University of BRISTOL

ORBITS

Imad Abziouad
HH Wills Physics Laboratory, University of Bristol, Bristol, BS8 1TL, UK
Ia17692@bristol.ac.uk

1 INTRODUCTION

The aim of this exercise was to utilise an improvement of the Euler method for integration and differentiation called the 4th order Runge-Kutta approach, to model orbits.

The second order differential equation for acceleration due to gravity was separated into two first order differential equations and then solved individually, for given values of time along an object's x and y components. By performing four evaluations at each time step of a function, large powers of step size cancel out and what you have left is a very accurate and efficient method of calculating velocities and positions by integration.

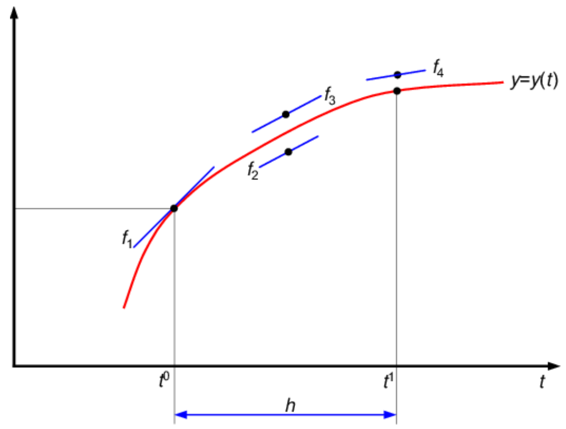

Figure. 1. Diagram showing the 4th order Runge-Kutta approach, where y is an arbitrary equation and h is the step size.

2 THEORY

2.1 The Orbit of a Rocket

To solve the motion of a rocket around the Earth, Earth was treated as being massive and stationary at the origin of a graph, while the rocket moves around it. This required solving the equation of motion for an object under gravitational force.

$$m\ddot{\mathbf{r}} = -\frac{mMG}{|\mathbf{r}|^2}\hat{\mathbf{r}} = -\frac{mMG}{|\mathbf{r}|^3}\mathbf{r} \tag{1}$$

Where M is the planetary mass, m the mass of the rocket, G the gravitational constant and r is the position of the rocket relative to the centre of the planet. Note how the mass of the rocket cancels in both sides of the equation, making the acceleration dependant only on the mass of the Earth.

The Runge-Kutta method can then be used to solve Eq. (1) for $\mathbf{r}$ and $\dot{\boldsymbol{r}}$, for 2D motion in the x-y plane. This can be done for any object orbiting a much larger celestial body.

2.2 The Launch

To launch a rocket from low-Earth orbit to the moon and back such that it slingshots around the moon, another numerical integration is necessary. The same Runge-Kutta approach is applied to the following equation of motion for a two-body problem.

$$m\ddot{\mathbf{r}} = -\frac{mM_E G}{|\mathbf{r}-\mathbf{R}_E|^3}(\mathbf{r}-\mathbf{R}_E) - \frac{mM_M G}{|\mathbf{r}-\mathbf{R}_M|^3}(\mathbf{r}-\mathbf{R}_M) \tag{2}$$

3 METHOD

3.1 Runge-Kutta method

The method used for all of the parts of the exercise involved the 4^{th} order Runge-Kutta, which required the calculation of 16 different variables to couple he first order differentials together to calculate the second order differential equation of motion.

4 RESULTS

4.1 The Orbits

This part of the exercise was used to plot the orbit of the Apollo 11 rocket around the Earth, the orbit of Europa around Jupiter, and finally the strange orbit of Mercury.

The figures below show the aforementioned orbits.

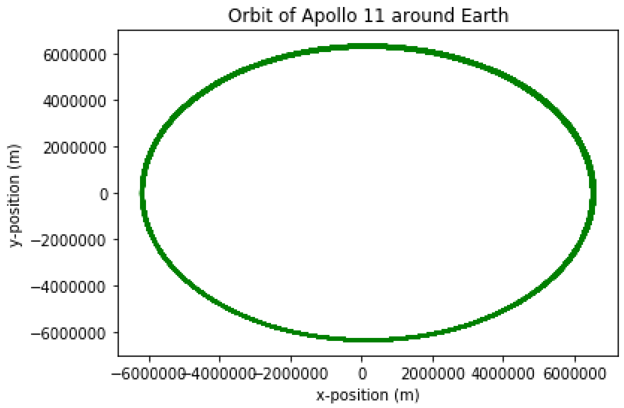

Figure 2 Orbit of Apollo 11 around the Earth.

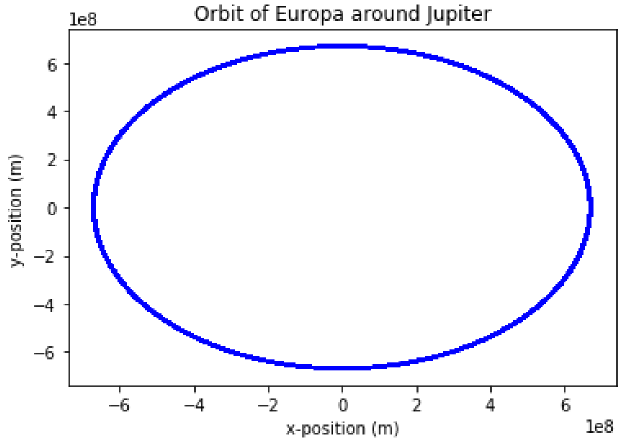

Figure 3 Orbit of Europa around Jupiter.

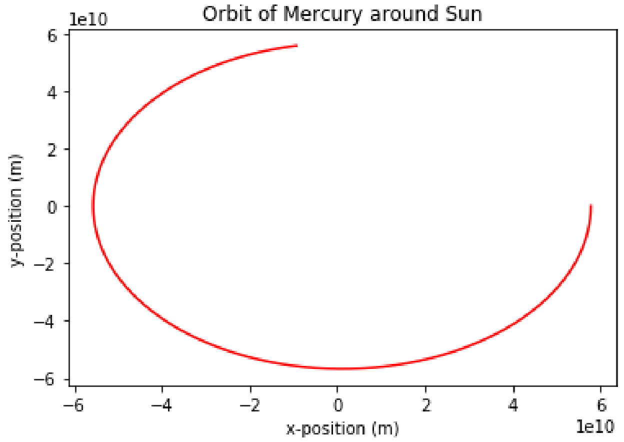

Figure 4 Orbit of Mercury around the Sun

4.2 The Launch

The rocket was launched from a height of 7000km in the positive y axis above the earth, with a positive x velocity of 110m/s. The results are as follows.

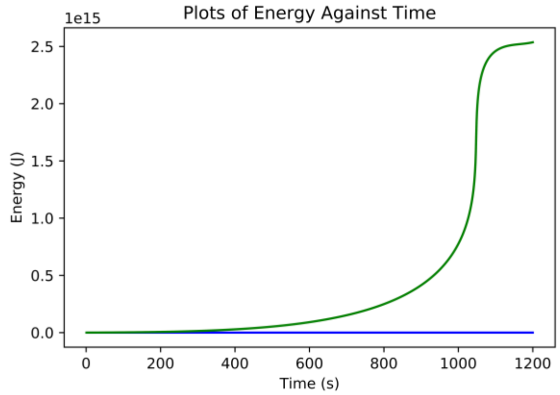

Figure 5 Plot of Kinetic and Potential Energy against time.

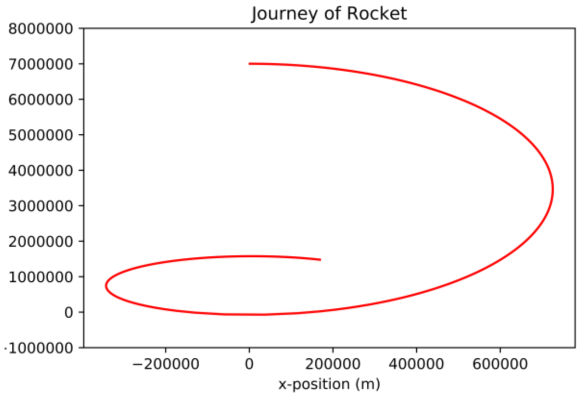

Figure 6 Plot of the motion of the Apollo 11 rocket around the moon.