THE UNIVERSITY OF SYDNEY

CONFIDENTIAL EXAM PAPER

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Computer Science

EXAMINATION

Semester 2 - Main, 2020

COMP2022 Models of Computation

EXAM WRITING TIME: 3 hours
READING TIME: 10 minutes

EXAM CONDITIONS:
This is an OPEN book examination. You are allowed to use passive information sources (i.e., existing written materials such as books and websites); however, you must not ask other people for answers or post questions on forums; always answer in your own words. You must not reveal the questions to anyone else. All work must be **done individually** in accordance with the University's "Academic Dishonesty and Plagiarism" policies.

INSTRUCTIONS TO STUDENTS:

1. Type your answers in your text editor and submit a **single PDF** via Canvas; all prose must be typed. Figures/diagrams (including tables, proofs, automata) can be rendered any way you like (hand drawn, latex, etc), as long as they are perfectly readable and part of the submitted PDF.
2. Start by typing your student ID at the top of the first page of your submission. Do **not** type your name.
3. Submit only your answers to the questions. Do **not** copy the questions. Start each of the five problems on a new page.
4. **You must always justify your answer/give reasons, unless explicitly stated otherwise.**

For examiner use only:

Problem	1	2	3	4	5	Total
Marks						
Out of	20	20	30	20	10	100

Problem 1. These questions are about Propositional Logic.

a) Are the formulas $(p \to q)$ and $(q \to p)$ logically equivalent? Give a short explanation/justification. [4 marks]

b) Using the equivalences in the provided tables, show that [6 marks]

$$((\neg p \wedge \neg q) \vee (\neg q \wedge \neg r)) \equiv \neg(q \vee (p \wedge r))$$

c) Is it the case that if F is satisfiable and G is satisfiable then $(F \wedge G)$ is satisfiable? Give a short explanation/justification. [4 marks]

d) Prove the following consequent in Natural Deduction: [6 marks]

$$(p \to \neg q), \neg\neg q \vdash \neg p$$

You may type your answer in a table or as a sequence of lines, or you may draw the table and insert it into your pdf. No marks will be awarded for proofs that do not use the rules taught in this course and summarised in the tables provided with the exam.

Problem 2. These questions are about Predicate Logic. The first two questions are about the structure

$$\mathcal{N} = (\mathbb{N} \cup \{0\}, f^{\mathcal{N}}, g^{\mathcal{N}}, E^{\mathcal{N}}, c^{\mathcal{N}}, d^{\mathcal{N}})$$

where $f^{\mathcal{N}}(x,y) = x + y$, $g^{\mathcal{N}}(x,y) = x \times y$, $E^{\mathcal{N}}(x,y)$ if $x = y$, and $c^{\mathcal{N}} = 0$ and $d^{\mathcal{N}} = 1$.

a) Express the following as a predicate-logic sentence in the vocabulary of $\mathcal{N}$: for every number x, if $x + x$ is equal to $x \times x$ then $x = 0$. Just give the sentence, no other justification needed. [2 marks]

b) Apply the recursive definition of truth-value to decide whether or not $\mathcal{N}$ satisfies the sentence $\exists x E(f(d,d), x)$. Show each step. [4 marks]

c) Apply the procedure learned in the course to put the following formula into Prenex Normal Form: [3 marks]

$$\forall w(\exists y P(y) \rightarrow \exists y Q(w,y))$$

Show each step.

d) Give a recursive definition NQ(F) that returns the total number of quantifiers in the predicate-logic formula F. For instance, [3 marks]

$$\text{NQ}(\exists x \forall y(F(x,y) \land \exists x P(x))) = 3.$$

Just give the recursive definition, no other justification is needed.

e) Prove the following consequent in Natural Deduction: [8 marks]

$$\forall x(\exists y E(y,x) \rightarrow \forall w E(x,w)) \vdash \forall x \forall y(E(y,x) \rightarrow E(x,y))$$

You may type your answer in a table or as a sequence of lines, or you may draw the table and insert it into your pdf. No marks will be awarded for proofs that do not use the rules taught in this course and summarised in the tables provided with the exam.

Problem 3. These questions are about Regular Languages.

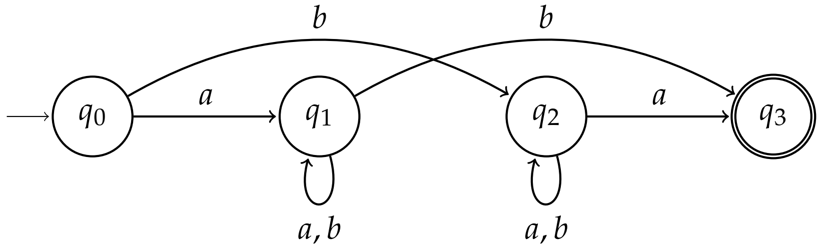
a) Provide a regular expression describing the language of strings over alphabet $\{a, b\}$ that do not contain ab as a substring. Explain in one sentence why your regular expression is correct. [4 marks]

b) Provide an NFA recognising the language [4 marks]

$$L = \{a^{i+1}(ba)^j : i, j \geq 0\} \subseteq \{a, b\}^*$$

Explain in one sentence why your NFA is correct.

c) Describe in words the language recognised by the following NFA (no additional justification is needed), and then give the DFA that results from applying the subset construction to the NFA: [6 marks]



You may draw or write the DFA, and no additional justification is needed. Note that you should only include the states in the DFA that are reachable from the initial state.

d) Show that the following language is regular: the set of strings of the form $0^n 1^m$ such that $n \equiv m \mod 3$ (i.e., if 3 divides $n - m$). Give a short explanation of your answer. [5 marks]

e) Prove, using a fooling argument, that the following language L is not regular: the set of strings of the form $1^a 0^b$ such that a divides b (where a, b are positive integers). Recall that a divides b if $b = a \times n$ for some integer n. [6 marks]

f) Show that if $L \subseteq \Sigma^*$ is regular then so is [5 marks]

$$SHIFT(L) = \{u \in \Sigma^* : \text{there exists } x, y \in \Sigma^* \text{ such that } u = xy \text{ and } yx \in L\}.$$

For instance, if $aba \in L$ then the following strings are in $SHIFT(L)$: aba, baa, aab.

Give a detailed justification of your argument.

Problem 4. These questions are about Context-free Languages.

a) Consider the following context-free grammar: [4 marks]

$$S \to 0S \mid 0S1S \mid \epsilon$$

For each of the following words, say whether or not it is generated by the grammar: 001, 0101, 0110. The only justification that is needed is a derivation for those words that are generated by the grammar.

Show that the grammar is ambiguous.

b) Put the following grammar into Chomsky-Normal Form using the technique from the course (the initial variable is S): [5 marks]

$$\begin{aligned} S &\to aSb \\ S &\to cTd \\ T &\to S \\ T &\to \epsilon \end{aligned}$$

Show each step.

c) Describe the language generated by the following context-free grammar: [5 marks]

$$\begin{aligned} S &\to X1Y \\ X &\to \epsilon \mid X0 \\ Y &\to \epsilon \mid 1Y \mid Y0 \end{aligned}$$

Is the grammar ambiguous? Give a short explanation of your answer.

d) Provide a context-free grammar for the following language L: the set of well-bracketed strings that use two different types of brackets, i.e., $()$ and $\{\}$. For instance, $(\{\})()$ is in the language, as is $(())$, as is $()\{\}$, but $(\}$ is not. Give a short explanation justifying why your grammar is correct. In particular, explain a) why your grammar only generates strings in L, and b) why your grammar generates all strings in L. [6 marks]

Problem 5. These questions are about Universal Models of Computation.

a) Is the following language Turing decidable? The set of all encodings $\langle \mathcal{A}, F \rangle$ where $\mathcal{A}$ is a *finite* structure and F is a sentence of predicate logic (over the vocabulary of $\mathcal{A}$) such that F is true in $\mathcal{A}$. Give a short explanation/justification. [3 marks]

b) Is it the case that if L_1, L_2 are Turing-recognisable then so is $L_1 \circ L_2$. Give a short explanation/justification. [3 marks]

c) Reduce $(\lambda y.(yy))(\lambda x.(\lambda y.(xy)))$ to beta-normal form. Show every step of the reduction. [4 marks]