The aim of this exercise is to analyse photographic plates from Eddington's 1919 eclipse expedition, to determine whether the data favour Newtonian gravity or Einstein's General Theory of Relativity. Light from stars that passes close to the Sun is deflected, and during an eclipse, these stars can be detected and their displacements measured, when compared with photographs taken when the Sun is far away.

General Relativity predicts that light passing a mass M at distance r will be bent through an angle

$$\theta_{\mathrm{GR}}(r) = \frac{4GM}{rc^2}$$

whereas an argument based on Newtonian gravity gives half this:

$$\theta_{\mathrm{N}}(r) = \frac{2GM}{rc^2}$$

We can either treat this as a parameter inference problem, modelling the bending as

$$\theta(r) = \frac{\alpha GM}{rc^2}$$

and inferring α, or as a model comparison problem. For this exercise we will do the former.

Data model

$$Dx^{\text{model exp.}}(\theta, x, y, E_x) = ax + by + c + \alpha E_x$$
$$Dy^{\text{model exp.}}(\theta, x, y, E_y) = dx + ey + f + \alpha E_y$$

where:

- x, y: coordinates of the stars
- E_x, E_y: coefficients of the gravitational displacement
- c, f: corrections to zero
- a, e: differences of scale value (caused e.g. by changes in temperature)
- b, d: depend on the orientiation of the two plates
- α: deflection at unit distance, i.e (50' from the Sun's centre)

Parameter of Interest (POI):

- α

Nuisance parameters (NP):

- a, b, c, d, e, f

Parameter vector:

$$\theta = (\alpha, a, b, c, d, e, f)$$

Combined likelihood:

$$\mathcal{L}(\theta, \text{all data}) = \prod_{i=star} \mathcal{L}_i(\theta, Dx^{\text{obs}}, Dy^{\text{obs}})$$

where $\mathcal{L}_i$ is:

$$\begin{aligned}\mathcal{L}_i =& \frac{1}{\sqrt{2\pi}\sigma_{Dx}} \exp\left\{ -\frac{1}{2} \frac{(Dx_i^{\text{obs}} - Dx^{\text{model exp.}}(\theta, x_i, y_i, E_{x,i}))^2}{\sigma_{Dx}^2} \right\} \cdot \\ & \frac{1}{\sqrt{2\pi}\sigma_{Dy}} \exp\left\{ -\frac{1}{2} \frac{(Dy_i^{\text{obs}} - Dy^{\text{model exp.}}(\theta, x_i, y_i, E_{y,i}))^2}{\sigma_{Dy}^2} \right\}\end{aligned}$$

where:

- $\sigma_{Dx} = 0.05$
- $\sigma_{Dy} = 0.05$