
- This assignment is **due in Week 13 on Wednesday 2 November, 11:59pm** on Gradescope.
- All work must be **done individually** without consulting anyone else's solutions in accordance with the University's "Academic Dishonesty and Plagiarism" policies.
- You will be evaluated not just on the correctness of your answers, but on your ability to present your ideas clearly and logically. **You should always explain how you arrived at your answer unless explicitly asked not to do so.** Your goal should be to convince the person reading your work that your answers are correct and your methods are sound.
- For clarifications, input formats, and more details on all aspects of this assignment (e.g., level of justification expected, late penalties, repeated submissions, what to do if you are stuck, etc.) you are expected to regularly monitor the Ed Forum post "Assignment FAQ".

Consider the domain of Halloween creatures with the following predicates:

- $G(x)$ means that x is a ghost.
- $Z(x)$ means that x is a zombie.
- $S(x)$ means that x is scary.
- $A(x, y)$ means that x is afraid of y.

This will be used in the first three problems.

Problem 1. (5 marks) The Halloween creatures want to scare people who have not taken this course by expressing statements about themselves in predicate logic.

1. There is a scary ghost.
2. Somebody is afraid of everything.
3. For every scary zombie, some ghost is afraid of them.
4. If everybody is afraid of a ghost, that ghost is scary.

No additional explanation is needed.

Problem 2. (10 marks) The creatures are developing *Logical Principles of Unnatural Philosophy*, a book that contains laws about monsters that are true in all possible universes. However, they need help determining which laws are valid or not.

Show the following using the equivalence laws taught in this course.

1. $\exists x(G(x) \rightarrow S(x)) \equiv \exists x \exists y(S(y) \vee \neg G(x))$
2. $\forall x S(x) \leftrightarrow \exists x S(x) \equiv \forall x(S(x) \rightarrow \forall y S(y))$

Problem 3. (10 marks) The proof-readers of the book are skeptical about two of the claimed equivalences. Show the following by providing, for each one, a counterexample over a finite domain.

1. $\forall x(\exists y A(y,x) \rightarrow S(x)) \not\equiv \forall x \exists y(A(y,x) \rightarrow S(x))$
2. $\forall x \exists y \forall z(A(x,y) \wedge A(y,z)) \not\equiv \exists x \forall y(A(x,y) \wedge A(y,x))$

Problem 4. (25 marks, 5/5/5/10) Prove the following consequents using the most horrifying thing of all, natural deduction.

1. $\forall x(P(x) \rightarrow \neg Q(x)),\ \forall x(R(x) \rightarrow P(x)) \ \vdash\ \forall x(Q(x) \rightarrow \neg R(x))$
2. $\forall x(P(x) \vee Q(y)) \vdash \forall x P(x) \vee Q(y)$
3. $\exists y \exists z \forall x(P(x,y) \vee P(x,z)) \ \vdash\ \forall x \exists y P(x,y)$
4. $$\begin{aligned}&\forall x \forall y \forall z((P(x,y) \wedge P(y,z)) \rightarrow P(x,z)),\\ &\exists x \exists y(P(x,y) \wedge \forall z(P(z,x) \wedge P(y,z))) \vdash \forall x \forall y P(x,y)\end{aligned}$$

Problem 5. (30 marks, 10/10/10) Let $\Sigma = \{a, b\}$. For each of the following languages, provide a Context-free Grrrrrammar that generates it:

1. $L(a^*b^*a^*)$.
2. $\{a^{2n+1}b^n : n \geq 0\}$.
3. $\{x : 2|x|_b \leq |x|_a \leq 3|x|_b\}$. Here $|x|_a$ is the number of a's in x, and $|x|_b$ is the number of b's in x. For instance $\epsilon, baa, aaab, ababaaa, abaaabbaabaaa$ are in the language while $a, b, aabaa, aaabb, ababababaaa$ are not in the language.

No additional explanation is needed.