

Assignment 2

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Download all python codes from

<https://github.com/ImNamitaKumari/Probability-and-Random-Variables/blob/main/Assignment1/codes/Assignment2.py>

and latex-tikz codes from

<https://github.com/ImNamitaKumari/Probability-and-Random-Variables/blob/main/Assignment1/Assignment2.tex>

1 PROBLEM

62) Suppose the random variable U has uniform distribution on $[0,1]$ and $X = -2 \log U$. The density of X is:

- (A) $f(x) = \begin{cases} e^{-x} & x > 0 \\ 0 & \text{otherwise} \end{cases}$
- (B) $f(x) = \begin{cases} 2e^{-2x} & x > 0 \\ 0 & \text{otherwise} \end{cases}$
- (C) $f(x) = \begin{cases} \frac{1}{2}e^{-\frac{x}{2}} & x > 0 \\ 0 & \text{otherwise} \end{cases}$
- (D) $f(x) = \begin{cases} \frac{1}{2} & x \in [0, 2] \\ 0 & \text{otherwise} \end{cases}$

2 SOLUTION

U has a uniform distribution in $[0,1]$.

$$\Rightarrow \text{PDF}(U) = f_U(x) = k, k \in \mathbf{N}, x \in [0, 1] \quad (2.0.1)$$

Now,

$$\int_0^1 f_U(x) dx = 1 \quad (2.0.2)$$

$$\Rightarrow \int_0^1 k dx = 1 \quad (2.0.3)$$

$$\Rightarrow k[x]_0^1 = 1 \quad (2.0.4)$$

$$\Rightarrow k = 1 \quad (2.0.5)$$

$$\Rightarrow f_U(x) = 1, \forall x \in [0, 1] \quad (2.0.6)$$

$$X = -2 \log U, U \in [0, 1] \quad (2.0.7)$$

$$\Rightarrow X \in [0, \infty) \quad (2.0.8)$$

CDF of X will be given by,

$$F_X(x) = \Pr(X \leq x), (x \in [0, \infty)) \quad (2.0.9)$$

$$(2.0.10)$$

Here, from (2.0.8),

$$x \in [0, \infty) \Rightarrow e^{-\frac{x}{2}} \in [0, 1] \quad (2.0.11)$$

$$\begin{aligned} \Rightarrow F_X(x) &= \Pr(-2 \log U \leq x) = \Pr(U \geq e^{-\frac{x}{2}}) \\ &= 1 - \Pr(U \leq e^{-\frac{x}{2}}) \\ &= 1 - F_U(e^{-\frac{x}{2}}) \end{aligned} \quad (2.0.12)$$

We know that,

$$\text{PDF} = f(x) = \frac{d[F(x)]}{dx} \quad (2.0.13)$$

From (2.0.11), x lies in the domain of values of X and $e^{-\frac{x}{2}}$ lies in domain in the values of U . Differentiating both sides of (2.0.12), we get

$$\frac{d[F_X(x)]}{dx} = -\frac{d[F_U(e^{-\frac{x}{2}})]}{dx} \quad (2.0.14)$$

$$\Rightarrow f_X(x) = -f_U(e^{-\frac{x}{2}}) \left(\frac{de^{-\frac{x}{2}}}{dx} \right) \quad (2.0.15)$$

From (2.0.6),

$$f_X(x) = -1 \times (e^{-\frac{x}{2}}) \left(-\frac{1}{2} \right) = \frac{1}{2} e^{-\frac{x}{2}}, x \in [0, \infty) \quad (2.0.16)$$

$$\text{Hence, } f_X(x) = \begin{cases} \frac{1}{2} e^{-\frac{x}{2}} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

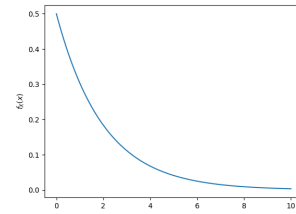


Fig. 0: PDF of random variable X