

- This assignment is **due in Week 11 on Wednesday 19 October, 11:59pm** on Gradescope.
- All work must be **done individually** without consulting anyone else's solutions in accordance with the University's "[Academic Dishonesty and Plagiarism](#)" policies.
- You will be evaluated not just on the correctness of your answers, but on your ability to present your ideas clearly and logically. **You should always explain how you arrived at your answer unless explicitly asked not to do so.** Your goal should be to convince the person reading your work that your answers are correct and your methods are sound.
- For clarifications, input formats, and more details on all aspects of this assignment (e.g., level of justification expected, late penalties, repeated submissions, what to do if you are stuck, etc.) you are expected to regularly monitor the Ed Forum post "[Assignment FAQ](#)".

Problem 1. (15 marks, 5 each)

For each of the following formulas, list the values of the formula under every assignment, state if the formula is valid or not, and state if the formula is satisfiable or not.

1. $(p \rightarrow q) \rightarrow (q \rightarrow p)$
2. $(p \leftrightarrow (q \leftrightarrow r))$
3. $(p \leftrightarrow (q \leftrightarrow r)) \leftrightarrow ((p \leftrightarrow q) \leftrightarrow r)$

Problem 2. (10 marks) Consider the following recursive procedure applied to formulas F of the basic syntax (i.e., the only connectives are \wedge, \vee, \neg):

$MyProc(F)$:

- If F is an atom, then return 0.
- If F is of the form $\neg G$ then return $MyProc(G) + 1$ (note: this is ordinary addition, not addition mod 2).
- If F is of the form $(G \vee H)$ then return $\max(MyProc(G), MyProc(H))$.
- If F is of the form $(G \wedge H)$ then return $\max(MyProc(G), MyProc(H))$.

1. (5 marks) What is the value of $MyProc$ on the following formulas:

- (a) $\neg q$
- (b) $p \wedge \neg \neg q$
- (c) $\neg(p \wedge \neg \neg q) \vee \neg p$
- (d) $(\neg q \wedge \neg(p \vee q)) \vee (\neg p \wedge \neg q)$
- (e) $\neg(\neg(\neg p \wedge (\neg q \vee \neg p)) \vee ((\neg p \vee q) \wedge \neg(p \wedge \neg \neg q)))$

2. State in a brief and precise sentence what $MyProc(F)$ does. (5 marks)

Do not simply "read off" the pseudocode (i.e., "if F is an atom then it returns 0, ...").

Problem 3. (15 marks, 5/10) Prove the following equivalences using the equivalence laws taught in the course.

1. $F \wedge \neg(G \vee H) \equiv \neg(\neg G \rightarrow (F \rightarrow H))$
2. $((F \rightarrow G) \rightarrow (H \rightarrow \neg G)) \rightarrow ((G \wedge F) \rightarrow \neg H) \equiv \neg \perp$

Problem 4. (10 marks) The following is a table for an ND proof with some entries replaced by "?". Complete the proof by providing the missing entries. None of the missing entries were originally blank.

Line	Assumptions	Formula	Justification	References
1	1	$(A \wedge \neg B)$	Asmp. I	
2	2	$(B \vee C)$	Asmp. I	
3	3	$(C \rightarrow D)$	Asmp. I	
4	4	B	Asmp. I	
5	5	C	Asmp. I	
6	?	$\neg B$?	?
7	?	?	\perp I	?
8	?	D	?	7
9	3, 5	?	?	5, 3
10	?	D	\vee E	?

Problem 5. (10 marks, 5 marks each) Prove the following in ND.

1. $(A \wedge B) \vee (\neg B \rightarrow B) \vdash B$
2. $(A \rightarrow B) \rightarrow C \vdash C \vee A$

Problem 6. (20 marks, 10 marks each) Prove the following in ND.

1. $\neg((A \vee B) \rightarrow (A \vee C)) \vdash (A \vee B) \wedge \neg(B \rightarrow C)$
2. $(A \vee B) \rightarrow (A \wedge B), (B \vee C) \wedge \neg(B \wedge C) \vdash (A \wedge \neg C) \vee (\neg A \wedge C)$