



Logic Design Assignment 1 - Solutions (TA)

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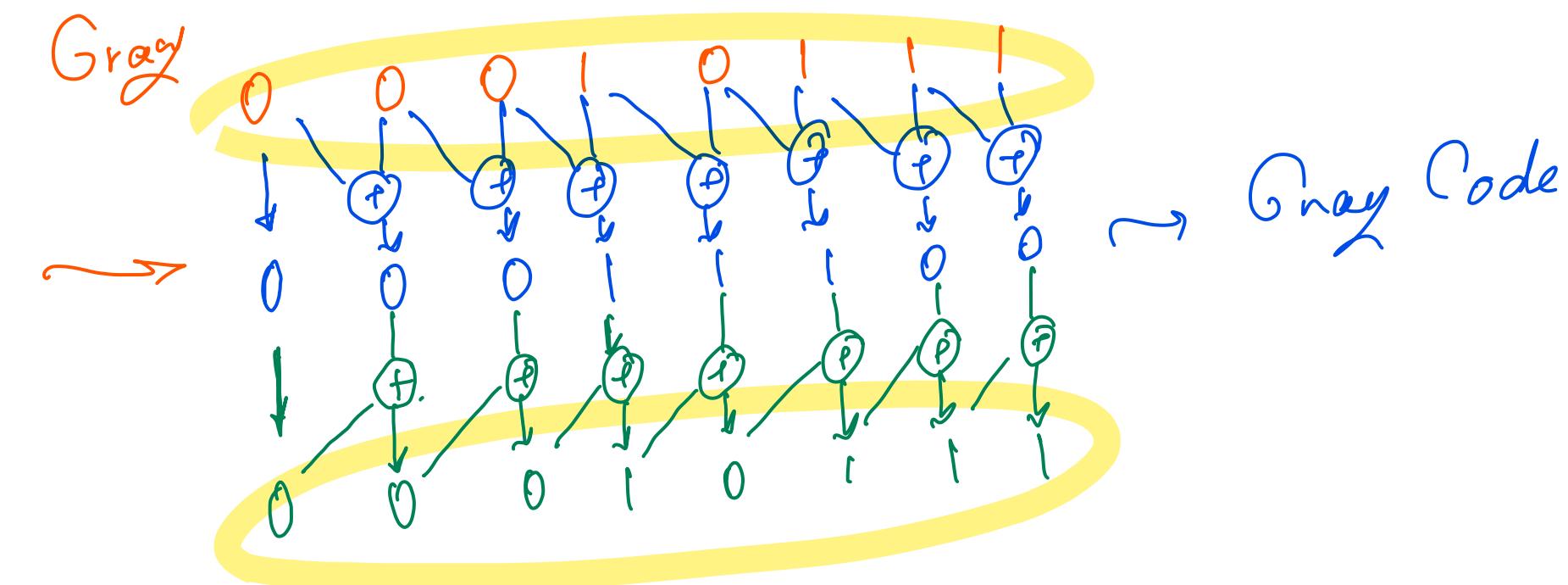
Fall Semester of 2024

Handwritten notes:

- Top row: 0001 0111 (highlighted in yellow)
- Middle row: 1110 1000
- Middle row: 1110 1001
- Bottom row: 10111
- Below the middle row: +1
- Below the bottom row: -10
- Bottom left: = 10
- Bottom left: = -10
- Bottom right: BCD : 0010 0011 = 23
- Bottom right: 10 23 11

Table Gray Code

$$\begin{array}{r} c \quad b \quad a \\ 0 \quad 0 \quad 0 \\ 0 \quad 0 \quad 1 \\ \hline 0 \quad 1 \quad 1 \\ 0' \quad 1 \quad 0 \\ \hline \vdots & \vdots & 0 \\ \vdots & 0 & \vdots \\ \vdots & 0 & 0 \end{array}$$



- ۱- عدد ۲۳ را در نظر بگیرید:

الف) آن را به باینری بنویسید (هشت بیتی علامت دار).

ب) مکمل ۱ آن را محاسبه کنید.

د) مکمل ۲ آن را محاسبه کنید.

ج) کد BCD آن را محاسبه کنید.

ه) کد گری (Gray Code) آن را محاسبه کنید.

Question - 2

۲- جداول درستی توابع ذیل را تهیه کنید.

x	y	z	F_1	F_2
0	0	0	0	1
0	0	1	0	1
0	1	0	1	1
0	1	1	0	1
1	0	0	0	0
1	0	1	1	0
1	1	0	0	1
1	1	1	1	0

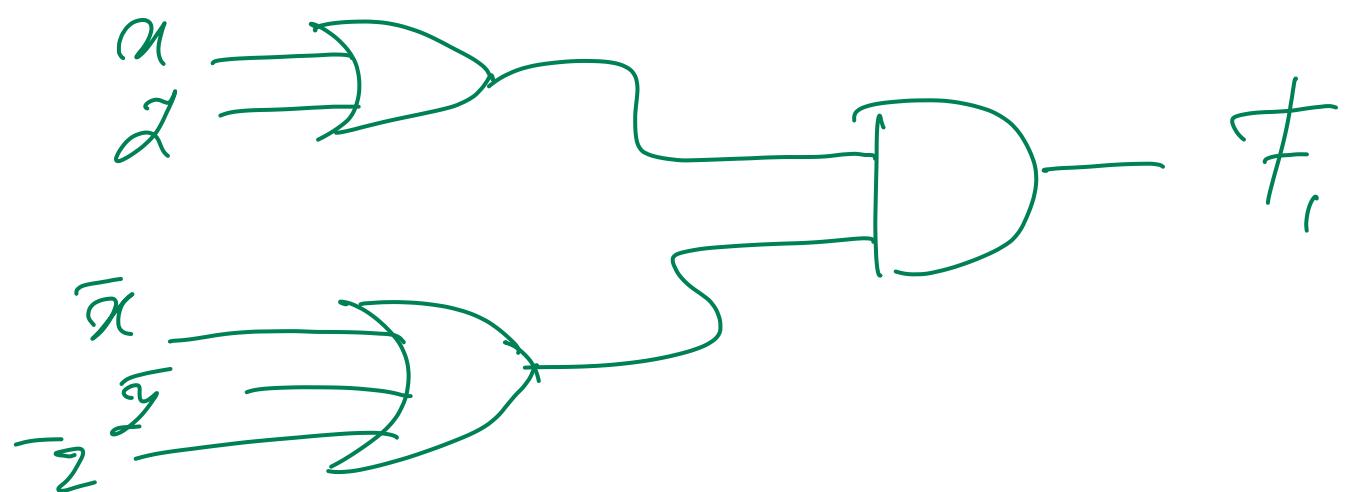
و

$$F_1 = (x + y) \cdot (x' + z) \cdot (x + y + z')$$

$$F_2 = x' + yz'$$

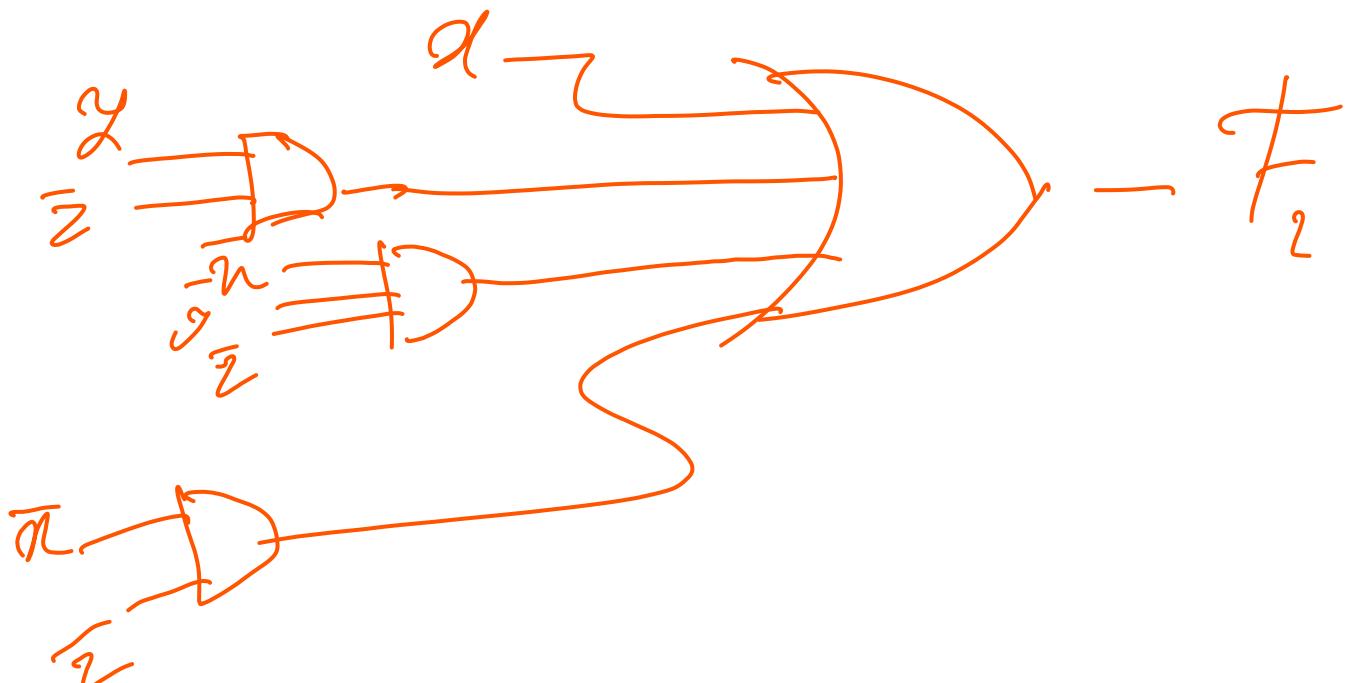
□

۳- نمودار منطقی عبارات ذیل را رسم کنید.



$$F_1 = (x + y) \cdot (\bar{x} + \bar{y} + z)$$

$$F_2 = x + (yz') + (\bar{x}\bar{y}\bar{z}') + x'z'$$



٤

تابع $x\bar{y} + x\bar{z}$ را به صورت مجموع مینترم‌ها و حاصل‌ضرب ماکسیموم‌ها بنویسید.

$$\begin{array}{c}
 \begin{array}{c|c}
 x & y & z & | & f \\
 \hline
 \end{array}
 \quad \text{POS} \quad \text{SOP} \\
 \begin{array}{c}
 m \\
 \hline
 \end{array} \quad \begin{array}{c}
 m \\
 \hline
 \end{array}
 \end{array}$$

SOP : $x\bar{y} + x\bar{z}$

$$\underbrace{x\bar{y}}_A (\bar{z} + z) = \underbrace{\cancel{x\bar{y}z}}_{101} + \underbrace{x\bar{y}\bar{z}}_{100}$$

$$x\bar{z} (\bar{y} + y) = \underbrace{x\bar{y}\bar{z}}_{110} + \underbrace{x\bar{y}z}_{100}$$

minterm $\rightarrow f \rightarrow 1$

$$A + A = A$$

$$f = x\bar{y}z + \cancel{x\bar{y}\bar{z}} + x\bar{y}\bar{z} + \cancel{x\bar{y}z}$$

$$f = \sum_m (4, 5, 6)$$

$$f = \prod_M (0, 1, 2, 3, 7)$$

Question - 5

۵- متمم توابع زیر را به صورت جمع جملات مینیمم بیان کنید.

$$\overline{F} = \sum_m (0, 1)$$

$$F(x,y) = \sum_m (0, 1)$$

$$\overline{F} = \overline{x}\overline{y} + \overline{x}y = \prod_m (0, 1)$$

POS $\rightarrow (x+y) \cdot (\overline{x}+\overline{y})$

x	y	M_0	M_1
0	0	0	1

$$F = \prod_m (0, 1)$$

$$\overline{F} = (x+y)(\overline{x}+\overline{y})$$

$$\overline{x}\overline{y} + \overline{x}y$$

الف) $F(A, B, C, D) = \sum(3, 5, 9, 11, 15)$

ب) $F(x, y, z) = \prod(2, 4, 5, 7)$

SOP

$$\overline{F} = \prod_m (3, 5, 9, 11, 15) = \sum_m (0, 1, 2, 4, 6, 7, 8, 10, 12, 13, 14)$$

$$F = \sum_m (2, 4, 5, 7)$$



$$\alpha (\alpha + \gamma) = \underbrace{\alpha \cdot \alpha}_{\alpha} + \alpha \cdot \gamma = \alpha (1 + \gamma) = \alpha$$

-۶ معادله‌های زیر را با استفاده از تئوری‌های جبر بول ساده کنید.

$$f_1 = (AB + C + D)(\bar{C} + \bar{D})$$

$$AB\bar{C} + \cancel{C\bar{C}} + \cancel{D\bar{C}} + \cancel{ABD} + \cancel{CD} + \cancel{DD}$$

$$AB\bar{C} + \cancel{D} + AB\bar{D} = AB\bar{C} + D$$

$$f_1 = AB\bar{C} + D$$

$$f_1(A, B, C, D, E) = (AB + C + D)(\bar{C} + \bar{D})(\bar{C} + \bar{D} + E) = AB\bar{C} + D$$

$$f_2(A, B, C) = ((B + \bar{A})(AB + C) + \cancel{ABA} + \cancel{ABC} + (A + B)(\bar{A} + C))$$

$$f_2 = B\bar{A}B + BC + \cancel{\bar{A}A\bar{B}} + \cancel{\bar{A}C} + \cancel{ABC} + \cancel{AC} + \cancel{B\bar{A}} + BC$$

$$AB + BC + \bar{A}C + (\bar{A} + \bar{B})\bar{C} + AC + B\bar{A} + BC$$

$$(\bar{A} + \bar{B} + A)\bar{C}$$

$$AB + BC + \cancel{\bar{A}C} + C + \cancel{B\bar{A}} + \cancel{BC} = B + \cancel{(BC)} + C = B + C$$

$$f_2 = \overline{B + C} = \bar{B}\bar{C}$$

-۷- معادلات بولی و نمودار مداری با خروجی‌های تعریف شده با جدول درستی زیر را رسم کنید.

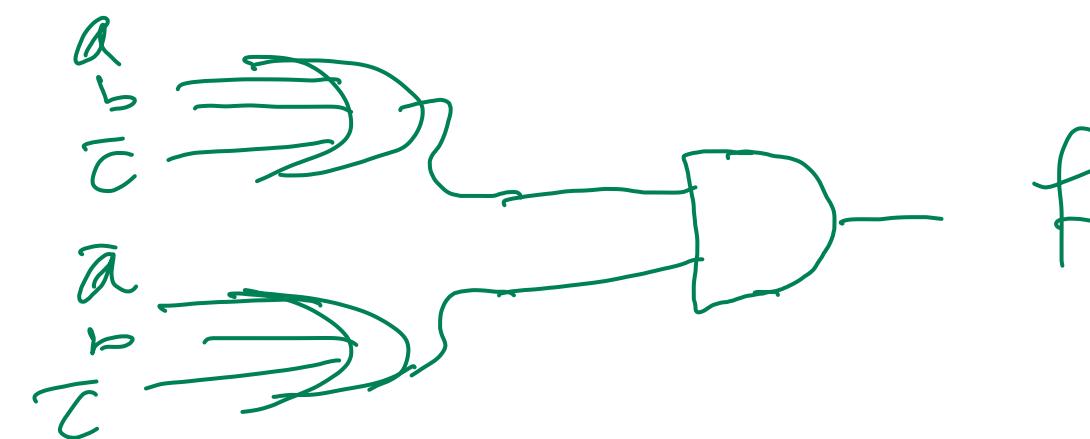
a	b	c	f₁	f₂
0	0	0	1	1
0	0	1	0	1
0	1	0	1	0
0	1	1	1	1
1	0	0	1	0
1	0	1	0	1
1	1	1	1	0

 M_1 M_5 M_2 M_4 M_7

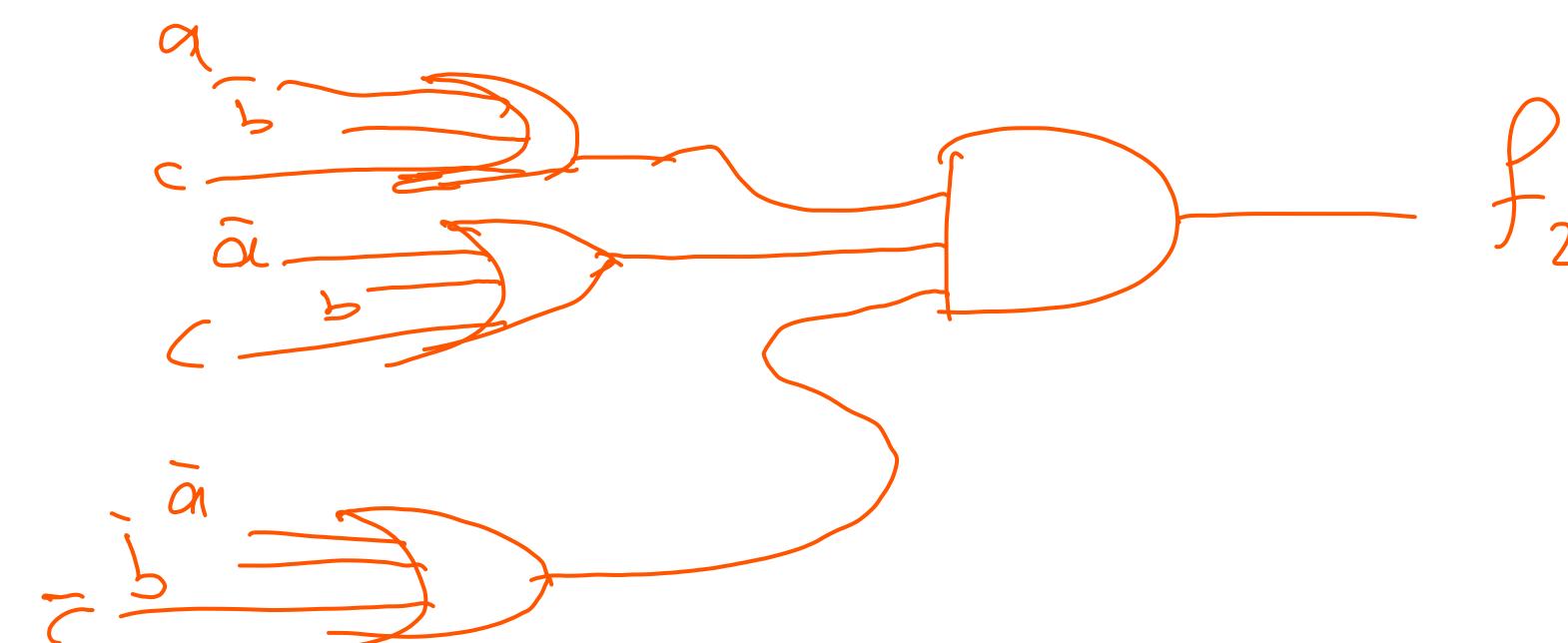
$$f_1 = T_M(1, 5)$$

$$f_2 = T_M(2, 4, 7)$$

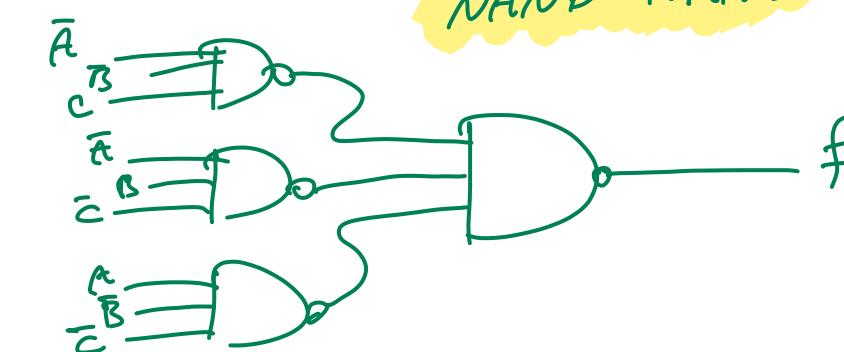
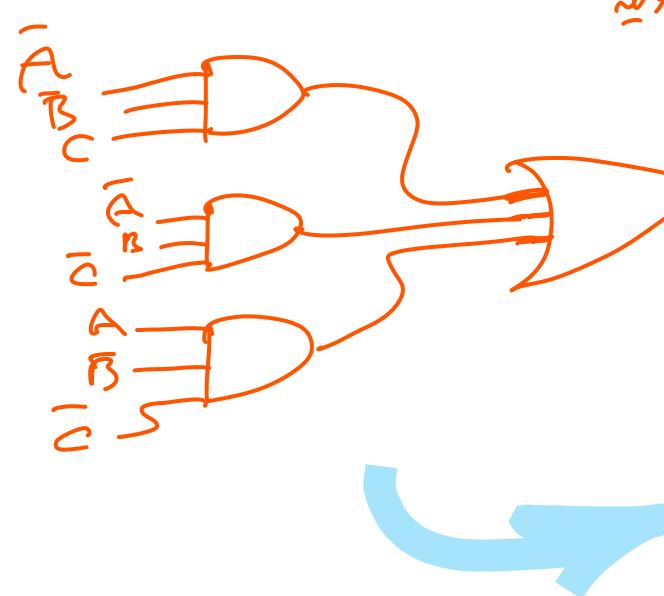
$$f_1 = (\bar{a} + b + \bar{c})(\bar{a} + b + c)$$



$$f_2 = (\bar{a} + \bar{b} + c)(\bar{a} + b + c)(\bar{a} + \bar{b} + \bar{c})$$



A	B	C	f
0	0	0	0
0	0	1	1 $\rightsquigarrow m_1$
0	1	0	1 $\rightsquigarrow m_2$
0	1	1	0
1	0	0	1 $\rightsquigarrow m_4$
1	0	1	0
1	1	0	0
1	1	1	0

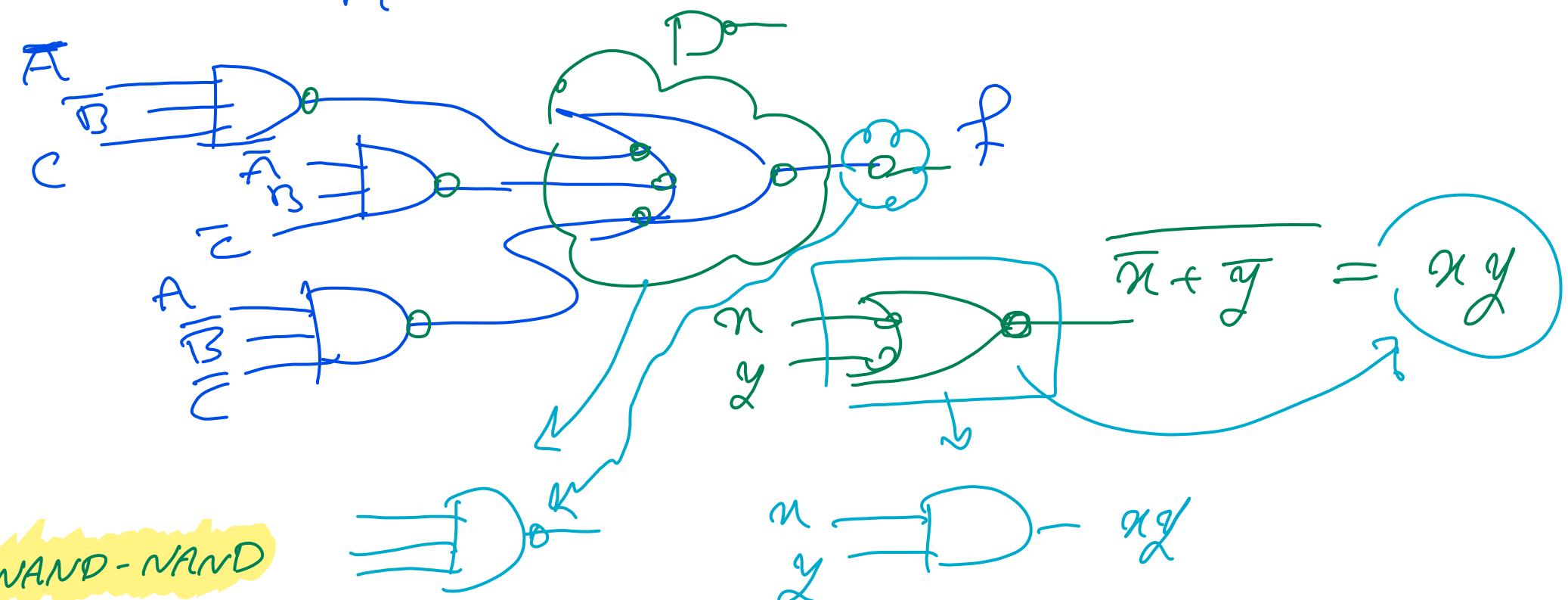


NAND-NAND

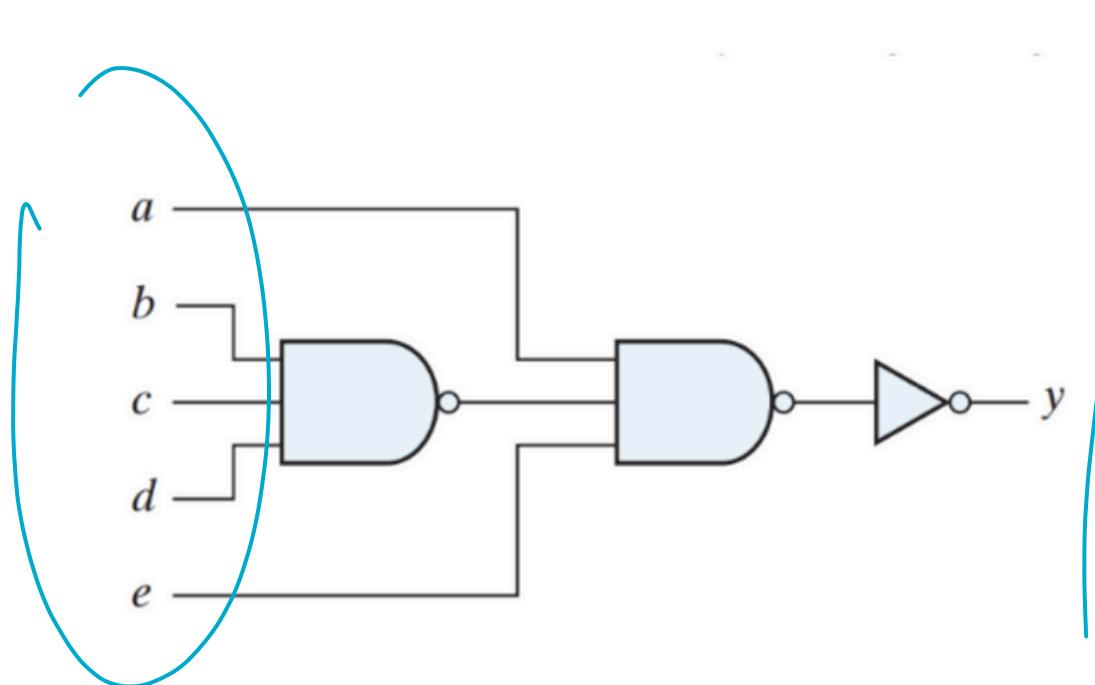
- معادله جبری و مدار متناظر با آن را طوری بدست آورید که دارای سه ورودی A, B, C باشد و خروجی فقط زمانی شود که فقط یک ورودی «1» باشد. (حال اختیاری: در پیاده سازی فقط از گیت NAND استفاده کنید).

SOP

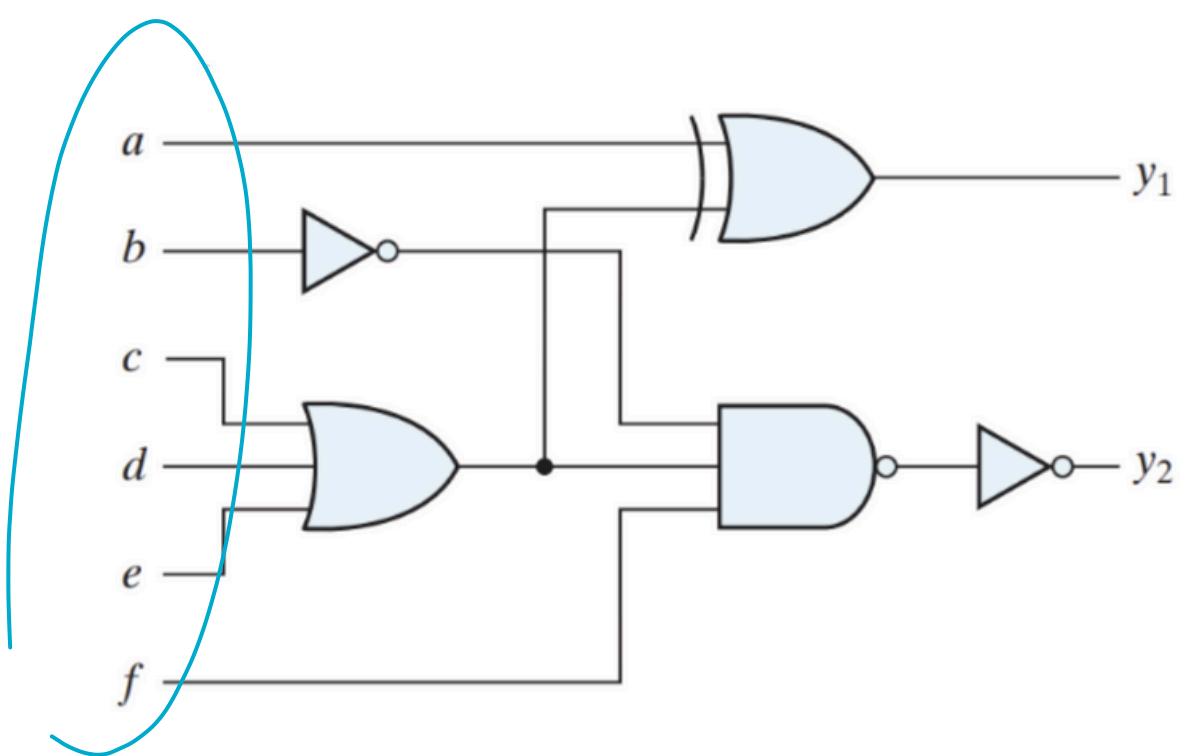
$$f = \sum_m (1, 2, 3) = \overline{\bar{A}\bar{B}C} + \overline{\bar{A}B\bar{C}} + \overline{A\bar{B}\bar{C}}$$



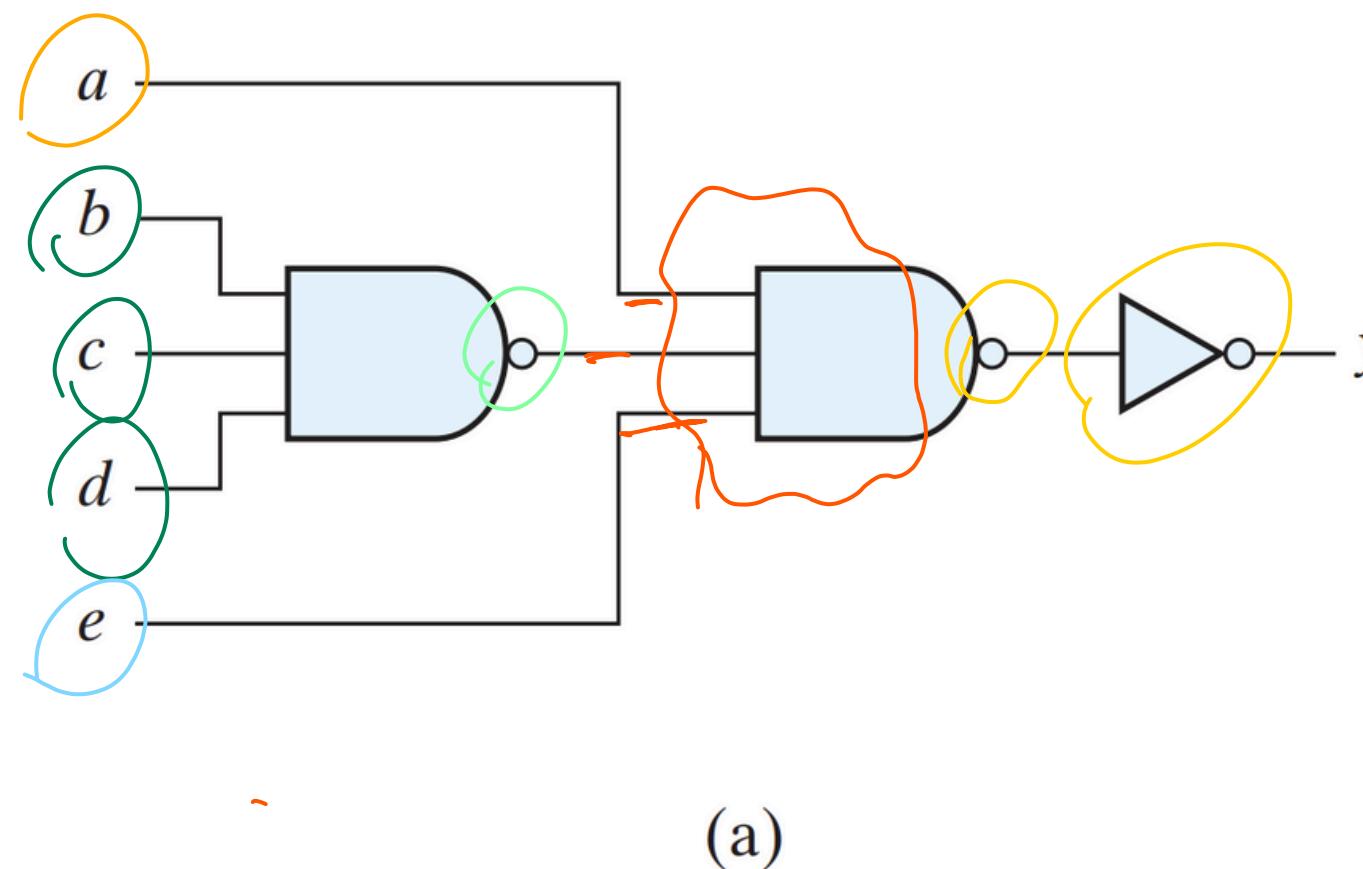
۹- عبارت‌های بولی و جدول درستی توصیف کننده خروجی یا خروجی‌های مدارهای منطقی زیر را بنویسید.



(ب)



(الف)



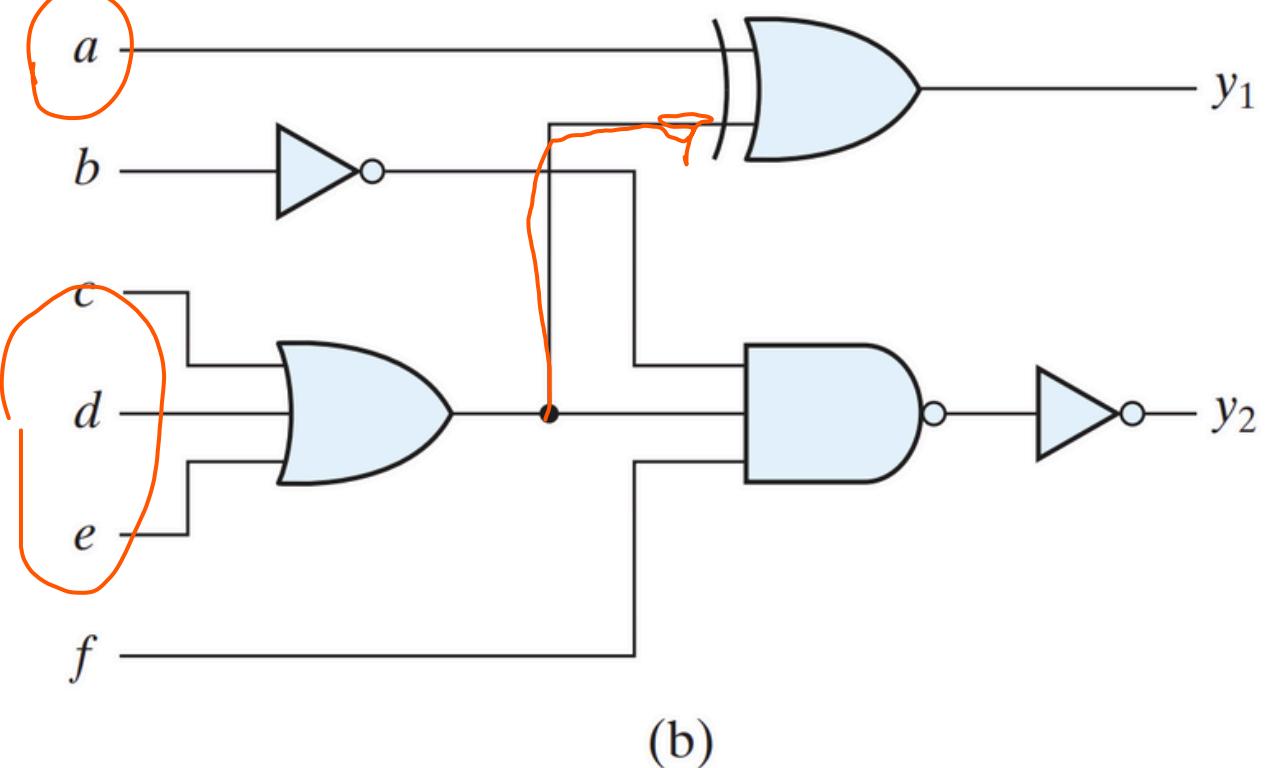
(a)

$$(a) y = \underbrace{a}_{\text{red}} \underbrace{(bcd)'e}_{\text{green}} = a(b' + c' + d')e$$

$$\begin{aligned} y &= a(b' + c' + d')e = ab'e + ac'e + ad'e \\ &= \Sigma(17, 19, 21, 23, 25, 27, 29) \end{aligned}$$

\overbrace{abcde}	y	\overbrace{abcde}	y
0 0 0 0 0	0	1 0 0 0 0	0
0 0 0 0 1	0	1 0 0 0 1	1
0 0 0 1 0	0	1 0 0 1 0	0
0 0 0 1 1	0	1 0 0 1 1	1
0 0 1 0 0	0	1 0 1 0 0	0
0 0 1 0 1	0	1 0 1 0 1	1
0 0 1 1 0	0	1 0 1 1 0	0
0 0 1 1 1	0	1 0 1 1 1	1
0 1 0 0 0	0	1 1 0 0 0	0
0 1 0 0 1	0	1 1 0 0 1	1
0 1 0 1 0	0	1 1 0 1 0	0
0 1 0 1 1	0	1 1 0 1 1	1
0 1 1 0 0	0	1 1 1 0 0	0
0 1 1 0 1	0	1 1 1 0 1	1
0 1 1 1 0	0	1 1 1 1 0	0
0 1 1 1 1	0	1 1 1 1 1	0

Question - 9 (b)



(b)

$ab\ cdef$	$y_1\ y_2$						
00 0000	0 0	01 0000	0 0	10 0000	1 0	11 0000	0 0
00 0001	0 0	01 0001	0 0	10 0001	1 0	11 0001	0 0
00 0010	1 0	01 0010	1 0	10 0010	1 0	11 0010	0 0
00 0011	1 1	01 0011	1 0	10 0011	1 1	11 0011	0 1
00 0100	0 0	01 0100	0 0	10 0100	0 0	11 0100	0 0
00 0101	0 0	01 0101	0 0	10 0101	0 0	11 0101	0 0
00 0110	1 0	01 0110	1 0	10 0110	0 0	11 0110	0 0
00 0111	1 1	01 0111	1 0	10 0111	0 1	11 0111	0 1
00 1000	1 0	01 1000	1 0	10 1000	0 0	11 1000	0 0
00 1001	1 1	01 1001	1 0	10 1001	0 1	11 1001	0 0
00 1010	1 0	01 1010	1 0	10 1010	0 0	11 1010	0 0
00 1011	1 0	01 1011	1 0	10 1011	0 1	11 1011	0 0
00 1100	1 0	01 1100	1 0	10 1100	0 0	11 1100	0 0
00 1101	1 1	01 1101	1 0	10 1101	0 1	11 1101	0 0
00 1110	1 0	01 1110	1 0	10 1110	0 0	11 1110	0 0
00 1111	1 1	01 1111	1 0	10 1111	0 1	11 1111	0 0

(b) $y_1 = a \oplus (c + d + e) = a'(c + d + e) + a(c'd'e') = a'c + a'd + a'e + ac'd'e'$
 $y_2 = b'(c + d + e)f = b'cf + b'df + b'ef$

$y_1 = a(c + d + e) = a'(c + d + e) + a(c'd'e') = a'c + a'd + a'e + ac'd'e'$
 $y_2 = b'(c + d + e)f = b'cf + b'df + b'ef$

$a'c---$
 $\begin{array}{ll} 001000 = 8 & 000100 = 8 \\ 001001 = 9 & 000101 = 9 \\ 001010 = 10 & 000110 = 10 \\ 001011 = 11 & 000111 = 11 \end{array}$
 $a'--d--$
 $\begin{array}{ll} 001100 = 12 & 001100 = 12 \\ 001101 = 13 & 001101 = 13 \\ 001110 = 14 & 001110 = 14 \\ 001111 = 15 & 001111 = 15 \end{array}$
 $a'---e-$
 $\begin{array}{ll} 001010 = 10 & 000010 = 2 \\ 000110 = 6 & 000011 = 3 \\ 000111 = 7 & 000110 = 6 \\ 000111 = 15 & 000111 = 7 \end{array}$
 $a-c'd'e'-$
 $\begin{array}{ll} 100000 = 32 & 100001 = 33 \\ 100001 = 33 & 110000 = 34 \\ 110001 = 35 & 110001 = 35 \end{array}$

$-b' c--f$
 $\begin{array}{ll} 011000 = 24 & 010100 = 20 \\ 011001 = 25 & 010101 = 21 \\ 011010 = 26 & 010110 = 22 \\ 011011 = 27 & 010111 = 23 \end{array}$
 $-b' -d-f$
 $\begin{array}{ll} 011100 = 28 & 011100 = 28 \\ 011101 = 29 & 011101 = 29 \\ 011110 = 30 & 011110 = 30 \\ 011111 = 31 & 011111 = 31 \end{array}$
 $-b' --ef$
 $\begin{array}{ll} 001001 = 9 & 001001 = 9 \\ 001011 = 11 & 001011 = 11 \\ 001101 = 13 & 001101 = 13 \\ 001111 = 15 & 001111 = 15 \\ 001011 = 19 & 001011 = 19 \\ 001101 = 22 & 001101 = 22 \\ 001111 = 23 & 001111 = 23 \\ 101001 = 41 & 101001 = 41 \\ 101011 = 43 & 101011 = 43 \\ 101101 = 45 & 101101 = 45 \\ 101111 = 47 & 101111 = 47 \end{array}$
 $000011 = 3 & 000111 = 7 \\ 001011 = 11 & 001101 = 11 \\ 001101 = 13 & 001011 = 11 \\ 001111 = 15 & 001111 = 15 \\ 100011 = 35 & 100111 = 39 \\ 101011 = 43 & 101101 = 45 \\ 101101 = 45 & 101011 = 51 \\ 101111 = 47 & 101111 = 55 \end{array}$

$y_1 = \Sigma (2, 3, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 18, 19, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35)$

$y_2 = \Sigma (3, 7, 9, 13, 15, 35, 39, 41, 43, 45, 47, 51, 55)$



TA Team and Contributions:

- **Abolfazl Ranjbar**
- **Ehsan Saberi**
- **Pourya Ardestani**

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