THE UNIVERSITY OF SYDNEY

CONFIDENTIAL EXAM PAPER

This paper is not to be removed from the exam venue.

Computer Science

EXAMINATION

Semester 2 - Main, 2022

COMP2022 Models of Computation

EXAM WRITING TIME: 3 hours
READING TIME: 10 minutes

EXAM CONDITIONS:
This is an OPEN book examination. You are allowed to use passive information sources (i.e., existing written materials such as books and websites); however, you must not ask other people for answers or post questions on forums; always answer in your own words. You must not reveal the questions to anyone else. All work must be **done individually** in accordance with the University's "Academic Dishonesty and Plagiarism" policies.

INSTRUCTIONS TO STUDENTS:

1. Type your answers in your text editor and submit a **single PDF** via Canvas; all prose must be typed. Figures/diagrams can be rendered any way you like (hand drawn, latex, etc), as long as they are perfectly readable and part of the submitted PDF.
2. Start by typing your student ID at the top of the first page of your submission. Do **not** type your name.
3. Submit only your answers to the questions. Do **not** copy the questions.
4. Start each of the five problems on a new page.
5. If the required level of justification is not stated, you should briefly justify your answer.

For examiner use only:

Problem	1	2	3	4	5	Total
Marks						
Out of	12	14	12	6	6	50

Problem 1. These questions are about Regular Languages. They are worth a total of 12 marks. If the required level of justification is not stated, you should briefly justify your answer.

a) Show that if S, T are regular expressions then there is a regular expression for $L(S) \cap L(T)$. [2 marks]

b) Give an NFA for the language of strings over alphabet $0, 1$ that have 001 as a substring. You may draw the NFA or type the transition relation. No additional explanation is needed. [2 marks]

c) Give a regular expression for the language $\{w \in \{0,1\}^* : w \neq 01\}$. No additional explanation is needed. [4 marks]

d) Fix $\Sigma = \{0, 1\}$. Consider the operation *del* that maps a string u to the string in which every 0 is deleted. So, e.g., $del(0) = \epsilon$, $del(01101) = 111$, $del(1) = 1$, and $del(\epsilon) = \epsilon$. For a language $L \subseteq \Sigma^*$, let $del(L) = \{del(u) : u \in L\}$. Explain why if L is regular, then $del(L)$ is regular. [4 marks]

Problem 2. These questions are about Turing Machines and Complexity. They are worth a total of 14 marks. If the required level of justification is not stated, you should briefly justify your answer. If you are describing a TM, give a high-level description unless otherwise specified.

a) Let M be a decider over input alphabet $\{0, 1\}$. Give a high-level description of a TM that decides the language $L(M) \cap L(0^*)$. [4 marks]

b) Explain why the **non-decidable** languages are closed under complement. [2 marks]

c) We know that every context-free language is in **P**. Explain why if L_1, L_2 are context-free languages, then $L_1 \cap L_2$ is in **P**. [4 marks]

d) Fix $\Sigma = \{0, 1\}$. Consider the operation *del* that maps a string u to the string in which every 0 is deleted. So, e.g., $del(0) = \epsilon$, $del(01101) = 111$, $del(1) = 1$, and $del(\epsilon) = \epsilon$. For a language $L \subseteq \Sigma^*$, let $del(L) = \{del(u) : u \in L\}$. Show that if L is decidable, then $del(L)$ is recognisable. [4 marks]

Problem 3. These questions are about Propositional Logic. They are worth a total of 12 marks. If the required level of justification is not stated, you should briefly justify your answer.

a) Are the formulas $p \wedge p$ and $p \vee p$ logically equivalent? Give a short explanation/justification of your answer. [2 marks]

b) Using the equivalence laws from the Table provided with the exam, show that $((A \wedge B) \wedge C) \rightarrow A \equiv \top$. [2 marks]

c) Is it true that if the formula $F \vee G$ is valid then either F is valid or G is valid? Give a short explanation/justification of your answer. [2 marks]

d) Write a CNF formula over variables p, q, r that says that the number of true variables is even. No additional explanation is needed. [2 marks]

e) Prove the following in Natural Deduction: [4 marks]

$$(p \vee q), (r \rightarrow \neg p) \vdash (q \rightarrow p) \rightarrow \neg r$$

Type your answer in a table or as a sequence of lines. No marks will be awarded for proofs that do not use the rules taught in this course and summarised in the Table provided with the exam.

Problem 4. These questions are about Predicate Logic. They are worth a total of 6 marks. If the required level of justification is not stated, you should briefly justify your answer.

a) Consider the units of study (UoS) domain, that includes the UoS `COMP2022`, `COMP2922`, and `INFO1103` (amongst others), and the following predicates: [4 marks]

1. `prerequisite`(x, y) is a binary predicate expressing that x is a pre-requisite for y.
2. `prohibition`(x, y) is a binary predicate saying that x is a prohibition of y.

Use this to write the following statements in predicate logic:

1. `COMP2922` is a prohibition of `COMP2022`.
2. Every UoS that is a pre-requisite for `COMP2022` is also a pre-requisite for `COMP2922`.
3. `INFO1103` has no pre-requisite.
4. If two UoS have the same pre-requisites then they have the same prohibitions.

No additional explanation is needed.

b) Prove the following in Natural Deduction: [2 marks]

$$\forall x(P(x) \to Q(x)), \exists x P(x) \vdash \exists x Q(x)$$

Type your answer in a table or as a sequence of lines. No marks will be awarded for proofs that do not use the rules taught in this course and summarised in the Table provided with the exam.

Problem 5. These questions are about Context-free Grammars. They are worth a total of 6 marks. If the required level of justification is not stated, you should briefly justify your answer.

a) Consider the following grammar: [4 marks]

$$S \to AS \mid SB \mid \epsilon$$
$$A \to Aa \mid \epsilon$$
$$B \to bB \mid \epsilon$$

Describe the language of the grammar, and show that the grammar is ambiguous.

b) Fix $\Sigma = \{0, 1\}$. Provide a context-free grammar for the language [2 marks]

$$\{u0v1w : |v| = |u| + |w|\}$$

For instance, taking $u = 01, w = 11, v = 1100$ we get that $u0v1w = 0101100111$ is in the language. No additional explanation is needed.