THE UNIVERSITY OF SYDNEY

CONFIDENTIAL EXAM PAPER

This paper is not to be removed from the exam venue.

Computer Science

EXAMINATION

Semester 2 - Main, 2021

COMP2022 Models of Computation

EXAM WRITING TIME: 3 hours
READING TIME: 10 minutes

EXAM CONDITIONS:
This is an OPEN book examination. You are allowed to use passive information sources (i.e., existing written materials such as books and websites); however, you must not ask other people for answers or post questions on forums; always answer in your own words. You must not reveal the questions to anyone else. All work must be **done individually** in accordance with the University's "Academic Dishonesty and Plagiarism" policies.

INSTRUCTIONS TO STUDENTS:

1. Type your answers in your text editor and submit a **single PDF** via Canvas; all prose must be typed. Figures/diagrams can be rendered any way you like (hand drawn, latex, etc), as long as they are perfectly readable and part of the submitted PDF.
2. Start by typing your student ID at the top of the first page of your submission. Do **not** type your name.
3. Submit only your answers to the questions. Do **not** copy the questions. Start each of the five problems on a new page.
4. If the required level of justification is not stated, you should briefly justify your answer.

For examiner use only:

Problem	1	2	3	4	5	Total
Marks						
Out of	13	11	14	8	4	50

Problem 1. These questions are about Propositional Logic. If the required level of justification is not stated, you should briefly justify your answer.

a) Is it the case that the formulas $p \wedge (p \vee q)$ and $p \vee (p \wedge q)$ are logically equivalent? [2 marks]

b) Using the equivalences from the tables provided with the exam, put the following formula into conjunctive normal form: $(p \vee \neg q) \rightarrow (\neg r \wedge s)$. [3 marks]

You may type your answer in a table or as a sequence of lines, or you may draw the table and insert it into your pdf. No marks will be awarded for proofs that do not use the rules taught in this course and summarised in the tables provided with the exam.

c) Prove the following consequent in natural deduction: [4 marks]

$$s \rightarrow \neg r, (\neg s \wedge r) \rightarrow m \vdash (s \vee \neg m) \rightarrow \neg r$$

You may type your answer in a table or as a sequence of lines, or you may draw the table and insert it into your pdf. No marks will be awarded for proofs that do not use the rules taught in this course and summarised in the tables provided with the exam.

d) Write a formula in CNF over variables x_1, x_2, x_3, that expresses that $x_1 + x_2 + x_3$ is even, i.e., that the number of variables assigned true is even. [2 marks]

e) The following algorithm transforms a formula F in CNF into a formula F'. What is the syntactic and semantic relationship between F and F'? In other words, what does the following algorithm achieve? [2 marks]

Given a formula F in CNF, return the formula F' that is formed by simultaneously replacing every $\vee$ by $\wedge$, and every $\wedge$ by $\vee$, and every literal by its negation (thus atoms x are replaced by $\neg x$, and literals $\neg x$ are replaced by x).

Problem 2. These questions are about Context-free Languages. If the required level of justification is not stated, you should briefly justify your answer.

a) Explain how to check if ϵ is derivable by a given CFG. [1 mark]

b) Given a CFG for L, show how to construct a CFG for L^+. Recall that L^+ is defined to be LL^*. [2 marks]

c) Consider the language generated by the following context-free grammar G: [4 marks]

$$\begin{aligned} S &\to AT \mid \epsilon \\ T &\to TA \mid TB \mid \epsilon \\ A &\to a \\ B &\to b \end{aligned}$$

Is the grammar G ambiguous?

Give a short English description of the language $L(G)$.

Is the language $L(G)$ regular?

d) Give a CFG that generates the following language, and give a brief justification of each rule: [4 marks]

$$\{u\#v\#w : u, v, w \in \{a, b\}^*, |v| \geq |u| + |w|\}$$

Problem 3. These questions are about Regular Languages. If the required level of justification is not stated, you should briefly justify your answer.

a) Is it the case that every NFA has more than one run on every string? [1 mark]

b) Consider the DFA M with states $Q = \{0, 1, 2\}$, $\Sigma = \{a, b\}$, $q_0 = 0$, $F = \{1\}$, and δ is as follows: [3 marks]

	a	b
0	0	1
1	2	0
2	2	2

Give a short English description of $L(M)$ and a regular expression for $L(M)$. No additional justifications are needed.

c) A *left-linear* grammar G is a CFG whose rules are of the form $A \to a$ and $A \to Ba$ where A, B are variables and a is a terminal or ϵ (you saw in tutorials that left-linear grammars only generate regular languages). Apply the following transformation to the grammar: keep every rule of the form $A \to a$, but replace every rule of the form $A \to Ba$ by $A \to aB$. Call the resulting grammar H. What is the relationship between $L(G)$ and $L(H)$? Give a detailed justification of your answer (e.g., comparing derivations in G with those in H). [3 marks]

d) What is wrong with the following argument that claims to show that every regular language is recognised by an NFA with exactly one accepting state and no ϵ-transitions: [3 marks]

1. Since L is regular, it is recognised by some DFA M, say with accepting states F.
2. Build an equivalent NFA N by adding a new state q to M, add ϵ-transitions from every state in F to q, and make q be the only accepting state.
3. Remove ϵ-transitions from N using the method taught in lectures.

e) Show that if $L \subseteq \Sigma^*$ is regular then also $L' = \{uv^R : uv \in L\} \subseteq \Sigma^*$ is regular. Here v^R is the reverse of the string v. For instance, if $abcd \in L$ (for $a, b, c, d \in \Sigma$) then $abcd, abdc, adcb, dcba \in L'$. [4 marks]

Problem 4. These questions are about Turing Machines. If the required level of justification is not stated, you should briefly justify your answer.

a) Is every language in **NP**? [1 mark]

b) Is the set of regular expressions over $\Sigma = \{a, b\}$ Turing-decidable? [2 marks]

c) Let L be the set of encodings $\langle G, K \rangle$ of undirected graphs $G = (V, E)$ and $K \in \mathbb{N}$ such that V can be partitioned into K non-empty pieces, each of which is a clique of G. Show that $L \in$ **NP**. [2 marks]

d) Let L be the set of encodings $\langle N, w \rangle$ where N is an NFA without ϵ transitions such that N accepts w. Show that L is in **P**. [3 marks]

Problem 5. These questions are about Predicate Logic. No additional explanation is needed.

Consider the domain of geographical regions, that includes the regions A, B and C (amongst others), and the following predicates:

- `in`(x, y) is a binary predicate expressing that region x is inside region y.
- `eq`(x, y) is a binary predicate saying that x and y are the same region.
- `border`(x, y) is a binary predicate saying that the regions x and y share a border.

Use this to write the following statements in predicate logic:

a) Every region that borders A or B is inside the region C. [1 mark]

b) Every region borders some other region. [1 mark]

c) If two regions share a border then one is inside the other. [1 mark]

d) If a region has every other region inside it, that region is C. [1 mark]