UNIVERSITY INSTITUTE OF ENGINEERING

Department of Computer Science & Engineering

(BE-CSE/IT-7th Sem)

Subject Name: NUMERICAL METHODS AND OPTIMIZATION USING PYTHON

Subject Code: 21CSH-459

Submitted to:

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Question: Performance Analysis of Root-Finding Algorithms in Python

Solution:

Root finding algorithm is a computational method used to determine the roots of a mathematical function. The root of a function is the value of x that makes the function equal to zero, i.e., f(x) = 0. These algorithms are essential in various fields of science and engineering because they help solve equations that cannot be easily rearranged or solved analytically. Examples of root-finding algorithms include the Bisection Method, Newton-Raphson Method, and Secant Method.

1. **Bisection Method:**

   The Bisection Method is a straightforward root-finding technique that works by repeatedly halving the interval in which the root lies. The method assumes that the function changes sign over the interval [a,b][a, b][a,b], which implies that a root exists between aaa and bbb. At each iteration, the interval is halved by computing the midpoint, and the function's value at the midpoint is used to determine the new interval. The process continues until the interval is sufficiently small, meaning the root has been approximated to a desired tolerance.

   <u>Code:</u>

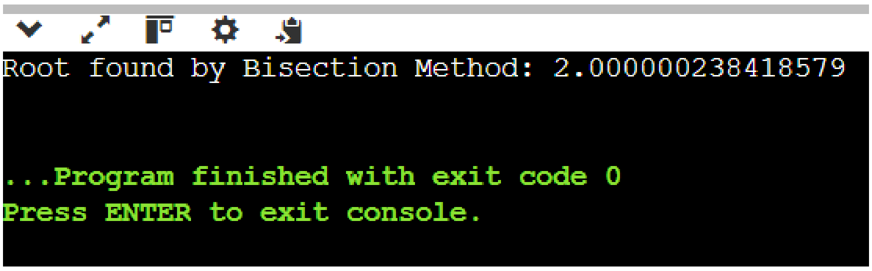
```
def bisection_method(f, a, b, tol=1e-6):
    if f(a) * f(b) >= 0:
        raise ValueError("The function must have different signs at a and b.")

    while (b - a) / 2 > tol:
        midpoint = (a + b) / 2.0
        if f(midpoint) == 0:
            return midpoint
        elif f(a) * f(midpoint) < 0:
            b = midpoint
        else:
            a = midpoint

    return (a + b) / 2.0

# Example usage:
f = lambda x: x**2 - 4
root = bisection_method(f, 0, 3)
print(f"Root found by Bisection Method: {root}")
```

   <u>Output:</u>



2. Newton-Raphson Method:

The Newton-Raphson Method is an iterative root-finding algorithm that uses the function's derivative to find successively better approximations to the root. Starting from an initial guess x0x_0x0, the method uses the formula:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

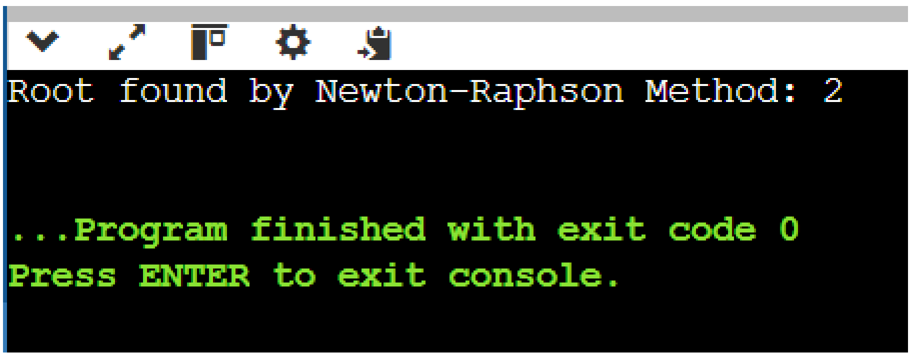
This process is repeated until the difference between consecutive approximations is less than a specified tolerance. The Newton-Raphson method is known for its fast convergence when the initial guess is close to the actual root. However, it requires the function to be differentiable and can fail or converge slowly if the initial guess is poor or if the derivative is zero at any iteration.

Code:

```
def newton_raphson_method(f, df, x0, tol=1e-6, max_iter=1000):
    x = x0
    for i in range(max_iter):
        fx = f(x)
        dfx = df(x)
        if abs(fx) < tol:
            return x
        x = x - fx / dfx
    raise ValueError("Maximum iterations exceeded without finding root.")

# Example usage:
f = lambda x: x**2 - 4
df = lambda x: 2 * x
root = newton_raphson_method(f, df, 2)
print(f"Root found by Newton-Raphson Method: {root}")
```

Output:



3. Secant Method:

The Secant Method is similar to the Newton-Raphson Method but does not require the computation of the derivative. Instead, it approximates the derivative using two previous points. The update formula is:

$$x_{n+1} = x_n - f(x_n) \cdot \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}$$

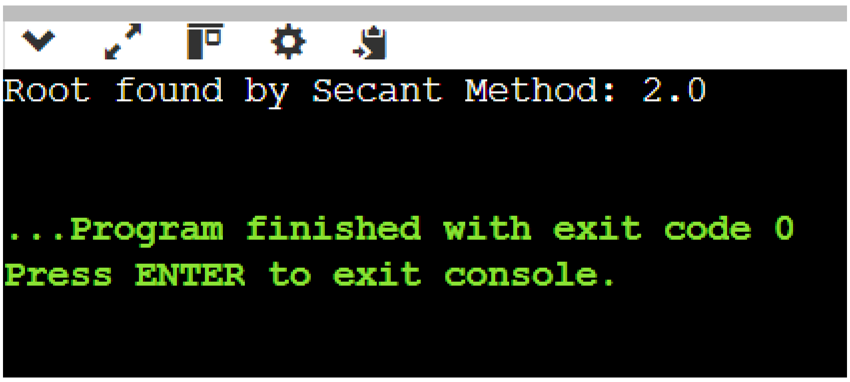
This method is particularly useful when the derivative of the function is difficult or impossible to calculate. The Secant Method generally converges faster than the Bisection Method but slower than the Newton-Raphson Method.

<u>Code:</u>

```
def secant_method(f, x0, x1, tol=1e-6, max_iter=1000):
    for i in range(max_iter):
        f_x0 = f(x0)
        f_x1 = f(x1)
        if abs(f_x1) < tol:
            return x1
        denominator = f_x1 - f_x0
        if denominator == 0:
            raise ValueError("Zero denominator encountered in Secant Method.")
        x2 = x1 - f_x1 * (x1 - x0) / denominator
        x0, x1 = x1, x2
    raise ValueError("Maximum iterations exceeded without finding root.")

# Example usage:
f = lambda x: x**2 - 4
root = secant_method(f, 2, 3)
print(f"Root found by Secant Method: {root}")
```

<u>Output:</u>



Performance analysis of all the mentioned three algorithms are:

Criteria	Bisection Method	Newton-Raphson Method	Secant Method
Convergence Speed	Slow	Fast (Quadratic convergence)	Faster than Bisection, slower than Newton
Order of Convergence	Linear (order 1)	Quadratic (order 2)	Superlinear (≈1.618)
Initial Guess Requirement	Two initial guesses	One initial guess and its derivative	Two initial guesses
Robustness	Very robust, always converges	Can fail if the derivative is zero or changes sign	Less robust, may fail if initial guesses are not close
Function Requirements	Only the function needs to be continuous	Function must be differentiable	Function needs to be continuous, but derivative not required
Error Bound	Error reduces by half each iteration	Error reduces quadratically	Error bound not well-defined
Computational Cost per Iteration	Low, requires two function evaluations	High, requires one function and one derivative evaluation	Moderate, requires two function evaluations
Ease of Implementation	Easy to implement	More complex due to derivative calculation	Moderate, simpler than Newton but more complex than Bisection