Temperature Dependence of T_1 Relaxation Time: An Inverse Problem Approach

Reza Madankan

Introduction

The process of Magentic Resonance Laser Induced Temperature Treatment (MRLITT) as a minimally invasive local treatment of cancerous tumors has received increasing attention in recent years. Performance of MRLITT crucially depends on parameters like tissue relaxation times, known as T_1 and T_2. One of the major challenges in MRLITT process is temperature dependence of these parameters. Hence, one needs to have an accurate approximation of changes in relaxation times as a function of tissue temperature. In thid manuscript, we study the effect of the temperature of T_1 relaxation time.

Problem Statement

It is well established that the T1 relaxation time changes as a function of temperature [1]. In general, it is assumed that T_1 relaxation time linearly changes as a function of temperature u:

$$T_1(u) = T_1^{ref}\left(1 + m(u - u^{ref})\right) \tag{1}$$

where, u^{ref} denotes the tissue temperature in normal conditions and T_1^{ref} represents the corresponding value of relaxation time. Coefficient m denotes the sensitivity of T_1 relaxation time with respect to temperature u and it is usually between 0.01 and 0.02 for different tissue types. Note that due to tissue dependencies of parameters m and T_1^{ref}, it is a reasonable assumption to consider these parameters to be uncertain.

On the other hand, a measurement data is given by the following observation model

$$\rho = h(u) = \frac{sin(\theta)\left(1 - e^{-\frac{T_r}{T_1(u)}}\right)}{sin(\beta)\left(1 - cos(\theta)e^{-\frac{T_r}{T_1(u)}}\right)} + \nu, \quad \nu \sim \mathcal{N}(0, \sigma) \tag{2}$$

where, θ and β are corresponding flip angles of the experiment and T_r denotes the repetition time. Note that β is usually considered to be a small angle and $\theta \gg \beta$ in general.

The key goal of this article is to find a good estimate for temperature dependencies of parameter T_1, given a set of data observations ρ_k, $k = 1, 2, \cdots, n$. Note that the uncertainty in T_1 is related to uncertain parameters m and T_1^{ref}. Hence, the problem of T_1 estimation is equivalent with estimating the parameters m and T_1^{ref}.

Parameter Estimation

We use a minimum variance framework for estimating the parameters T_1^{ref} and m. To do so, lets first concatenate these parameters in a parameter vector Θ, i.e.

$$\Theta = [T_1^{ref}, \quad m]^T$$

Note that there exist some prior statistics,e.g. mean and covariance, for the parameter Θ:

$$\hat{\Theta} \equiv \mathcal{E}[\Theta] = \int_{\Theta} \Theta p(\Theta) d\Theta \tag{2}$$

$$\Sigma \equiv \mathcal{E}[(\Theta - \mathcal{E}[\Theta])(\Theta - \mathcal{E}[\Theta])^T] = \int_{\Theta} (\Theta - \mathcal{E}[\Theta])(\Theta - \mathcal{E}[\Theta])^T p(\Theta) d\Theta \tag{2}$$

Now, based on the minimum variance framework, posterior statistics of Θ are given as [2, 3]:

$$\begin{aligned}\hat{\Theta}^+ &= \hat{\Theta}^- + \mathbf{K}\left(\mathbf{z} - \mathcal{E}^-[h(u)]\right) \\ \mathbf{\Sigma}^+ &= \mathbf{\Sigma}^- + \mathbf{K}\mathbf{\Sigma}_{hh}\mathbf{K}^T\end{aligned}$$

where, $\mathbf{z} = \rho_1, \rho_2, \cdots, \rho_n$ is the measurement vector of n observations and the gain matrix $\mathbf{K}$ is given by

$$\mathbf{K} = \mathbf{\Sigma}_{\Theta h}^T \left(\mathbf{\Sigma}_{hh}^- + \mathbf{R}\right)^{-1} \tag{2}$$

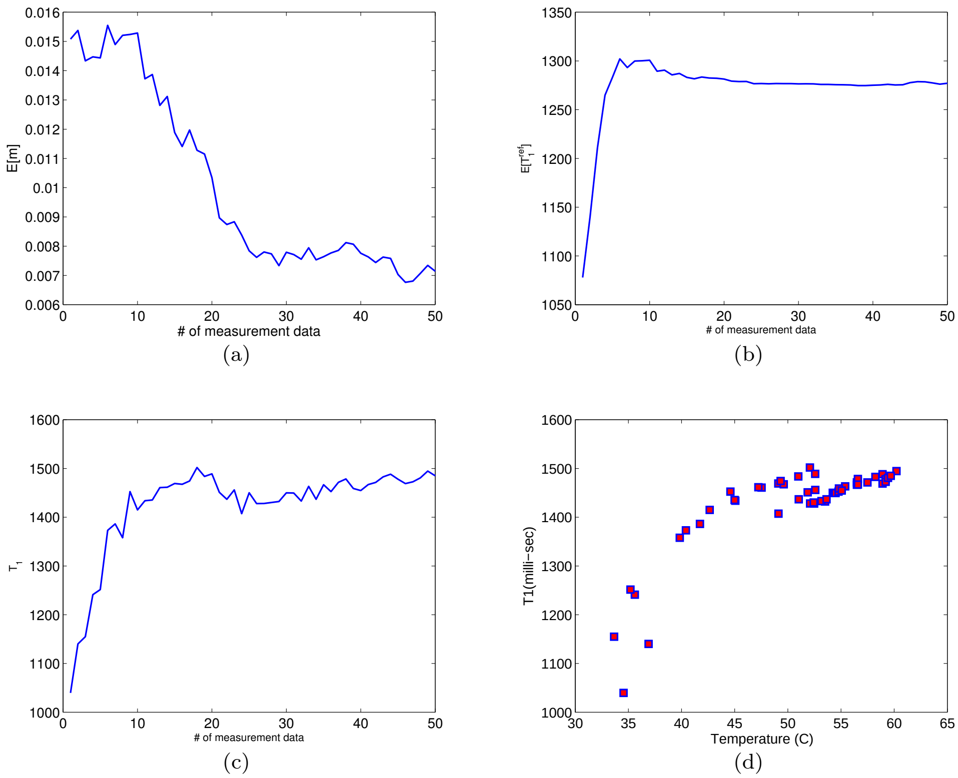
and $\mathbf{R}$ represent covariance matrix of the noise signal ν. The matrices $\mathbf{\Sigma}_{\Theta h}$ and $\mathbf{\Sigma}_{hh}$ are defined as:

$$\begin{aligned}\hat{h}^- &\triangleq \mathcal{E}^-[h(u, \mathbf{\Theta})] = \int_{\Theta} \underbrace{h(u, \Theta)}_{h} p(\mathbf{\Theta}) d\mathbf{\Theta} \\ \mathbf{\Sigma}_{\Theta h} &\triangleq \mathcal{E}^-[(\mathbf{\Theta} - \hat{\mathbf{\Theta}})(h - \hat{h}^-)^T] \\ \mathbf{\Sigma}_{hh}^- &\triangleq \mathcal{E}^-[(h - \hat{h}^-)(h - \hat{h}^-)^T]\end{aligned}$$

Note that in above equations, superscripts $+$ and $-$ are used to represent posterior and prior values of the corresponding statistics.

Numerical Simulations

For simulation purpose, we have considered the problem of estimating T_1 for a set of experimental data, resulted from MRLITT.



(a) (b) (c) (d)

Figure 1: a-c) convergence of parameter estimates versus different number of data observations a) m, b) T_1^{ref}, and c) T_1. Fig d) shows the variations of the estimated T_1 versus changes of tissue temperature.

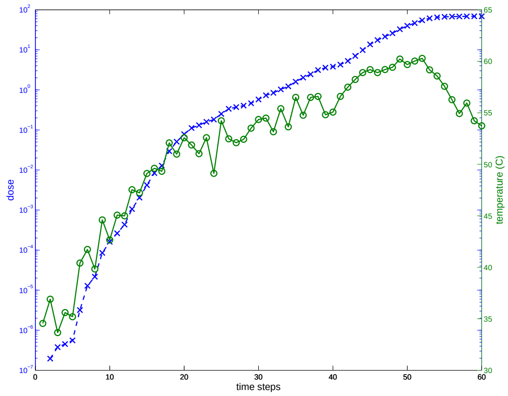
Figure 2: Variation of T_1 field during the heating

References

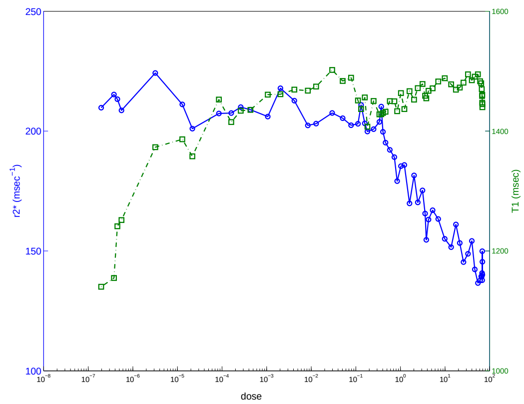
[1] V. Rieke and K. Butts Pauly, "Mr thermometry," *Journal of Magnetic Resonance Imaging*, vol. 27, no. 2, pp. 376–390, 2008.

[2] R. Madankan, S. Pouget, P. Singla, M. Bursik, J. Dehn, M. Jones, A. Patra, M. Pavolonis, E. Pitman, T. Singh, *et al.*, "Computation of probabilistic hazard maps and source parameter estimation for volcanic ash transport and dispersion," *Journal of Computational Physics*, vol. 271, pp. 39–59, 2014.

[3] R. Madankan, P. Singla, T. Singh, and P. D. Scott, "Polynomial-chaos-based bayesian approach for state and parameter estimations," *Journal of Guidance, Control, and Dynamics*, vol. 36, no. 4, pp. 1058–1074, 2013.



(a)



(b)

Figure 3: a) variations of temperature and necrosis dose over time, b) variations of $r2*$ and $T1$ versus necrosis dose

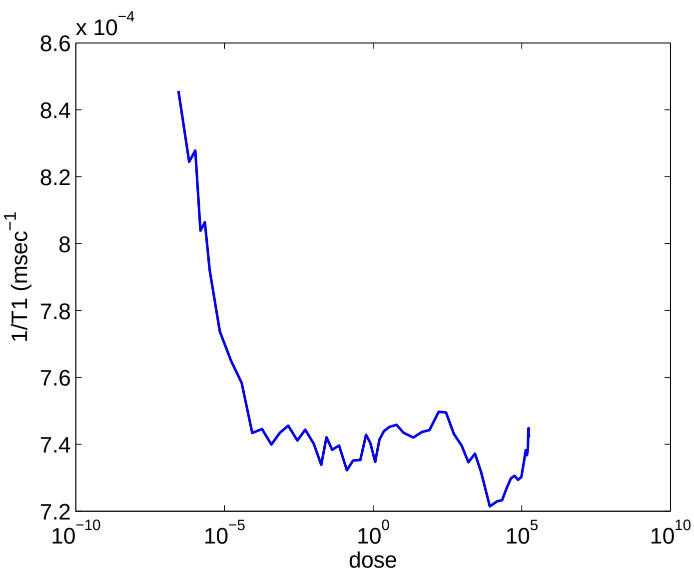


Figure 4: variations of $r1$ versus necrosis dose