, Max Born, translated by J. W. Fisher and D. R. Hartree (Bell, London, 1927; Ungar, New York, 1960). The original edition was completed in November 1924, just before the great breakthrough to the new quantum mechanics. The classical mechanics of multiply periodic motion is discussed in general terms, as the basis for the spectra and the interaction with the electromagnetic field. Examples include the Zeeman and Stark effects, and the single electron in the potential of two fixed nuclei; perturbation theory is used for a first attempt at the helium atom. (I)
37. **The Quantum Theory of the Atom**, George Birtwistle (Cambridge U.P., Cambridge, 1926). A contemporary textbook for the student who

is familiar with basic physics and wants to learn the best explanations then available, starting with radiation theory and classical mechanics to deal with the spectral features of atoms and molecules. An excellent account. (I)

38. **Quantum Principles and Line Spectra**, J. H. Van Vleck, Bulletin of the National Research Council, Vol. 10 part 4, published by the National Academy of Sciences, Washington DC, 1926. A very complete presentation of the ''old'' quantum theory, its explanations and results, with special emphasis on the correspondence principle of Bohr for the interpretation of the spectral intensities. Its unusually large set of references to contemporary work also includes the first reference on page 44 to Einstein's 1917 paper on the quantization conditions in a non-separable system; I have found only one more reference (by Lanczos 1949 in Sec. VII A; Ref. 49) until Keller's 1958 paper in Sec. VII C; Ref. 75. (A)

39. **Zeemaneffekt und Multiplettstruktur der Spektrallinien**, E. Back und A. Landé (Springer-Verlag, Berlin, 1925). A status report by the experts on both experiments and theory concerning atomic spectroscopy, on the eve of the big breakthrough. The detailed knowledge of the facts is impressive, and is reminiscent of the present condition in nuclear and particle physics where a consistent theory is also missing right now. (I)

2. Articles in scientific journals

40. ''Classical View of the Stark Effect in Hydrogen Atoms,'' T. P. Hezel, C. E. Burkhardt, M. Ciocca, and J. J. Leventhal, Am. J. Phys. **60**, 324–328 (1992). Old quantum theory done in explicit detail. (E)

41. ''Classical View of the Properties of Rydberg Atoms: Application of the Correspondence Principle,'' T. P. Hezel, C. E. Burkhardt, M. Ciocca, J. J. Leventhal, and L.-W. He, Am. J. Phys. **60**, 329–335 (1992). Follow-up to the preceding article. (E)

D. Molecules

42. ''Über das Modell des Wasserstoffmolekülion,'' W. Pauli, Jr., Ann. Phys. (Vierte Folge) **68**, 177–240 (1922). This published version of Pauli's Ph.D. dissertation with Sommerfeld is remarkable because it fails to predict the simplest case of molecular binding in spite of its impressive mastery of the old quantum theory. The reason is the erroneous handling of the angular momentum quantization, cf. Strand and Reinhardt in Sec. 4.11; Ref. 282. (I)

E. Thermodynamics

1. Textbooks and monographs

43. **The New Heat Theorem—Its Foundation in Theory and Experiment**, W. Nernst, translated from the second edition by Guy Barr (Methuen, London, 1926; reprinted by Dover, New York, 1969). The definitive account and textbook of the so-called third law of thermodynamics, the vanishing of the entropy at the absolute zero of the temperature scale, which turned out to have its explanation in quantum mechanics. (I)

F. Statistical mechanics

44. **Statistical Mechanics—The Theory of the Properties of Matter in Equilibrium (based on an Essay awarded the Adams Prize in the University of Cambridge 1923–1924)**, R. H. Fowler (Cambridge U.P., London, 1929). A marvelous survey including the early quantum results of Planck, Debye, Nernst, including interatomic forces, Brownian motion, meteorology, astrophysics, and the interaction with radiation. (E)

45. **Report on Radiation and the Quantum-Theory** (2nd ed.), J. H. Jeans (The Physical Society of London, London, 1924). Covers the development of the statistical foundations of the radiation theory and the theory of specific heats, as well as the relation with Bohr's correspondence principle. (E)

1. Articles in scientific journals

46. ''Modern and Old View in the Dynamical Foundations of Classical Statistical Mechanics,'' L. Galgani, in *Applications of Modern Dynamics to Celestial Mechanics and Astrodynamics*, edited by V. Szebehely (Reidel, Dordrecht, 1982). A fascinating report on the work of Boltzmann and Nernst trying to explain the blackbody radiation classically, and almost succeeding. (I)

47. ''Zero-Point Energy in Early Quantum Theory,'' P. W. Milonni and M.-L. Shih, Am. J. Phys. **59**, 684–698 (1991). A careful discussion of the historical background and the physical foundations for a crucial early input into modern physics. (E)

VII. MODERN PERIOD

A. Classical mechanics with a modern viewpoint

The emphasis in classical mechanics has changed greatly since Newton, and the arrival of quantum mechanics in the 1920s can be seen in the choice of the methods and the examples that are discussed. But the new ideas from the work of Poincaré have not penetrated into the awareness of most physicists. The solutions of problems in classical mechanics, and quantum mechanics as well, are still presented as a matter of finding the right transformation of coordinates.

1. Textbooks and monographs

48. **Classical Mechanics**, Herbert Goldstein (Addison–Wesley, Reading, MA, 1950). A generation of physicists has been brought up with this textbook. It emphasizes subjects that are close to the problems of quantum mechanics such as central forces, rigid-body rotation, small oscillations, special relativity, the Hamiltonian formalism, and some simple field theory. There are many references with short comments to related monographs. (I)

49. **The Variational Principles of Mechanics**, Cornelius Lanczos (University of Toronto Press, Toronto, 1949; reprinted by Dover, New York, 1986). A very useful, well organized, and quite explicit presentation of classical mechanics for physicists rather than mathematicians, starting from the variational principle in its different garbs, as well as the resulting canonical formalism, and some beginner's field theory. It contains the second reference to Einstein's 1917 paper on quantization conditions, Ref. 30. (I)

50. **A Treatise on Analytical Dynamics**, L. A. Pars (Heinemann, London, 1965). A valuable textbook where traditional mechanics is developed systematically from general principles, emphasizing direct insight rather than strict mathematical analysis, but with many applications at each stage. There is still no awareness of the unavoidable irregularities in the typical motions; all the methods and examples allow explicit solutions in terms of relatively simple formulas. (I)

51. **Variational Principles in Dynamics and Quantum Theory** (3rd ed.), W. Yourgrau and S. Mandelstam (Saunders, Philadelphia, 1968). A fine historical survey starting with Fermat and his successors in classical mechanics, going on to the ''old'' and ''new'' quantum mechanics including Feynman and Schwinger. By way of contrast even hydrodynamics is covered; there is a general emphasis on some of the philosophical meaning. (A)

52. **Mechanics: Classical and Quantum**, T. T. Taylor (Pergamon, Oxford, 1976). Although this textbook was written in the ''Post-Modern'' Period, the author decided to adopt the views of the earlier ''Modern'' Period. The connection between classical and quantum mechanics is found in the similarities of the mathematical formalisms rather than the phenomena. (I)

B. The new wave mechanics

No serious student in physics can do without any knowledge of wave mechanics as it can be found in a vast number of textbooks. The intended audience for these books covers a wide range, from pure mathematicians who are interested in proving general theorems, to engineers and chemists who want to concentrate on limited and well-defined applications. We will mention only the relatively few books where the transition from classical to quantum mechanics gets more than just a perfunctory treatment. A number of monographs can be considered classics in QM because of their independent approach and depth.

Finally, some authors have tried to present the subject without the help of its almost inevitable mathematical apparatus, but rather by appealing to the reader's intuition with the help of ingenious illustrations. This kind of effort is of great importance for physics, however, because it helps in making the origin and the practice of quantum mechanics more understandable to the lay people. A teacher might find some inspiration to get away from a purely formal discussion of the important concepts.

1. Qualitative discussions for the general reader

a. Textbooks and monographs

53. **The Strange Story of the Quantum**, Banesh Hoffmann (Harper, 1947, Dover, New York, 1959). ''An account for the general reader of the growth of ideas underlying our present atomic knowledge.'' Written in a colloquial style, with a minimum of mathematics and few pictures, for a reader who needs constant encouragement and enjoys the little devices of live conversation. (E)
54. **Are Quanta Real? A Galilean Dialogue**, J. M. Jauch (Indiana U.P., Bloomington/London, 1973). The main issues concerning classical and quantum mechanics are discussed in a fictional trialogue between the three persons of Galileo's **Dialogue on the Two Major Systems of the World** of 1632 and his **Discourses and Demonstrations Concerning Two New Sciences** of 1638. Salviati represents the modern scientist, Sagredo is the intelligent layman, and Simplicio the defender of the conventional wisdom. A clever imitation of Galileo's learned and polemic language. (E)
55. **In Quest of the Quantum**, Leonid Ponomarev, translated from the Russian by Nicholas Weinstein (Mir, Moscow, 1973). A very entertaining and well-written introduction with hardly any mathematical formulas, but numerous whimsical drawings to emphasize some of the simple arguments. Short biographies of the main actors are found throughout. (E)
56. **Taking the Quantum Leap; the New Physics for Non-Scientists**, Fred Alan Wolf (Harper & Row, San Francisco, 1981). A very attractive, general introduction for the layperson into the difficulties that haunt quantum mechanics. The questions concerning the wave nature of a particle are emphasized with the help of many sketches, and occasional works of art, as well as pictures and biographical notes on the main actors. (E)
57. **The Conceptual Development of Quantum Mechanics** (2nd ed.), Max Jammer (American Institute of Physics, New York, 1989). An understandable and extremely well documented history that explains the origins of quantum mechanics starting with Planck's work on blackbody radiation and going into some of the controversies in the interpretation of wave mechanics. The author is remarkably even-handed and stays away from unwarranted generalizations and formalistic arguments. A unique source of information.
58. **The Historical Development of Quantum Mechanics** (5 volumes), J. Mehra and H. Rechenberg (Springer-Verlag, New York, 1982 to 1987). A report that reads more like a story for the general public with many anecdotal details, but contains a lot of information not generally found, e.g., the discovery of QM (Vol. 2), the formulation of matrix mechanics and its modifications (Vol. 3), Erwin Schrödinger and the rise of wave mechanics (Vol. 5). (I)

2. General discussion with a philosophical bent

a. Textbooks and monographs

59. **The New Science—Where is Science going? The Universe in the Light of Modern Physics, The Philosophy of Physics**, Max Planck, with a preface by A. Einstein and an introduction by James Franck (Meridian, 1959). This volume 3 of **Planck's Complete Works** contains many of the popular talks that its illustrious author gave throughout his life, after his original discovery had completely transformed classical physics, and his personal life ended in tragedy during and after World War II. (E)
60. **The Physicist's Conception of Nature (Symposium in honor of the seventieth birthday of P. A. M. Dirac)**, edited by Jagdish Mehra (Reidel, Dordrecht, 1973). A large and remarkable collection of articles by many of the most outstanding participants in the development of QM. Hardly any special problems in physics are discussed, although the contrast between CM and QM is often mentioned in many different contexts, such as the classical electron, indeterminacy, symmetry, irreversibility, the measurement process. (A)

3. Fundamental treatises for the beginning expert

a. Textbooks and monographs

61. **Problems of Atomic Dynamics**, Max Born (MIT, Cambridge, 1926). Reprinted as a paperback by MIT Press in 1970. These 30 lectures were given at MIT between November 14, 1925 and January 22, 1926; the first 20 lectures are concerned with the structure of atoms, and the last 10 with the lattice theory of solids. This is the first account of the operator version of quantum mechanics as discovered by Heisenberg, and elaborated by Born, Jordan, and Pauli, including the latter's theory of the hydrogen spectrum; its preface is dated January 22, 1926, so that this text predates Schrödinger's first paper. The arguments that led from the ''old'' to the ''new'' quantum theory, i.e., from the reliance on classical mechanics to the use of matrices, are discussed in a way that is no longer found in the textbooks, unfortunately. (I)
62. **The Principles of Quantum Mechanics**, P. A. M. Dirac (Oxford U.P., London, 1930, 1935, 1947). This monograph is the first comprehensive text of the new quantum mechanics, and each new edition presents some important new view of the author. It is in a class by itself because it records the progress in his systematic development that starts from his insights into classical mechanics. The second edition presents the first approach to the path integral, and the third is phrased in the formalism of bra's $\langle$ and ket's $\rangle$ whose juxtaposition forms the bracket $\langle , \rangle$ or scalar product. (A)
63. **Lectures on Quantum Mechanics**, Paul A. M. Dirac (Belfer Graduate School of Science, Yeshiva University, New York, 1964). A slender volume of four lectures where Dirac exposes his views on the process that leads from the classical Hamiltonian mechanics through the study of Poisson brackets to the quantization in both curved and flat spaces. Although the ideas are general and abstract, the formalism is simple in typical Dirac fashion. (I)
64. **The Feynman Lectures on Physics. III. Quantum Mechanics**, Richard P. Feynman, Robert B. Leighton, and Matthew Sands (Addison–Wesley, Reading, MA, 1965). This textbook on QM is unique because it tries to motivate everything for an audience that is not familiar with the use of mathematical language. There is an abundance of simple sketches with explanations based on intuition, and that very often means pictures from classical mechanics. Special attention is devoted to the effects of symmetry, which are usually hard to grasp except through formal arguments. (E)

b. Articles in scientific journals

65. ''The Correspondence Principle Revisited,'' R. L. Liboff, Phys. Today 50–55 (February 1984). Discussion of Planck's quantum going to zero versus the quantum number going to infinity. (E)

C. WKB wave functions and path integrals

1. Textbooks and monographs

66. **An Introduction to Phase-Integral Methods**, J. Heading (Wiley, New York, 1962). A short and very useful introduction to the problems of the transition from CM to QM for the motion in one dimension. After a good historical survey, the WKBJ solutions and the Stokes phenomenon are discussed with the help of the functions in the complex plane. The behavior near transition points (from classically allowed to forbidden regions) as well as various applications including waves in the ionosphere are treated. (I)
67. **Quantum Mechanics and Path Integrals**, R. P. Feynman and A. R. Hibbs (McGraw–Hill, New York, 1965). Introductory textbook that treats many examples very explicitly, but only when they are amenable to complete solutions, i.e., integrable. (I)
68. **JWKB Approximation: Contributions to the Theory**, Nanny Froeman and Per Olof Froeman (North-Holland, Amsterdam, 1965). A short but systematic treatise where this approximation to Schrödinger's equation in one dimension is discussed in the complex plane with some mathematical sophistication, including higher-order terms in powers of Planck's constant. (A)

69. **Semi-Classical Approximations in Quantum Mechanics**, V. P. Maslov and M. V. Feodoriuk (Reidel, Boston, 1981). This is the first English version of a book in Russian by the first author from the early 1960s that was translated into French in 1972. Although the phase-integral solution of Schrödinger's equation is treated in more than one dimension, and many clever models are discussed, the emphasis lies on the mathematical justification rather than on the physical ideas. The main challenge of dealing with the classically chaotic motions is not taken up. Maslov's name seems forever tied to the phase losses at caustics. (A)

70. **Classical Dynamics and its Quantum Analogues**, David Park (Springer, New York, 1979). Although less than 20 years old, these lecture notes give the standard version of the classical-quantal connection, with all the familiar examples worked out in detail. The explanations and the mathematics are kept at a level for undergraduates, and include discussions of Hamiltonian dynamics, perturbation theory, rigid body, and simple continuous systems like strings. (E)

2. Articles in scientific journals

71. ''Space-Time Approach to Non-Relativistic Quantum Mechanics,'' R. P. Feynman, Rev. Mod. Phys. **20**, 367–387 (1948). [Reprinted in *Selected Papers on Quantum Electrodynamics*, edited by Julian Schwinger (Dover, New York, 1958).] Dirac's idea is carried a step further by interpreting the propagator (or probability amplitude) in quantum mechanics as an integral over all the different paths between the given end points where the integrand is the exponential of the classical action integral. Classical mechanics comes out immediately because the path of stationary action with respect to small variations is the one to suffer least from the destructive interference among the paths. Feynman's view has become the starting point for most of the more refined models in QM. (I)

72. ''On the Definition and Approximation of Feynman's Path Integral,'' C. Morette, Phys. Rev. **81**, 848–852 (1951). The first attempt of a mathematical proof to show that the path integral is a solution of Schrödinger's equation for sufficiently short times. (A)

73. ''Path Concepts in Hamilton–Jacobi Theory,'' Rolf Landauer, Am. J. Phys. **20**, 363–367 (1952). An isolated precursor in the efforts to understand the relation between classical and quantum mechanics beyond the WKB approximation. (E)

74. ''Reflections in One-Dimensional Wave Mechanics,'' R. Landauer, Phys. Rev. **82**, 80–83 (1951). Correction to the WKB approximation by including multiple reflections in one dimension. (E)

75. ''Corrected Bohr–Sommerfeld Quantum Condition for Nonseparable Systems,'' J. B. Keller, Ann. Phys. (N.Y.) **4**, 180–188 (1958). The seminal paper of Einstein from 1917 is interpreted quantum mechanically: Each invariant torus in phase space is covered by a simple wave function which is then projected onto position space, and leads to the quantization conditions, including the effect of caustics. (I)

76. ''Asymptotic Solutions for Eigenvalue Problems,'' J. B. Keller and S. I. Rubinow, Ann. Phys. (N.Y.) **9**, 24–75 (1960). The method of Keller's paper above is used to treat a whole collection of integrable, but not easily separable, systems with two degrees of freedom: The possibility of classical chaos is not discussed at all. (A)

D. Atomic spectra

1. Textbooks and monographs

77. **''Quantum Mechanics of One- And Two-Electron Atoms''**, Hans A. Bethe and Edwin E. Salpeter (Springer-Verlag, Berlin, 1957). Second enlarged edition of Bethe's article of the same title in the *Handbuch der Physik*, Vol. XXIV in 1933. It is meant as a reference work pertaining to the calculations and their comparison with experiment, and is intended for ''graduate students who wish to learn 'applied quantum mechanics','' giving'' 'low-brow' explicit derivations.'' ''Specific application to atomic systems of general field-theoretic results is described in detail.'' A unique source of information in an area where semiclassical methods have recently been used with some success. (I)

78. **Atomic Spectra and Atomic Structure**, Gerhard Herzberg, translated from **Atomspektren und Atomstruktur** by J. W. T. Spinks (Prentice–Hall, New York, 1937). The introductory volume to a series of monographs on spectroscopy for the specialist in the field. The author discusses the experimental techniques as well as the results in a systematic manner using the most direct theoretical approach, including classical arguments. (E)

79. **The Spectrum of the Atomic Hydrogen**, G. W. Series (Oxford U.P., New York, 1957). A short technical history of this central subject, from the earliest results in the late nineteenth century to the latest triumphs of the quantum electrodynamics. The newest high-precision data are quoted and discussed, but not fully explained. (I)

80. **Atomic Spectra**, H. G. Kuhn (Longmans, London, 1962). An elementary introduction that uses the classical pictures whenever useful, and includes a discussion of the most important experimental evidence. (E)

81. **Structure of Matter**, W. Finkelnburg, translation from German by O. Matossi-Riechemeier (Springer-Verlag, New York, 1964). A very successful introductory text into nuclear and atomic, as well as some molecular and condensed matter physics, on the basis of simplified quantum pictures. (E)

82. ''The Photoionization of One- and Two-Electron Atoms,'' L. M. Branscomb, in **Physics of the One- and Two-Electron Atoms** (North-Holland, Amsterdam, 1969) pp. 660–699. A very readable survey covering mostly experiments, including the astrophysical implications. (I)

E. Spin and statistics

83. ''Exclusion Principle and Spin,'' B. L. Van der Waerden in **Theoretical Physics in the Twentieth Century, A Memorial Volume to Wolfgang Pauli**, edited by M. Fierz and V. F. Weisskopf (Interscience, New York, 1960), p. 199. Gives the best account of the difficult process of getting Pauli to accept the electron spin in the critical year of 1925; his objections were exactly the quasiclassical picture that was used by the discoverers, Kronig as well as Goudsmit and Uhlenbeck. (I) [cf. the remarks by A. Herrmann, co-editor of **Pauli's Correspondence**, Volume 1. 1919–1929 (Springer-Verlag, Berlin, 1979), p. XV].

F. Scattering

84. **The Theory of Atomic Collisions**, N. F. Mott and H. S. W. Massey (Oxford U.P., London, 1933) (first edition, with a second edition concentrating more on nuclear problems in 1949). The first systematic introduction into the practical solution of problems in the scattering of particles. Starting with electrons in a Coulomb field, including spin, proceeding to whole atoms hit by electrons elastically and inelastically, ending with massive particles in gas and solids, as well as nuclear phenomena, including relativistic two-body problems and radiation. (A)

G. Molecular bond

85. **Band Structure and Molecular Structure**, R. de L Kronig (Cambridge U.P., London, 1930). A primer on the analysis of molecular structure with the help of rotational and vibrational spectra, on the basis of quantum mechanics. The previously semiempirical results can now be explained quite systematically and completely. (E)

86. **Introduction to Quantum Mechanics with Applications to Chemistry**, Linus Pauling and E. Bright Wilson (McGraw–Hill, New York, 1935). The textbook for a generation of chemists trying to learn an important subject with an unexpectedly heavy use of mathematics. Short chapters on classical mechanics and the old quantum theory are followed by the regular fare and occasional excursions into the semiclassical regime. Subjects of interest to chemists such as the molecular bond, the rotation, vibration, as well as excited states of molecules, and the Franck–Condon principle are emphasized. (I)

87. **Molecular Spectra and Molecular Structure—I. Spectra of the Diatomic Molecules,—II. Infrared and Raman Spectra of Polyatomic Molecules**, Gerhard Herzberg (Van Nostrand, Princeton, NJ, 1939 for first volume; second volume and enlarged edition of the first, 1950). The basic and, at its time, complete monograph on spectra of diatomic molecules. It can fairly be proclaimed as the book that proved quantum mechanics to be the foundation of molecular physics, and therefore of all chemistry. But the experimental results are way ahead of any detailed calculations, so that explanations have to remain qualitative, using simple, sometimes classical arguments. Both volumes are an impressive demonstration of the amount of knowledge in this special field that was already known, and understood in a qualitative way, some 50 years ago. (A)

H. Electrons in solids

1. Textbooks and monographs

88. **The Theory of the Properties of Metals and Alloys**, N. F. Mott and H. Jones (Clarendon, Oxford, 1936; reprinted by Dover, New York, 1958). The conduction of electrons in a solid is always approached classically to the extent that external fields, both electric and magnetic, are applied, and a Boltzmann-like transport equation is solved. (I)

89. **Quantum Theory of Solids**, R. E. Peierls (Clarendon, Oxford, 1955). Introductory text of unusual clarity and simplicity of mathematical tools by one of the original contributors in this field. (I)

90. **Solid State Physics**, N. W. Ashcroft and N. D. Mermin (Holt, Rinehart and Winston, Philadelphia, 1976). A comprehensive textbook for graduate students in physics that starts out with three valuable sections on the old Drude and Sommerfeld theories of metals, as well as their failures. The remainder of this impressive work gives frequent discussions of the limits for many of the standard methods in the field, as well as extensive experimental data. (I)

I. Second quantization and field theory

1. Textbooks and monographs

91. **The Quantum Theory of Radiation**, W. Heitler (Oxford U.P., London, 1936). The first monograph on this topic, and the reference work until the renewal of the radiation theory in the late 1940s by Schwinger, Tomonaga, Feynman, and Dyson. The classical theory is first treated in its Hamiltonian form, which is then quantized first *in vacuo* and then in interaction with matter. This represents the prelude to the great explosion of quantum field theory that now dominates high-energy physics as well as other areas like superconductivity, magnetism, and statistical mechanics. All these developments are included in the much larger third edition which never became as important as this first one. (A)

92. **Quantum Electrodynamics—34 selected papers**, edited by Julian Schwinger (Dover, New York, 1958). A collection of the important papers that led to the development of the modern theory in the late 1940s including the radiative corrections in atomic physics. (A)

93. **QED—The Strange Story of Light and Matter**, R. P. Feynman (Princeton U.P., Princeton, NJ, 1985). A short and very readable introduction for the novice. (E)

94. **The Quantum Theory of Light**, R. Loudon (Clarendon, Oxford, 1973). An introduction for the beginner in this area that leads through atomic physics and the quantized radiation field into photon optics, lasers, light scattering, and nonlinear optics. (I)

J. Quantum statistics

1. Textbooks and monographs

95. **The Theory of Electric and Magnetic Susceptibilities**, J. H. Van Vleck (Oxford U.P., London, 1932). Possibly the first monograph on the electromagnetic properties of gases, liquids, and solids based on statistical mechanics, both classical and quantum. An excellent survey of the progress and new applications under the recent developments. (I)

96. **Thermodynamics and Statistical Mechanics**, A. H. Wilson (Cambridge U.P., London, 1957). A very useful monograph that covers wide areas and discusses many applications in simple mathematical language without much fuss about the philosophical foundations. Classical as well as quantum mechanical aspects of statistical mechanics are treated in some detail, including the third law of thermodynamics, electric and magnetic phenomena. (I)

97. **Ergodic Theory in Statistical Mechanics**, I. E. Farquhar (Interscience, London, 1964). Survey of both the classical and quantal ergodic theory in the form that was originally proposed by Boltzmann, i.e., without the sophisticated techniques of functional analysis, emphasizing coarse graining, classical phase-space theorems, averaging over initial states. (I)

98. **The Principles of Statistical Mechanics**, Richard C. Tolman (Oxford U.P., London, 1938). A broadly based effort to ''give a reasonably clear and complete picture,'' ''from a modern point of view,'' of ''the more powerful methods of Gibbs'' both in the classical and quantal domain. Only relatively simple examples are treated as illustrations of the fundamental ideas, which are presented in verbal rather than abstract mathematical terms. (A)

99. **Statistical Mechanics**, Kerson Huang (Wiley, New York, 1963). Standard text for the beginning student in this area, before the more formal methods from field theory became dominant. (A)

2. Articles in scientific journals

100. ''On the Quantum Correction for Thermodynamic Equilibrium,'' E. Wigner, Phys. Rev. **40**, 749–759 (1932). The ''Wigner distribution function'' for a quantum system is defined in this paper, and shown to satisfy an equation reminiscent of Boltzmann's classical distribution in phase space. Wigner's idea has inspired countless applications where the statistical features are inherent in quantum mechanics rather than coming from thermodynamics. (I)

VIII. POST-MODERN PERIOD

The last 30 years in almost every area of physics have seen an accumulation of results that far surpasses what any individual is able to appreciate. Whereas this growth seems exponential at a relatively steady rate in many well-established fields, it looks as if a sleeping giant had been awakened in the special area of this Resource Letter.

The sudden explosion is intimately connected with the growing awareness of what is now called chaos, for the lack of both a better word and of any description, let alone explanation, of this general phenomenon. The various quarters where quantum mechanics is the fundamental tool have responded quite differently. Nuclear, atomic, and molecular physicists have been in the forefront of trying out new ideas for making the connection with classical mechanics. Condensed-matter physicists are jumping on the band wagon with a vengeance right now, but the main practitioners of high-energy and even of statistical physics have as yet to take notice.

Some of the fields with a long history, going back more than a century, experience this development differently because quantum mechanics is not part of their foundation. Therefore, fluid dynamics and acoustics will not be mentioned although they face similar problems in reconciling the ray and the wave picture. The same holds true for all of optics over its wide range of frequencies, with an additional complication. The electromagnetic field has to be ''second-quantized'' to allow for the photons as classical particles with Bose–Einstein statistics. The resulting behavior of many nonlinear devices in quantum optics is again chaotic, but it is described by the same theory as the chaos in classical mechanics, and is hardly mentioned in this Resource Letter.

The vast literature has been organized somewhat artificially into separate groups, but many textbooks, monographs, and conference proceedings spread out into more than one of these classifications. More serious, however, is a large amount of duplication where the same author contributes very similar papers to different conference proceedings. Many summary descriptions may sound alike, so that one collection of contributed articles may be just as good as another. The reader suffers no great loss if only one such collection is available at the local library. Yet many of these proceedings provide a good starting point to find out what is happening and who is active in any of the specialties.

It would have been impossible to give even a cursory survey of the papers that appear in the archival journals, because so many people are working hard and publish their results as soon as they have something to say. I have made a choice of articles that I consider particularly interesting because of some new result or novel idea. Such an article may not always provide the most efficient route for the newcomer, compared with some later discussion of the same topic, but it may offer a better insight into the discovery process at the price of some extra effort. But on the whole, the list of publications below is no more than a sampling from a vast set, with some inevitable personal bias.

A. The new classical mechanics

1. Textbooks and monographs

101. **Dynamical Systems**, George D. Birkhoff (American Mathematical Society Colloquium Publications, volume IX, published by the AMS, Providence, RI, 1927, with a new edition in 1966). ''It represents essentially a continuation of Poincaré's profound and extensive work on Celestial mechanics. ...the style may appear less formal and rigorous than is now (1966) customary. But just the informal and lively manner of writing has been inspiring to many mathematicians.'' (I)

102. **Mathematical Methods of Classical Mechanics**, V. I. Arnold, translated by K. Vogtmann and A. Weinstein (Springer-Verlag, New York, 1978). A very readable textbook, although in mathematical language, with many simple sketches and numerous examples. A lengthy set of appendices explains without necessarily proving the main propositions of modern mechanics. (I)

103. **Introduction to Dynamics**, I. Percival and D. Richards (Cambridge U.P., Cambridge, 1982). A very useful short textbook to get an undergraduate student acquainted with the modern ideas and simple examples (and many problems) in classical mechanics that will lead eventually to (post-modern) quantum mechanics. (E)

104. **Regular and Chaotic Dynamics**, A. J. Lichtenberg and M. A. Lieberman (Springer-Verlag, New York, 1st ed. 1983, 2nd ed. 1992). An up-to-date monograph covering nonlinear oscillations, perturbation theory, stability, stochastic processes in Hamiltonian and dissipative systems, with many numerical examples and figures. (A)

105. **Chaos in Dynamical Systems**, G. M. Zaslavsky, translated by V. I. Kisin (Harwood, Chur, Switzerland, 1985). Introductory discussion of stochastic behavior in different circumstances with many examples about oscillations, nonlinear waves, and quantum systems. (I)

106. **Chaos—Making a New Science**, J. Gleick (Penguin, New York, 1987). A popular best-seller that describes both the new problems of classical chaos in simple nontechnical language and the personal stories of the main protagonists. (E)

107. **Chaotic Dynamics—An Introduction**, G. L. Baker and J. P. Gollub (Cambridge U.P., New York, 1990). An introduction for undergraduates in the sciences with an emphasis on practical computations, and some discussion of simple physical systems from mechanics and fluid dynamics. (E)

108. **Newton's Clock—Chaos in the Solar System**, I. Peterson (Freeman, New York, 1993). A very entertaining and highly instructive introduction to chaotic phenomena for the general reader. The history of coping with the astronomical observations of the motions in the solar system is told with many interesting sidelights, as well as illustrations, diagrams, and explicit data. (E)

2. Conference proceedings

109. ''The Theory of Orbits in the Solar System and in Stellar Systems,'' edited by G. Contopoulos, **Symposium No. 25 of the International Astronomical Union**, Thessaloniki, Greece, August 17–22, 1964 (Academic, London and New York, 1966). A marvelous collection of contributions in celestial mechanics showing a complete awareness of the ''new classical mechanics'' at a time when most physicists had no idea of it. (I)

110. ''Order in Chaos,'' **Proceedings of a Conference at the Center for Nonlinear Studies** in Los Alamos, New Mexico 1982, edited by D. Campbell and H. Rose [Physica D **7** (1983)]. One of the first get-togethers on chaos covering many aspects from experimental observations, mathematical properties and model systems, fractal structures, transition to chaos in circle maps, fluids and vortices, with a special section (for the first time) on quantum chaos. (I)

111. ''Physics of Chaos and Related Problems,'' **Proceedings of Nobel Symposium 59** in Graeftavallen, Sweden 1984, edited by S. Lundqvist [Phys. Scr. **T9** (1985)]. A rather general survey of the field by the experts covering the behavior of low-dimensional classical systems, pattern formation in large systems such as hydrodynamics, turbulence, computational techniques, and quantum systems. (I)

112. **Hamiltonian Dynamical Systems**, a reprint selection compiled and introduced by R. S. MacKay and J. D. Meiss (Adam Hilger, Bristol, 1987). A substantial collection containing articles from various scientific journals, about equilibria and periodic orbits, quasiperiodic orbits and the breakup of invariant tori, chaotic behaviors and many applications. (A)

3. Articles in scientific journals

113. ''The Applicability of the Third Integral of Motion: Some Numerical Experiments,'' M. Hénon and C. Heiles, Astron. J. **69**, 73–79 (1964). Two harmonic oscillators are coupled by the simplest third-order coupling, which leads to the most popular model for the transition from integrability at low energies to classical chaos at higher energies. (I)

114. ''The Anisotropic Kepler Problem in Two Dimensions,'' M. C. Gutzwiller, J. Math. Phys. **14**, 139–152 (1973). The first example of a realistic physical system where all the classical trajectories can be described by a simple binary code. (I)

115. ''A Universal Instability of Many—Dimensional Oscillator Systems,'' B. V. Chirikov, Phys. Rep. **52**, 263–379 (1979). The nonlinear coupling between linear oscillators is studied in detail with many numerical examples in order to establish the instability due to resonance overlap, various mechanisms of effective diffusion in phase space, and the meaning of entropy. (A)

116. ''Triple Collisions in the Planar Isosceles Three Body Problem,'' R. L. Devaney, Invent. Math. **60**, 249–267 (1980). A simple mathematical discussion of the main features in the classical model that comes close to yielding the ground state of the helium atom. (I)

117. ''How Random is a Coin Toss?'' Joseph Ford, Phys. Today **36** (4), 40–47 (April 1983). A general discussion of randomness in physics by an author who is intimately familiar with, and bases his ideas on the chaos in classical mechanics. (E)

118. ''Chaotic Scattering modelled by an Inclined Billiard,'' M. Hénon, Physica D **33**, 132–156 (1988). A very simple model for a classical particle that bounces off a corrugated boundary, and where a complete symbolic dynamics can be defined. (E)

119. ''A General Model for Motion bound to an Impurity in an anisotropic semiconductor,'' J. Casasayas, A. Nunes, and A. M. Ozorio de Almeida, Physica D **48**, 311–321 (1991). The chaos that is found in the anisotropic Kepler problem is shown to be a very general feature for a whole class of motions in potentials of similar kind. (A)

120. ''Global Behavior of the Charged Isosceles Three-Body Problem,'' P. Atela and R. I. McLachlan, Int. J. Bifurcation Chaos **4**, 865–884 (1994). The many periodic orbits in this system are explained in terms of two ''sources,'' the usual Kepler motion and the collision-ejection trajectories of the collision with the origin. (A)

B. Integrable versus chaotic features in quantum mechanics

1. Textbooks and monographs

121. **Hamiltonian Systems—Chaos and Quantization**, Alfredo M. Ozorio de Almeida (Cambridge U.P., Cambridge, 1988). An introduction for theoretical physicists starting with classical mechanics and its techniques in the nonintegrable cases, and continuing on to some of the novel methods in quantum mechanics, such as Wigner functions, random matrices, and periodic orbits. (I)

122. ''Quantum Mechanics of Classically Non-Integrable Systems,'' Bruno Eckhardt, Phys. Rep. (Review Section of Physics Letters) **163**, 205–297 (1988). A well-organized introduction covering the basic results at the time and giving a large list of references. The discussion is limited to basic principles and interesting model problems. (I)

123. **Chaos in Classical and Quantum Mechanics**, Martin C. Gutzwiller (Springer-Verlag, New York, 1990). The first half is concerned with a modern approach to classical mechanics with few degrees of freedom, including different types of approximation methods, periodic orbits and surfaces of section, entropy, and the discussion of instructive examples that are usually left to the specialists. The second half deals with the quantum mechanics of integrable versus chaotic classical systems, in particular their wave functions and energy-level statistics, the connection to periodic orbits in the trace formula, and again some important examples in detail. (I)

124. **Quantum Signatures of Chaos**, Fritz Haake (Springer-Verlag, Berlin, 1991). The statistics of the energy levels in QM gets a careful and quite complete discussion on the basis of rather abstract and general ideas, but with only few concrete examples. This monograph uses simple and yet quite elaborate mathematics; it ends with a discussion of dissipative systems. (A)

125. **The Transition to Chaos (in Conservative Classical Systems: Quantum Manifestations)**, Linda E. Reichl (Springer-Verlag, New York, 1992). A very complete and exhaustive treatise that covers the whole development from classical mechanics in its conventional form including area-preserving maps and the diffusion of trajectories, all the way to spectral statistics, semiclassical theory, driven and stochastic systems in QM. A well-organized and largely self-contained reference work in these areas with many illustrations and examples. (A)

126. **Semiclassical Physics**, M. Brack and R. K. Bhaduri (Addison–Wesley, Reading, MA, 1997). This textbook starts with some simple examples of the relation between classical and quantum mechanics, and goes on to explain how the density of states as well as individual energy levels are obtained in nonintegrable systems, with the help of many practical illustrations. (I)

2. Conference proceedings and collections

127. ''Stochastic Behavior in Classical and Quantum Hamiltonian Systems,'' **Volta Memorial Conference** in Como, Italy, 1977, edited by G. Casati and J. Ford (Springer-Verlag, Berlin, 1979). The first conference in this area assembled some of the pioneers coming from mathematics, astronomy, and theoretical physics. This volume is a source for some of the original work, and presents the various viewpoints and motivations. In particular, the editors demonstrate the absence of quantum diffusion in the kicked-rotator model. (I)

128. ''Chaotic Behavior in Quantum Systems—Theory and Applications,'' **Proceedings of the NATO Advanced Research Workshop** in Como, 1983, edited by G. Casati (Plenum, New York 1985). This sequel to the first Como meeting covers a spread of topics that is more narrowly focused on the relations between classical and quantal systems, but it gives a good picture of the field at this, still early date, covering all the fundamental, theoretical and experimental, problems that have since become branches of their own. (I)

129. ''Dynamical Chaos,'' **Proceedings of Royal Society Discussion Meeting**, edited by M. V. Berry, I. C. Percival, and N. O. Weiss, Proc. R. Soc. London, Ser. A **413**, 1–199 (1987), as well as Cambridge U.P. and Princeton U.P. An interesting set of talks including biology, hydrodynamics, physics, and mathematics, with Berry's Bakerian Lecture ''Quantum Chaology.'' (E)

130. ''Chaos and Quantum Physics,'' **Proceedings of the 1989 Summer School in Les Houches**, edited by M.-J. Giannoni, A. Voros, and J. Zinn-Justin (North-Holland, Amsterdam, 1991). A rather complete picture of the state-of-the-art in this area where chaos in classical mechanics gets directly connected with quantum mechanics. The most active authors in this area get enough space to explain their views, including Percival, Bohigas, Gutzwiller, Berry, Colin de Verdiere, Schmit, Smilansky, Chirikov, Heller, Delande, and Muehlschlegel. (A)

131. ''Quantum Chaos,'' **Proceedings of the International School of Physics ''Enrico Fermi,''** edited by G. Casati, I. Guarnieri, and U. Smilansky (North-Holland, Amsterdam, 1993). Good survey articles on periodic-orbit theory, He-atom, Riemann zeta-function, localization, level statistics, scattering, time dependence, as well as some dissipation and noise. (I)

132. ''Quantum Chaos—Quantum Measurement,'' **Proceedings of the NATO Advanced Workshop**, edited by P. Cvitanovic, I. Percival, and A. Wirzba (Kluwer, Dordrecht, 1992). A collection of articles that gives some idea about the extent of the renewed interest in relatively simple, but mathematically difficult problems in quantum mechanics, as well as their effect on the old discussion of the measurement process. (I)

133. ''Chaos and Quantum Chaos,'' **South African Summer School in Theoretical Physics 1992**, edited by W. Dieter Heiss (Springer-Verlag, Berlin, 1992). Excellent introductory and in-depth articles on the main problems in this area, in particular on random-matrix models, various billiards, experiments in atomic and molecular physics, classical and quantum maps, localization of wave functions. (I)

134. ''Periodic Orbit Theory in Classical and Quantum Mechanics,'' Focus issue edited by P. Cvitanovic, Chaos **2**, 1–158 (1992). An outstanding collection of contributions from the NORDITA program on ''Physics of Quantum Chaos and Measurement'' and the NATO Advanced Research Workshop on ''Quantum Chaos—Theory and Experiment'' in Copenhagen. The special role of the classical periodic orbits in various mathematical problems and physical experiments is explored; the importance of this topic had been first recognized only 20 years earlier. (I)

135. ''Chaotic Scattering,'' Focus Issue of Chaos edited by T. Tel and E. Ott, Chaos **3**, 417–706 (1993). The second half has a fine collection of articles dealing with quantum chaotic scattering. (I)

136. ''Quantum and Chaos: How Incompatible?'' **Proceedings of the Fifth Yukawa International Seminar** held in Kyoto 1993, edited by K. Ikeda [Prog. Theor. Phys. Suppl. No. 116 (1994)]. A number of carefully written survey articles covering chaos in both classical and quantum mechanics, expansion in powers of h, scattering, adiabatic invariants, few-body problems, time scales in relaxation phenomena, is followed by many contributed papers of various length in the same areas. (I)

137. ''Quantum Chaos: Present and Future,'' Special Issue edited by K. Nakamura, Chaos, Solitons Fractals—Interdisciplinary Journal of Nonlinear Science **5** (7) (1995), Pergamon. A wide spectrum of articles by some of the best authors covering fundamental issues as well as special applications such as driven systems, statistical mechanics, semiclassical quantization, and quantum measurements. (I)

138. **Quantum Chaos—between Order and Disorder**, edited by G. Casati and B. Chirikov (Cambridge U.P., Cambridge, 1995). The most up-to-date collection with a number of reviews rather than reports on recent results. (I)

3. Articles in scientific journals

139. ''Semiclassical Approximations in Wave Mechanics,'' M. V. Berry and K. E. Mount, Rep. Prog. Phys. **35**, 315–397 (1972). The earliest survey of the post-modern efforts to reconcile classical and quantum mechanics. (I)

140. ''Time-Dependent Approach to Semiclassical Dynamics,'' E. J. Heller, J. Chem. Phys. **62**, 1544–1555 (1975). Schrödinger's equation is solved in terms of a plane wave with a Gaussian spread whose profile is given by the displacement from a classical trajectory. (E)

141. ''Quantum Corrections to a Classical Photo-Dissociation Model,'' E. J. Heller, J. Chem. Phys. **68**, 2066–2075 (1978). The cross section for photoabsorption is written as a propagator in time that can be approximated with the help of classical trajectories. (E)

142. ''Semi-Classical Quantization on Adiabatically Generated Tori, or Einstein on the Brink,'' W. P. Reinhardt and R. E. Gillilan, in **Path Integrals from meV to MeV**, edited by M. C. Gutzwiller *et al.* (World Scientific, Singapore, 1986). The well-known adiabatic principle of quantum mechanics is used to find the quantization in the chaotic region by starting in the regular region where the conventional quantization conditions hold. (E)

143. ''The Reality of the Quantum World,'' A. Shimony, Sci. Am. **258**, 46–53 (January 1988). A modern discussion of the particle-wave duality. (E)

144. ''Dynamical Aspects of Quantum-Classical Correspondence in Quantum Chaos,'' M. Toda, S. Adachi, and K. Ikeda, Prog. Theor. Phys. Suppl. **98**, 323–375 (1989). A systematic and fairly complete, but sometimes quite technical discussion of the main issues and problems. (I)

145. ''Canonical Transformation in Quantum Mechanics,'' Y. S. Kim and E. P. Wigner, Am. J. Phys. **58**, 439–448 (1990). The role of classical phase space is re-examined more than 50 years after the first suggestion of the second author, cf. Ref. 100.

146. ''On the Quantization of the Three-Particle Toda Lattice,'' S. Isola, H. Krantz, and R. Livi, J. Math. Phys. A **24**, 3061–3076 (1991). Comparison of various quantizations of this system on the border between integrable and chaotic. (I)

147. ''Back to the Quantum Future—Evading Chaos to Make Quantum Predictions,'' I. Peterson, Sci. News **140**, 282–284 (November 2, 1991), including a beautiful picture of a ''Chaotic Quantum Chase'' by Eric Heller on the cover of the issue. An intriguing introduction for the general reader to Heller's results on the quantum motion in a stadium-shaped billiard. (E)

148. ''Quantum Chaos,'' M. C. Gutzwiller, Sci. Am. **206**, 78–84 (January 1992). Does chaos lurk in the smooth, wavelike quantum world? Recent work shows that the answer is yes—symptoms of chaos enter even into the wave patterns associated with atomic energy levels. (E)

149. ''Quantum Chaos,'' Roderick V. Jensen, Nature (London) **355**, 311–318 (1992). This review article explains the issues involved for the general reader with the help of a few well-chosen examples related to atomic physics. (E)

150. ''Classical Kronig–Penney Model,'' U. Oseguera, Am. J. Phys. **60**, 127–130 (1992). A classical analog for the quantal band structure in a one-dimensional lattice is proposed. (E)

151. ''Does Quantum Mechanics Obey the Correspondence Principle? Is It Complete?'' J. Ford and G. Mantica, Am. J. Phys. **60**, 1086–1098 (1992). One of Ford's many efforts to explain why he thinks that ''quantum mechanics is much too simple a theory to adequately describe a complex world.'' His arguments are well worth considering. (E)

152. ''Semiclassical Propagation: How long can it last?'' M. A. Sepulveda, S. Tomsovic, and E. J. Heller, Phys. Rev. Lett. **69**, 402–405 (1992). The propagator in semiclassical theory is shown to yield valid solutions for quantum mechanics well past the time classical chaos has mixed phase space on a scale smaller than Planck's length. (I)

153. ''Postmodern Quantum Mechanics,'' E. Heller and S. Tomsovic, Phys. Today 38–46 (July 1993). A well-written report on the origins of the renewed interest in semiclassical methods as illustrated by the authors work on quantum waves spreading inside a stadium-shaped billiard. (E)

154. ''Classical geometric forces of reaction: An exactly solvable Model; Chaotic Classical and Half-Classical Adiabatic Reactions: Geometric Magnetism and Deterministic Friction,'' M. V. Berry and J. M. Robbins, Proc. R. Society London Ser. A **442**, 641–658 (1993), 659–672. A fast motion (spin) coupled to a slow motion (particle translation) produces a force that depends on an integral over the trajectory. When the fast motion is quantal and the slow motion classical, there results an effective magnetic force and a deterministic friction in the slow motion. (A)

155. ''Manifestations of Classical Phase Structures in Quantum Mechanics,'' O. Bohigas, S. Tomsovic, and D. Ullmo, Phys. Rep. **223**, 43–133 (1993). Two coupled quartic oscillators realize Percival's distinction between regular and irregular spectra which then leads to the distribution of energy levels and their quantum fluctuations. (I)

156. ''Semiquantal Dynamics of Fluctuations: Ostensible Quantum Chaos,'' A. K. Pattanayak and W. Schieve, Phys. Rev. Lett. **72**, 2855–2858 (1994). The Ehrenfest theorem is refined to include the change in a Gaussian wave packet when moving in a double well. (E)

157. ''Moyal Quantum Mechanics: The Semiclassical Heisenberg Dynamics,'' T. A. Osborn and F. H. Molzahn, Ann. Phys. (N.Y.) **241**, 79–127 (1995). Develops a mathematical formalism to get a systematic expansion of expectation values in powers of h without the usual singularities near the caustics of classical mechanics. (A)

C. Path integrals

1. Textbooks and monographs

158. **Morse Theory**, J. Milnor, based on lecture notes by M. Spivak and R. Wells (Princeton U.P., Princeton, NJ, 1963). Marston Morse laid the foundations in the 1930s for a general theory to understand the global behavior of variational quantities, like the integral over the Lagrangian in mechanics. His discussion of the second variation for the extremal curves (Morse index) is essential for the phases in all the semiclassical approximations. (A)

159. **Techniques and Applications of Path Integration**, L. S. Schulman (Wiley-Interscience, New York, 1981). A very well organized general introduction to the idea of path integrals in all its many manifestations. The developments are carried out very explicitly with explanations concerning the motivation of each step. The applications cover first of all quantum mechanics, and go on to optics, polarons, spins, and multiply connected spaces, statistical mechanics, droplets, plus some mention of more difficult subjects from these areas. (I)

160. **Trajectories and Rays—The Path-Summation in Quantum Mechanics and Optics**, A. Ranfagni, D. Mugnai, P. Moretti, and M. Cetica (World Scientific, Singapore, 1990). Fine introduction to this whole circle of ideas requiring some mathematical proficiency for doing explicit calculations. It covers semiclassical quantum mechanics, tunneling, dissipation, and Maxwell's equations for optics. (I)

2. Conference proceedings

161. ''Path Integrals and Their Applications in Quantum, Statistical, and Solid State Physics,'' **Proceedings of the NATO ASI in Antwerpen**, Belgium, 1977, edited by G. J. Papadopoulos and J. T. Devreese (Plenum, New York, 1978). Probably the first gathering devoted exclusively to Feynman's idea of the path integral and many of its applications. Topics range from the classical-quantal relations, to statistical mechanics, diffusion, polymers, polarons, dissipation, hydrodynamics. (I)

162. **Path Integrals from meV to MeV**, papers from the symposium held in Bielefeld, Germany, 1985, edited by M. C. Gutzwiller, A. Inomata, J. R. Klauder, and L. Streit (World Scientific, Singapore, 1986). Covers a wide range of subjects like classical waves, semiclassical quantization, nonequilibrium systems, lattice physics, spin and relativity, as well as mathematical techniques. (I)

163. ''Path Summation: Achievements and Goals,'' **Proceedings of the Adriatico Research Conference in Trieste**, Italy, 1987, edited by S. Lundqvist, A. Ranfagni, V. Sa-yakanit, and L. S. Schulman (World Scientific, Singapore, 1988). A collection of status reports from many different users of the path-integral idea, mostly outside the application in ordinary quantum-mechanics, such as in statistical mechanics, semiconductors, polymers, field theory, dissipation, tunneling, and various mathematical techniques. (E)

164. ''Path Integrals from meV to MeV,'' **Proceedings of the Fourth Meeting of Path Integrals from meV to MeV**, held in Tutzing, Bavaria, 1992, edited by H. Grabert, A. Inomata, L. S. Schulman, and U. Weiss (World Scientific, Singapore, 1993). Emphasis on the relevance in a wide variety of different fields (e.g., polymers, solid-state physics, disordered background), including quantum chaos. The path integral is seen as an indispensable aid to our intuition. (I)

3. Articles in scientific journals

165. ''Gaussian Path Integrals,'' G. J. Papadopoulos, Phys. Rev. D **11**, 2870–2875 (1975). Gives the simplest practical method to obtain the semiclassical approximation to Feynman's path integral. (E)

166. ''Oscillatory Integrals and the Method of Stationary Phase in Infinitely Many Dimensions, with Application to the Classical Limit of Quantum Mechanics,'' S. Albeverio and R. Hoegh-Kron, Invent. Math. **40**, 59–106 (1977). One of the more successful schemes to put Feynman's path integral on a firm mathematical basis. (A)

167. ''Solution of the Path Integral for the H-Atom,'' H. Duru and H. Kleinert, Phys. Lett. B **84**, 185–188 (1979). With the help of a new time-like variable a formal nonlinear transformation reduces the path integral to harmonic oscillators, but this procedure is not shown to be legitimate. (I)

168. ''Exact-Path-Integral Treatment of the Hydrogen Atom,'' R. Ho and A. Inomata, Phys. Rev. Lett. **48**, 231–234 (1982). The nonlinear transformation of the coordinates to avoid the singularity in the trajectory, due to Kustanheimo and Stiefel, is carried out inside the path integral at the price of some rescaling of time; this exact evaluation of the path integral is now justified. (I)

D. Negative curvature surfaces and Riemann zeta function

1. Textbooks and monographs

169. **Scattering Theory for Automorphic Functions**, P. D. Lax and R. S. Phillips (Princeton U.P., Princeton, NJ, 1976). The classical motion on surfaces of negative curvature (along geodesics) is ideally chaotic; this monograph discusses the scattering of waves under these conditions, and provides the mathematical foundations for their treatment. (A)

170. ''The Selberg Trace Formula for PSL(2,R),'' D. A. Hejhal, **Lecture Notes in Mathematics 548 and 1001** (Springer-Verlag, Berlin, 1976). The definitive monograph on the Selberg trace formula and all its variations in the special case of the two-dimensional surfaces of constant negative curvature which is the ideal case of purely hyperbolic (''hard'') classical chaos. The mathematics is rigorous, but not excessively abstract. (I)

171. ''Chaos on the Pseudosphere,'' N. L. Balazs and A. Voros, Phys. Rep. **143** (3), 109–240 (1986). A theoretical physicist's rather than a mathematician's view of the propagation of particles and waves on a surface of constant negative curvature, still trying to be correct while emphasizing the geometric pictures. (I)

2. Articles in scientific journals

172. ''Stochastic Behavior in Quantum Scattering,'' M. C. Gutzwiller, Physica D **7**, 341–355 (1983). The model of a two-dimensional box (torus) with an exceptional point at infinity is realized with the help of the modular region in Poincare's hyperbolic plane. The scattering phase shift for a wave is given by Riemann's zeta function on the line $x=1$, and represents the essence of quantum chaos: analytically smooth and yet effectively unpredictable. (E)

173. ''On the Distribution of Spacings between Zeros of the Zeta Function,'' A. M. Odlyzko, Math. Comput. 273–308 (January 1987). A unique state-of-the-art calculation verifying the Riemann hypothesis for the first billion zeros, and getting the first hundred thousand zeros beyond the lowest trillion to eight-figure accuracy. The zeta function serves as abstract model for the trace formula. (A)

174. ''Quantum Chaos of the Hadamard–Gutzwiller Model,'' R. Aurich, M. Sieber, and F. Steiner, Phys. Rev. Lett. **61**, 483–487 (1988). The chaotic motion of a particle on a surface of constant negative curvature has an energy spectrum that is obtained from the sum over periodic orbits. (I)

175. ''A New Asymptotic Representation for zeta (1/2+it) and Quantum Spectral Invariants,'' M. V. Berry and J. P. Keating, Proc. R. Soc. London, Ser. A **437**, 151–173 (1992). The famous method of Riemann and Siegel for computing the zeta function is interpreted in the light of the trace formula, and this idea leads to a remarkable improvement that avoids some of the well-known difficulties in lowest order. (A)

176. ''Chaotic Billiards Generated by Arithmetic Groups,'' E. B. Bogomolny, B. Georgeot, M.-J. Giannoni, and C. Schmit, Phys. Rev. Lett. **69**, 1477 (1992). The hidden symmetries of some special triangles in the hyperbolic plane yield the Poisson-type distribution of energy levels just as in integrable systems. (I)

177. ''Prime Formula Weds Number Theory and Quantum Physics,'' B. Cipra, Science **274**, 2014 (December 20, 1996). News report about the significance of Riemann's zeta function for the energy spectrum of a chaotic system. (E)

E. Energy levels and eigenfunctions

1. Articles in scientific journals

178. ''Can One Hear the Shape of a Drum?'' Mark Kac, Am. Math. Monthly **73**, 1–23 (1966). A jewel of a mathematical survey concerning the asymptotic distribution for large wave numbers of the eigenvalues of the Laplacian. Proofs are carried out by appealing to the (heat) diffusion equation, and the first correction to Weyl's law is obtained. Since it depends on the length of the boundary, it tells something about the shape of the domain. (I)

179. ''Distribution of Eigenfrequencies for the Wave Equation in a Finite Domain. I. Three-Dimensional Problem with Smooth Boundary Surface.—II. Electromagnetic Field. Riemannian Spaces.—Asymptotic Evaluation of the Green's Function for Large Quantum Numbers,'' R. Balian and C. Bloch, Ann. Phys. (NY) **60**, 401–447 (1970), **64**, 271–307, 63 (1971), 592–602, Errata **84**, 559–563 (1974). Weyl's law for the asymptotic distribution of eigenvalues at large wave numbers (frequencies, energies) is sharply improved by getting the higher-order corrections in these three pioneering papers. Various geometric and physical conditions are treated, but the general relation with the periodic orbits is not recognized because Green's function is always damped by adding an imaginary part to the wave number in order to even out local fluctuations. (A)

180. ''Distribution of Energy Eigenvalues in the Irregular Spectrum,'' P. Pechukas, Phys. Rev. Lett. **51**, 943 (1983). The eigenvalues on the energy axis are shown to act as so many particles with a repulsive interaction that depends on the off-diagonal elements in the Hamiltonian. (I)

181. ''Characterization of Chaotic Quantum Spectroscopy and Universality of Level Fluctuation Laws,'' O. Bohigas, M.-J. Giannoni, and C. Schmit, Phys. Rev. Lett. **52** 1–4 (1984). The level distribution in the quantum Sinai billiard is consistent with the predictions of the Gaussian orthogonal ensemble which is known from the level distribution in nuclear physics. (E)

182. ''Bound-State Eigenfunctions of Classically Chaotic Hamiltonian Systems: Scars of Periodic Orbits,'' E. J. Heller, Phys. Rev. Lett. **53**, 1515–1518 (1987). Many eigenfunctions of the stadium billiard are found to have large amplitudes along certain classical periodic orbits. (E)

183. ''Adiabatic Switching: A Tool for Semiclassical Quantization and a New Probe of Classically Chaotic Phase Space,'' W. P. Reinhardt, Adv. Chem. Phys. **73**, 925–978 (1989). The very slow change of an external parameter generally maintains a discrete eigenstate, even if its energy is not conserved and there is a transition from regular to chaotic in the classical system. (I)

184. ''Periodic Orbits, Bifurcations, and Quantum Mechanical Eigenfunctions and Spectra,'' M. Founargiotakis, S. G. Farantos, G. Contopoulos, and C. Polymilis, J. Chem. Phys. **91**, 1389–1402 (1989). A fairly complete study of various spectral features in a typical potential of two harmonic oscillators that are coupled by third-order terms. (I)

185. ''You Can't Hear the Shape of a Drum,'' Barry Cipra, Science **255**, 1642 (March 27, 1992). Short Report for the general reader of recent mathematical results showing two different shapes of drums yielding the same spectrum. (E)

186. ''Semiclassical Calculation of Scars for a Smooth Potential,'' D. Provost and M. Baranger, Phys. Rev. Lett. **71**, 662–665 (1993). Bogomolnyi's construction of the wave function in the neighborhood of an unstable periodic orbit is found to be in good agreement with exact results. (I)

187. ''Vibrations of Strongly Irregular or Fractal Resonators,'' B. Sapoval and Th. Gobron, Phys. Rev. E **47**, 3013–3024 (1993). The effect of the boundary smoothness on the distribution of energy levels is explored numerically in order to find any scaling related to fractal dimension. (E)

188. ''Experiments on Not 'Hearing the Shape' of Drums,'' S. Sridhar and A. Kudrolli, Phys. Rev. Lett. **72**, 2175 (1994). Experiments in two different microwave cavities show them to have the lowest 54 resonances of the same frequencies with four-figure accuracy. (E)

189. ''High Orders of the Weyl Expansion for Quantum Billiards; Resurgence of Periodic Orbits, and the Stokes Phenomenon,'' M. V. Berry and C. J. Howls, Proc. R. Soc. London, Ser. A **447**, 527–555 (1994). Weyl's law is vastly expanded by constructing many higher order terms in the number of energy levels $N(E)$ up to the energy E as an asymptotic series in $1/E$. The divergence is shown to be related to the shortest periodic orbit. (A)

190. ''Gaussian Orthogonal Ensemble Statistics in a Microwave Stadium Billiard with Chaotic Dynamics: Porter–Thomas Distribution and Algebraic Decay of Time Correlations,'' H. Alt, H.-D. Gräf, H. L. Harney, R. Hofferbert, H. Lengeler, A. Richter, P. Schardt, and W. H. Weidenmüller, Phys. Rev. Lett. **74**, 62–65 (1995). 950 resonances on a microwave cavity. (E)

191. ''Chaotic Dynamics and GOE-GUE transition,'' O. Bohigas, M.-J. Giannoni, A. M. Ozorio de Almeida, and C. Schmit, Nonlinearity **8**, 203–221 (1995). The transition of the energy spectrum from the Gaussian orthogonal to Gaussian unitary ensemble when time reversal is broken, follows explicitly from the trace formula for the two-point correlation function. (I)

192. ''Wave Packet Propagation, Nonlinear Dynamics, and Constructing Chaotic Eigenstates,'' S. Tomsovic, in **Chaos–The Interplay Between Stochastic and Deterministic Behaviour**, edited by P. Garbaczewski, M. Wolf, and A. Weron, Lecture Notes in Physics (Springer-Verlag, Berlin, 1995), pp. 331–353. Semiclassical wave function depending on time for the stadium billiard yielding very good eigenstates by Fourier analysis. (E)

F. Periodic orbits and the trace formula

1. Articles in scientific journals

193. ''Energy spectrum according to Classical Mechanics,'' M. C. Gutzwiller, J. Math. Phys. **11**, 1791–1806 (1970). The energy levels for integrable systems (particle in a rectangular box and electron in a spherically symmetric potential) are shown to result directly from the classical periodic orbits. (I)

194. ''Periodic Orbits and Classical Quantization Conditions,'' M. C. Gutzwiller, J. Math. Phys. **12**, 343–358 (1971). The trace of Green's

function for an arbitrary system is shown to become a sum over classical periodic orbits in the semiclassical limit, thus yielding what is now called the general trace formula, in particular when the orbits are unstable as in the anisotropic Kepler problem. (I)

195. ''Eigenfrequency Density Oscillations,'' R. Balian and C. Bloch, Ann. Phys. (N.Y.) **69**, 76–160 (1972). The investigation of the oscillations in parallelpipeds and spheres leads to the use of closed polygons, and the qualitative discussion of the classical periodic orbits. But neither their stability, nor the effect of the caustics, nor finally the trace formula are mentioned. (I)

196. ''Semiclassical Quantization of Nonseparable Systems: A New Look at Periodic Orbit Theory,'' W. H. Miller, J. Chem. Phys. **63**, 996–999 (1975). The trace formula is interpreted for the case of a stable periodic orbit in terms of an oscillating degree of freedom in its neighborhood transverse to the periodic orbit. The same idea is proposed by A. Voros in **Géométrie symplectique et Physique Mathématique** (Aix-en, Provence, 1974), Editions du CNRS, Paris, 1975. (I)

197. ''Closed Orbits and the Regular Bound Spectrum,'' M. V. Berry and M. Tabor, Proc. R. Soc. London, Ser. A **349**, 101–123 (1976). The trace formula for the energy levels in an integrable system is discussed more generally but on the same basis as in Ref. 193. (I)

198. ''The Classical Quantization of a Hamiltonian with Ergodic Behavior,'' M. C. Gutzwiller, Phys. Rev. Lett. **45**, 150–153 (1980). The trace formula is shown to agree with Selberg's for a particle on a surface of constant negative curvature, and is used to calculate the energy levels of the anisotropic Kepler problem by adding over the binary code for the classical periodic orbits. (A)

199. ''A Rule for Quantizing Chaos?'' M. V. Berry and J. P. Keating, J. Phys. A **23**, 4839–4849 (1990). The trace formula is rewritten to look like the Riemann zeta function, so that the Riemann–Siegel method for computing can be used. That leads to a drastic reduction of the infinite sum (or product) and suggests a duality between short and long periodic orbits. (A)

200. ''Geometric Properties of Maslov Indices in the Semiclassical Trace Formula for the Density of States,'' S. C. Creagh, J. M. Robbins, and R. G. Littlejohn, Phys. Rev. A **42**, 1907–1922 (1990). The Maslov (or Morse) index, which is important for semiclassical quantization in the presence of caustics, is related to the topology of the stable and unstable manifolds in phase space. (A)

201. ''Quantum Eigenstates from Classical Periodic Orbits,'' G. Tanner, P. Scherer, E. B. Bogomolny, B. Eckhardt, and D. Wintgen, Phys. Rev. Lett. **67**, 2410–2413 (1991). The formula for finding the energy levels from the periodic orbits is technically improved to the point where relatively few orbits yield highly excited states. (E)

202. ''Semiclassical Quantization of Multidimensional Systems,'' E. B. Bogomolny, Nonlinearity **5**, 805–866 (1992). The quantization of an arbitrary system with k degrees of freedom is reduced to a map of a $(k-1)$-dimensional surface on itself, which can be directly expressed in classical terms and leads to the periodic orbits. (A)

203. ''Prebifurcation Periodic Ghost Orbits in Semi-Classical Quantization,'' M. Kus, F. Haake, and D. Delande, Phys. Rev. Lett. **71**, 2167–2171 (1993). Corrections to the trace formula are necessary in the neighborhood of a classical bifurcation in order to smooth out the singularity; these are explained in terms of complex solutions in phase space, and confirmed in the case of the ''kicked top.''

204. ''$\hbar$ Expansion for the Periodic-Orbit Quantization of Hyperbolic Systems,'' P. Gaspard and D. Alonso, Phys. Rev. A **47**, R3468–R3471 (1993). The individual terms in the trace formula are expanded to higher order in $\hbar$; cf. related paper in the special issue of Chaos **3**, 1–12 (1993). (A)

205. ''Analytical Tests of Gutzwiller's Trace Formula for Harmonic-Oscillator Potentials,'' M. Brack and S. R. Jain, Phys. Rev. A **51**, 3462–3468 (1995). The difficulties of the trace formula for the purely harmonic oscillator in more than one dimension are resolved by taking the appropriate limits to deal with the degenerate periodic orbits. (I)

206. ''Periodic Orbits of Nonscaling Hamiltonian Systems from Quantum Mechanics,'' M. Baranger, M. R. Haggerty, B. Lauritzen, D. C. Meredith, and D. Provost, Chaos **5**, 261–270 (1995). The relation between quantal energy levels and classical periodic orbits suggests a quantal energy-versus-period plot that reveals bifurcations and stability transitions. (A)

207. ''Periodic Orbit Quantization beyond the Semiclassical Theory,'' G. Vattay and P. E. Rosenqvist, Phys. Rev. Lett. **76**, 335–339 (1996). Theory with $\hbar$ corrections and comparison with scattering on three circular disks. (A)

208. ''Ray Splitting and Quantum Chaos,'' R. Blümel, T. M. Antonsen, B. Georgeot, E. Ott, and R. E. Prange, Phys. Rev. E **53**, 3284–3302 (1996). The close connection between periodic orbits and energy levels (or resonances) persists even if rays are allowed to split on interfaces. (I)

G. Maps and billiards

1. Articles in scientific journals

209. ''Quantizing a Classically Ergodic System: Sinai's Billiard and the KKR method,'' M. V. Berry, Ann. Phys. (N.Y.) **131**, 163–216 (1981). A pioneering article in the quantum-mechanical treatment of ergodic billiards, including not only the first numerical computations but also a new derivation of the trace formula for billiards of the Sinai type. (A)

210. ''Chaos, Quantum Recurrences, and Anderson Localization,'' S. Fishman, D. R. Grempel, and R. E. Prange, Phys. Rev. Lett. **49**, 509–512 (1982). The periodically kicked rotator is mapped into the motion of a particle through a one-dimensional lattice with a random local potential so that its eigenstates become localized à la Anderson rather than the usual Bloch waves in a regular lattice. This unexpected idea demonstrates that the energy in the quantal kicked rotator does not increase linearly with time as it would classically. (A)

211. ''The Eigenfunctions of Classically Chaotic Systems,'' E. J. Heller, P. W. O'Connor, and J. Gehlen, Phys. Scr. **40**, 354–359 (1989). Several series of consecutive, highly excited states for the stadium billiard demonstrate their nonrandom character, although the system is chaotic classically. (E)

212. ''The Quantized Baker's Transformation,'' N. L. Balazs and A. Voros, Ann. Phys. (N.Y.) **190**, 1–31 (1989). The first successful attempt to quantize the most elementary model of chaos in classical mechanics. (A)

213. ''Classical Structures in the Quantized Baker Transformation,'' M. Saraceno, Ann. Phys. (N.Y.) **199**, 37–60 (1990). The eigenfunctions are represented as probability distributions in phase space where they reveal quite clearly the importance of periodic orbits. (A)

214. ''Simple Models of Quantum Chaos: Spectrum and Eigenfunctions,'' F. M. Izraelev, Phys. Rep. **196**, 299–392 (1990). Everything about the kicked rotator where one has to deal with the quasispectrum of a periodically driven system. (I)

215. ''Semiclassical Dynamics of Chaotic Motion: Unexpected Long-Time Accuracy,'' S. Tomsovic and E. J. Heller, Phys. Rev. Lett. **67**, 664–667 (1991). The quantum mechanical wave function starting in a fixed point at time 0 is constructed with the help of the classical trajectories in the stadium billiard, and the result is found to follow the exact wave for a long time in spite of the classical chaos. (E)

216. ''Periodic Orbits of an Almost Integrable System,'' S. R. Jain and H. D. Parab, J. Phys. A **25**, 6669–6683 (1992). All the periodic orbits are enumerated and classified for the $\pi/3$ rhombus with reflecting walls. (I)

217. ''Extensive Numerical Study of Spectral Statistics for Rational and Irrational Polygonal Billiards,'' A. Shudo and Y. Shimizu, Phys. Rev. E **47**, 54–62 (1993). The spectrum for a particle inside a rhombus-shaped box has a complex dependence on the angle at the vertex. (E)

218. ''Microwave Billiards with Broken Time Reversal Symmetry,'' U. Stoffregen, J. Stein, H.-J. Stoeckmann, M. Kus, and F. Haake, Phys. Rev. Lett. **74**, 2666 (1995). Inserting a ferrite strip unsymmetrically into a microwave cavity destroys the time reversal symmetry of the underlying equations, and therefore changes the statistical properties of the spectrum. (I)

219. ''Exact and Quasiclassical Fredholm Solutions of Quantum Billiards,'' B. Georgeot and R. E. Prange, Phys. Rev. Lett. **74**, 2851–2854 (1995). Although the frequency distribution in vibrating membranes was first established by H. Weyl in 1912 with the help of integral equations, this seems to be the first mathematically correct treatment with the help of the Fredholm method, and yields convergent expressions for the trace formula. (A)

220. ''Quantization of the Three-Dimensional Sinai Billiard,'' H. Primack and U. Smilansky, Phys. Rev. Lett. **74**, 4831 (1995). The first large-scale study of the quantum-mechanical spectrum in a classically chaotic system of three dimensions. (I)

221. ''Microwave Studies of Billiard Green Functions and Propagators,'' J. Stein, H.-J. Stoeckmann, and U. Stoffregen, Phys. Rev. Lett. **75**, 53

(1995). Detailed experimental investigations of the electromagnetic waves that propagate inside a stadium-shaped microwave cavity, both at a given frequency and as function of time. (I)

222. ''Diffractive Orbits in Quantum Billiards,'' N. Pavloff and C. Schmit, Phys. Rev. Lett. **75**, 61–64 (1995). The diffraction of waves in the corners of an enclosure leads to additional periodic orbits that show up in the energy apectrum. (E)

223. ''Studies of Chaotic Dynamics in a Three-Dimensional Superconducting Microwave Billiard,'' H. Alt, H. D. Gräf, R. Hofferbert, C. Rangacharyulu, H. Rehfeld, A. Richter, P. Schardt, and A. Wirzba, Phys. Rev. E **54**, 2303–2312 (1996). The first time such a spectrum in a cavity has been measured; the special interest comes because the semiclassical Green's function in empty three-dimensional space is exact, whereas it is only approximate in the two-dimensional examples so far. (E)

224. ''Playing Billiards with Microwaves—Quantum Manifestations of Classical Chaos,'' A. Richter, in **Emerging Applications of Number Theory**, edited by D. Hejhal, IMA Volumes in Mathematics and its Applications (Springer-Verlag, New York, 1998). This survey article describes the outstanding experimental work on resonances in microwave cavities and their interpretation, which was carried out at the Institute for Nuclear Physics in Darmstadt, Germany. (I)

H. General atomic physics

1. Textbook and monograph

225. **Atoms in Strong Magnetic Fields**, H. Ruder, G. Wunner, H. Herold, and F. Geyer (Springer-Verlag, Berlin, 1994). A general survey with extensive listing of theoretical and experimental results, including a discussion of the relevance of classical mechanics and chaos. (I)

226. **Spectra of Atoms and Molecules**, P. F. Bernath (Oxford U.P., New York, 1995). General introduction with a special chapter on Semiclassical Theory of Photoelectric Detection of Light. (I)

2. Conference proceedings and collections

227. ''Quantum Coherence—30 Years of the Aharonov–Bohm Effect,'' **Conference at the University of South Carolina**, edited by J. S. Anandan (World Scientific, Singapore, 1990). A fine survey of the state-of-the-art concerned with geometric phases, experiments on the AB effect in resistive devices, and philosophical as well as practical issues related to Bell's inequality. (A)

228. **Irregular Atomic Systems and Quantum Chaos**, edited by J.-C. Gay, Comments At. Mol. Phys., volume 25; and as book by Gordon and Breach, Philadelphia, 1992. A fairly complete and up-to-date collection of articles, written for this volume, and centered around the quantum-mechanical few-body problem in atomic physics. The connection with recent high-precision experiments is brought out more than usual. (A)

229. **Classical, Semiclassical and Quantum Dynamics in Atoms**, edited by H. Friedrich and B. Eckhardt (Springer-Verlag, Berlin, 1997). A collection of articles that is dedicated to the memory of Dieter Wintgen, all concentrated on the post-modern use of classical mechanics in the understanding of atomic phenomena. (I)

230. **Chaos in Atomic Physics**, R. Blümel and W. P. Reinhardt (Cambridge U.P., New York, 1997). This textbook starts with some typical examples of classical chaos, and goes on to discuss in detail the main manifestations in atomic physics, especially, in hydrogen and helium as well as scattering in simple molecules including the latest research. (I)

3. Articles in scientific journals

231. ''Phase-Integral Approximation in Momentum Space and the Bound States of an Atom—I and II,'' M. C. Gutzwiller, J. Math. Phys. **8**, 1979–2000 (1967) and **10**, 1004–1020 (1969). A fairly systematic effort to get the semiclassical approximation of the Feynman path integral in different coordinate systems, and apply the results to the energy levels and eigenstates in a spherically symmetric potential; the relation between caustics in classical mechanics and special phases in quantum mechanics is established. (A)

232. ''The Quantization of a Classically Ergodic System,'' M. C. Gutzwiller, Physica D **5**, 183–207 (1982). The energy levels for the anisotropic Kepler problem in two dimensions are calculated by adding over the periodic orbits, including the effect of reflection symmetries. (A)

233. ''Celestial Mechanics on a Microscopic Scale,'' T. Uzer, D. Farelly, J. A. Milligan, P. E. Raines, and J. P. Skelton, Science **253**, 42–53 (July 5, 1991). Introduction to the simplest examples of post-modern atomic physics, with a very useful list of references and notes. (E)

234. ''Quantization of Chaotic Systems,'' G. Tanner and D. Wintgen, Chaos **2**, 53–59 (1992). Review of the trace formula and its technical realizations with application to the anisotropic Kepler problem. (I)

235. ''Paths to Chaos,'' H. Friedrich, Phys. World 32–36 (April 1992). Introduction for newcomers to the chaotic features in atomic physics. (E)

236. ''Cavity Quantum Electrodynamics,'' S. Haroche and J.-M. Raimond, Sci. Am. 54–62 (April 1993). Atoms and photons in small cavities behave completely unlike those in free space. Their quirks illustrate some of the principles of quantum physics and make possible the development of new sensors. (E)

237. ''Resonances and Recurrences in the Absorption Spectrum of an Atom in an Electric Field,'' J. Gao and J. B. Delos, Phys. Rev. A **49**, 869–880 (1994). Since the electric field does not destroy integrability, the old quantization rules apply, and their connection with periodic orbits is used as first pointed out by Gutzwiller, and then by Berry and Tabor. (I)

238. ''The Classical Limit of an Atom,'' M. Nauenberg, C. Stroud, and J. Yeazell, Sci. Am. **270**, 44–49 (June 1994). Pulses of laser light make giant atoms whose properties come from both worlds, classical and quantal (cf. further references to articles by the authors at the end). (E)

239. ''Recurrence Spectroscopy: Observation and Interpretation of Large-Scale Structure in the Absorption Spectra of Atoms in Magnetic Fields,'' J. Main, G. Wiebush, and K. Welge, with J. Shaw and J. B. Delos, Phys. Rev. A **49**, 847–868 (1994). Experiments are analyzed in terms of closed orbits, and are shown to demonstrate their bifurcations as functions of the magnetic field. (I)

240. ''Threshold Ionization of Atoms by Electron and Positron Impact,'' J.-M. Rost, J. Phys. B **28**, 3003–3026 (1995). The total ionization probabilities are obtained by calculating the S matrix semiclassically on the basis of Feynman's path integral, and come out in excellent agreement with experiment. (I)

I. Hydrogen and Rydberg atoms

1. Textbooks and monographs

241. ''Rydberg Atoms,'' T. F. Gallagher, Rep. Prog. Phys. **51**, 143–188 (1988). Review of experiments and simple explanations with a large bibliography. (E)

242. ''The Hydrogen Atom in a Uniform Magnetic Field,'' H. Friedrich and D. Wintgen, Phys. Rep. **183**, 37–79 (1989). Review of the progress in this area by two of the main contributors to the semiclassical theory. (I)

243. ''Classical and Quantal Chaos in the Diamagnetic Kepler Problem,'' H. Hasegawa, M. Robnik, and G. Wunner, Prog. Theor. Phys. Suppl. **98**, 198–286 (1989). A systematic report that starts with the standard result on classical chaos and goes on to describe the latest results in the experiment and theory of the hydrogen atom in a strong magnetic field. (I)

2. Conference proceedings and collections

244. ''Relevance of a Classical Chaos in Quantum Mechanics: The Hydrogen Atom in a Monochromatic Field,'' G. Casati, B. V. Chirikov, D. L. Shepelyansky, and I. Guarneri, Phys. Rep. **154** (2), 77–123 (1987). An exhaustive discussion of the first experiment that demonstrates classically chaotic features in a very simple quantum system. (A)

245. **The Spectrum of the Hydrogen Atom—Advances**, edited by G. W. Series (World Scientific, Singapore, 1988). Several contributions including some recognition of the chaos in a large magnetic field. (A)

246. **The Hydrogen Atom**, edited by G. F. Bassani, M. Inguscio, and T. W. Hänsch (Springer-Verlag, Berlin, 1989). Mostly refinements of experiment and theory, but some recognition of chaos in magnetic fields. (I)

247. ''Atoms in Strong Fields,'' **NATO Advanced Study Institute on the Island of Kos**, Greece, 1988, edited by C. A. Nicolaides, C. W. Clark, and M. H. Nayfeh (Plenum, New York, 1990). An early collection

concerned with the effects of large (dc and ac) electric and magnetic fields on the spectrum of the hydrogen atom, including traps and high-intensity lasers. (I)

3. Articles in scientific journals

248. ''Hydrogen Atom in a strong Magnetic Field,'' Y. Yafet, R. W. Keyes, and E. N. Adams, J. Phys. Chem. Solids **1**, 137–142 (1956). Although this research was carried out during the ''modern'' period, its results are important for understanding the recent experiments. (E)

249. ''Effects of High Magnetic Fields on Electronic States in Semiconductors—The Rydberg Series and the Landau Levels,'' H. Hasegawa, in **Physics of Solids in Intense Magnetic Fields** (Pergamon, New York, 1969), pp. 246–270. A readable introduction to the spectrum of an impurity in a semiconductor with special attention to the effect of strong magnetic fields; cf. Ref. 238. (E)

250. ''Multiphoton Ionization of Highly Excited Hydrogen Atoms,'' J. Bayfield and P. Koch, Phys. Rev. Lett. **53**, 258–261 (1974). The excitation of H atoms from a quantum level in the 1960s by strong microwave fields is interpreted in terms of classical trajectories rather than a (very unlikely) multiphoton process. (I)

251. ''Semiclassical Quantization of Three-Dimensional Quasi-Landau Resonances Under Strong-Field Mixing,'' J. Main, A. Holle, G. Wiebusch, and K. H. Welge, Z. Phys. D **6**, 295–302 (1987). The ''quasi-Landau'' resonances near the ionization limit are explained by the old-fashioned quantization rules that are valid for integrable systems. (E)

252. ''Studies of the Sinusoidally Driven Weakly Bound Atomic Electron in the Threshold Region for Classically Stochastic Behavior,'' J. E. Bayfield, in **Quantum Measurement and Chaos**, edited by E. R. Pike and S. Sarkar (Plenum, New York, 1987). Report on one of the first experiments to demonstrate the symptoms of chaos in a quantum-mechanical system. (I)

253. ''Effects of Closed Orbits on Quantum Spectra: Ionization of Atoms in a Magnetic Field. I. Physical Picture and Calculations; II. Derivation of Formulas,'' M. L. Du and J. B. Delos, Phys. Rev. A **38**, 1896–1912, 1913–1930 (1988). A quite complete theory of the hydrogen spectrum in a magnetic field on the basis of classical closed orbits. (A)

254. ''Symbolic Description of Periodic Orbits for the Quadratic Zeeman Effect,'' B. Eckhardt and D. Wintgen, J. Phys. B **23**, 355–363 (1990). The classically chaotic trajectories for the hydrogen-atom in a magnetic field are fully described by a ternary code. (E)

255. ''Positive-Energy Spectrum of the Hydrogen Atom in a Magnetic Field,'' D. Delande, A. Bommier, and J.-C. Gay, Phys. Rev. Lett. **66**, 141 (1991). (A)

256. ''Long-Period Orbits in the Stark Spectrum of Lithium,'' M. Courtney, H. Jiao, N. Spellmeyer, and D. Kleppner, Phys. Rev. Lett. **73**, 1340–1343 (1994). Recurrence spectra show some very long periodic orbits along the direction of the electric field. (I)

257. ''Closed Orbit Bifurcations in Continuum Stark Spectra,'' M. Courtney, H. Jiao, N. Spellmeyer, D. Kleppner, J. Gao, and J. B. Delos, Phys. Rev. Lett. **74**, 1538–1541 (1995). The bifurcations provide a basis for understanding the spectra as a function of the electric field. (I)

258. ''The Importance of Resonances in Microwave 'Ionization' of Excited Hydrogen Atoms,'' P. M. Koch and K. A. H. van Leuwen, Phys. Rep. **255**, 289–403 (1995). The experiments on the ionization of highly excited H atoms by microwaves are reviewed in detail; they are the first where the effects of chaos in the classical system could be demonstrated. (I)

259. ''Nonstationary, Nondispersive Wave Packets in a Rydberg Atom,'' A. F. Brunello, T. Uzer, and D. Farrelly, Phys. Rev. Lett. **76**, 2874–2877 (1996). Wave functions in a circularly polarized microwave field and a magnetic field yield eigenstates that are scarred by periodic orbits from the classically chaotic system. (I)

260. ''Bifurcation of Periodic Orbits of Hamiltonian Systems: Analysis Using Normal Forms,'' D. A. Sadowski and J. B. Delos, Phys. Rev. E **54**, 2033–2070 (1996). Mathematical study of the periodic orbits in the diamagnetic Kepler problem in view of the spectrum on H atoms where bifurcations can be seen in a lower magnetic field than they occur classically. (A)

J. Helium and planetary atoms

1. Textbook

261. Analytic Perturbation Theory for the Two-Electron Problem—from Perturbation Theory for Linear Operators, T. Kato (Springer-Verlag, New York, 1966), pp. 410–413. The higher-order terms in the perturbation expansion for the eigenvalues of the atomic two-electron problem are shown to converge provided the nuclear charge is larger than 4. (A)

2. Articles in scientific journals

262. ''Planetary Atoms,'' I. C. Percival, Proc. R. Soc. London, Ser. A **353**, 289–297 (1977). General introduction to doubly excited atoms and the most simple-minded explanation of the energy levels in terms of the old quantum theory. (E)

263. ''Observation of Doubly Excited Resonances in the negative H-ion,'' P. G. Harris, H. C. Bryant, A. H. Mohaghegi, R. A. Reeder, and C. Y. Tang, with J. B. Donahue and C. R. Quick, Phys. Rev. A **42**, 6443–6465 (1990). A fascinating experiment where using the relativistic effects in a high-energy beam of negative H ions provides the analog for the more easily available double excitations in a He atom. The theory is done in the old-fashioned way. (I)

264. ''Semiclassical Cycle Expansion for the Helium Atom,'' G. S. Ezra, K. Richter, G. Tanner, and D. Wintgen, J. Phys. B **24**, L413–L420 (1991). The series of doubly excited states is obtained with high accuracy from the classical periodic orbits of the collinear He atom. (I)

265. ''The Semiclassical Helium Atom,'' D. Wintgen, K. Richter, and G. Tanner, Chaos **2**, 19–32 (1992). Review of the semiclassical approach to the ground state of the helium atom. (E)

266. ''Classical Dynamics of s-Wave Helium,'' G. Handke, M. Draeger, and H. Friedrich, Physica A **197**, 113–129 (1993). Simple exposition with pictures of the chaos in the classical motion of the two electrons. (E)

267. ''Photodetachment Cross Section of H^- in Crossed Electric and Magnetic Fields—Closed-Orbit Theory,'' A. D. Peters and S. B. Delos, Phys. Rev. A **47**, 3020–3042 (1993). (I)

268. ''Photoabsorption Spectra of Atoms in Parallel Electric and Magnetic Fields,'' J.-M. Mao, K. A. Rapelje, S. J. Blodgett-Ford, J. B. Delos, A. Koenig, and H. Rinneberg, Phys. Rev. A **48**, 2117–2126 (1993). Experiments on the Ba atoms reveal the development of classical periodic orbits by bifurcation as function of the electric field. (I)

K. Molecular physics

The items in this and some of the following sections came to the attention of the author without any systematic effort on his part to understand all the problems in this area that can be treated with the help of classical mechanics. They are presented here in order to encourage the reader to go to the library of the physics and chemistry departments, and find out the many fields where the quantal nature of physics and chemistry can be illustrated in terms of classical concepts.

1. Textbook

269. Semiclassical Mechanics with Molecular Applications, M. S. Child (Clarendon, Oxford, 1991). A fairly systematic introduction with many examples, but except for the short Appendix D there is no awareness of classical chaos. (I)

2. Conference proceedings and collections

270. Atom-Molecule Collision Theory: A Guide for the Experimentalist, edited by R. S. Bernstein (Plenum, New York, 1974), contains ''Rotational Excitation. III. Classical Trajectory Methods,'' by M. D. Pattengill, pp. 359–376; ''Vibrational Excitation. II. Classical and Semiclassical Methods,'' by W. R. Gentry, pp. 391–426; ''Reactive Scattering Cross-Sections. III. Quasiclassical and Semiclassical Methods,'' by D. G. Truhlar and J. T. Muckerman, pp. 505–566. (I)

271. Dynamics of Molecular Collisions (2 Volumes), edited by W. H. Miller (Plenum, New York, 1976). Part B contains articles on: ''Classical Trajectory Methods in Molecular Collisions,'' by R. N. Proter and L. M. Raff, pp. 1–52; ''Dynamics of Unimolecular Reactions,''

by W. L. Hase, pp. 121–170, concerned with RRKM (Rice–Ramsperger–Kassel–Marcus) theory; ''Semiclassical Methods in Molecular Collision Theory,'' by M. S. Child, pp. 171–216; ''Nonadiabatic Processes in Molecular Collisions,'' by J. C. Tully, pp. 217–258; ''Statistical Approximations in Collision Theory,'' by P. Pechukas, pp. 269–322. (A)

272. **Theory of Chemical Reaction Dynamics** (Vol. III), edited by M. Baer (CRC, Boca Raton, FL, 1985) consists of three large chapters, ''The Classical Trajectory Approach to Reactive Scattering,'' by L. M. Raff and D. L. Thompson; ''Periodic Orbits and the Theory of Reactive Scattering,'' by Eli Pollak; and ''Semiclassical Reactive Scattering,'' by M. S. Child. (A)

273. **Quantum Chemistry and Technology on the Mesoscopic Level**, Conference in Fukui City 1993, edited by Hiroshi Hasegawa, J. Jpn. Phys. Soc. Appl. A **63** (1994). Contributions by mostly Japanese authors concerning classical mechanics and semiclassical quantization, Rydberg atoms, level statistics, and mesoscopic devices. (A)

274. **Molecular Dynamics and Spectroscopy by Stimulated Emission Pumping (SEP)**, edited by H.-L. Dai and R. W. Field (World Scientific, Singapore, 1995). Collection of reviews containing articles on: ''Dynamical Analysis of Highly Excited Vibrational Spectra: Progress and Prospects,'' by M. E. Kellman; ''Vibrational Energy Flow in Rotating Molecules,'' by T. Uzer and T. Carrington Jr.; ''The Extraction of Dynamics from Complex Molecular Spectra in Case where the Classical Motion is Chaotic,'' by H. S. Taylor. (I)

3. Surveys

275. ''Classical-Limit Quantum Mechanics and the Theory of Molecular Collisions,'' W. H. Miller, Adv. Chem. Phys., edited by I. Prigogine and S. A. Rice, **25**, 69–177 (1974). Generally considered the basic text for the semiclassical formalism in molecular collisions. (A)

276. ''Quasiperiodic and Stochastic Behavior in Molecules,'' D. W. Noid, M. L. Koszykowski, and R. A. Marcus, Annu. Rev. Phys. Chem. **32**, 267–309 (1981). Very useful, descriptive review covering classical and semiclassical approaches. (E)

277. ''Theories of Intramolecular Vibrational Energy Transfer,'' T. Uzer with an appendix by W. H. Miller, Phys. Rep. **199**, 73–146 (1991). Extensive review with a large list of related references, emphasizing the connection between classical and quantum, aspects. (I)

278. ''Classical Trajectory Studies of Intramolecular Dynamics: Local Mode Dynamics, Rotation-Vibration Interaction and the Structure of Multidimensional Phase Space,'' G. S. Ezra, in **Advances in Classical Trajectory Methods** (JAI, city, 1992), Vol. 1, pp. 1–40. Survey for specialists. (A)

279. ''Correspondence Principle, Semiclassical Quantization, and Chaos in the Dynamics of Triatomic Molecules,'' F. Borondo, in **Computational Chemistry—Structure, Interactions, and Reactivity**, edited by S. Fraga, Studies in Physical and Theoretical Chemistry A **77**, 592–620 (1992). A general introduction for the beginner using the example of the vibrations in the molecule LiCN. (I)

4. Articles in scientific journals

280. ''A Spectral Analysis Method of Obtaining Molecular Spectra from Classical Trajectories,'' D. W. Noid, M. Koszykowski, and R. A. Marcus, J. Chem. Phys. **67**, 404–408 (1977). The trajectories of an integrable and many-dimensional system are Fourier analyzed, and its natural frequencies are directly used to find its quantum spectrum. (A)

281. ''Dynamical Effects in Unimolecular Decomposition: A Classical Trajectory Study of the Dissociation of C_2H_6,'' E. R. Grant and D. Bunker, J. Chem. Phys. **68**, 628–636 (1978). Comparison with the results based on statistical assumptions in phase space; cf. also J. Santamaria *et al.* in Chem. Phys. Lett. **56**, 170–174 (1978). (I)

282. ''Semiclassical Quantization of the Low Lying Electronic States of H_2^+,'' M. P. Strand and W. P. Reinhardt, J. Chem. Phys. **70**, 3812–3827 (1979). Pauli's Ph.D. thesis (cf. Ref. 42) is finally done correctly, including the phase corrections on caustics. (I)

283. ''Uniform Semiclassical Theory of Avoided Crossings,'' T. Uzer, D. W. Noid, and R. A. Marcus, J. Chem. Phys. **79**, 4412–4425 (1983). (E)

284. ''On rotation and vibration motions of molecules,'' A. Guichardet, Ann. Inst. Henri Poincaré **40**, 329–342 (1984). A mathematical formulation for the basic coupling between rotational and vibrational degrees of freedom in molecules. (I)

285. ''Nonlinear Dynamics of Vibration-Rotation Interactions: Rigid Bender H_2O,'' J. H. Frederick and G. M. McClelland, J. Chem. Phys. **84**, 4347–4363 (1986). Classical-trajectory analysis to find resonances and region of chaos as function of energy and angular momentum. (I)

286. ''Classical Dynamics of intramolecular energy flow and overtone-induced dissociation in HOOH and HOOD,'' T. Uzer, J. T. Hynes, and W. P. Reinhardt, J. Chem. Phys. **85**, 5791–5804 (1986). Classical trajectories are calculated to get dissociation lifetimes following OH local-mode overtone absorption. (I)

287. ''Chaos and Dynamics on .5–300 ps Time Scales in Vibrationally Excited Acetylene: Fourier Transform of Stimulated-Emission Pumping Spectrum,'' J. P. Pique, Y. Chen, R. W. Field, and J. L. Kinsey, Phys. Rev. Lett. **58**, 475 (1987). The very complex, high-resolution spectrum is shown to make a transition from regular to chaotic at 26 500 cm^{-1}. (A)

288. ''Recurrences in the Autocorrelation Function Governing the Ultraviolet absorption spectra of O_3,'' B. R. Johnson and J. L. Kinsey, J. Chem. Phys. **91**, 7638–7653 (1989). The Fourier transform reveals time-dependent features that are associated with periodic orbits. (A)

289. ''Theoretical Methods for the Analysis of Spectra of Highly Vibrationally Excited Polyatomic Molecules,'' F. Borondo, J. M. Gomez, L. Lorente, and R. M. Benito, Laser Chem. **12**, 85–102 (1992). A good general introduction. (E)

290. ''Periodic Orbits and Molecular Photoabsorption,'' O. Zobay and G. Alber, J. Phys. B **26**, L539–L546 (1993). Comparison of exact quantum mechanics with semiclassical calculations for collinear CO_2. (I)

291. ''The Correspondence between Classical Nonlinear Resonances and Quantum Mechanical Fermi Resonances,'' F. L. Roberts and C. Jaffe, J. Chem. Phys. **99**, 2495–2505 (1993). A model is discussed in detail because there is no good understanding as yet, cf. the bibliography. (E)

292. ''Gateway States and Bath States in the Vibrational Spectrum of H_3^+,'' C. Ruth LeSueur, J. R. Henderson, and J. Tennyson, Chem. Phys. Lett. **206**, 429–436 (1993). The correlations in an exceedingly dense spectrum of metastable states are reduced to simple closed orbits. (A)

293. ''Classical Atom-Diatom Scattering: Self-Similarity, Scaling Laws, and Renormalization,'' A. Tiyapan and C. Jaffé, J. Chem. Phys. **99**, 2765–2780 (1993). Study of chaos in a classical model. (I)

294. ''Extraction of Dynamics from the Resonance Structure of HeH_2^+ spectra,'' V. A. Mandelshtam, H. S. Taylor, C. Jung, H. F. Bowen, and D. J. Kouri, J. Chem. Phys. **102**, 7988–8000 (1995). A chemist's search for the relevant mechanical model starting from the experimental spectrum. (I)

L. Nuclear physics and spins

1. Monograph and survey

295. ''Aspects of Chaos in Nuclear Physics,'' O. Bohigas and H. A. Weidenmüller, Annu. Rev. Nucl. Particle Sci. **38**, 421–453 (1988) (I).

296. **Quantum Chaos—A New Paradigm of Nonlinear Dynamics**, K. Nakamura (Cambridge U.P., Cambridge, 1993). In spite of its general title this monograph concentrates on the chaotic motion of small spin systems as well as spin waves in solids. The treatment is quite systematic and complete with many references, but it requires a fair degree of mathematical sophistication. (I)

2. Conference proceedings

297. ''Semiclassical Descriptions of Atomic and Nuclear Collisions,'' **Proceedings of the Niels Bohr Centennial Conference**, Copenhagen, 25–28 March 1985, edited by J. Bang and J. de Boer (North-Holland, Amsterdam, 1985). Remarkably, this interesting collection of articles is completely unaware of modern developments in classical mechanics.

298. ''Quantum Chaos and Statistical Nuclear Physics,'' **Proceedings of a Meeting in Cuernavaca, Mexico 1986**, edited by T. H. Seligman and H. Nishioka (Springer-Verlag, Berlin, 1986). An early collection of contributions with an emphasis on applications in nuclear physics, resonances, shapes and time dependence of wave functions. (A)

3. Articles in scientific journals

299. ''Quantum Tunnelling in the Deformed Region of the LMG Model,'' A. Vourdas and R. F. Bishop, J. Phys. G **11**, 95–101 (1985). Collective monopole vibrations of nuclei as well as ground state correlations are amenable in this approach. (A)

300. ''Nature of Quantum Chaos in Spin Systems,'' G. Müller, Phys. Rev. A **34**, 3345–3355 (1986). (I)
301. ''Friction in Nuclear Dynamics,'' W. J. Swiatecki, **Proceedings of the Symposium on Semiclassical Descriptions of Atomic and Nuclear Collisions**, edited by J. de Boer and J. Bang (North-Holland, Amsterdam, 1986). Friction is associated with the transition to a chaotic regime. (I)
302. ''Integrable and Nonintegrable Classical Spin Clusters—Integrability Criteria and Analytic Structure of Invariants,'' E. Magyari, H. Thomas, R. Weber, C. Kaufman, and G. Mueller, Z. Phys. B **65**, 363–374 (1987). A general introduction to the idea of coupled classical spins, which then will help understanding the behavior of the better known quantum spins. (E)
303. ''Nuclear Dissipation and the Order to Chaos Transition,'' W. J. Swiatecki, Nucl. Phys. A **488**, 375c–393c (1988). Chaos is responsible for the disappearance of the shell effect and for the dissipative behavior of the nucleus. (E)
304. ''Quantum Chaos in a Schematic Shell Model,'' D. C. Meredith, S. E. Koonin, and M. R. Zirnbauer, Phys. Rev. A **37**, 3499–3513 (1988). Comparison of classical criteria for chaos and the distribution of quantum energy levels in this model for nuclear structure. (I)
305. ''Semiclassical Theory of Spin-Orbit Coupling,'' R. G. Littlejohn and W. G. Flynn, Phys. Rev. A **45**, 7697–7217 (1992). The semiclassical treatment of Schrödinger's equation is generalized to include the multicomponent wave functions of particles with spin. (A)
306. ''Semiclassical Periodic Orbit Theory for Identical Particles,'' Hans A. Weidenmüller, Phys. Rev. A **48**, 1819–1823 (1993). The trace formula is generalized to include a system of several particles interacting while they obey the Pauli exclusion principle. (E)
307. ''Semiclassical Analysis of the Supershell Effect in Reflection-Asymmetric Superdeformed Oscillator,'' K.-i. Arita and K. Matsuyanagi, Prog. Theor. Phys. **91**, 723–746 (1994). (I)
308. ''Classical Bifurcation and Enhancement of Quantum Shells: Systematic Analysis of Reflection-Asymmetric Deformed Oscillator,'' K.-i. Arita and K. Matsuyanagi, Nucl. Phys. A **592**, 9–32 (1995). (I)

M. Field theory

Instantons are a special manifestation of the much older solitons that go back to some simple hydrodynamics at the beginning of the nineteenth century. Mathematicians discussed them as solitary waves due to gravity on the interface between water and air. Physicists picked up the idea in the 1950s and 1960s, and it finally entered particle physics with a 1974 paper listed below. Their implicit hope was that elementary particles would be directly related to soliton-like solutions of their classical nonlinear field theories. But Derrick's theorem (cf. below) shows that solitons occur only in $1+1$ dimensions, and, quite generally, they are tied to the possibility of completely integrating the equations of motion, which is a very unlikely circumstance at best (cf. below).

1. Conference proceedings

309. ''The Principles of Instanton Calculus: A Few Applications,'' J. Zinn-Justin, in **Recent Advances in Field Theories and Statistical Mechanics**, edited by J. B. Zuber and R. Stora. Les Houches Lectures XXXIX (Elsevier, Amsterdam, 1984), pp. 39–172. Rather down-to-earth collection of significant examples with explicit calculations that lead to instantons, such as Phi-Four, i.e., the scalar field phi to the fourth power, in various dimensions, the ''false vacuum,'' large-order perturbation theory. (I)

2. Articles in scientific journals

310. ''Comments on Nonlinear Wave Equations as Models for Elementary Particles,'' G. H. Derrick, J. Math. Phys. **5**, 1252–1254 (1964). Scaling argument to show that there are no stable and localized solutions for a large class of equations in two and more spatial dimensions. (E)
311. ''Nonperturbative Methods and Extended-Hadron Models in Field Theory. I. Semiclassical Functional Methods; II. Two-Dimensional Models and Extended Hadron; III. Four-Dimensional Non-Abelian Models,'' R. F. Dashen, B. Hasslacher, and A. Neveu, Phys. Rev. D **10**, 4114–4129, 4130–4136, and 4136–4142 (1974). A first try at generalizing the semiclassical treatment for a system with few degrees of freedom to a field theory. (A)
312. ''Quantum Meaning of Classical Field Theories,'' R. Jackiw, Rev. Mod. Phys. **49**, 681–706 (1977). Review of the then recent work on quantizing the vibrations around the simplest solution of a classical field theory. (I)
313. ''WKB Wave Function for Systems with Many Degrees of Freedom: A Unified View of Solitons and Pseudoparticles,'' J. L. Gervais and B. Sakita, Phys. Rev. D **16**, 3507–3514 (1977). The semiclassical solutions along the imaginary time axis become the solitons and pseudoparticles that are important for vacuum tunneling in field theories. (A)

N. Clusters

1. Articles in scientific journals

314. ''Cooling, Stopping, and Trapping Atoms,'' W. D. Phillips, P. L. Gould, and P. D. Lett., Science **239**, 877–884 (19 February 1988). Using lasers and magnetic traps for the manipulations of single atoms. (E)
315. ''Seeing Chaos in a Simple System,'' R. Pool, Science **241**, 787 (August 12, 1988). The complicated motion of barium ions held in an electromagnetic trap is an example of chaotic behavior. (E)
316. ''Observation of Quantum Supershell in Clusters of Sodium Atoms,'' J. Pedersen, S. Bjornholm, J. Borggreen, K. Hansen, T. P. Martin, and H. D. Rasmussen, Nature (London) **353**, 24 (October 1991). (E)
317. ''Model Nuclei in the Form of Metal Clusters,'' S. Bornholm and J. Pedersen, Nucl. Phys. News **1**, 18–22 (1991). The asymptotic distribution of energy levels in a finite volume is used to explain the occurrence of small clusters in certain sizes and shapes. (I)
318. ''Semiclassical Analysis of the Electronic Shell Structure in Metal Clusters. Influence of Surface Softness on Supershell Structure of Metal Clusters: Application to Gallium,'' J. Lermé, Ch. Bordas, M. Pellarin, B. Baguenard, J. L. Vialle, and Broyer, Phys. Rev. B **48**, 9028–9044 and 12110–12122 (1993). The quantized motion of the itinerant electrons in a cluster determines their sizes and shapes. (I)

O. Statistical mechanics

1. Textbooks and monographs

319. **Introduction to Modern Statistical Mechanics**, David Chandler (Oxford U.P., London, 1987). A short, direct, and excellent introduction to Monte Carlo methods, classical fluids, chemical kinetics, diffusion, fluctuations, and phase transition. (E)

2. Conference proceedings

320. ''Monte Carlo Methods in Quantum Problems,'' **NATO Advanced Research Workshop in Paris 1982**, edited by Malvin H. Kalos (Reidel, Dordrecht, 1982). An early and technically understandable collection of calculations in statistical mechanics, quantum chemistry, solid-state physics, various lattice models including hadrons. (I)

3. Articles in scientific journals

321. ''Influence of Dissipation on Quantum Tunneling in Macroscopic Systems,'' A. O. Caldeira and A. J. Leggett, Phys. Rev. Lett. **46**, 211–214 (1981). Linear coupling of a particle in a double well to an infinite set of harmonic oscillators produces a reduction in the tunneling rate. (A)
322. ''Supersymmetry and the theory of disordered metals,'' K. B. Efetov, Adv. Phys. **32**, 53–127 (1983). The random element now is the distribution of impurities (not the motion due to heat); the modes of the electron diffusion, the distribution of energy levels, and the localization of electron states is investigated with the help of a new approach that is called the supersymmetry method, or also the nonlinear sigma model, and has an obvious connection with classical mechanics. (A)
323. ''Crossover from Thermal Hopping to Quantum Tunneling,'' H. Grabert and U. Weiss, Phys. Rev. Lett. **53**, 1787 (1984). Kramers theory of thermal activation is brought down to lower temperatures where quantum tunneling becomes competitive. (A)
324. ''Grassmann Integration in Stochastic Quantum Physics: The Case of Compound-Nucleus Scattering,'' J. J. M. Verbarschoot, H. A. Weidenmüller, and M. Zirnbauer, Phys. Rep. **129**, 367–438 (1985). An inde-

pendent, and formally somewhat different method that is close to the supersymmetry method, and yields similar results. (A)

325. ''Influence of Friction and Temperature on Coherent Quantum Tunneling,'' U. Weiss, H. Grabert, and S. Linkwitz, J. Low Temp. Phys. **68**, 213–244 (1987). When a quantum particle tunnels through a barrier of an almost symmetric double-well potential, the coherent quantum oscillations are shown to be strongly affected by the interaction with a heat-bath environment. (A)

326. ''Dynamics of the Dissipative Two-State System,'' A. J. Leggett, S. Chakravarty, A. T. Dorsey, M. P. A. Fisher, A. Garg, and W. Zwerger, Rev. Mod. Phys. **59**, 1–85 (1987). The system is modeled by a spin that is coupled to a Boson field, and gets a very detailed treatment. (A)

327. ''Periodic Orbit Approach to the Quantum Kramers Rate,'' P. Hänggi and W. Hontscha, Ber. Bunsenges. Phys. Chem. **95**, 379–385 (1991). Reaction rate in the presence of multidimensional tunneling for the dissipative environment; cf. same authors in J. Chem. Phys. **88**, 4094–4095 (1988). (I)

328. ''Universalities in the spectra of disordered and chaotic systems,'' B. D. Simons and B. L. Altshuler, Phys. Rev. B **48**, 5422–5438 (1993). The supersymmetry method is elevated to even greater generality, and applied to the strong correlation in many-electron systems. (A)

329. ''Orbital Magnetism in Ensembles of Ballistic Billiards,'' D. Ullmo, K. Richter, and R. A. Jalabert, Phys. Rev. Lett. **74**, 383 (1995). The magnetic susceptibility for small two-dimensional structures is calculated in terms of periodic orbits. (E)

330. ''A Semiclassical Approach to Dissipation in Quantum Mechanics,'' F. Grossmann, J. Chem. Phys. **103**, 3696–3704 (1995). The suppression of quantum behavior in an anharmonic oscillator is shown semiclassically to be due to its coupling to many harmonic oscillators (many references to earlier papers along this line). (I)

331. ''Semiclassical Density Matrix near the Top of a Potential Barrier,'' Physica A **223**, 193–213 (1996); ''Exact and Semiclassical Density Matrix of a Particle Moving in a Barrier Potential with Bound States,'' Phys. Rev. E **104**, 7526–7538 (1996), F. J. Weiper, J. Ankerhold, and Hermann Grabert. Path integrals are used at high temperatures which are then lowered to a critical temperature where the fluctuations become large because of a bifurcation in the trajectories and presence of bound states. (I)

P. Tunneling

1. Articles in scientific journals

332. ''Fractal Analysis of Chaotic Tunneling of Squeezed States in a Double-Well Potential,'' H. Dekker, Phys. Rev. A **35**, 1825–1837 (1987). The energy splitting in the almost degenerate states leads to a time dependence like a pathological (Weierstrass) function. (A)

333. ''Classical Chaos versus Quantum Dynamics: KAM Tori and Cantori as Dynamical Barriers,'' G. Radons, T. Geisel, and T. Rubner, Adv. Chem. Phys. **73**, 891–923 (1989). Although barriers in phase space can prevent classical trajectories from becoming completely ergodic, examples show how quantum dynamics overcomes this restriction by tunneling. (I)

334. ''Traversal Reflection and Dwell Time for Quantum Tunneling,'' M. Büttiker, in **Electronic Properties of Multilayers and Low-Dimensional Semiconductor Structures**, edited by J. M. Chamberlain *et al.* (Plenum, New York, 1990). (E)

335. ''Chaotic Quantum Phenomena without Classical Analog,'' G. Jona-Lasinio, C. Presilla, and F. Capasso, Phys. Rev. Lett. **68**, 2269 (1992). The many-electron wave function in a one-dimensional two-barrier potential, treated in Hartree–Fock approximation, yields chaotic motion for some observables such as the charge between the two barriers. (I)

336. ''Quantum Tunneling and Regular and Irregular Quantum Dynamics of a Driven Double-Well Oscillator,'' W. A. Lin and L. E. Ballentine, Phys. Rev. A **45**, 3637–3645 (1992). Following the motion of a Gaussian wave packet. (I)

337. ''Manifestations of Classical Phase Space Structures in Quantum Mechanics,'' O. Bohigas, S. Tomsovic, and D. Ullmo, Phys. Rep. **223**, 43–133 (1993). The division of the quantum spectrum into regular and chaotic components corresponding to classical phase space is refined, and put on a physical footing, including the tunneling through different regions. (I)

338. ''Barrier Interaction Time in Tunneling,'' R. Landauer and Th. Martin, Rev. Mod. Phys. **66**, 217–228 (1994). Former theories on this subject are reviewed; measuring an extra degree of freedom for the tunneling particle is recommended as the physically most relevant concept. (E)

339. ''Wentzel–Kramers–Brillouin Theory of Multidimensional Tunneling: General Theory for Energy Splitting,'' S. Takada and H. Nakamura, J. Chem. Phys. **100**, 98–113 (1994). The treatment of tunneling in more than one dimension, though still open, is studied in the context of molecular dissociations and intramolecular conversions. (I)

340. ''Chaos-Induced Avoided Level Crossing and Tunneling,'' M. Latka, P. Grigolini, and B. J. West, Phys. Rev. A 50, 1071–1081 (1994). Floquet states in the driven pendulum are investigated. (I)

341. ''Magnetotunneling Spectroscopy of a Quantum Well in the Regime of Classical Chaos,'' T. M. Fromhold, L. Eaves, F. W. Theard, M. L. Leadbeater, T. J. Foster, and P. C. Main, Phys. Rev. Lett. **72**, 2608–2611 (1994). Resonances in the current-voltage measurements are related to unstable closed orbits. (E)

342. ''Tunneling versus Chaos in the Kicked Harper Model,'' R. Roncaglia, L. Bonci, F. West, B. J. West, and P. Grigolini, Phys. Rev. Lett. **73**, 802 (1994). Tunneling between the two islands depends on the area of chaotic phase space; complications arise when its size equals Planck's quantum. (I)

343. ''Chaotic Landau Level Mixing in Classical and Quantum Wells,'' D. L. Shepelyanski and A. D. Stone, Phys. Rev. Lett. **74**, 2098–2011 (1995). A charged particle moves through a barrier in an electric field normal to the barrier and a magnetic field at an arbitrary angle. (I)

344. ''Transition State Theory for Ballistic Electrons,'' B. Eckhardt, J. Phys. A **28**, 3469–3475 (1995). Use of the RRKM theory from chemical physics to get the conductance including tunneling and magnetic field. (E)

Q. Mesoscopic systems

1. Collections and surveys

345. **Mesoscopic Phenomena in Solids**, edited by B. L. Altshuler, P. A. Lee, and R. A. Webb (North-Holland, Amsterdam, 1991). Collection of articles written by the leading experts including the Aharonov–Bohm effect in gold loops, electric currents in disordered structures, conduction fluctuations, and correlation involving single electrons or pairs. (I)

346. ''Quantum Transport in Small Disordered Samples from the Diffusive to the Ballistic Regime,'' S. Washburn and R. A. Webb, Rep. Prog. Phys. **55**, 1311–1383 (1992). Both quantum confinement and quantum interference in impurity scattering perturb the classical Drude conductance in small samples at low temperatures. (I)

347. ''Mesoscopic Physics,'' special issue edited by G. Jona-Lasinio, J. Math. Phys. **37**, 4773–5268 (October 1996). About 20 articles ranging from highly abstract to fairly down-to-earth for many different experimental arrangements by some of the most active groups in this area. (A)

2. Conference proceedings

348. ''Quantum Coherence in Mesoscopic Systems,'' **NATO Advanced Study Institute in Les Arcs, France 1990**, edited by B. Krenner (Plenum, New York, 1991). One of the first gatherings of the experts in the area of conductance fluctuations, persistent currents, localization of electron wave functions, all under the general roof of quantum chaos. (I)

349. ''Physics of Nanostructures,'' **NATO ASI in St. Andrews, Scotland 1991**, edited by J. H. Davies and A. R. Long [Scottish Universities Summer School in Physics (SUSSP), Edinburgh, 1992]. Covers the formation of low-dimensional heterostructures, coherent quantum transport, and semiclassical motion in conductors, and tunnel junctions. (I)

350. ''Nanostructures and Mesoscopic Systems,'' **Proceedings of an International Symposium in Santa Fe, New Mexico, 1991**, edited by W. P. Kirk and M. A. Reed (Academic, New York, 1992). Many articles on ballistic transport, coherence, tunneling, Coulomb blockade, large arrays of electric and optical devices. (I)

351. ''Quantum Fluctuations in mesoscopic and macroscopic systems,'' **Adriatico Research Conference**, Trieste, 1990, edited by H. A. Cerdeira, F. Guinea Lopez, and U. Weiss (World Scientific, Singapore, 1991). Collection of articles on quantum transport, periodic orbits, path integrals, various devices such as junctions, rings, and arrays. (I)

352. **Single Charge Tunneling**, Special Issue of Z. Phys. B **85**, 317–467 (1991), edited by H. Grabert and H. Horner. Mostly up-to-date experiments and some theory concerning the observation of a single electronic charge tunneling into and out of a small device. (I)

353. **Quantum Complexity in Mesoscopic Systems**, Papers from the conference at the Center for Nonlinear Studies at Los Alamos, New Mexico 1994, edited by Alan R. Bishop, Robert E. Ecke, and Ronnie Mainieri, Physica D **83** (1995). The quantum manifestations of classical chaos are presented along with the recent results from the experiments on mesoscopic systems, going from the question of decoherence to the quantum Hall effect, and including the quantum corrals on the surface of a copper crystal. (I)

354. ''Quantum Dynamics of Submicron Structures,'' **NATO ASI in Trieste, Italy, 1994**, edited by Hilda A. Cerdeira, Bernhard Kramer, and Gerd Schoen (Kluwer, Dordrecht, 1995). A diverse collection of articles on mesoscopic fluctuations, quantum Hall states, quantum dots, ac effects, superconductors, tunneling and transport phenomena. (I)

355. ''Mesoscopic Quantum Physics,'' **1994 Les Houches Summer School**, edited by E. Akkermans, G. Montambaux, and J. L. Pichard (Elsevier, Amsterdam, 1995). Some surveys and some contributed articles concerned with localization and transport in small two-dimensional quasimetallic samples, cf. in particular the surveys by Altshuler and Simons, Stone, and Smilansky. (I)

3. Articles in scientific journals

356. ''Quenching of the Hall Effect,'' C. W. J. Beenakker and H. van Houten, Phys. Rev. Lett. **60**, 2406–2409 (1988). Ballistic transport through microscopic four-terminal structure for measuring the Hall effect leads to quenching in sufficiently strong magnetic fields. (E)

357. ''Conductance Fluctuations in the Ballistic Regime: A Probe for Quantum Chaos?'' R. A. Jalabert, H. U. Baranger, and A. D. Stone, Phys. Rev. Lett. **65**, 2442–2445 (1990). The geometric features in a ballistic conductor cause random-looking fluctuations. (E)

358. ''Classical and Quantum Ballistic-Transport Anomalies in Microjunctions,'' H. U. Baranger, D. P. diVincenzo, R. A. Jalabert, and A. D. Stone, Phys. Rev. B **44**, 10637–10675 (1991). Extreme sensitivity to the geometry of the junctions is found in both classical and quantum calculations. (I)

359. ''Chaotic Scattering, Unstable Periodic Orbits, and Fluctuations in Quantum Transport,'' R. V. Jensen, Chaos **1**, 101–109 (1991). Early review of this area with some suggestions. (E)

360. ''Science at the Atomic Scale,'' P. Ball and L. Garwin, Nature (London) **355**, 761–766 (1992). A review for the outsider with special attention to the fabrication of the new devices. (E)

361. ''Next Electron, Please...,'' K. Harmans, Phys. World 50–53 (March 1992). Applying certain voltages to a simple nanoelectronic circuit does not always cause a flow of charge yet offers the possibility of a new quantum standard for current. (E)

362. ''Statistical Theory of Coulomb Blockade Oscillations: Quantum Chaos in Quantum Dots,'' R. A. Jalabert, A. D. Stone, and Y. Alhassid, Phys. Rev. Lett. **68**, 3468–3471 (1992). A numerical test of random matrix theory. (A)

363. ''Conductance Fluctuations and Chaotic Scattering in Ballistic Microstructures,'' C. M. Marcus, A. J. Rimberg, R. M. Westervelt, P. M. Hopkins, and A. C. Gossard, Phys. Rev. Lett. **69**, 506–509 (1992). A marvelous set of experiments with very intriguing results. (I)

364. ''Quantum Dots,'' M. A. Reed, Sci. Am. 118–123 (January 1993). Nanotechnologists can now confine electrons to point-like structures. Such ''designer atoms'' may lead to new electronic and optical devices. (E)

365. ''Quantized Periodic Orbits in Large Antidot Arrays,'' D. Weiss, K. Richter, A. Menschig, R. Bergmann, H. Schweitzer, and K. von Klitzing, Phys. Rev. Lett. **70**, 4118–4121 (1993). Magnetoresistance oscillations are attributed to certain periodic orbits rather than the Aharonov–Bohm effect. (I)

366. ''When Electrons Go with the Flow,'' P. Main, New Sci. 30–33 (12 June 1993). Remove the obstacles that create electrical resistance, and you get ballistic electrons and a quantum surprise. (E)

367. ''Imaging Standing Waves in a Two-dimensional Electron Gas,'' M. F. Crommie, C. P. Lutz, and D. M. Eigler, Nature (London) **363**, 524–527 (1993). A scanning tunneling microscope (STM) on a Cu (111) surface reveals standing wave patterns of the electrons in surface states. (E)

368. ''Quantum-Chaotic Scattering Effects in Semiconductor Microstructures,'' H. U. Baranger, R. A. Jalabert, and A. D. Stone, Chaos **3**, 665–682 (1993). Detailed discussion of many different examples. (A)

369. ''Weak Localization in Chaotic versus Non-chaotic Cavities: A Striking Difference in the Line Shape,'' A. M. Chang, H. U. Baranger, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **73**, 2111–2114 (1994). Experimental evidence for Lorentzian versus linear decrease in resistance with magnetic field according as the shape of conductor. (I)

370. ''Precursors and Transitions to Chaos in a Quantum Well in a Tilted Magnetic Field,'' G. Müller, G. S. Boebinger, H. Mathur, L. N. Pfeiffer, and K. West, Phys. Rev. Lett. **75**, 2875–2878 (1995). Peak doublings and triplings in the I-V characteristic as the magnetic tilt increases correspond to bifurcations of periodic orbits in the transition region to classical chaos. (I)

371. ''Non-Gaussian Distribution of Coulomb Blockade Peak Heights in Quantum Dots,'' A. M. Chang, H. U. Baranger, L. N. Pfeiffer, K. W. West, and T. Y. Chang; and ''Statistics and Parametric Correlations of Coulomb Blockade Fluctuations in Quantum Dots,'' J. A. Folk, S. R. Patel, S. F. Godijn, A. G. Huibers, S. M. Cronewett, and C. M. Marcus, Phys. Rev. Lett. **76**, 1695–1698 and 1699–1702 (1996). Numerical tests of random matrix theory are shown to be consistent with quantum chaos in weakly coupled, closed systems in zero and nonzero magnetic fields.

R. Quantum optics

1. Textbooks and monographs

372. ''Chaos in Quantum Optics,'' J. R. Ackerhalt, P. W. Milonni, and M. L. Shih, Phys. Rep. **128**, 205–300 (1985). An early introduction to the nonlinear behavior of lasers that leads inevitably to classical chaos; with many practical examples. (E)

373. **Elements of Quantum Optics** (2nd ed.), P. Meystre and M. Sargent III (Springer-Verlag, Berlin, 1990). Readable textbook for graduate students willing to face some algebra, covering wide variety of topics from bistability to squeezed states. (I)

374. **Optical Coherence and Quantum Optics**, L. Mandel and E. Wolf (Cambridge U.P., Cambridge, 1995). The most up-to-date and complete textbook in this area, explaining all the mathematical details very explicitly, e.g., Chap. 9 on Semiclassical Theory of Photoelectric Detection of Light. (A)

2. Collections

375. ''Instabilities and Chaos in Quantum Optics,'' edited by F. T. Arecchi and R. G. Harrison (Springer-Verlag, Berlin, 1987). Phenomena in Active and Passive Systems, e.g., ring lasers, optical ''turbulence,'' and pulsations in resonators. (I)

376. ''Cavity Quantum Electrodynamics,'' edited by P. R. Berman (Academic, Boston, 1994). Collection of articles on the state-of-the-art: atoms in a cavity, one-electron emission by moving atoms, optical resonator, Casimir effect, micro-masers. (I)

3. Articles in scientific journals

377. ''Ray and Wave Chaos in Asymmetric Resonant Optical Cavities,'' J. U. Nöckel and A. D. Stone, Nature (London) (1997) 45–47, 385. The output of light waves from a cavity with an oval (rather than circular) shape is explained in terms of chaos in the classical ray optics. (E)

S. The interplay between classical and quantum mechanics

List of titles for a Reprint Book from the Resource Letters in AJP

30. ''On the Quantization Condition of Sommerfeld and Epstein,'' Albert Einstein, translation by Charles Jaffe of the article in Verhandlungen der Deutschen Physikalischen Gesellschaft **19**, 82–92 (1917), Report No. 116 (1980) of the Joint Institute for Laboratory Astrophysics (JILA). (I)

129. ''Quantum Chaology,'' M. V. Berry, The Bakerian Lecture 1987, Proc. R. Soc. London, Ser. A **413**, 183–198 (1987); also published separately in **Dynamical Chaos**, edited by M. V. Berry, I. C. Percival, and N. O. Weiss (Princeton U.P., Princeton, NJ, 1987). (I)

137. ''The Many Faces of Quantum Chaos,'' Asher Peres, in a Special Issue edited by K. Nakamura, Chaos, Solitons Fractals **5** (7), 1069–1075 (1995), also published as a separate volume by Pergamon. (I)

148. ''Quantum Chaos,'' M. C. Gutzwiller, Sci. Am. **266**, 78–84 (January 1992). (E)

153. ''Postmodern Quantum Mechanics,'' E. Heller and S. Tomsovic, Phys. Today, 38–46 (July 1993). (E)

182. ''Bound-State Eigenfunctions of Classically Chaotic Hamiltonian Systems: Scars of Periodic Orbits,'' E. J. Heller, Phys. Rev. Lett. **52**, 1515–1518 (1987). (E)

233. ''Celestial Mechanics on a Microscopic Scale,'' T. Uzer, D. Farrelly, J. A. Milligan, P. E. Raines, and J. P. Skelton, Science **253**, 42–52 (1991). (E)

234. ''Quantization of Chaotic Systems,'' G. Tanner and D. Wintgen, Chaos **2**, 53–59 (1992), in Chaos Focus Issue on Periodic Orbit Theory, edited by P. Cvitanovic, **2** (1), 1–156 (1992). (I)

251. ''Semiclassical Quantization of Three-Dimensional Quasi-Landau Resonances Under Strong-Field Mixing,'' J. Main, A. Holle, G. Wiebush, and K. H. Welge, Z. Phys. D **6**, 295–302 (1987). (E)

257. ''Closed Orbit Bifurcations in Continuum Stark Spectra,'' M. Courtney, H. Jiao, N. Spellmeyer, D. Kleppner, J. Gao, and J. B. Delos, Phys. Rev. Lett. **74**, 1538–1541 (1995). (I)

265. ''The Semiclassical Helium Atom,'' D. Wintgen, K. Richter, and G. Tanner, Chaos **2**, 18–32 (1992), same issue as 134. (E)

280. ''A Spectral Analysis Method of Obtaining Molecular Spectra from Classical Trajectories,'' D. W. Noid, M. Koszykowski, and R. A. Marcus, J. Chem. Phys. **67**, 404–408 (1977). (I)

290. ''Periodic Orbits and Molecular Photoabsorption,'' O. Zobay and G. Alber, J. Phys. B **26**, L539–L546 (1993). (I)

306. ''Semiclassical Periodic Orbit Theory for Identical Particles,'' H. A. Weidenmueller, Phys. Rev. A **48**, 1819–1823 (1993). (A)

317. ''Model Nuclei in the Form of Metal Clusters,'' S. Bjornholm and J. Pedersen, Nucl. Phys. News **1** (6), 18–22 (1991). (I)

329. ''Orbital Magnetism in Ensembles of Ballistic Billiards,'' D. Ullmo, K. Richter, and R. A. Jalabert, Phys. Rev. Lett. **74**, 383–386 (1995). (E)

363. ''Conductance Fluctuations and Chaotic Scattering in Ballistic Microstructures,'' C. M. Marcus, A. J. Rimberg, R. M. Westervelt, P. M. Hopkins, and A. C. Gossard, Phys. Rev. Lett. **69**, 506–509 (1992). (I)

365. ''Quantized Periodic Orbits in Large Antidot Arrays,'' D. Weiss, K. Richter, A. Menschig, R. Bergmann, H. Schweitzer, and K. von Klitzing, Phys. Rev. Lett. **70**, 4118–4121 (1993).