Assignment 2

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Download all python codes from

https://github.com/ImNamitaKumari/Probability-and-Random-Variables/blob/main/Assignment1/codes/Assignment2.py

and latex-tikz codes from

https://github.com/ImNamitaKumari/Probability-and-Random-Variables/blob/main/Assignment1/Assignment2.tex

1 Problem

62) Suppose the random variable U has uniform distribution on [0,1] and $X = -2\log U$. The density of X is:

(A) $f(x) = \begin{cases} e^{-x} & x > 0 \\ 0 & \text{otherwise} \end{cases}$

(B) $f(x) = \begin{cases} 2e^{-2x} & x > 0 \\ 0 & \text{otherwise} \end{cases}$

(C) $f(x) = \begin{cases} \frac{1}{2}e^{-\frac{x}{2}} & x > 0 \\ 0 & \text{otherwise} \end{cases}$

(D) $f(x) = \begin{cases} \frac{1}{2} & x \in [0,2] \\ 0 & \text{otherwise} \end{cases}$

2 Solution

U has a uniform distribution in [0,1].

$$\implies \text{PDF(U)} = f_U(x) = k, k \in \mathbf{N}, x \in [0,1] \tag{2.0.1}$$

Now,

$$\int_0^1 f_U(x)\, dx = 1 \tag{2.0.2}$$

$$\implies \int_0^1 k\, dx = 1 \tag{2.0.3}$$

$$\implies k[x]_0^1 = 1 \tag{2.0.4}$$

$$\implies k = 1 \tag{2.0.5}$$

$$\implies f_U(x) = 1, \forall x \in [0,1] \tag{2.0.6}$$

CDF of X will be given by,

$$F_X(x) = \Pr(X \le x), (x \in [0,\infty)) \tag{2.0.7}$$

$$\implies e^{-\frac{x}{2}} \in [0,1] \tag{2.0.8}$$

$$\begin{aligned}\implies F_X(x) = \Pr(-2\log U \le x) &= \Pr\left(U \ge e^{-\frac{x}{2}}\right) \\ &= 1 - \Pr\left(U \le e^{-\frac{x}{2}}\right) \\ &= 1 - F_U(e^{-\frac{x}{2}}) \end{aligned} \tag{2.0.9}$$

We know that,

$$\text{PDF} = f(x) = \frac{d[\text{F}(x)]}{dx} \tag{2.0.10}$$

Differentiating both sides of (2.0.9), we get

$$\frac{d[\text{F}_X(x)]}{dx} = -\frac{d[\text{F}_U(e^{-\frac{x}{2}})]}{dx} \tag{2.0.11}$$

$$\implies f_X(x) = -f_U(e^{-\frac{x}{2}})\left(\frac{de^{-\frac{x}{2}}}{dx}\right) \tag{2.0.12}$$

From (2.0.8) and (2.0.6),

$$f_X(x) = -1 \times \left(e^{-\frac{x}{2}}\right)\left(-\frac{1}{2}\right) = \frac{1}{2}e^{-\frac{x}{2}}, x \in [0,\infty) \tag{2.0.13}$$

Hence, $f_X(x) = \begin{cases} \frac{1}{2}e^{-\frac{x}{2}} & x > 0 \\ 0 & \text{otherwise} \end{cases}$

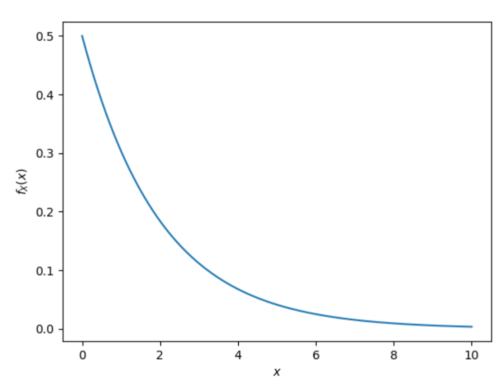

Fig. 0: PDF of random variable X