

Assignment 3

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Download all latex-tikz codes from

<https://github.com/ImNamitaKumari/Probability-and-Random-Variables/blob/main/Assignment3/Assignment3.tex>

1 PROBLEM

Let $P(E)$ denote the probability of the event E . Given $P(A)=1$, $P(B)=\frac{1}{2}$, the values of $P(A|B)$ and $P(B|A)$ respectively are

- 1) $\frac{1}{4}, \frac{1}{2}$
- 2) $\frac{1}{2}, \frac{1}{4}$
- 3) $\frac{1}{2}, 1$
- 4) $1, \frac{1}{2}$

2 SOLUTION

Let $X \in \{0,1\}$ be the random variable denoting the non-occurrence or occurrence of event A and $Y \in \{0,1\}$ be the random variable denoting the non-occurrence or occurrence of event B .

| X | $\Pr(X)$ | Y | $\Pr(Y)$ |
|-----|----------|-----|---------------|
| 0 | 0 | 0 | $\frac{1}{2}$ |
| 1 | 1 | 1 | $\frac{1}{2}$ |

TABLE I: Random Variables and their probabilities

As evident from the venn diagram,

$$\Pr(\{X = 1\}\{Y = 1\}) = P(AB) = \frac{1}{2} \quad (2.0.1)$$

$$\begin{aligned} P(A|B) &= \Pr(X = 1|Y = 1) = \frac{\Pr(\{X = 1\}\{Y = 1\})}{\Pr(Y = 1)} \\ &= \frac{\frac{1}{2}}{\frac{1}{2}} = 1 \end{aligned} \quad (2.0.2)$$

$$\begin{aligned} P(B|A) &= \Pr(Y = 1|X = 1) = \frac{\Pr(\{Y = 1\}\{X = 1\})}{\Pr(X = 1)} \\ &= \frac{\frac{1}{2}}{1} = \frac{1}{2} \end{aligned} \quad (2.0.3)$$

Hence the correct answer is option 4).

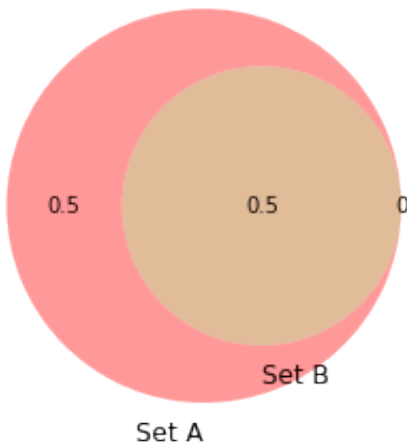


Fig. 0: Venn diagram