ESCUELA POLITÉCNICA NACIONAL

MÉTODOS NUMÉRICOS

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[Tarea 12] Ejercicios Unidad 05-A | ODE Método de Euler

```
%load_ext autoreload
import numpy as np
import math
from src import ODE_euler, graphics, ODE_euler_nth
```

```
The autoreload extension is already loaded. To reload it, use:
  %reload_ext autoreload
```

EJERCICIO UNO

Use el método de Euler para aproximar las soluciones para cada uno de los siguientes problemas de valor inicial.

PARTE A

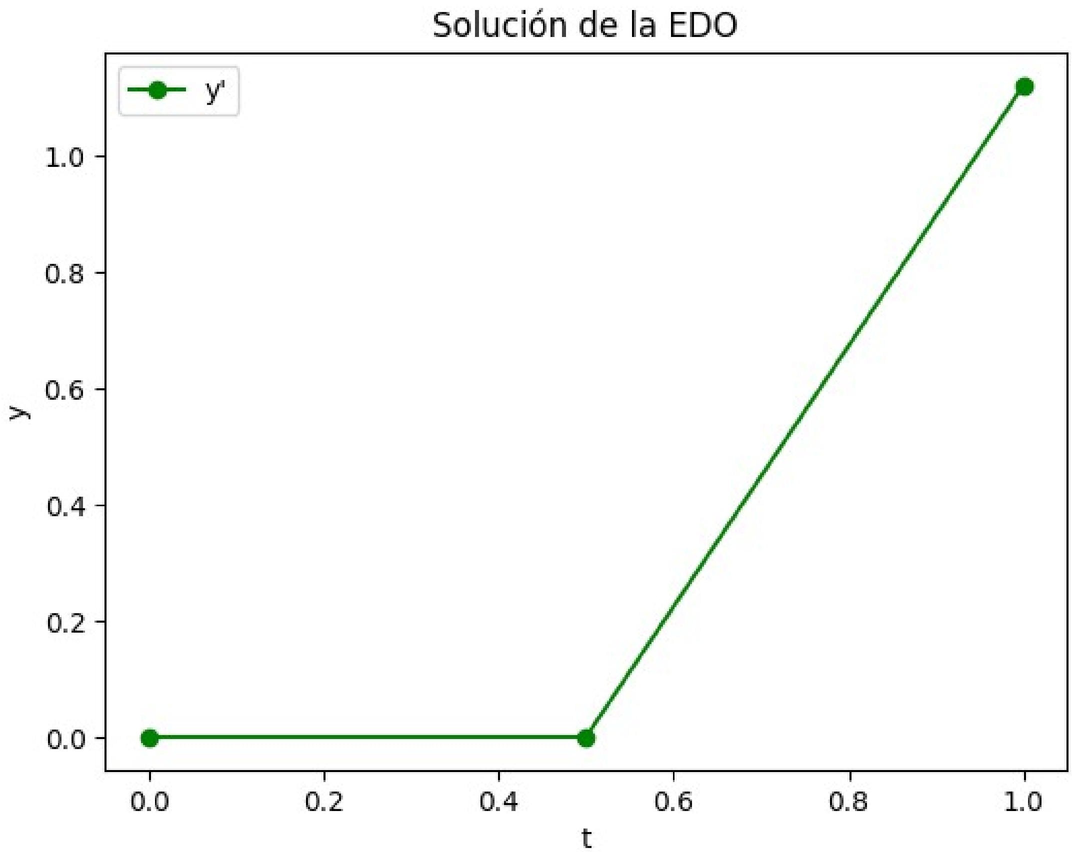
$$\begin{pmatrix} y' = t e^{3t} - 2y, & 0 \le t \le 1 \\ y(0) = 0, & h = 0.5 \end{pmatrix}$$

```
y_der = lambda t, y: t*math.exp(3*t) - 2*y
y_init = 0

ys1a, ts1a, h = ODE_euler(a = 0, b = 1, f = y_der, y_t0 = y_init, N =
2)

print(f"El valor de h es: {h}")
graphics(ts1a, ys1a)
```

```
El valor de h es: 0.5
```



PARTE B

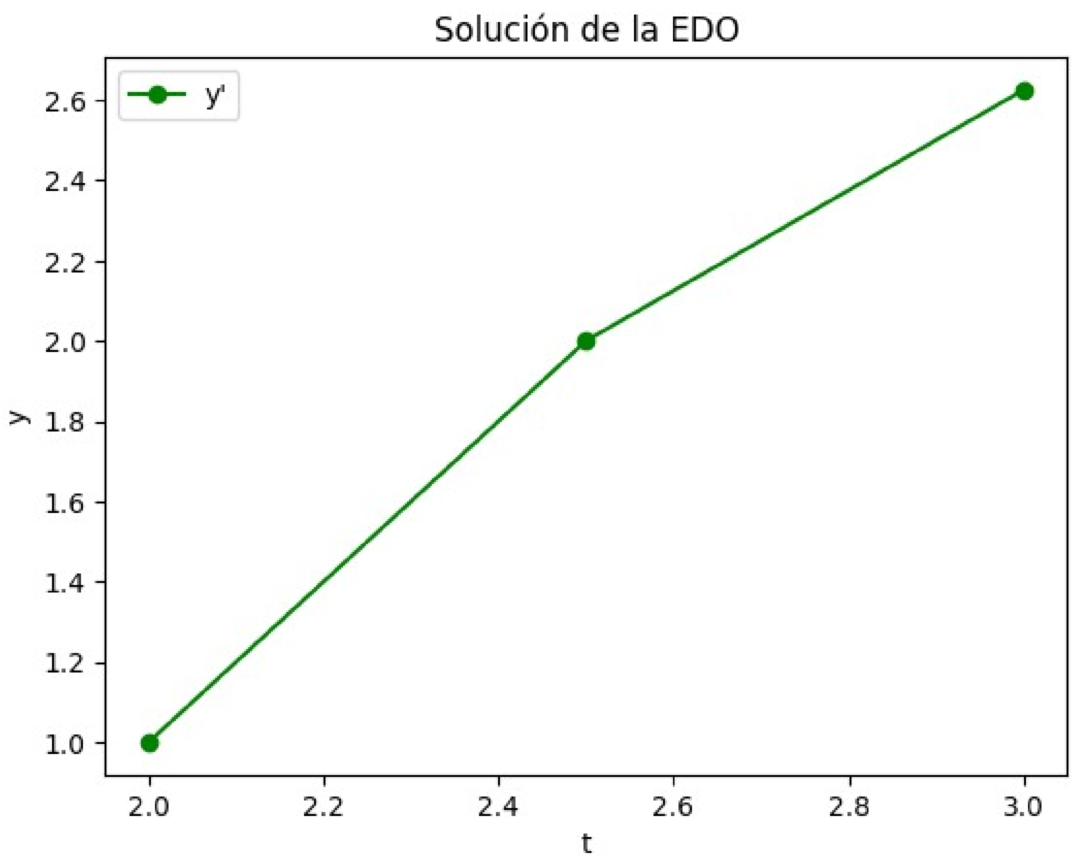
$$\begin{cases} y' = 1 + (t - y)^2, & 2 \le t \le 3 \\ y(2) = 1, & h = 0.5 \end{cases}$$

```
y_der = lambda t, y: 1 + (t - y)**2
y_init = 1

ys1b, ts1b, h = ODE_euler(a = 2, b = 3, f = y_der, y_t0 = y_init, N = 
2)

print(f"El valor de h es: {h}")
graphics(ts1b, ys1b)

El valor de h es: 0.5
```



PARTE C

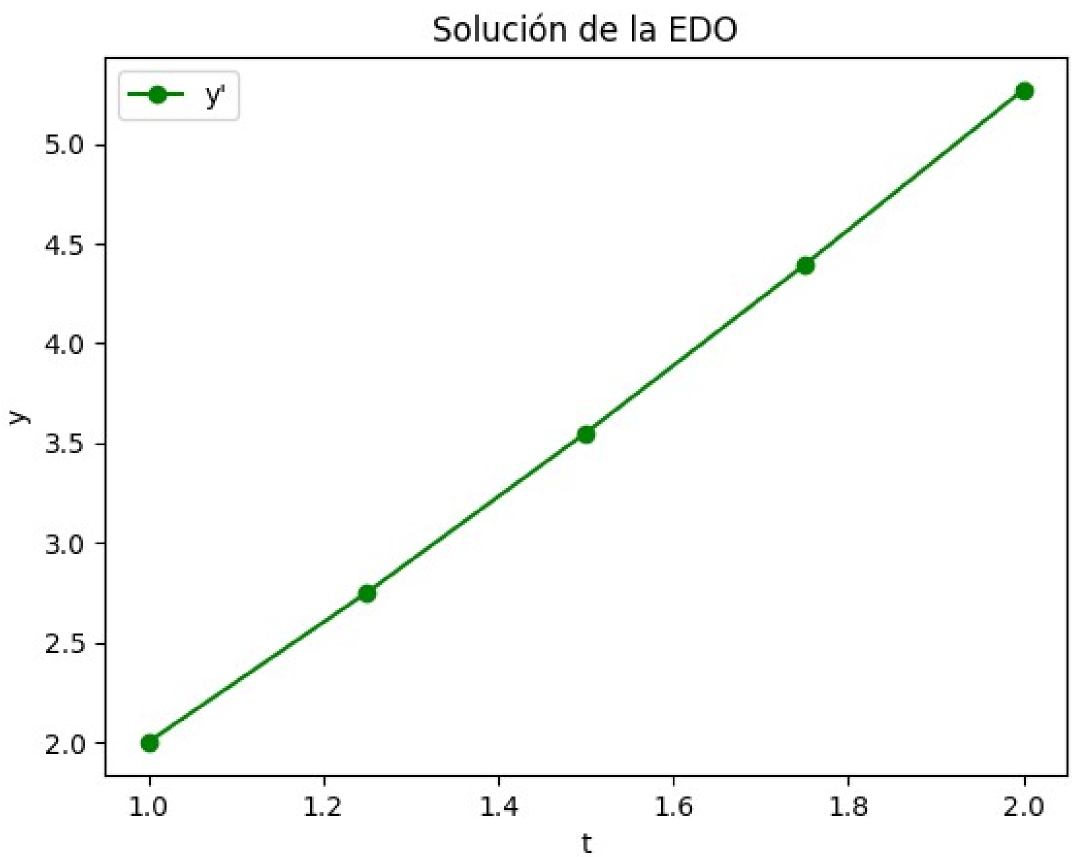
$$\begin{cases} y' = 1 + \dfrac{y}{t}, & 1 \le t \le 2 \\ y(1) = 2, & h = 0.25 \end{cases}$$

```
y_der = lambda t, y: 1 + y/t
y_init = 2

ys1c, ts1c, h = ODE_euler(a = 1, b = 2, f = y_der, y_t0 = y_init, N =
4)

print(f"El valor de h es: {h}")
graphics(ts1c, ys1c)

El valor de h es: 0.25
```



PARTE D

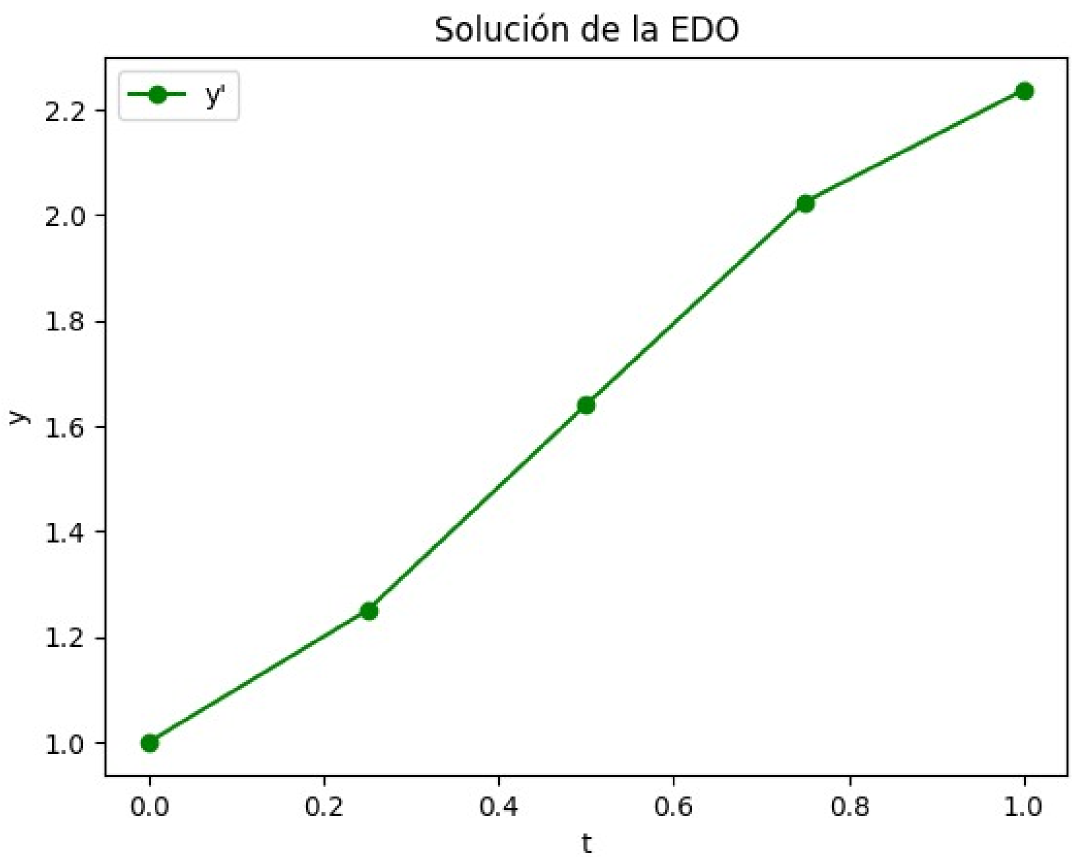
$$\begin{cases} y' = \cos(2t) + \sin(3t), & 0 \le t \le 1 \\ y(0) = 1, & h = 0.25 \end{cases}$$

```
y_der = lambda t, y: math.cos(2*t) + math.sin(3*t)
y_init = 1

ys1d, ts1d, h = ODE_euler(a = 0, b = 1, f = y_der, y_t0 = y_init, N =
4)

print(f"El valor de h es: {h}")
graphics(ts1d, ys1d)
```

```
El valor de h es: 0.25
```



EJERCICIO DOS

Las soluciones reales para los problemas de valor inicial en el ejercicio 1 se proporcionan aquí. Compare el error real en cada paso

PARTE A

$$y(t)=\frac{1}{5}te^{3t}-\frac{1}{25}e^{3t}+\frac{1}{25}e^{-2t}$$

```
def y(t):
    return 1/5*t*math.exp(3*t) - 1/25*t*math.exp(3*t) + 
1/25*t*math.exp(-2*t)

errorReal = np.mean([abs(y(t) - y_aprox) / abs(y(t)) for y_aprox, t in 
zip(ys1a, ts1a)])
print(f"El error real es: {errorReal}")

---------------------------------------------------------------------
-----
ZeroDivisionError                         Traceback (most recent call
```

```
last)
Cell In[12], line 4
      1 def y(t):
      2     return 1/5*t*math.exp(3*t) - 1/25*t*math.exp(3*t) +
1/25*t*math.exp(-2*t)
----> 4 errorReal = np.mean([abs(y(t) - y_aprox) / abs(y(t)) for
y_aprox, t in zip(ys1a, ts1a)])
      5 print(f"El error real es: {errorReal}")

ZeroDivisionError: float division by zero
```

PARTE B

$$y(t)=t+\frac{1}{1-t}$$

```
def y(t):
    return t + 1/(1 - t)

errorReal = np.mean([abs(y(t) - y_aprox) / abs(y(t)) for y_aprox, t in
zip(ys1b, ts1b)])
print(f"El error real es: {errorReal}")

El error real es: 0.04696969696969694
```

PARTE C

$$y(t)=t\ln t+2t$$

```
def y(t):
    return t * math.log(t) + 2*t

errorReal = np.mean([abs(y(t) - y_aprox) / abs(y(t)) for y_aprox, t in
zip(ys1c, ts1c)])
print(f"El error real es: {errorReal}")

El error real es: 0.013575458924045315
```

PARTE D

$$y(t)=\frac{1}{2}\sin(2t)-\frac{1}{3}\cos(3t)+\frac{4}{3}$$

```
def y(t):
    return 1/2*math.sin(2*t) - 1/3*math.cos(3*t) + 4/3

errorReal = np.mean([abs(y(t) - y_aprox) / abs(y(t)) for y_aprox, t in
```

```
zip(ys1d, ts1d)])
print(f"El error real es: {errorReal}")

El error real es: 0.035265188624637164
```

EJERCICIO TRES

Utilice el método de Euler para aproximar las soluciones para cada uno de los siguientes problemas de valor inicial.

PARTE A

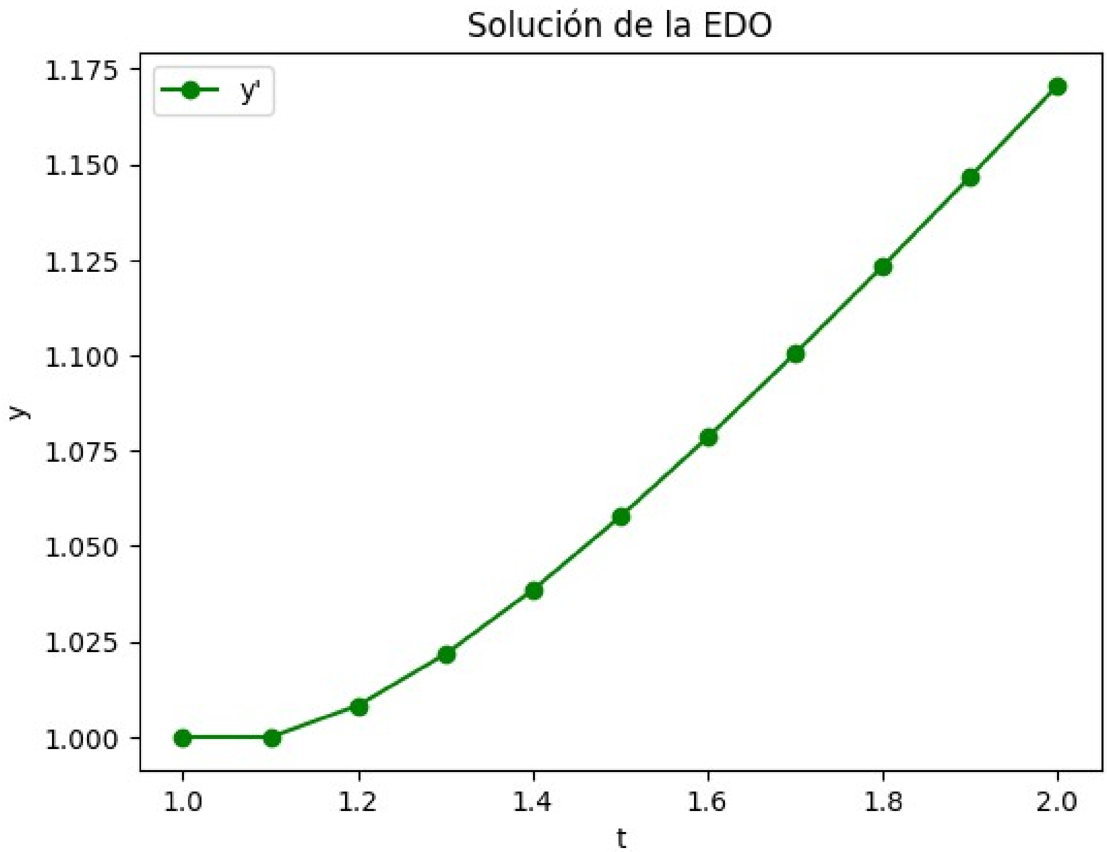
$$\left\{ \begin{array}{ll} y' = \frac{y}{t} - \left(\frac{y}{t}\right)^2, & 1 \le t \le 2 \\ y(1) = 1, & h = 0.1 \end{array} \right)$$

```
y_der = lambda t, y: y/t - (y/t)**2
y_init = 1

ys2a, ts2a, h = ODE_euler(a = 1, b = 2, f = y_der, y_t0 = y_init, N =
10)

print(f"El valor de h es: {h}")
graphics(ts2a, ys2a)

El valor de h es: 0.1
```



PARTE B

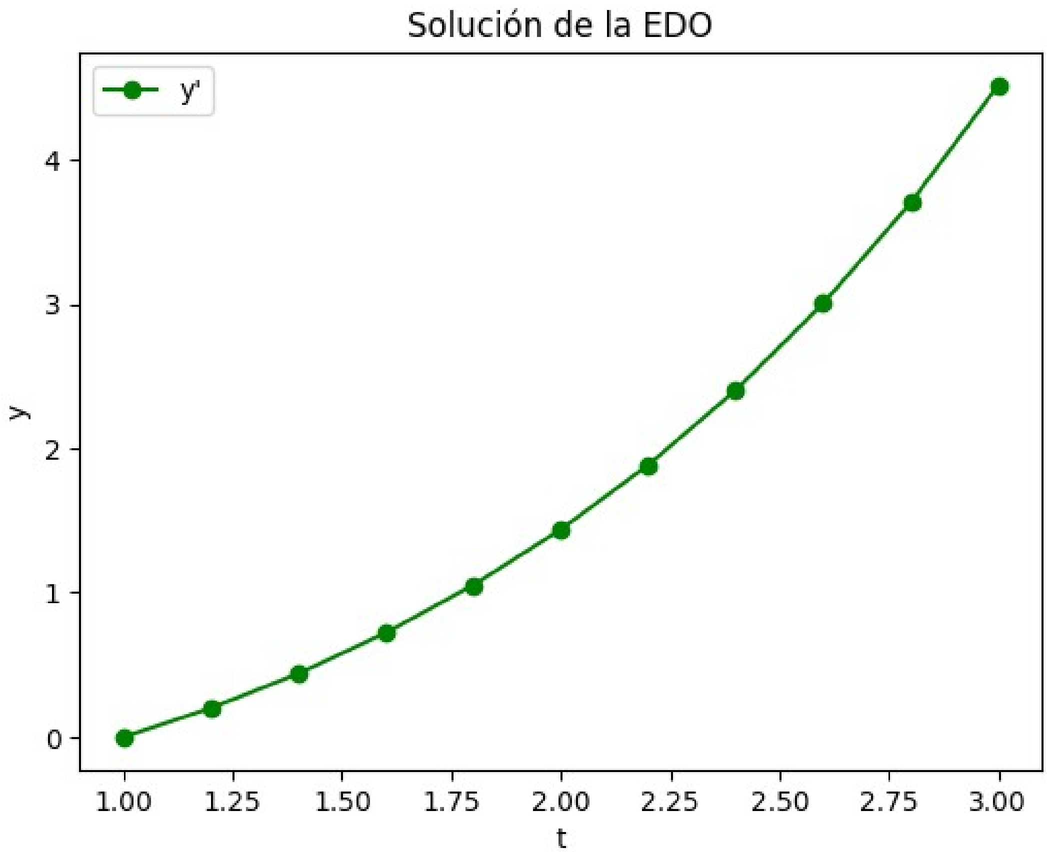
$$\begin{cases} y' = 1 + \dfrac{y}{t} + \left(\dfrac{y}{t}\right)^2, & 1 \le t \le 3 \\ y(1) = 0, & h = 0.2 \end{cases}$$

```
y_der = lambda t, y: 1 + y/t + (y/t)**2
y_init = 0

ys2b, ts2b, h = ODE_euler(a = 1, b = 3, f = y_der, y_t0 = y_init, N =
10)

print(f"El valor de h es: {h}")
graphics(ts2b, ys2b)

El valor de h es: 0.2
```



PARTE C

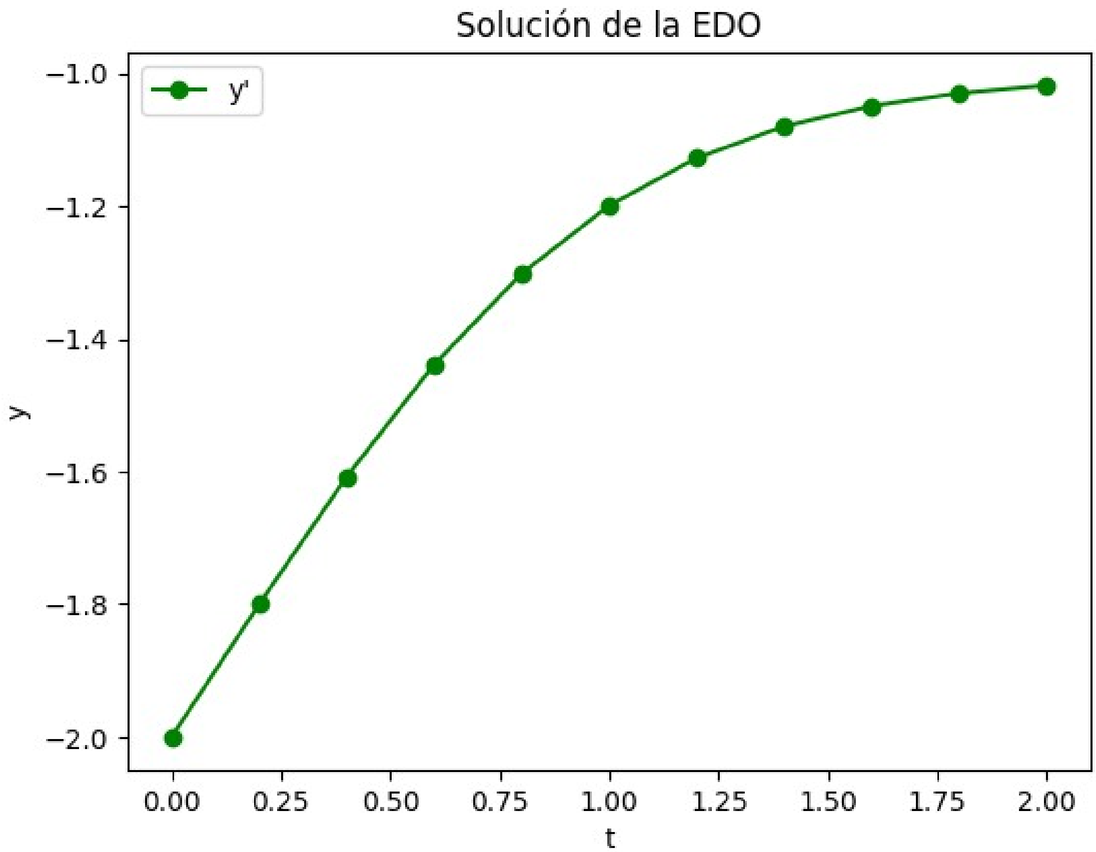
$$\begin{cases} y' = -(y+1)(y+3), & 0 \le t \le 2 \\ y(0) = -2, & h = 0.2 \end{cases}$$

```
y_der = lambda t, y: -(y + 1)*(y + 3)
y_init = -2

ys2c, ts2c, h = ODE_euler(a = 0, b = 2, f = y_der, y_t0 = y_init, N =
10)

print(f"El valor de h es: {h}")
graphics(ts2c, ys2c)

El valor de h es: 0.2
```



PARTE D

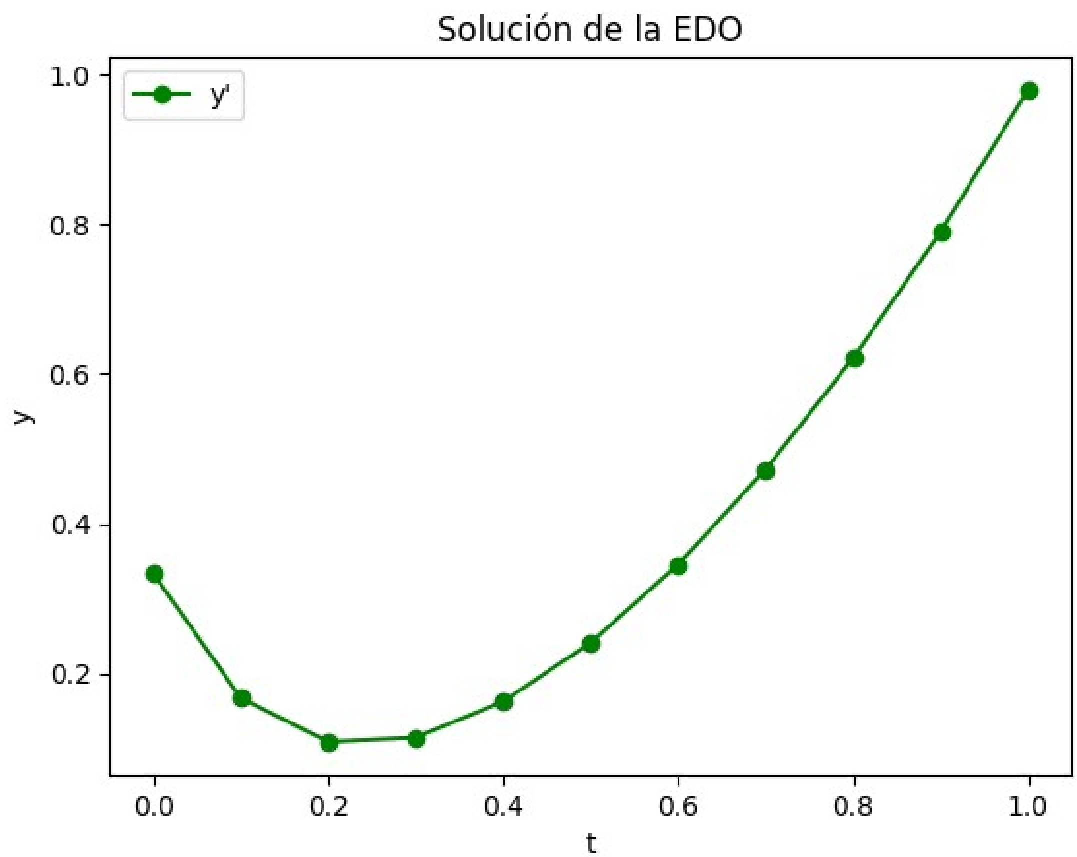
$$\begin{cases} y' = -5y + 5t^2 + 2t, & 0 \le t \le 1 \\ y(0) = \frac{1}{2}, & h = 0.1 \end{cases}$$

```
y_der = lambda t, y: -5*y + 5*t**2 + 2*t
y_init = 1/3

ys2d, ts2d, h = ODE_euler(a = 0, b = 1, f = y_der, y_t0 = y_init, N =
10)

print(f"El valor de h es: {h}")
graphics(ts2d, ys2d)

El valor de h es: 0.1
```



EJERCICIO CUATRO

Aquí se dan las soluciones reales para los problemas de valor inicial en el ejercicio 3. Calcule el error real en las aproximaciones del ejercicio 3.

PARTE A

$$y(t)=\frac{t}{1+\ln t}$$

```
def y1(t):
    return t/(1 + math.log(t))

errorReal = np.mean([abs(y(t) - y_aprox) / abs(y(t)) for y_aprox, t in
zip(ys2a, ts2a)])
print(f"El error real es: {errorReal}")

El error real es: 0.4026114748989524
```

PARTE B

$$y(t) = t\tan(\ln t)$$

```
def y2(t):
    return t*math.tan(math.log(t))

errorReal = np.mean([abs(y(t) - y_aprox) / abs(y(t)) for y_aprox, t in zip(ys2b, ts2b)])
print(f"El error real es: {errorReal}")

El error real es: 1.4857714189452615
```

PARTE C

$$y(t) = -3 + \frac{2}{1+e^{-2t}}$$

```
def y3(t):
    return - 3 + 2/(1 + math.exp(-2*t))

errorReal = np.mean([abs(y(t) - y_aprox) / abs(y(t)) for y_aprox, t in zip(ys2c, ts2c)])
print(f"El error real es: {errorReal}")

El error real es: 2.0191941754493365
```

PARTE D

$$y(t) = t^2 + \frac{1}{3}e^{-5t}$$

```
def y4(t):
    return t**2 + (1/3)*math.exp(-5*t)

errorReal = np.mean([abs(y(t) - y_aprox) / abs(y(t)) for y_aprox, t in zip(ys2d, ts2d)])
print(f"El error real es: {errorReal}")

El error real es: 0.7773952281750381
```

EJERCICIO CINCO

Utilice los resultados del ejercicio 3 y la interpolación lineal para aproximar los siguientes valores de $y(t)$. Compare las aproximaciones asignadas para los valores reales obtenidos mediante las funciones determinadas en el ejercicio

PARTE A

$$y(0.25) \text{ y } y(0.93)$$

```
res = y1(0.25)
print(res)

res = y1(0.93)
print(res)
```

```
-0.6471748623905226
1.0027718477462106
```

PARTE B

$$y(t)=y(1.25) \text{ y } y(1.93)$$

```
res = y2(1.25)
print(res)

res = y2(1.93)
print(res)
```

```
0.2836531261952289
1.4902277738186658
```

PARTE C

$$y(2.10) \text{ y } y(2.75)$$

```
res = y3(2.1)
print(res)

res = y3(2.75)
print(res)
```

```
-1.0295480633865461
-1.008140275431792
```

PARTE D

$$y(t)=y(0.54) \text{ y } y(0.94)$$

```
res = y4(0.54)
print(res)

res = y4(0.94)
print(res)
```

```
0.3140018375799166
0.8866317590338986
```

EJERCICIO SEIS

Use el método de Taylor de orden 2 para aproximar las soluciones para cada uno de los siguientes problemas de valor inicial.

PARTE A

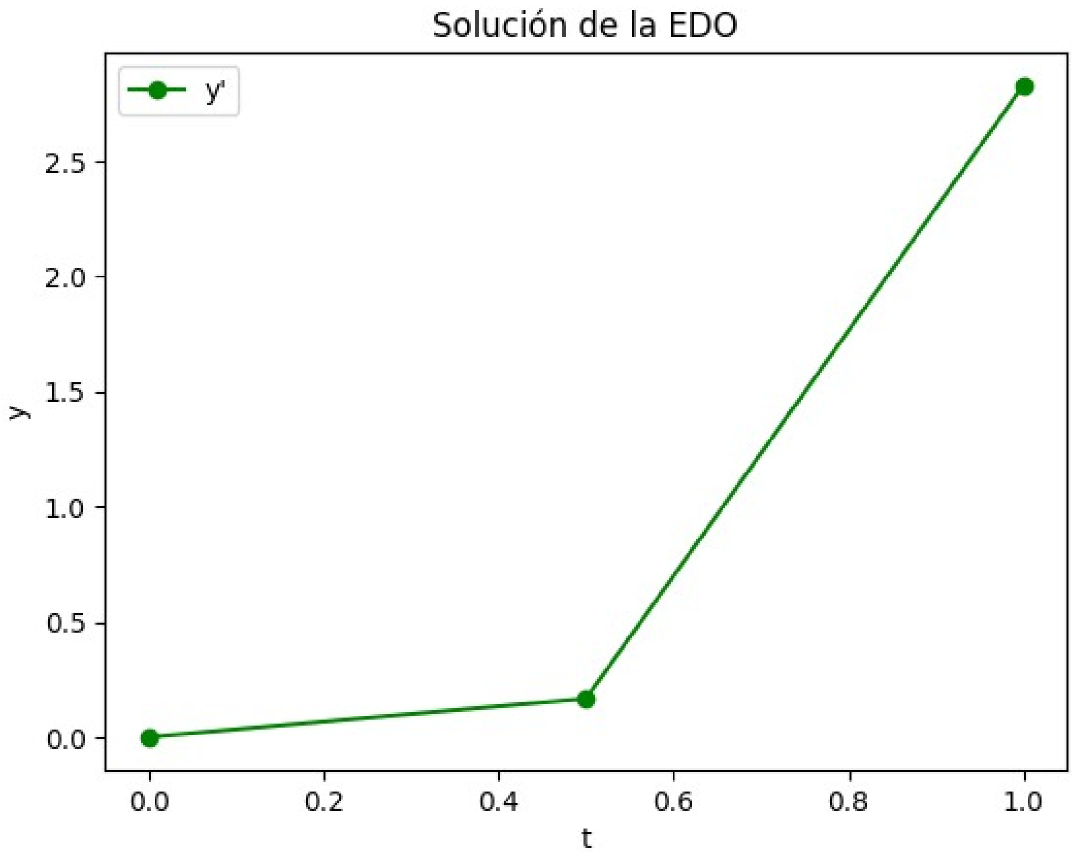
$$\left\{ \begin{array}{cc} y' = t e^{3t} - 2y, & 0 \leq t \leq 1 \\ y(0) = 0, & h = 0.5 \end{array} \right)$$

```
y_der = lambda t, y: t*math.exp(3*t) - 2*y
y_der_2 = lambda t, y: -2*y_der(t, y) + math.exp(3*t) +
3*t*math.exp(3*t)
y_der_3 = lambda t, y: -2*y_der_2(t, y) + 3 * math.exp(3*t) +
9*t*math.exp(3*t)
y_init = 0

ys3a, ts3a, h = ODE_euler_nth(a = 0, b = 1, f = y_der,
                              f_derivatives = [y_der_2, y_der_3],
                              y_t0 = y_init, N = 2)

print(f"El valor de h es: {h}")
graphics(ts3a, ys3a)
```

```
El valor de h es: 0.5
```



PARTE B

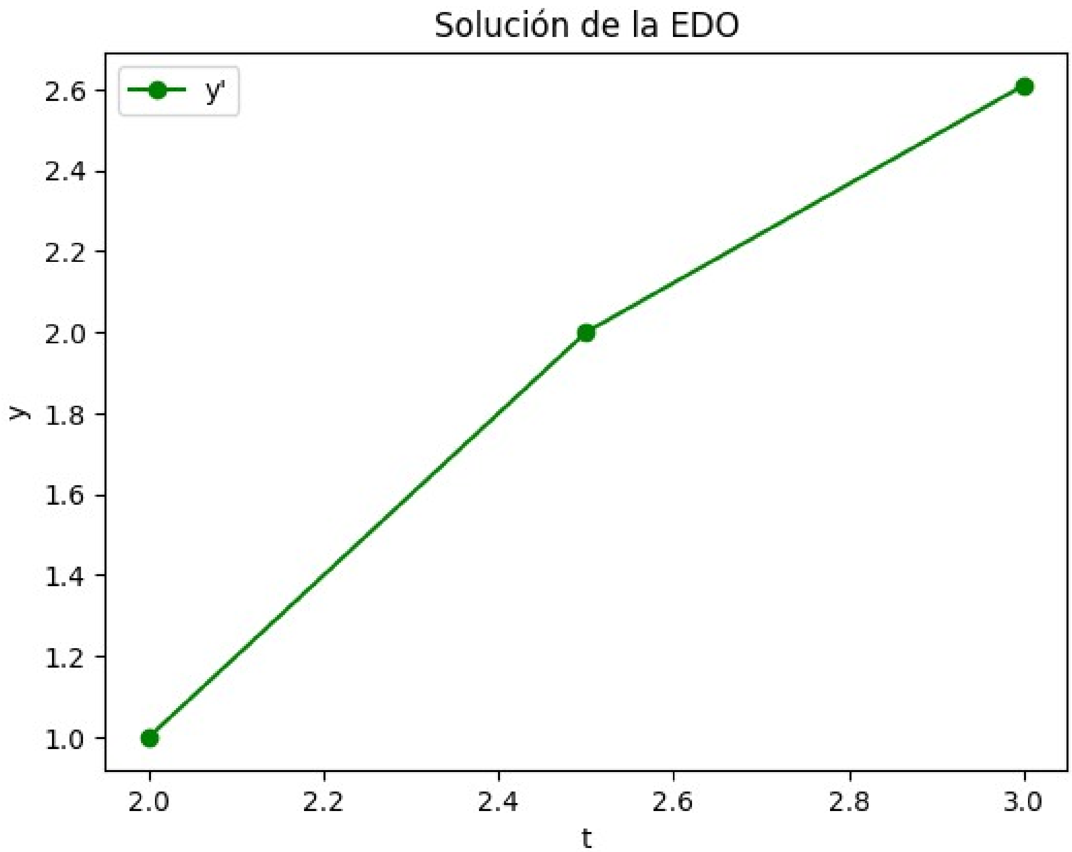
$$\begin{cases} y' = 1 + (t - y)^2, & 2 \le t \le 3 \\ y(2) = 1, & h = 0.5 \end{cases}$$

```
y_der = lambda t, y: 1 + (t - y)**2
y_der_2 = lambda t, y: 2*(t - y)*(1 - y_der(t,y))
y_der_3 = lambda t, y: 2*(1 - y_der(t, y))**2 - 2*(t - y)*y_der_2(t,y)
y_init = 1

ys3b, ts3b, h = ODE_euler_nth(a = 2, b = 3, f = y_der,
                          f_derivatives = [y_der_2, y_der_3],
                          y_t0 = y_init, N = 2)

print(f"El valor de h es: {h}")
graphics(ts3b, ys3b)

El valor de h es: 0.5
```



PARTE C

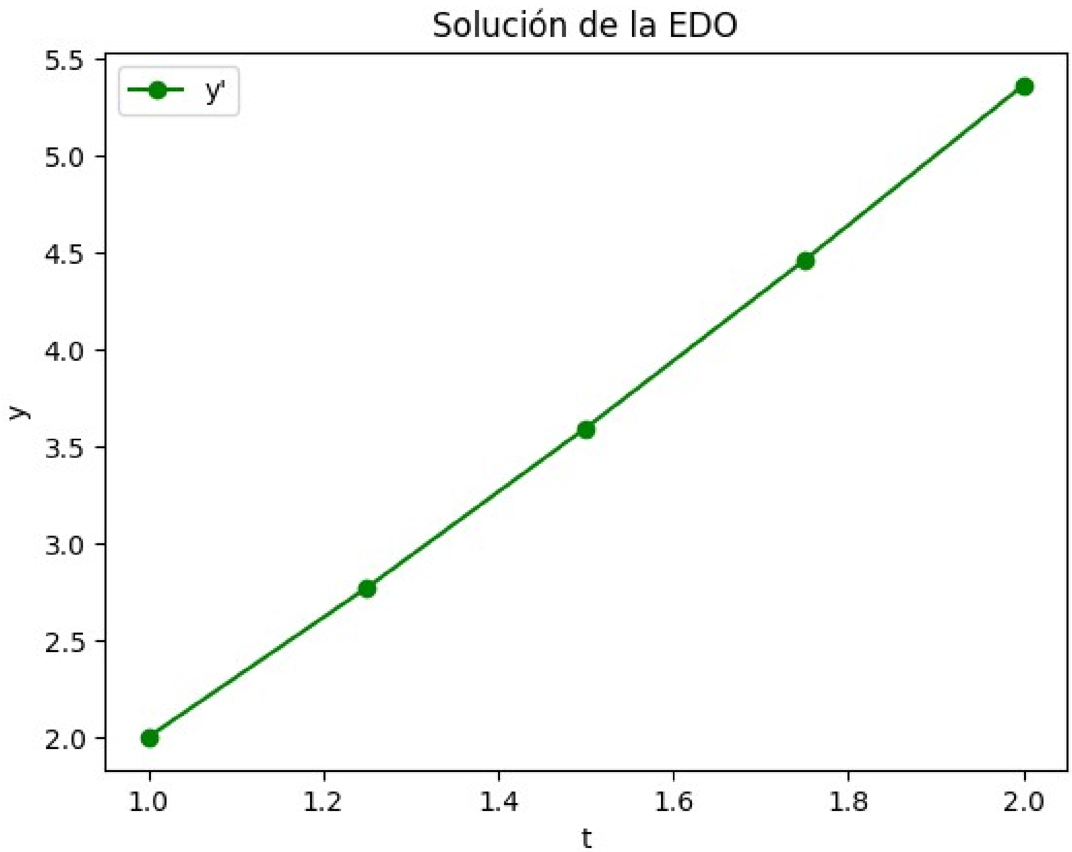
$$\begin{cases} y' = 1 + \dfrac{y}{t}, & 1 \leq t \leq 2 \\ y(1) = 2, & h = 0.25 \end{cases}$$

```
y_der = lambda t, y: 1 + y/t
y_der_2 = lambda t, y: (t * y_der(t, y) - y)/t**2
y_der_3 = lambda t, y: -1/t**2
y_init = 2

ys3c, ts3c, h = ODE_euler_nth(a = 1, b = 2, f = y_der,
                              f_derivatives = [y_der_2, y_der_3],
                              y_t0 = y_init, N = 4)

print(f"El valor de h es: {h}")
graphics(ts3c, ys3c)

El valor de h es: 0.25
```



PARTE D

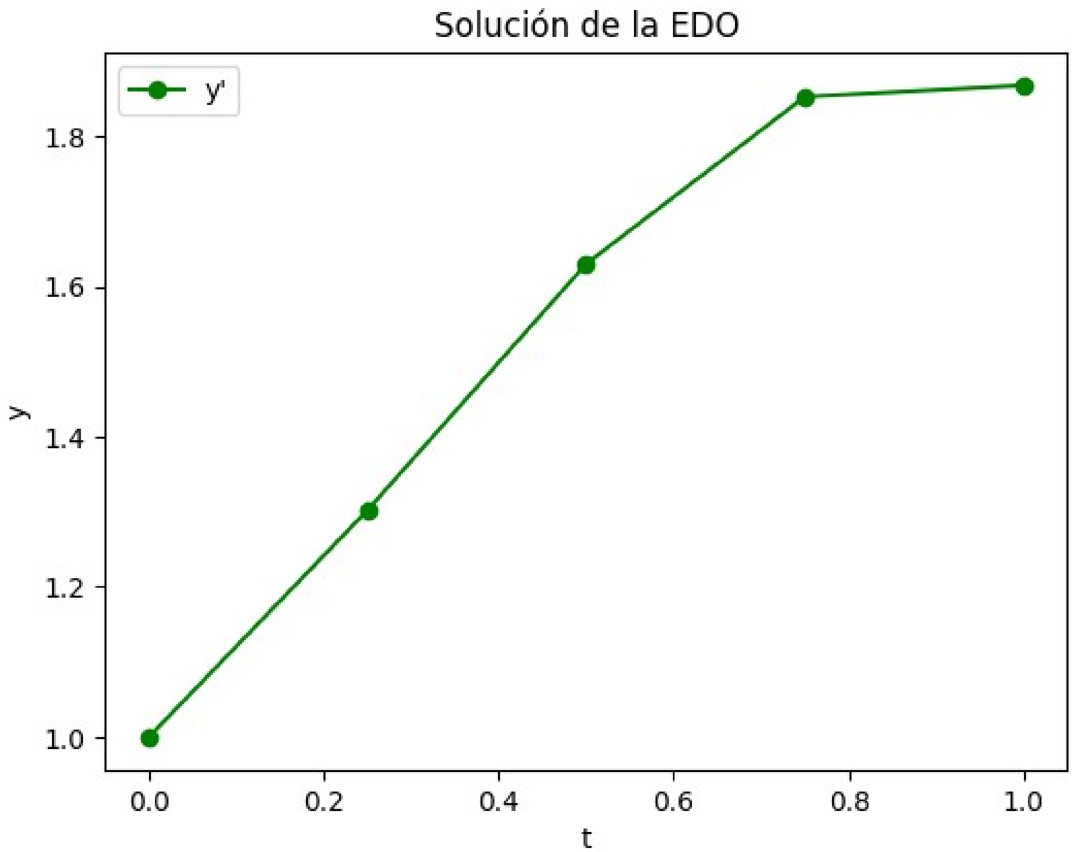
$$\begin{cases} y' = \cos(2t) + \sin(3t), & 0 \le t \le 1 \\ y(0) = 1, & h = 0.25 \end{cases}$$

```
y_der = lambda t, y: math.cos(2*t) + math.sin(3*t)
y_der_2 = lambda t, y: -2*math.sin(2*t) + 3*math.cos(3*t)
y_der_3 = lambda t, y: -4*math.cos(2*t) - 9*math.sin(3*t)
y_init = 1

ys3d, ts3d, h = ODE_euler_nth(a = 0, b = 1, f = y_der,
                              f_derivatives = [y_der_2, y_der_3],
                              y_t0 = y_init, N = 4)

print(f"El valor de h es: {h}")
graphics(ts3d, ys3d)

El valor de h es: 0.25
```



EJERCICIO SIETE

Repita el ejercicio 6 con el método de Taylor de orden 4

PARTE A

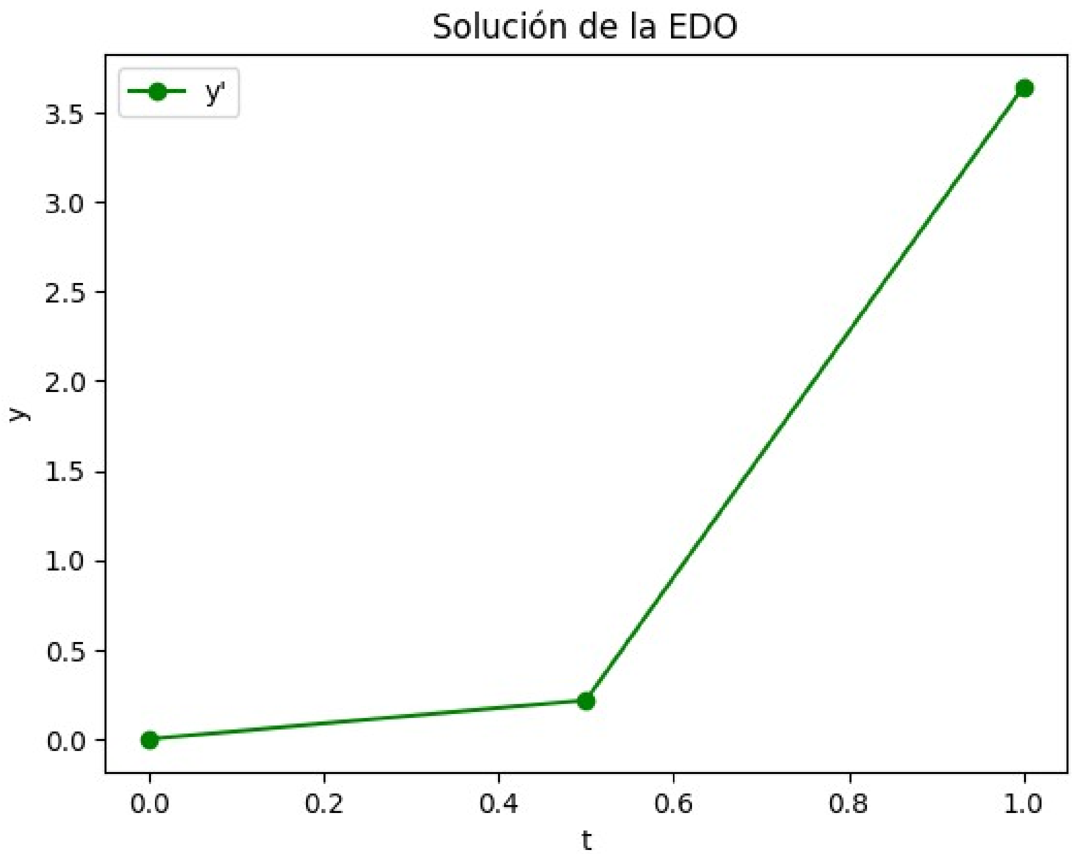
$$\begin{pmatrix} y' = t e^{3t} - 2y, & 0 \le t \le 1 \\ y(0) = 0, & h = 0.5 \end{pmatrix}$$

```
y_der = lambda t, y: t*math.exp(3*t) - 2*y
y_der_2 = lambda t, y: -2*y_der(t, y) + math.exp(3*t) +
3*t*math.exp(3*t)
y_der_3 = lambda t, y: -2*y_der_2(t, y) + 3*math.exp(3*t) +
9*t*math.exp(3*t)
y_der_4 = lambda t, y: -2*y_der_3(t, y) + 6*math.exp(3 * t) +
27*t*math.exp(3*t)
y_der_5 = lambda t, y: -2*y_der_4(t, y) + 12*math.exp(3 * t) +
81*t*math.exp(3*t)
y_init = 0
```

```
ys4a, ts4a, h = ODE_euler_nth(a = 0, b = 1, f = y_der,
                              f_derivatives = [y_der_2, y_der_3, y_der_4, 
y_der_5],
                              y_t0 = y_init, N = 2)

print(f"El valor de h es: {h}")
graphics(ts4a, ys4a)
```

```
El valor de h es: 0.5
```



PARTE B

$$\left\{ \begin{array}{ll} y' = 1 + (t - y)^2, & 2 \le t \le 3 \\ y(2) = 1, & h = 0.5 \end{array} \right)$$

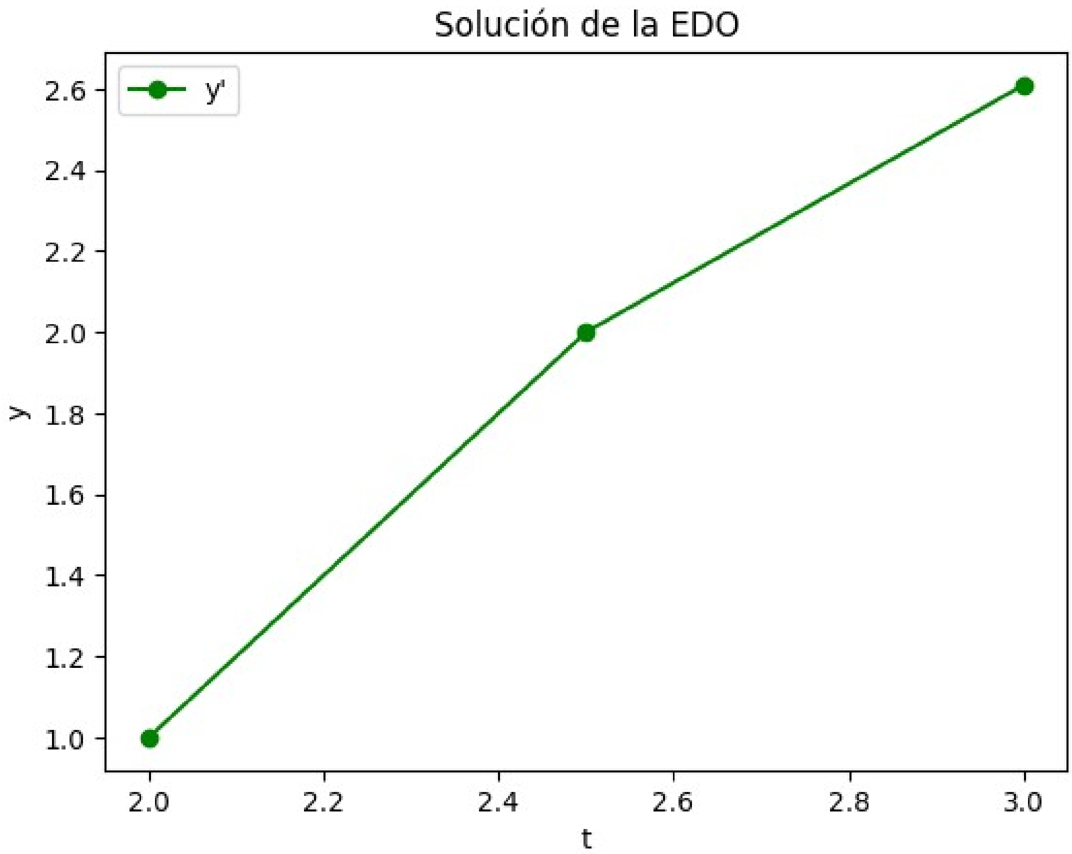
```
y_der = lambda t, y: 1 + (t - y)**2
y_der_2 = lambda t, y: 2*(t - y)*(1 - y_der(t,y))
y_der_3 = lambda t, y: 2*(1 - y_der(t, y))**2 - 2*(t - y)*y_der_2(t,y)
y_init = 1

ys4b, ts4b, h = ODE_euler_nth(a = 2, b = 3, f = y_der,
                              f_derivatives = [y_der_2, y_der_3],
```

```
                                    y_t0 = y_init, N = 2)

print(f"El valor de h es: {h}")
graphics(ts4b, ys4b)

El valor de h es: 0.5
```



PARTE C

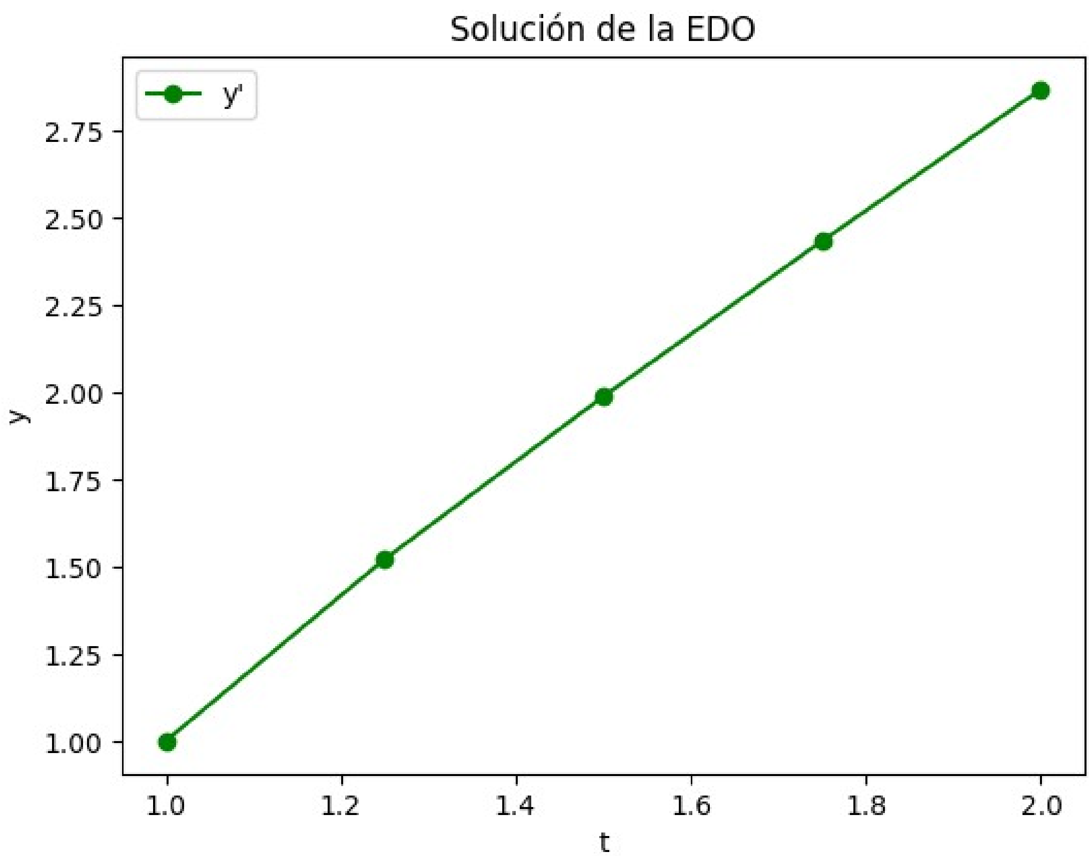
$$\begin{cases} y'=1+\dfrac{y}{t}, & 1\le t\le 2 \\ y(1)=2, & h=0.25 \end{cases}$$

```
y_der = lambda t, y: 1 + (t / y)
y_der_2 = lambda t, y: (t * y_der(t, y) - y) / t**2
y_der_3 = lambda t, y: ((t**2) * y_der_2(t, y) - (2 * t * (y_der(t, y)
- (y / t)))) / t**3
y_init = 1

ys4c, ts4c, h = ODE_euler_nth(a = 1, b = 2, f = y_der,
                              f_derivatives = [y_der_2, y_der_3],
                              y_t0 = y_init, N = 4)
```

```
print(f"El valor de h es: {h}")
graphics(ts4c, ys4c)
```

```
El valor de h es: 0.25
```



PARTE D

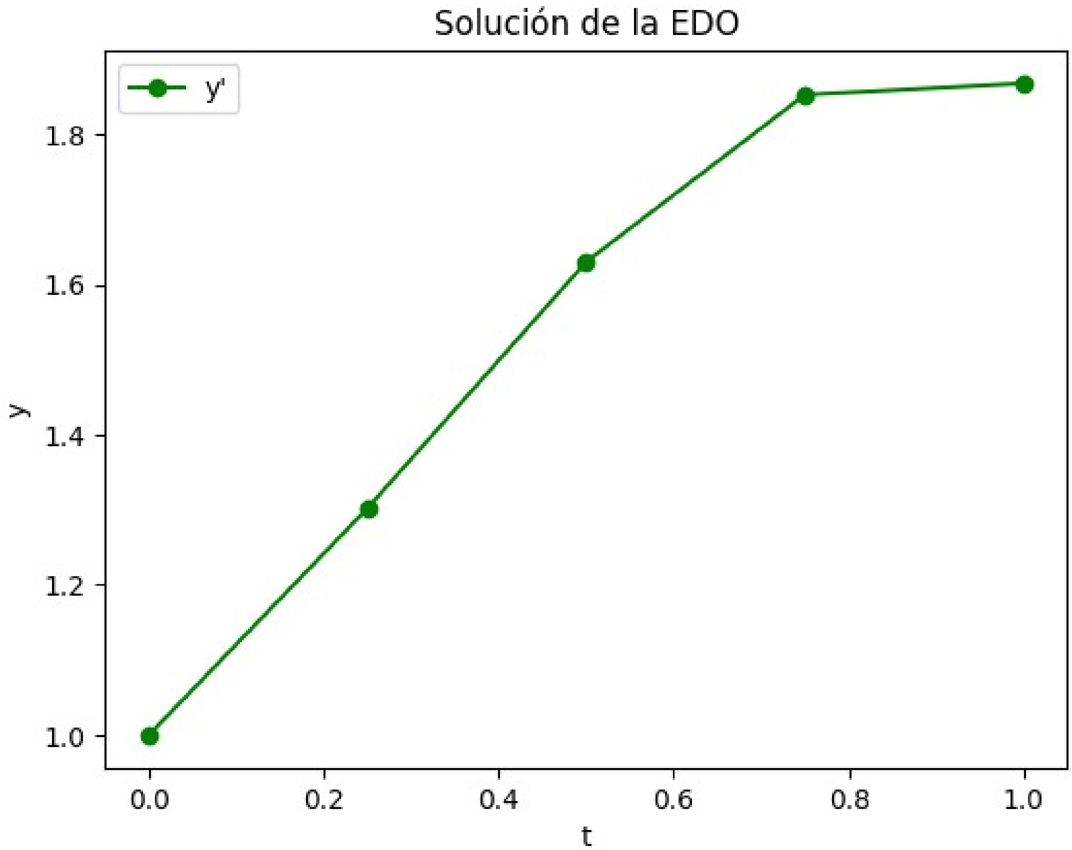
$$\begin{cases} y' = \cos(2t) + \sin(3t), & 0 \le t \le 1 \\ y(0) = 1, & h = 0.25 \end{cases}$$

```
y_der = lambda t, y: math.cos(2*t) + math.sin(3*t)
y_der_2 = lambda t, y: -2 * math.sin(2*t) + 3 * math.cos(3*t)
y_der_3 = lambda t, y: -4 * math.cos(2*t) - 9 * math.sin(3*t)
y_init = 1

ys4d, ts4d, h = ODE_euler_nth(a = 0, b = 1, f = y_der,
                              f_derivatives = [y_der_2, y_der_3],
                              y_t0 = y_init, N = 4)

print(f"El valor de h es: {h}")
graphics(ts4d, ys4d)
```

```
El valor de h es: 0.25
```



REPOSITORIO:

https://github.com/ImYasid/METODOS_NUMERICOS.git

REFERENCIAS BIBLIOGRÁFICAS:

[1] Richard L. Burden, 2017. Análisis Numérico. Lugar de publicación: 10ma edición. Editorial Cengage Learning.

DECLARACIÓN DEL USO DE INTELENGIA ARTIFICIAL

Se utilizo IA para la optimización de código adicional al mejoramiento de la gramática del texto para un mejor entendimiento.