

ECS230 HW4

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The data points (x,y) were read from the **data.dat** file, and the polynomial fit(least squares fit) for the points was calculated in the C program **hw4.c**. The data points and the polynomial fit was plotted in **hw4_plot.py** along with residual calculation. To make the execution easier, a bash script(**hw4_run.sh**) was added which would run both the codes together. Command: sh hw4_run.sh <degree of polynomial>

On running the program for all the degrees ranging from 3 to 9, we found that the polynomial fit was better for higher degrees. It achieves the best fit for degree 9 (number of data points - 1). This is due to the fact that least squares error keeps on decreasing with the number of coefficients used, to the point where the polynomial overfits (for degree 9).

This can also be explained using the rank of the matrix X. The matrix $X^T X$ is invertible only when its rank is equal to the number of data points. If this is not the case, then there will be linearly dependent rows that cause a loss of accuracy while approximating the inverse of $X^T X$. This also explains the case for polynomials of degrees greater or equal to the number of data points (10). The linearly dependent rows forces the matrix X to be singular and hence causes the coefficients for the higher degrees to be approximately 0. The coefficients of the lower degrees are the same as the polynomial of degree 9 and hence the fitted polynomial is also the same as degree 9 for all higher degrees.

From various runs of the program using different degrees, we can see that the residual decreases as the degree increases suggesting a better polynomial fit. This can be seen in Figure 8.

We can see the quality of the fit obtained for polynomials of all the degrees in the figures below.

Degrees 5 and 9 are in Figures 3 and 7. The corresponding polynomials(with the coefficients) are:

$$P_5(x) = 3.302285 + 0.548540x + 1.698495x^2 - 1.062627x^3 + 0.254667x^4 - 0.021502x^5$$

$$P_9(x) = 248.965958 - 2335.344129x + 7418.507300x^2 - 10986.641009x^3 + 8904.162047x^4 - 4275.885028x^5 + 1251.408627x^6 - 219.339567x^7 + 21.169436x^8 - 0.865455x^9$$

The complete output for degrees 5 and 9 are included at the end of the report. Also, the coefficients for the other polynomial degrees along with the matrix L can be found at the end of the report.

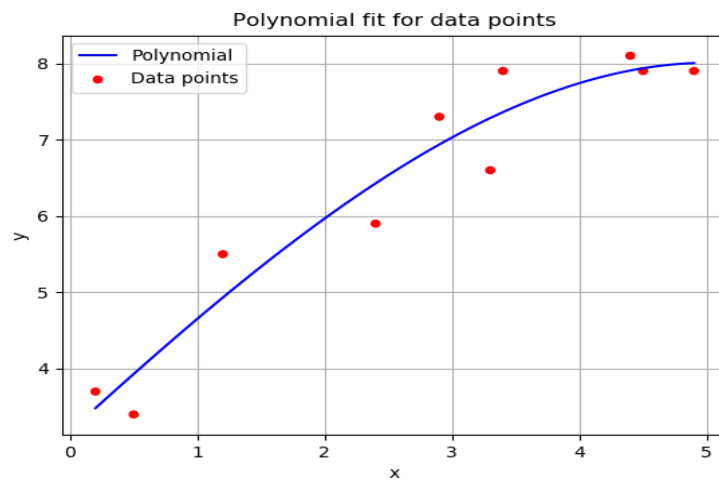


Figure 1: Polynomial fit for degree 3

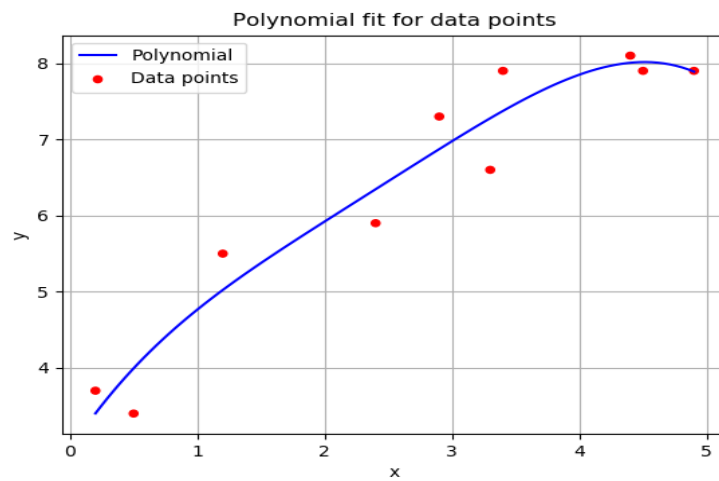


Figure 2: Polynomial fit for degree 4

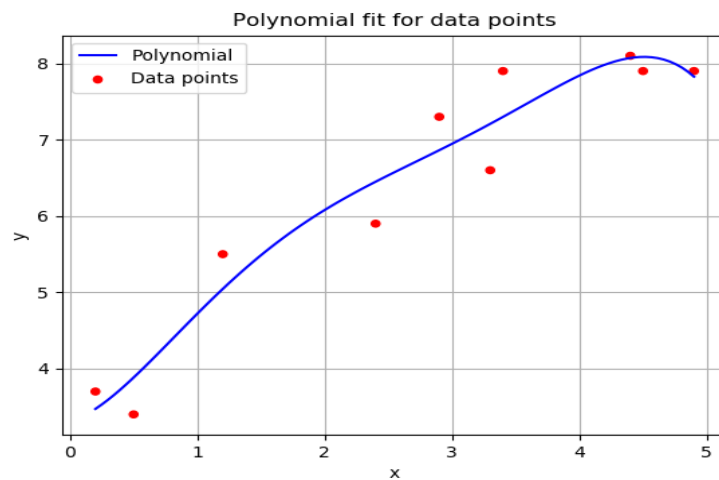


Figure 3: Polynomial fit for degree 5

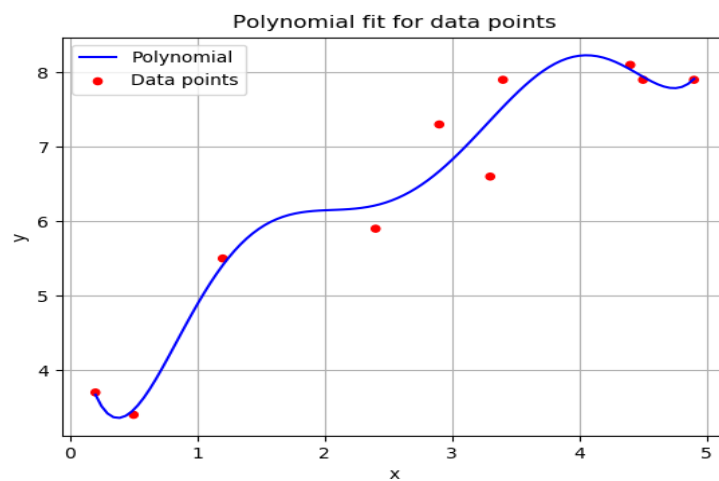


Figure 4: Polynomial fit for degree 6

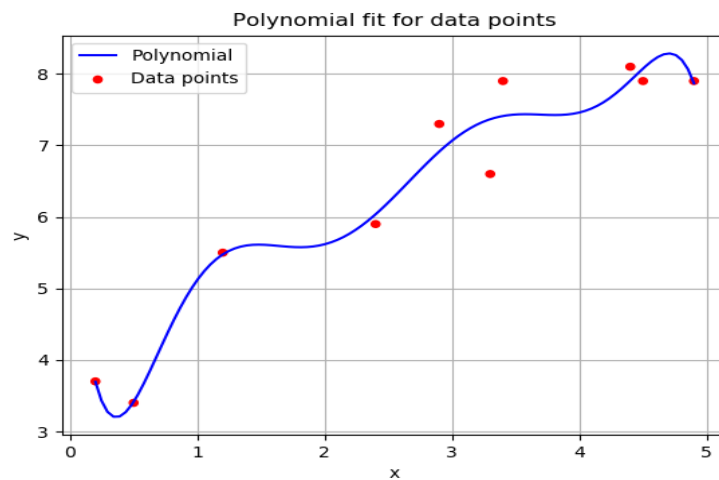


Figure 5: Polynomial fit for degree 7

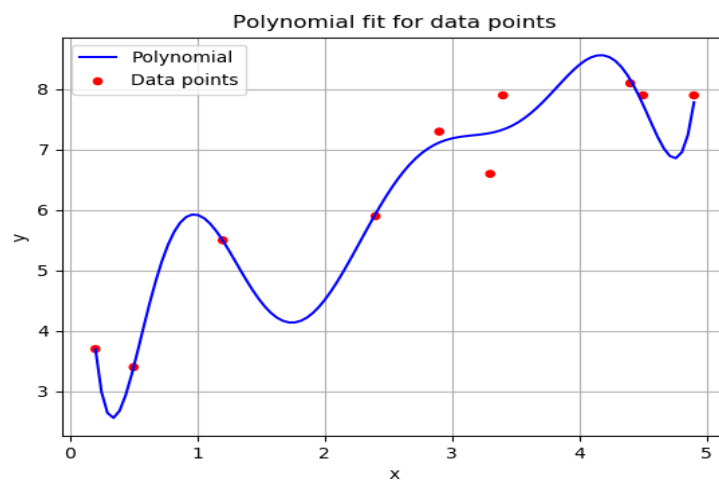


Figure 6: Polynomial fit for degree 8

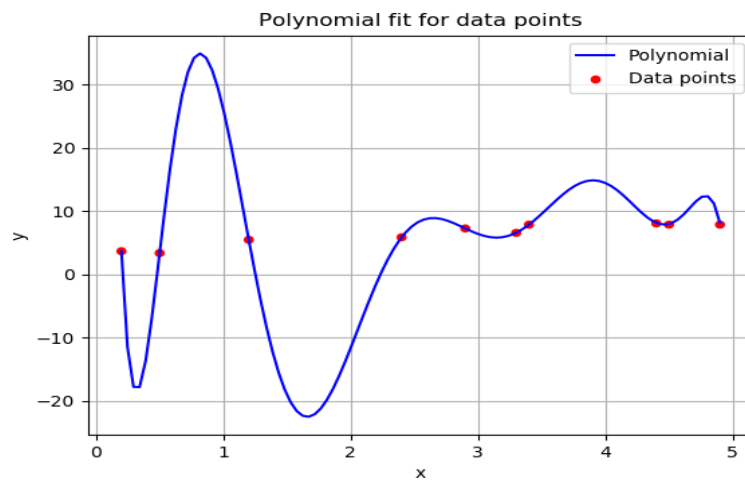


Figure 7: Polynomial fit for degree 9

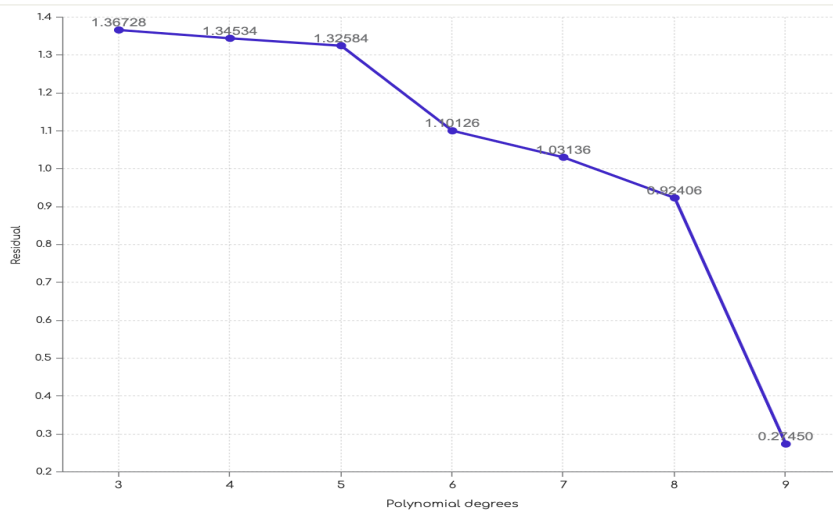


Figure 8: Residuals for various polynomial degrees

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~ /workspace/ECS230 sh hw4_run.sh 5
Matrix X:
1.000000 0.500000 0.250000 0.125000 0.062500 0.031250
1.000000 0.200000 0.040000 0.008000 0.001600 0.000320
1.000000 1.200000 1.440000 1.728000 2.073600 2.488320
1.000000 2.900000 8.410000 24.389000 70.728100 205.111490
1.000000 3.400000 11.560000 39.304000 133.633600 454.354240
1.000000 2.400000 5.760000 13.824000 33.177600 79.626240
1.000000 4.400000 19.360000 85.184000 374.809600 1649.162240
1.000000 4.500000 20.250000 91.125000 410.062500 1845.281250
1.000000 4.900000 24.010000 117.649000 576.480100 2824.752490
1.000000 3.300000 10.890000 35.937000 118.592100 391.353930

-----
Matrix XTX:
10.000000 27.700000 101.970000 409.273000 1719.621300 7452.161770
27.700000 101.970000 409.273000 1719.621300 7452.161770 33026.567037
101.970000 409.273000 1719.621300 7452.161770 33026.567037 148818.438655
409.273000 1719.621300 7452.161770 33026.567037 148818.438655 678992.342083
1719.621300 7452.161770 33026.567037 148818.438655 678992.342083 3127498.672417
7452.161770 33026.567037 148818.438655 678992.342083 3127498.672417 14512038.543277

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Matrix XTY:
64.200000
202.910000
773.963000
3156.550700
13391.743790
58404.717011

-----
Matrix L:
3.162278 0.000000 0.000000 0.000000 0.000000 0.000000
8.759509 5.024042 0.000000 0.000000 0.000000 0.000000
32.245745 25.241846 6.533178 0.000000 0.000000 0.000000
129.423486 116.626228 51.268111 6.784688 0.000000 0.000000
543.792002 535.188731 303.441456 68.546818 8.957628 0.000000
2356.580469 2464.963161 1623.792326 481.578786 117.436679 10.613064

-----
Matrix b:
3.302285
0.548540
1.698495
-1.062627
0.254667
-0.021502

-----
Residual for the polynomial fit - 1.325843052048077

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Figure 9: Complete output for degree 5

```

Matrix X:
1.000000 0.500000 0.250000 0.125000 0.062500 0.031250 0.015625 0.007812 0.003906 0.001953
1.000000 0.200000 0.040000 0.008000 0.001600 0.000320 0.000064 0.000013 0.000003 0.000001
1.000000 1.200000 1.440000 1.728000 2.073600 2.488320 2.985984 3.583181 4.290817 5.159780
1.000000 2.900000 8.410000 24.309000 70.728100 205.111490 594.823321 1724.987631 5002.464130 14507.145976
1.000000 3.400000 11.560000 39.304000 133.633600 454.354240 1544.804416 5252.335014 17857.939049 60716.992766
1.000000 2.400000 5.760000 13.824000 33.177600 79.626240 191.102976 458.647142 1100.753142 2641.807540
1.000000 4.400000 19.360000 85.184000 374.809600 1649.162240 7256.312856 31927.780966 140482.236252 618121.839510
1.000000 4.500000 20.250000 91.125000 410.062500 1845.281250 8380.765625 37366.945312 168151.253906 756680.642578
1.000000 4.900000 24.010000 117.649000 576.480100 2824.752490 13841.287201 67822.307285 332329.305696 1628413.597910
1.000000 3.300000 10.890000 35.937000 118.592100 391.353930 1291.467969 4261.844298 14064.086182 46411.404402

Matrix XT:
10.000000 27.700000 101.970000 409.273000 1719.621300 7452.161770 33026.567037 148818.438655 678992.342083 3127498.672417
27.700000 101.970000 409.273000 1719.621300 7452.161770 33026.567037 148818.438655 678992.342083 3127498.672417 14512038.543277
101.970000 409.273000 1719.621300 7452.161770 33026.567037 148818.438655 678992.342083 3127498.672417 14512038.543277 67732371.152119
409.273000 1719.621300 7452.161770 33026.567037 148818.438655 678992.342083 3127498.672417 14512038.543277 67732371.152119 317632499.957908
1719.621300 7452.161770 33026.567037 148818.438655 678992.342083 3127498.672417 14512038.543277 67732371.152119 317632499.957908 1495443977.308277
7452.161770 33026.567037 148818.438655 678992.342083 3127498.672417 14512038.543277 67732371.152119 317632499.957908 1495443977.308277 7064473457.161751
33026.567037 148818.438655 678992.342083 3127498.672417 14512038.543277 67732371.152119 317632499.957908 1495443977.308277 7064473457.161751 33470793953.852802
148818.438655 678992.342083 3127498.672417 14512038.543277 67732371.152119 317632499.957908 1495443977.308277 7064473457.161751 33470793953.852798 158995811147.602264
678992.342083 3127498.672417 14512038.543277 67732371.152119 317632499.957908 1495443977.308277 7064473457.161751 33470793953.852798 158995811147.602264 757053992476.260010
3127498.672417 14512038.543277 67732371.152119 317632499.957908 1495443977.308277 7064473457.161751 33470793953.852802 158995811147.602264 757053992476.260010 3612429064743.778320

Matrix XTv:
64.200000
202.910000
773.963000
3156.550700
13391.743790
58404.717011
259935.897116
1174549.902276
5368639.315758
24756528.655340

Matrix L:
3.162278 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000
8.759509 5.024042 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000
32.245745 25.241846 6.533178 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000
129.423406 116.626228 51.268111 6.784688 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000
543.792002 535.188731 303.441456 68.546818 0.957628 0.000000 0.000000 0.000000 0.000000 0.000000
2356.580469 2464.963161 1623.792326 481.578786 117.436679 10.613064 0.000000 0.000000 0.000000 0.000000
10443.917513 11412.095224 8289.700126 2927.706354 1001.530335 165.427909 12.143240 0.000000 0.000000 0.000000
47060.522398 53097.736152 41283.456271 16535.495790 7073.046800 1610.271754 209.050765 7.294516 0.000000 0.000000
214716.231479 248144.787781 202768.251954 89512.168065 45008.406820 12607.957140 2187.198088 148.400997 5.064407 0.000000
909001.918399 1164175.576363 908083.660915 471920.750926 268462.145230 87058.156497 18076.109928 1781.693568 123.323487 1.044920

Matrix b:
248.965958
-2335.344129
7418.507300
-10986.641009
8904.162047
-4275.885028
1251.408627
-219.339567
21.169436
-0.865455

Residual for the polynomial fit - 0.274495660605805

```

Figure 10: Complete output for degree 9

For polynomials of degree 3:

$$P_3(x) = -0.01593x^3 - 0.03132x^2 + 1.521x + 3.179$$

$$L = \begin{bmatrix} 3.162278 & 0.000000 & 0.000000 & 0.000000 \\ 8.759509 & 5.024042 & 0.000000 & 0.000000 \\ 32.245745 & 25.241846 & 6.533178 & 0.000000 \\ 129.423486 & 116.626228 & 51.268111 & 6.784688 \end{bmatrix}$$

For polynomials of degree 4:

$$P_4(x) = -0.02723x^4 + 0.2592x^3 - 0.9256x^2 + 2.527x + 2.931$$

$$L = \begin{bmatrix} 3.162278 & 0.000000 & 0.000000 & 0.000000 & 0.000000 \\ 8.759509 & 5.024042 & 0.000000 & 0.000000 & 0.000000 \\ 32.245745 & 25.241846 & 6.533178 & 0.000000 & 0.000000 \\ 129.423486 & 116.626228 & 51.268111 & 6.784688 & 0.000000 \\ 543.792002 & 535.188731 & 303.441456 & 68.546818 & 8.957628 \end{bmatrix}$$

For polynomials of degree 5:

$$P_5(x) = -0.0215x^5 + 0.2547x^4 - 1.063x^3 + 1.698x^2 + 0.5485x + 3.302$$

$$L = \begin{bmatrix} 3.162278 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 \\ 8.759509 & 5.024042 & 0.000000 & 0.000000 & 0.000000 & 0.000000 \\ 32.245745 & 25.241846 & 6.533178 & 0.000000 & 0.000000 & 0.000000 \\ 129.423486 & 116.626228 & 51.268111 & 6.784688 & 0.000000 & 0.000000 \\ 543.792002 & 535.188731 & 303.441456 & 68.546818 & 8.957628 & 0.000000 \\ 2356.580469 & 2464.963161 & 1623.792326 & 481.578786 & 117.436679 & 10.613064 \end{bmatrix}$$

For polynomials of degree 6:

$$P_6(x) = 0.0608x^6 - 0.9693x^5 + 5.882x^4 - 16.88x^3 + 22.87x^2 - 11.21x + 5.124$$

$$L = \begin{bmatrix} 3.162278 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 \\ 8.759509 & 5.024042 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 \\ 32.245745 & 25.241846 & 6.533178 & 0.000000 & 0.000000 & 0.000000 & 0.000000 \\ 129.423486 & 116.626228 & 51.268111 & 6.784688 & 0.000000 & 0.000000 & 0.000000 \\ 543.792002 & 535.188731 & 303.441456 & 68.546818 & 8.957628 & 0.000000 & 0.000000 \\ 2356.580469 & 2464.963161 & 1623.792326 & 481.578786 & 117.436679 & 10.613064 & 0.000000 \\ 10443.917513 & 11412.095224 & 8289.700126 & 2927.706354 & 1001.530335 & 165.427909 & 12.143240 \end{bmatrix}$$

For polynomials of degree 7:

$$P_7(x) = -0.05299x^7 + 0.9731x^6 - 7.149x^5 + 26.74x^4 - 53.51x^3 + 54.7x^2 - 23.09x + 6.512$$

$$L = \begin{bmatrix} 3.162278 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 \\ 8.759509 & 5.024042 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 \\ 32.245745 & 25.241846 & 6.533178 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 \\ 129.423486 & 116.626228 & 51.268111 & 6.784688 & 0.000000 & 0.000000 & 0.000000 & 0.000000 \\ 543.792002 & 535.188731 & 303.441456 & 68.546818 & 8.957628 & 0.000000 & 0.000000 & 0.000000 \\ 2356.580469 & 2464.963161 & 1623.792326 & 481.578786 & 117.436679 & 10.613064 & 0.000000 & 0.000000 \\ 10443.917513 & 11412.095224 & 8289.700126 & 2927.706354 & 1001.530335 & 165.427909 & 12.143240 & 0.000000 \\ 47060.522398 & 53097.736152 & 41283.456271 & 16535.495790 & 7073.046800 & 1610.271754 & 209.050765 & 7.294516 \end{bmatrix}$$

For polynomials of degree 8:

$$P_8(x) = 0.09472x^8 - 1.98x^7 + 17.09x^6 - 78.46x^5 + 205.7x^4 - 306.4x^3 + 242x^2 - 85.42x + 13.25$$

$$L = \begin{bmatrix} 3.162278 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 \\ 8.759509 & 5.024042 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 \\ 32.245745 & 25.241846 & 6.533178 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 \\ 129.423486 & 116.626228 & 51.268111 & 6.784688 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 \\ 543.792002 & 535.188731 & 303.441456 & 68.546818 & 8.957628 & 0.000000 & 0.000000 & 0.000000 & 0.000000 \\ 2356.580469 & 2464.963161 & 1623.792326 & 481.578786 & 117.436679 & 10.613064 & 0.000000 & 0.000000 & 0.000000 \\ 10443.917513 & 11412.095224 & 8289.700126 & 2927.706354 & 1001.530335 & 165.427909 & 12.143240 & 0.000000 & 0.000000 \\ 47060.522398 & 53097.736152 & 41283.456271 & 16535.495790 & 7073.046800 & 1610.271754 & 209.050765 & 7.294516 & 0.000000 \\ 214716.231479 & 248144.787781 & 202768.251954 & 89512.168065 & 45008.406820 & 12607.957140 & 2187.198088 & 148.400997 & 5.064407 \end{bmatrix}$$

For polynomials of degree 9:

$$P_9(x) = -0.8655x^9 + 21.17x^8 - 219.3x^7 + 1251x^6 - 4276x^5 + 8904x^4 - 1.099e + 04x^3 + 7419x^2 - 2335x + 249$$

$$L = \begin{bmatrix} 3.162278 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 \\ 8.759509 & 5.024042 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 \\ 32.245745 & 25.241846 & 6.533178 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 \\ 129.423486 & 116.626228 & 51.268111 & 6.784688 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 \\ 543.792002 & 535.188731 & 303.441456 & 68.546818 & 8.957628 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 \\ 2356.580469 & 2464.963161 & 1623.792326 & 481.578786 & 117.436679 & 10.613064 & 0.000000 & 0.000000 & 0.000000 & 0.000000 \\ 10443.917513 & 11412.095224 & 8289.700126 & 2927.706354 & 1001.530335 & 165.427909 & 12.143240 & 0.000000 & 0.000000 & 0.000000 \\ 47060.522398 & 53097.736152 & 41283.456271 & 16535.495790 & 7073.046800 & 1610.271754 & 209.050765 & 7.294516 & 0.000000 & 0.000000 \\ 214716.231479 & 248144.787781 & 202768.251954 & 89512.168065 & 45008.406820 & 12607.957140 & 2187.198088 & 148.400997 & 5.064407 & 0.000000 \\ 989001.918399 & 1164175.576363 & 988083.660915 & 471920.750926 & 268462.145230 & 87058.156497 & 18076.109928 & 1781.693568 & 123.323487 & 1.044920 \end{bmatrix}$$