

# 1 Electric conductivity $\sigma$ estimates

Faraday's law in integral form and Ampere's law in differential form may be combined to yield an expression for the complex permittivity,  $\kappa$  [12].

$$\frac{\oint_{\partial A} \nabla \times \vec{H} \cdot d\vec{l}}{\mu\omega^2 \int_A \vec{H} \cdot d\vec{a}} \approx \kappa = \epsilon - i \sigma / \omega \quad \vec{H} = H_x \hat{x} + H_y \hat{y} + H_z \hat{z} \quad (1)$$

The magnetic field applied by the transmit coil is denoted  $\vec{H}$ ,  $\omega$  is the Larmor frequency, and  $\mu$  is the permeability. A non-axial plane such as the coronal plane,  $y = \text{constant}$ , is preferred because the axial component,  $H_z$  is not

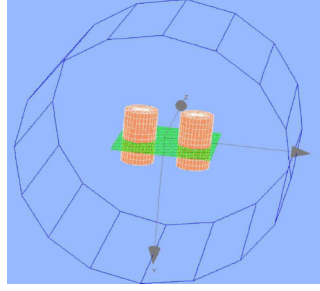


Figure 1: Coronal Imaging Plane [12].

measurable. For reference coronal imaging plane with respect to a right hand coordinate system is shown in Figure 1, In this case (1) may be reduced to an expression for the conductivity,  $\sigma$ .

$$\frac{\oint_{\partial A_{xz}} \left\{ \underbrace{\partial_y}_{=0 \text{ by assumption}} \underbrace{H_z}_{\text{}} - \partial_z H_y, \partial_x H_y - \partial_y H_x \right\} \cdot d\vec{l}}{\mu\omega^2 \int_{A_{xz}} H_y \cdot dxdz} \approx \kappa = \epsilon - i \sigma / \omega \quad \Rightarrow \quad \sigma \approx \frac{\oint_{\partial A_{xz}} \partial_x H_y - \partial_y H_x ds}{\mu\omega \int_{A_{xz}} H_y \cdot dxdz} \quad (2)$$

The integrals yield a scalar valued average estimate of conduction,  $\sigma \in \mathbb{R}$  with the ROI of interest,  $A$ .

Paraphrasing [12]: "The use of a birdcage-type RF coil is advantageous, i.e., a quadrature body coil (QBC) or corresponding head coil. A birdcage coil allows choosing the coil polarization during transmission and reception. In particular, the structure of the birdcage coil imposes that transmit sensitivity,  $H_+ = \frac{H_x + iH_y}{2}$ , is substantially larger than receive sensitivity,  $H_- = \frac{H_x - iH_y}{2}$ . The determination of  $H_+$ , is the so-called B1 mapping. A double angle method [11] is appropriate. However, these approaches only describe the determination of the modulus  $|H_+|$ , from the signal ratio observed at the two flip angles  $\theta_i$ .

$$|H_+| = \frac{\cos^{-1}\left(\frac{1}{2r}\right)}{\omega t} \quad r = \frac{1}{2 \cos \theta_1} = \frac{\sin \theta_1}{\sin \theta_2} \quad \theta_2 = 2 \theta_1 \quad (3)$$

The use of a switched birdcage coil allows also the estimation of the B1 phase."

## References

- [11] EK Insko and L Bolinger. Mapping of the radiofrequency field. *Journal of Magnetic Resonance, Series A*, 103(1):82–85, 1993.
- [12] Ulrich Katscher, Tobias Voigt, Christian Findelee, Peter Vernickel, Kay Nehrke, and O Dossel. Determination of electric conductivity and local sar via b1 mapping. *Medical Imaging, IEEE Transactions on*, 28(9):1365–1374, 2009.