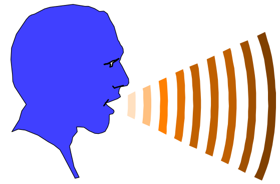
   

EE3414

Multimedia Communication Systems - I

Image Filtering: Noise Removal, Sharpening, Deblurring

Yao Wang

Polytechnic University, Brooklyn, NY11201

http://eeweb.poly.edu/~yao

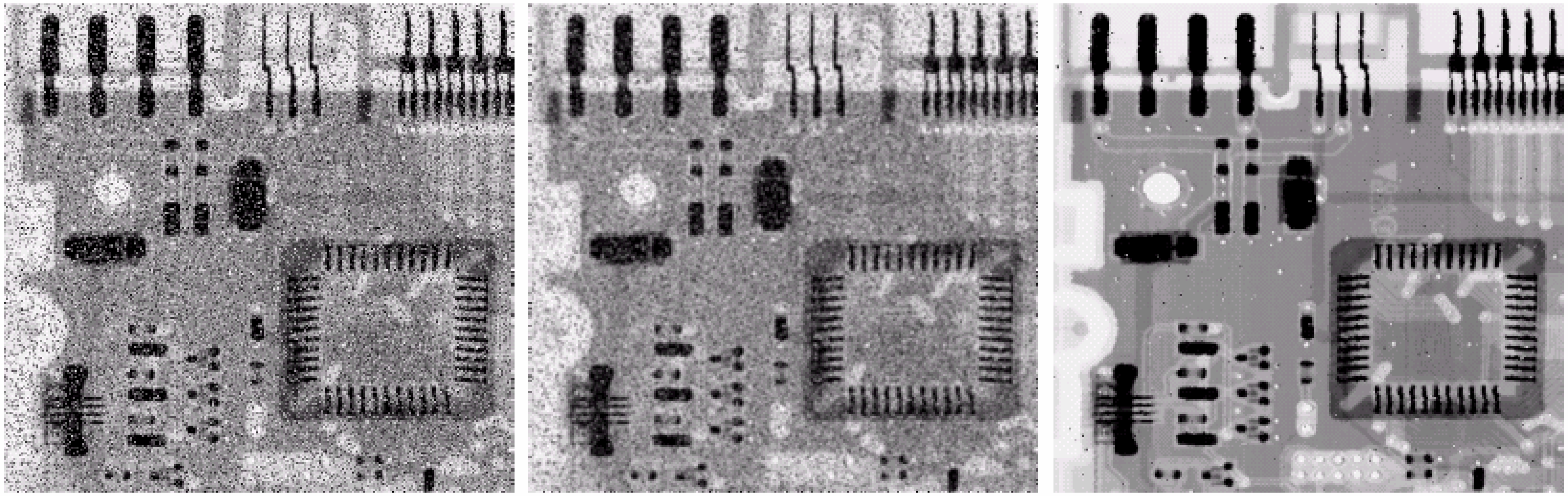
Outline

- Noise removal by averaging filter
- Noise removal by median filter
- Sharpening (Edge enhancement)
- Deblurring

Noise Removal (Image Smoothing)

- An image may be “dirty” (with dots, speckles,stains)
- Noise removal:
  - To remove speckles/dots on an image
  - Dots can be modeled as impulses (salt-and-pepper or speckle) or continuously varying (Gaussian noise)
  - Can be removed by taking mean or median values of neighboring pixels (e.g. 3x3 window)
  - Equivalent to low-pass filtering
- Problem with low-pass filtering
  - May blur edges
  - More advanced techniques: adaptive, edge preserving

Example



a b c

FIGURE 3.37 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3×3 averaging mask. (c) Noise reduction with a 3×3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Averaging Filter

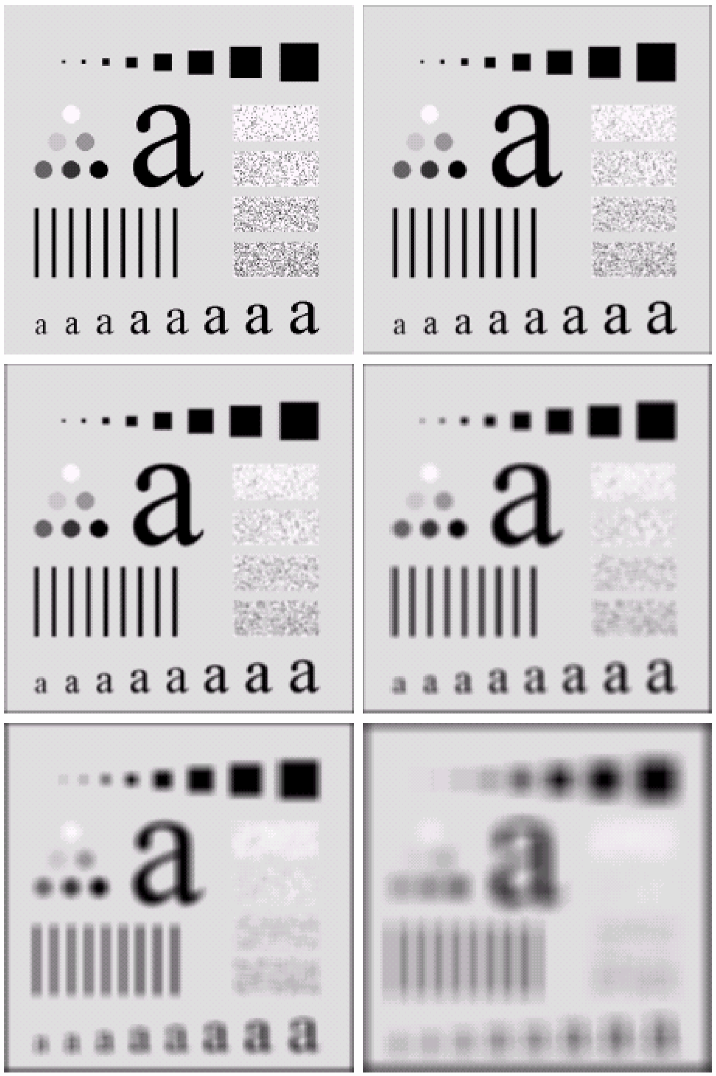
- Replace each pixel by the average of pixels in a square window surrounding this pixel
- Trade-off between noise removal and detail preserving:
  - Larger window -> can remove noise more effectively, but also blur the details/edges

Example: 3x3 average

100	100	100	100	100
100	200	205	203	100
100	195	200	200	100
100	200	205	195	100
100	100	100	100	100

100	100	100	100	100
100	144	167	145	100
100	167	200	168	100
100	144	166	144	100
100	100	100	100	100

Example



a b
c d
e f

FIGURE 3.35 (a) Original image, of size 500 × 500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes $n = 3, 5, 9, 15$, and 35, respectively. The black squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their gray levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50 × 120 pixels.

Weighted Averaging Filter

- Instead of averaging all the pixel values in the window, give the closer-by pixels higher weighting, and far-away pixels lower weighting.

$$g(m,n) = \sum_{l=-L}^{L} \sum_{k=-L}^{L} h(k,l) s(m-k, n-l)$$

- This type of operation for arbitrary weighting matrices is generally called “2-D convolution or filtering”. When all the weights are positive, it corresponds to weighted average.
- Weighted average filter retains low frequency and suppresses high frequency = low-pass filter

Graphical Illustration

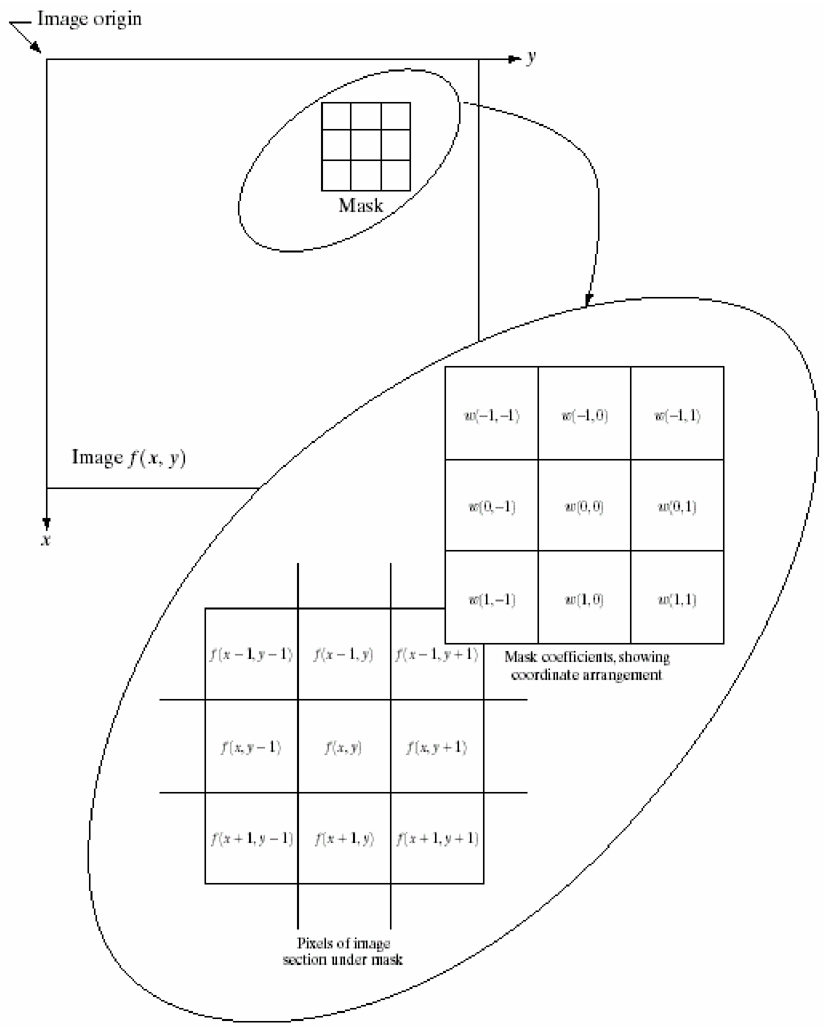


FIGURE 3.32 The mechanics of spatial filtering. The magnified drawing shows a 3×3 mask and the image section directly under it; the image section is shown displaced out from under the mask for ease of readability.

Example Weighting Mask

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \qquad \frac{1}{16} \times \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

All weights must sum to one

Example: Weighted Average

100	100	100	100	100
100	200	205	203	100
100	195	200	200	100
100	200	205	195	100
100	100	100	100	100

$$\frac{1}{9}\times\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

100	100	100	100	100
100	144	167	145	100
100	167	200	168	100
100	144	166	144	100
100	100	100	100	100

$$\frac{1}{16}\times\begin{bmatrix}1&2&1\\2&4&2\\1&2&1\end{bmatrix}$$

100	100	100	100	100
100	156	176	158	100
100	174	201	175	100
100	156	175	156	100
100	100	100	100	100

Filtering in 1-D: a Review

- Continuous-Time Signal

$$\text{Time Domain}: \quad g(t) = s(t) * h(t) = \int_{-\infty}^{\infty} h(\tau) s(t-\tau) d\tau$$

$$\text{Frequency Domain}: \; G(f) = S(f) H(f)$$

$$\text{Filter frequency response}: \; H(f) = \int_{-\infty}^{\infty} h(t) e^{-j2\pi f t} dt$$

- Extension to discrete time 1D signals

$$\text{Time Domain (linear convolution)}: \quad s(n) = s(n) * h(n) = \sum_m h(m) s(n-m)$$

$$\text{Frequency Domain}: \; G(f) = S(f) H(f)$$

$$\text{Filter frequency response (DTFT)}: \; H(f) = \sum_n h(n) e^{-j2\pi f n}$$

$H(f)$ is periodic, only needs to look at the range $f \in (-1/2, 1/2)$

$1/2$ corresponds to $f_s / 2$

Filtering in 2D

Weighted averaging = 2D Linear Convolution

$$g(m,n) = \sum_{l=l_0}^{l_1} \sum_{k=k_0}^{k_1} h(k,l)s(m-k,n-l)$$

In 2D frequency domain $G(f_1, f_2) = S(f_1, f_2)H(f_1, f_2)$

Frequency response of the 2D Filter

$$H(f_1, f_2) = \sum_{m=l_0}^{l_1} \sum_{n=k_0}^{k_1} h(m,n)e^{-j2\pi(f_1 m + f_2 n)}$$

$H(f_1, f_2)$ is periodic, only needs to look at the square region

$f_1 \in (-1/2, 1/2), f_2 \in (-1/2, 1/2).$

Frequency Response of Averaging Filters

- Averaging over a 3x3 window

$$h(m,n)=\frac{1}{9}\begin{bmatrix}0&0&0&0&0\\0&1&1&1&0\\0&1&1&1&0\\0&1&1&1&0\\0&0&0&0&0\end{bmatrix}$$

$$H(f_1,f_2)=\frac{1}{9}\left(e^{-j2\pi(f_1\cdot(-1)+f_2\cdot(-1))}+e^{-j2\pi(f_1\cdot(0)+f_2\cdot(-1))}+e^{-j2\pi(f_1\cdot(1)+f_2\cdot(-1))}\right)$$

$$+\frac{1}{9}\left(e^{-j2\pi(f_1\cdot(-1)+f_2\cdot(0))}+e^{-j2\pi(f_1\cdot(0)+f_2\cdot(0))}+e^{-j2\pi(f_1\cdot(1)+f_2\cdot(0))}\right)$$

$$+\frac{1}{9}\left(e^{-j2\pi(f_1\cdot(-1)+f_2\cdot(1))}+e^{-j2\pi(f_1\cdot(0)+f_2\cdot(1))}+e^{-j2\pi(f_1\cdot(1)+f_2\cdot(1))}\right)$$

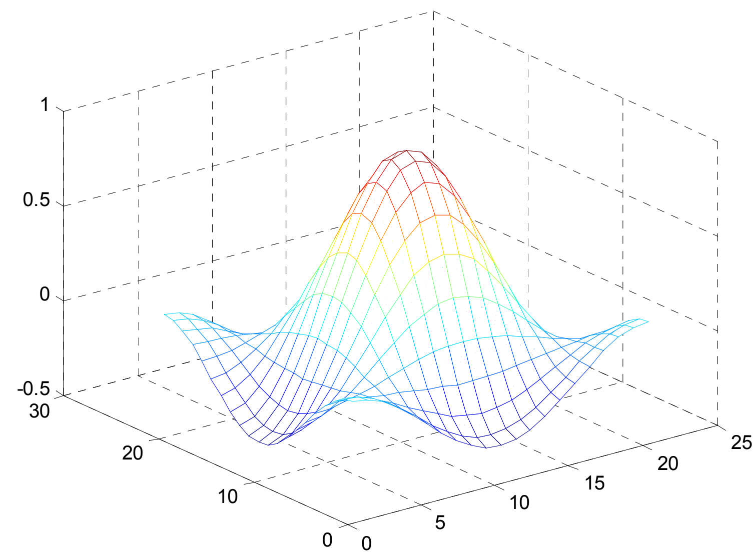
$$=\frac{1}{9}\left(e^{j2\pi f_1}+1+e^{-j2\pi f_1}\right)e^{j2\pi f_2}+\frac{1}{9}\left(e^{j2\pi f_1}+1+e^{-j2\pi f_1}\right)\cdot 1+\frac{1}{9}\left(e^{j2\pi f_1}+1+e^{-j2\pi f_1}\right)e^{-j2\pi f_2}$$

$$=\frac{1}{9}\left(e^{j2\pi f_1}+1+e^{-j2\pi f_1}\right)\left(e^{j2\pi f_2}+1+e^{-j2\pi f_2}\right)=\frac{1}{9}\left(1+2\cos 2\pi f_1\right)\left(1+2\cos 2\pi f_2\right)$$

$$H(f_1, f_2) = H(f_1)H(f_2)$$

$$H(f_1) = \frac{1}{3}\left(1 + 2\cos 2\pi f_1\right), H(f_2) = \frac{1}{3}\left(1 + 2\cos 2\pi f_2\right)$$

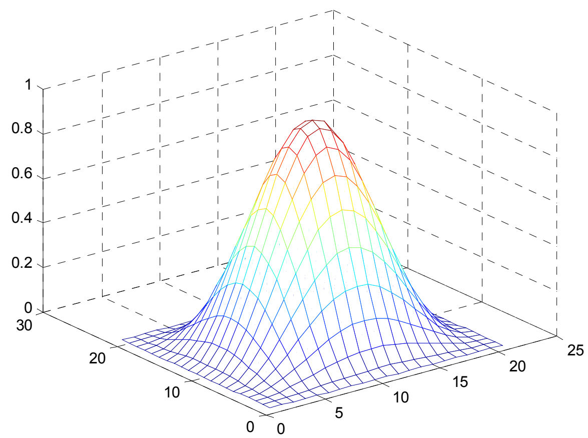
Sketch H(f1)



Frequency Response of Weighted Averaging Filters

$$H = \frac{1}{(1+b)^2}\begin{bmatrix} 1 & b & 1 \\ b & b^2 & b \\ 1 & b & 1 \end{bmatrix} = \frac{1}{(1+b)^2}\begin{bmatrix} 1 \\ b \\ 1 \end{bmatrix}\begin{bmatrix} 1 & b & 1 \end{bmatrix};$$

$$H(u,v) = \big(b + 2\cos(2\pi u)\big)\big(b + 2\cos(2\pi v)\big)/(1+b)^2$$



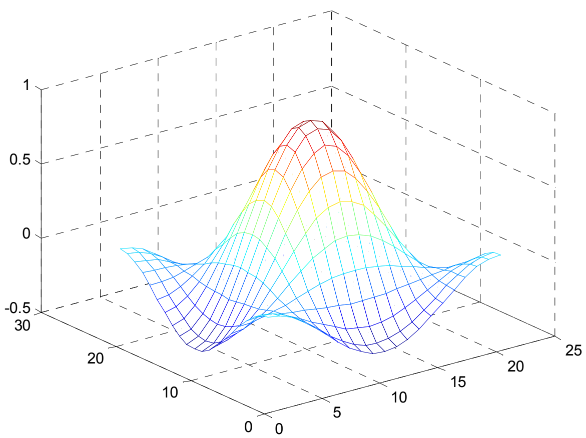
Averaging vs. Weighted Averaging

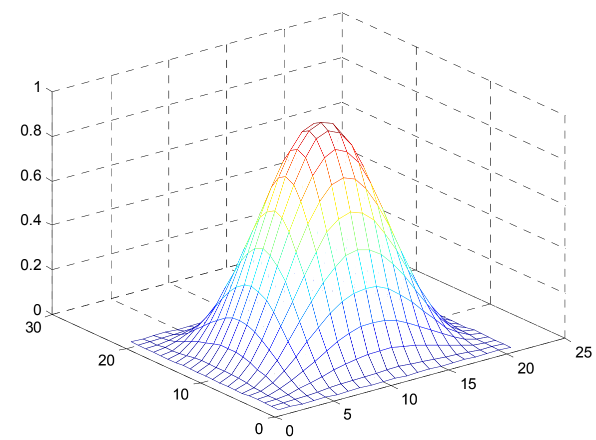
$$H = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \end{bmatrix};$$

$$H(u,v) = \left(1 + 2\cos(2\pi u)\right)\left(1 + 2\cos(2\pi v)\right)/9$$

$$H = \frac{1}{(1+b)^2}\begin{bmatrix} 1 & b & 1 \\ b & b^2 & b \\ 1 & b & 1 \end{bmatrix} = \frac{1}{(1+b)^2}\begin{bmatrix} 1 \\ b \\ 1 \end{bmatrix}\begin{bmatrix} 1 & b & 1 \end{bmatrix};$$

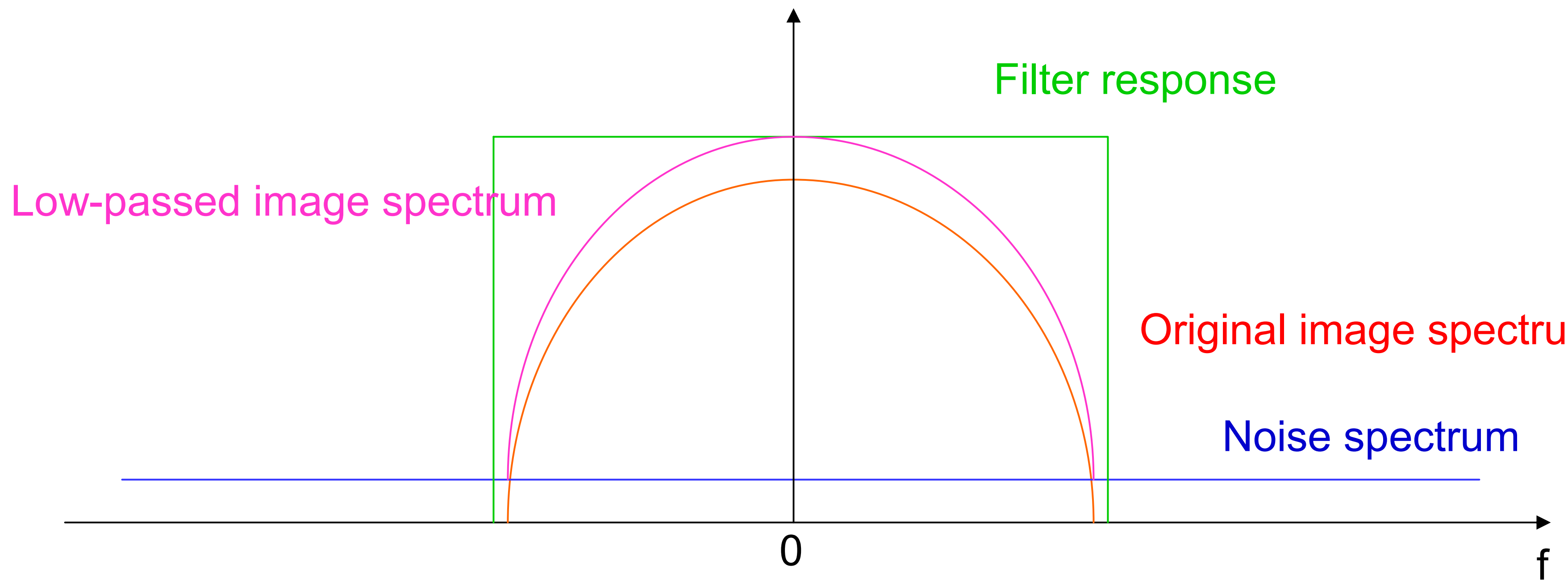
$$H(u,v) = \left(b + 2\cos(2\pi u)\right)\left(b + 2\cos(2\pi v)\right)/(1+b)^2$$





b=2

Interpretation in Freq Domain



Noise typically spans entire frequency range, where as natural images have predominantly lower frequency components

Median Filter

- Problem with Averaging Filter
  - Blur edges and details in an image
  - Not effective for impulse noise (Salt-and-pepper)
- Median filter:
  - Taking the median value instead of the average or weighted average of pixels in the window
    - Median: sort all the pixels in an increasing order, take the middle one
  - The window shape does not need to be a square
  - Special shapes can preserve line structures
- Order-statistics filter
  - Instead of taking the mean, rank all pixel values in the window, take the n-th order value.
  - E.g. max or min

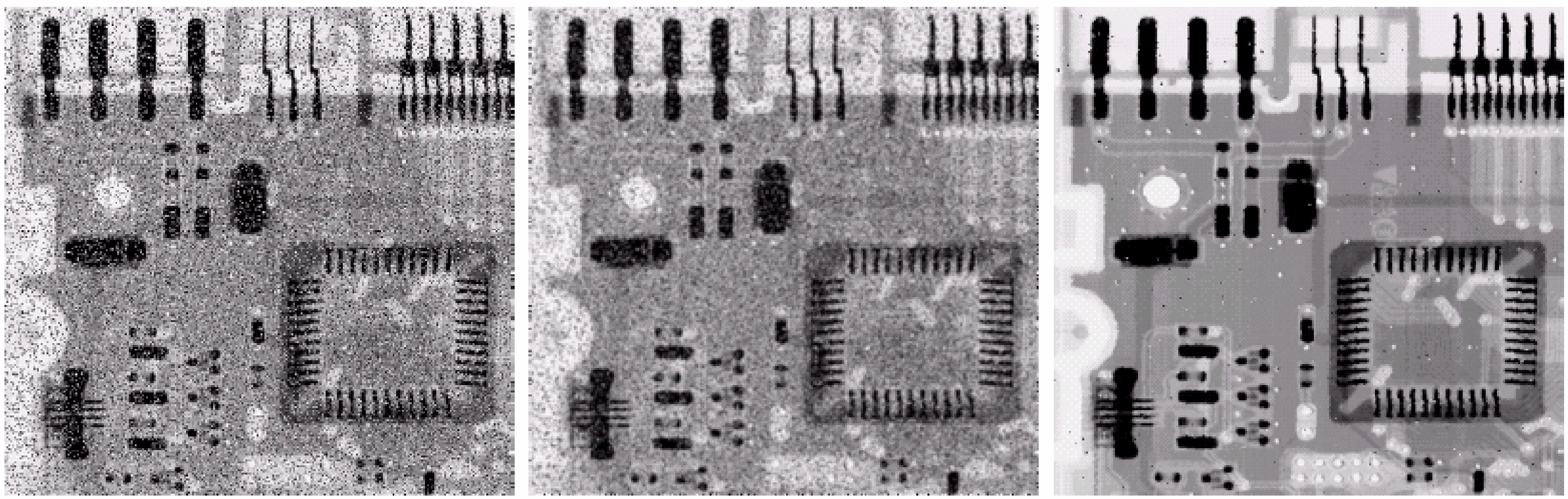
Example: 3x3 Median

100	100	100	100	100
100	200	205	203	100
100	195	200	200	100
100	200	205	195	100
100	100	100	100	100

100	100	100	100	100
100	100	200	100	100
100	200	200	200	100
100	100	195	100	100
100	100	100	100	100

Matlab command: medfilt2(A,[3 3])

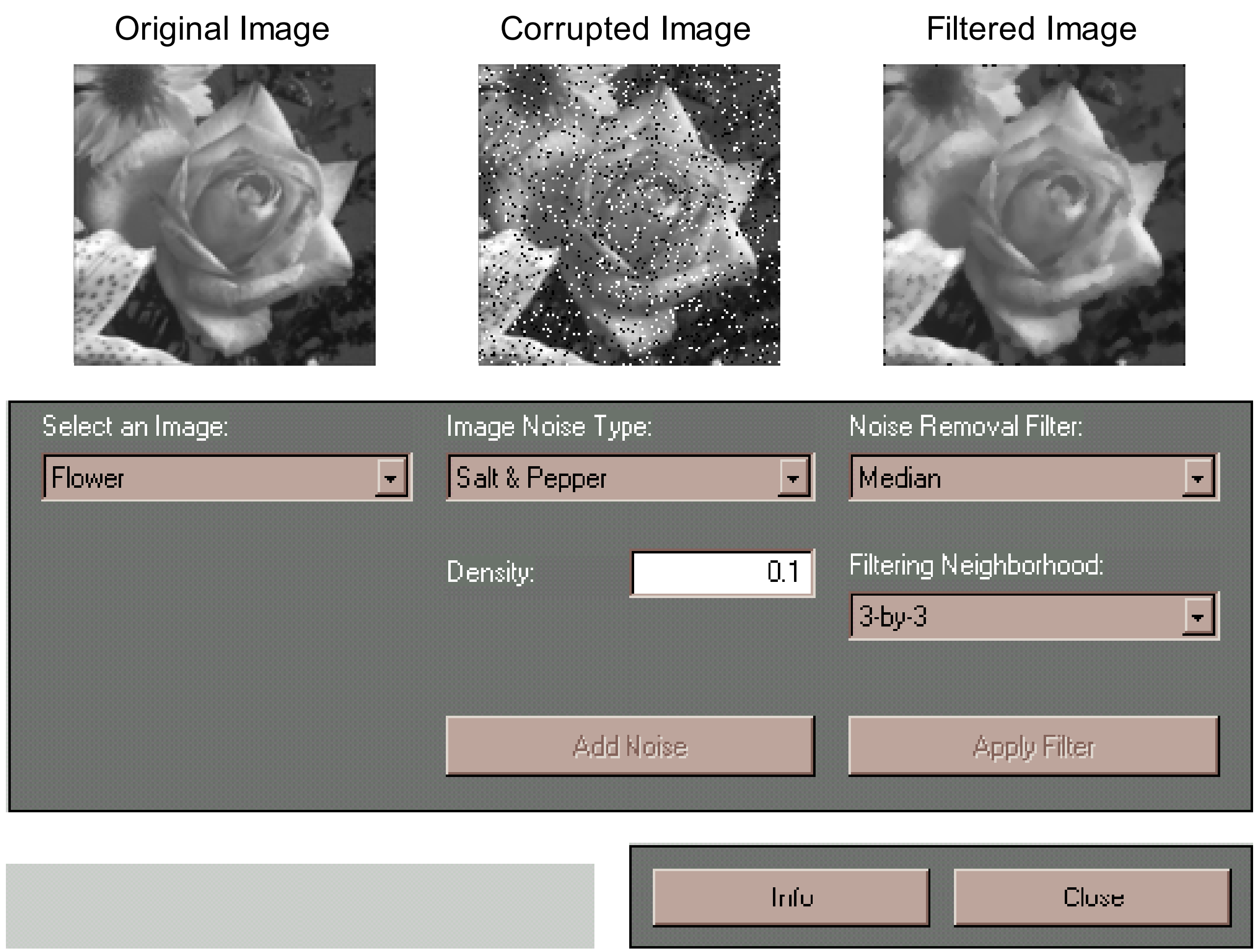
Example



a b c

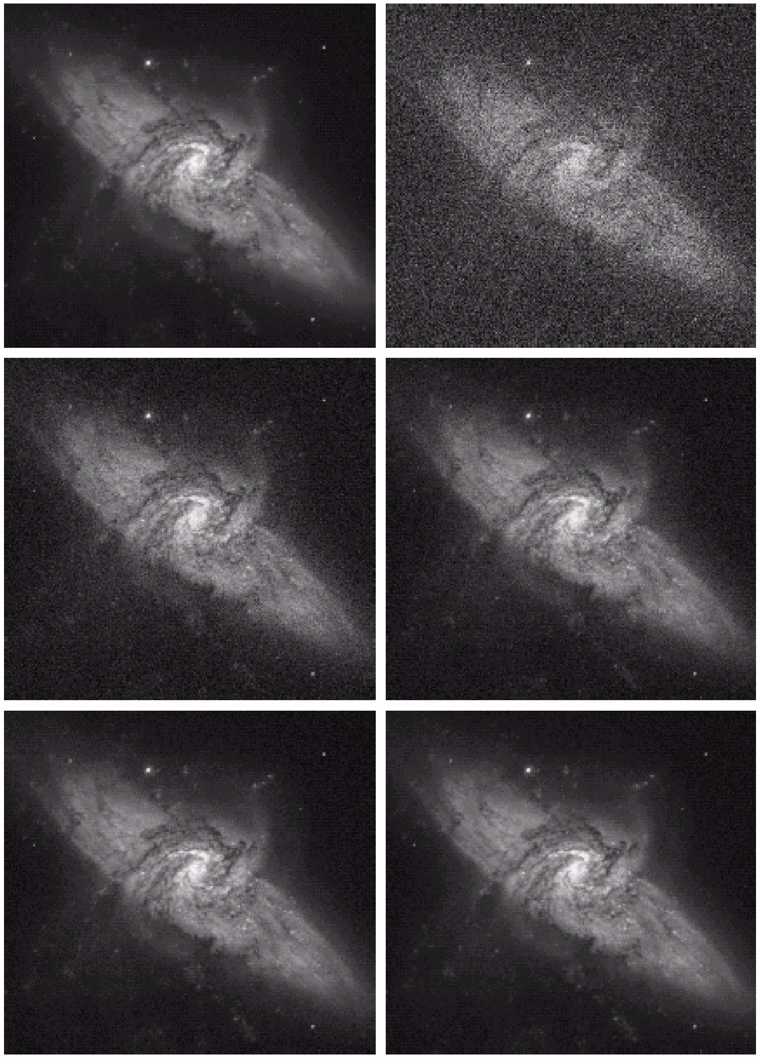
FIGURE 3.37 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3×3 averaging mask. (c) Noise reduction with a 3×3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Matlab Demo: nrfiltdemo



Can choose between mean, median and adaptive (Wiener) filter with different window size

Noise Removal by Averaging Multiple Images



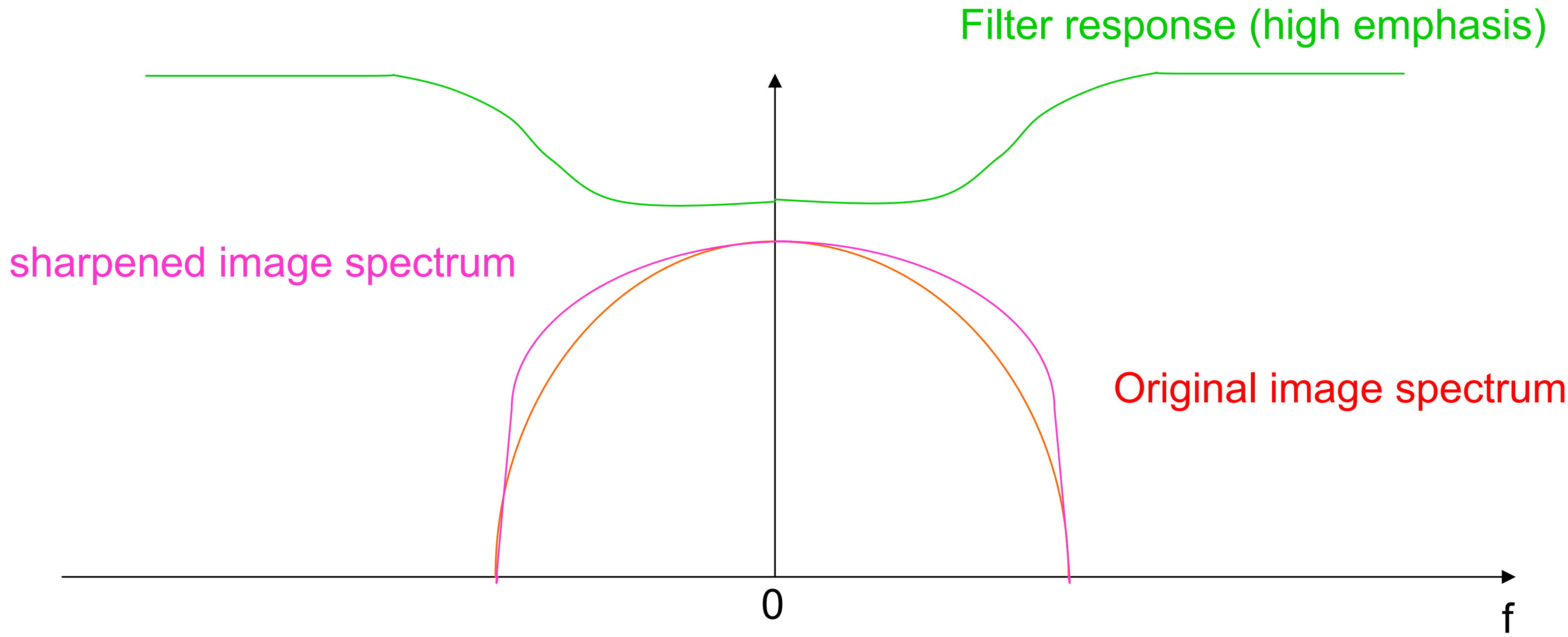
a b
c d
e f

FIGURE 3.30 (a) Image of Galaxy Pair NGC 3314. (b) Image corrupted by additive Gaussian noise with zero mean and a standard deviation of 64 gray levels. (c)–(f) Results of averaging $K = 8, 16, 64$, and 128 noisy images. (Original image courtesy of NASA.)

Image Sharpening

- Sharpening : to enhance line structures or other details in an image
- Enhanced image = original image + scaled version of the line structures and edges in the image
- Line structures and edges can be obtained by applying a difference operator (=high pass filter) on the image
- Combined operation is still a weighted averaging operation, but some weights can be negative, and the sum=1.
- In frequency domain, the filter has the “high-emphasis” character

Frequency Domain Interpretation



Highpass Filters

- Spatial operation: taking difference between current and averaging (weighted averaging) of nearby pixels
  - Can be interpreted as weighted averaging = linear convolution
  - Can be used for edge detection
- Example filters

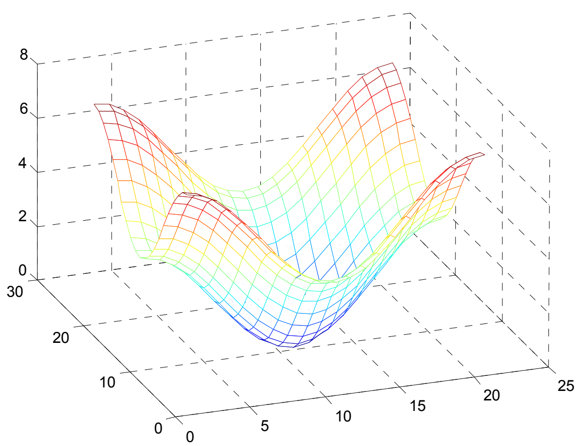
$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}; \quad \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}; \quad \begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}; \quad \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix};$$

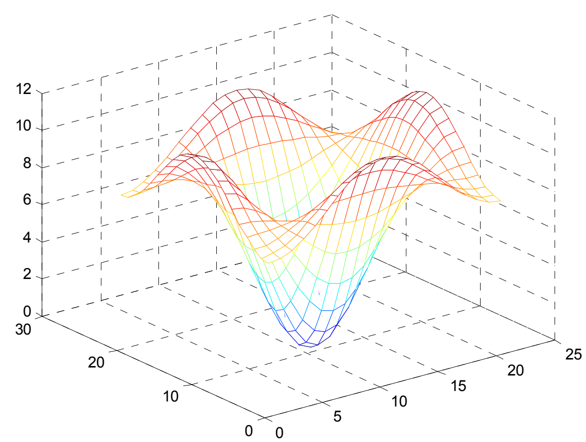
  - All coefficients sum to 0!

Example Highpass Filters

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

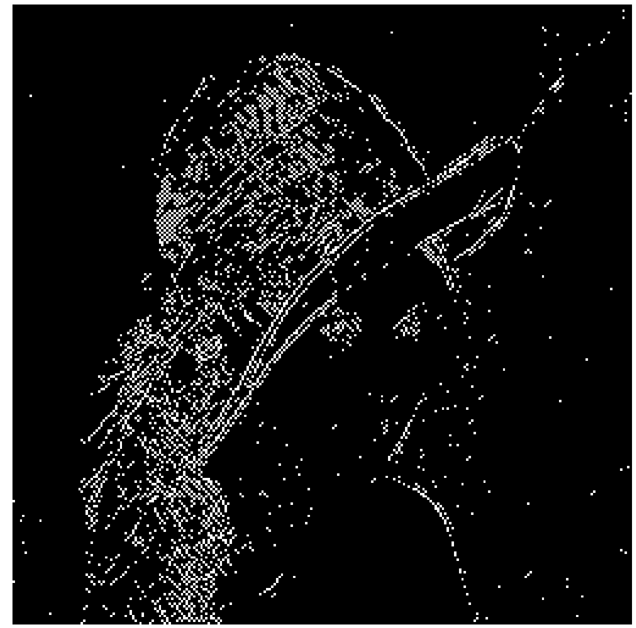




Example of Highpass Filtering

Original image

Isotropic edge detection



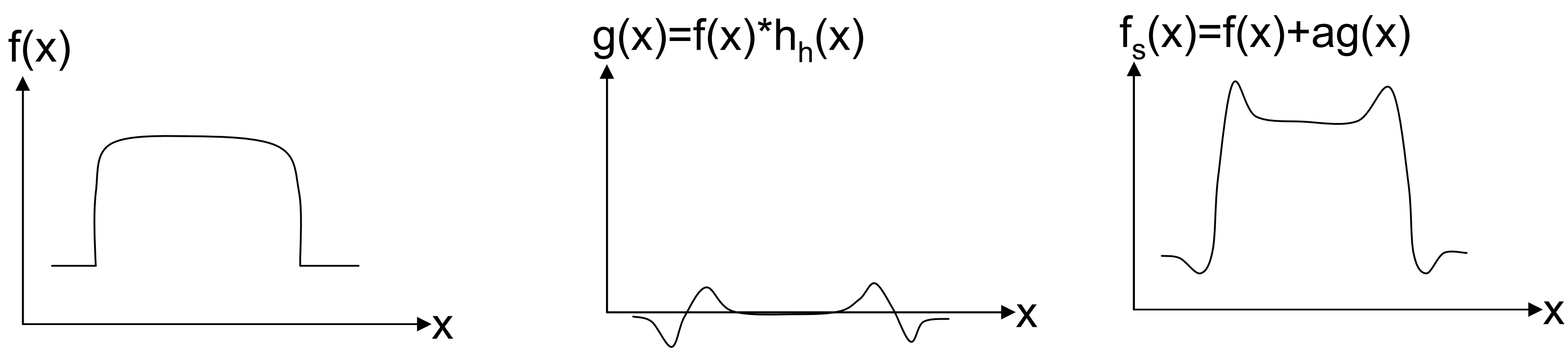
Binary image

Designing Sharpening Filter Using High Pass Filters

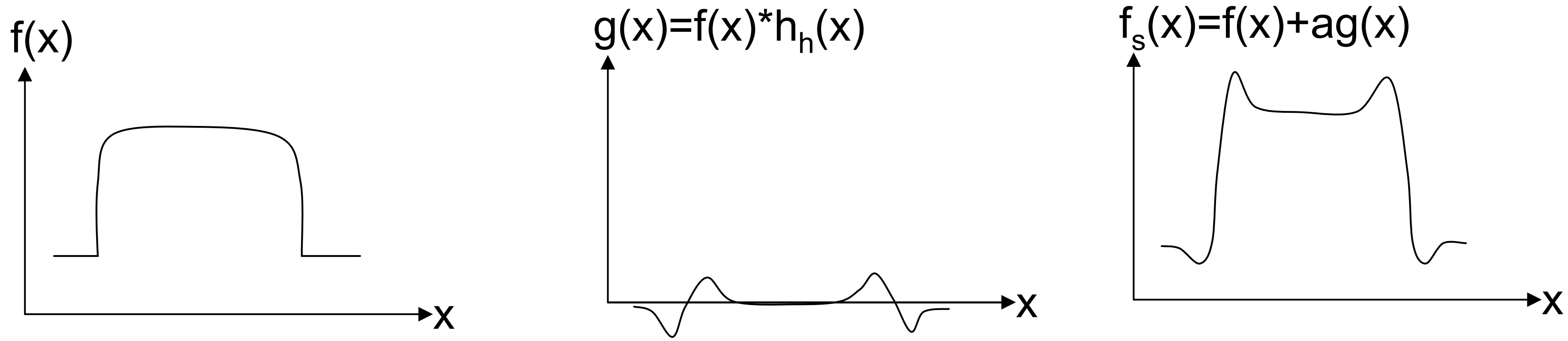
- The desired image is the original plus an appropriately scaled high-passed image
- Sharpening filter

$$f_s = f + \lambda f_h$$

$$h_s(m,n) = \delta(m,n) + \lambda h_h(m,n)$$



Example Sharpening Filters



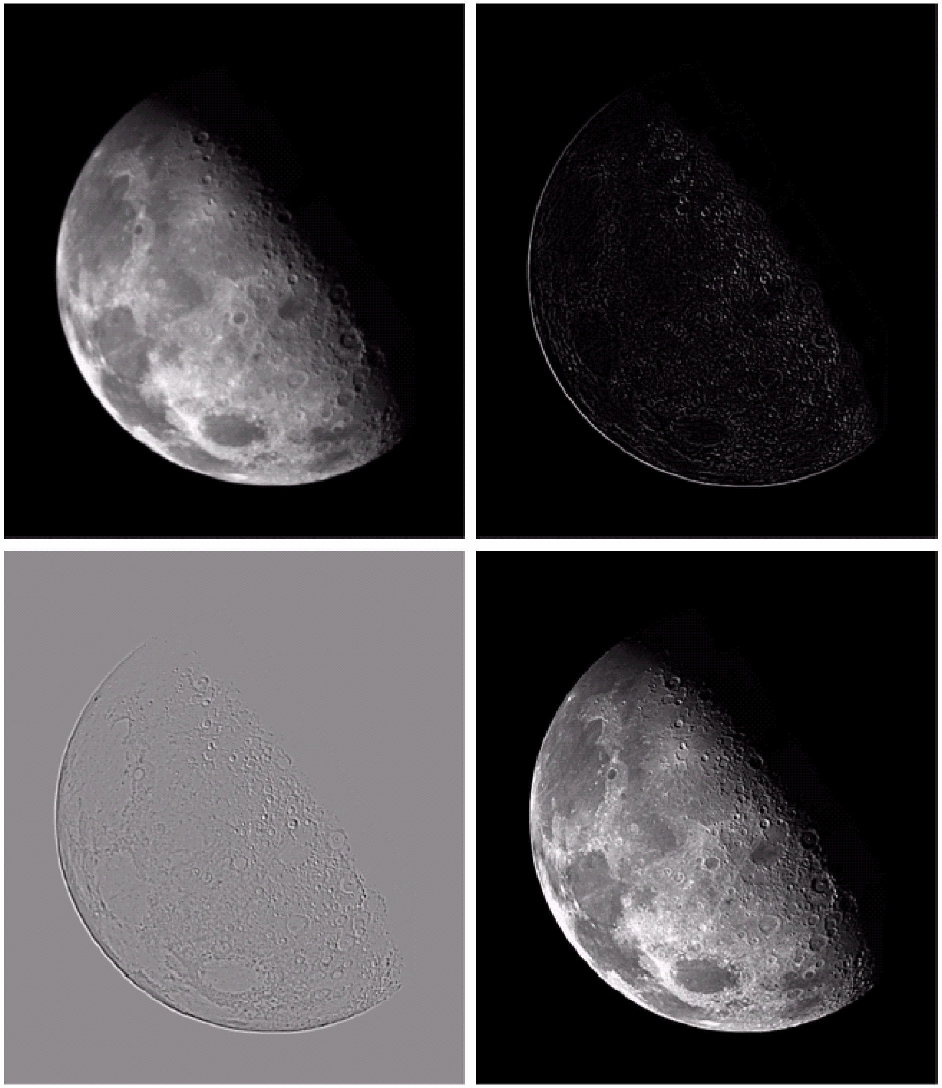
$$H_h = \frac{1}{4}\begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix} \Rightarrow H_s = \frac{1}{4}\begin{bmatrix} 0 & -1 & 0 \\ -1 & 8 & -1 \\ 0 & -1 & 0 \end{bmatrix} \quad with \quad \lambda = 1.$$

$$H_h = \frac{1}{8}\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix} \Rightarrow H_s = \frac{1}{8}\begin{bmatrix} -1 & -1 & -1 \\ -1 & 16 & -1 \\ -1 & -1 & -1 \end{bmatrix} \quad with \quad \lambda = 1.$$

Example of Sharpening

a b
c d

FIGURE 3.40
(a) Image of the North Pole of the moon.
(b) Laplacian-filtered image.
(c) Laplacian image scaled for display purposes.
(d) Image enhanced by using Eq. (3.7-5).
(Original image courtesy of NASA.)



$$H_h = \frac{1}{4}\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

$$H_s = \frac{1}{8}\begin{bmatrix} -1 & -1 & -1 \\ -1 & 16 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

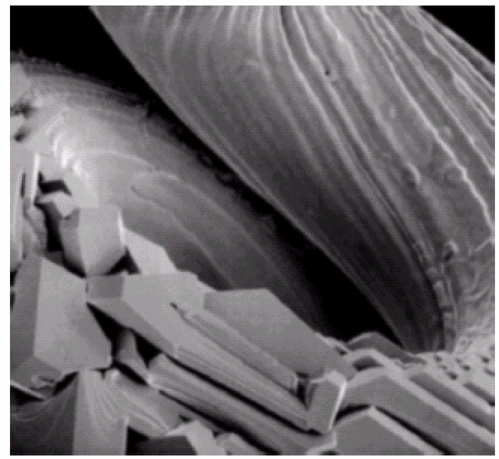
Example of Sharpening

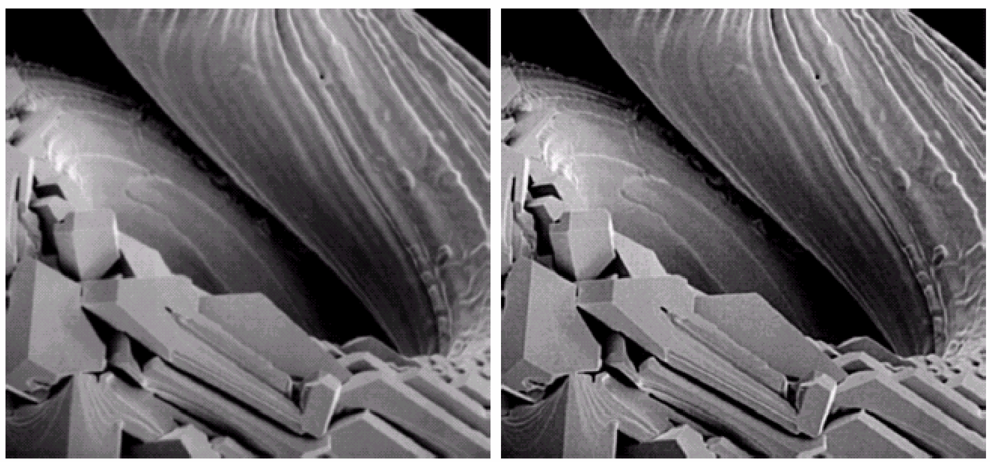
$\lambda = 4$

0	–1	0
–1	5	–1
0	–1	0

$\lambda = 8$

–1	–1	–1
–1	9	–1
–1	–1	–1





a b c
d e

FIGURE 3.41 (a) Composite Laplacian mask. (b) A second composite mask. (c) Scanning electron microscope image. (d) and (e) Results of filtering with the masks in (a) and (b), respectively. Note how much sharper (e) is than (d). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

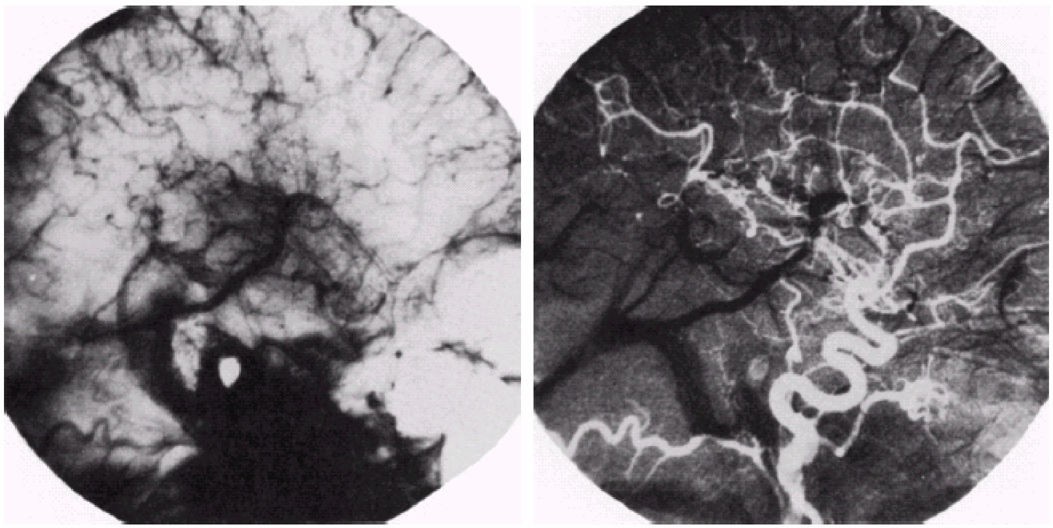
Challenges of Noise Removal and Image Sharpening

- How to smooth the noise without blurring the details too much?
- How to enhance edges without amplifying noise?
- Still a active research area

Wavelet-Domain Filtering

Courtesy of Ivan Selesnick

Feature Enhancement by Subtraction



a b

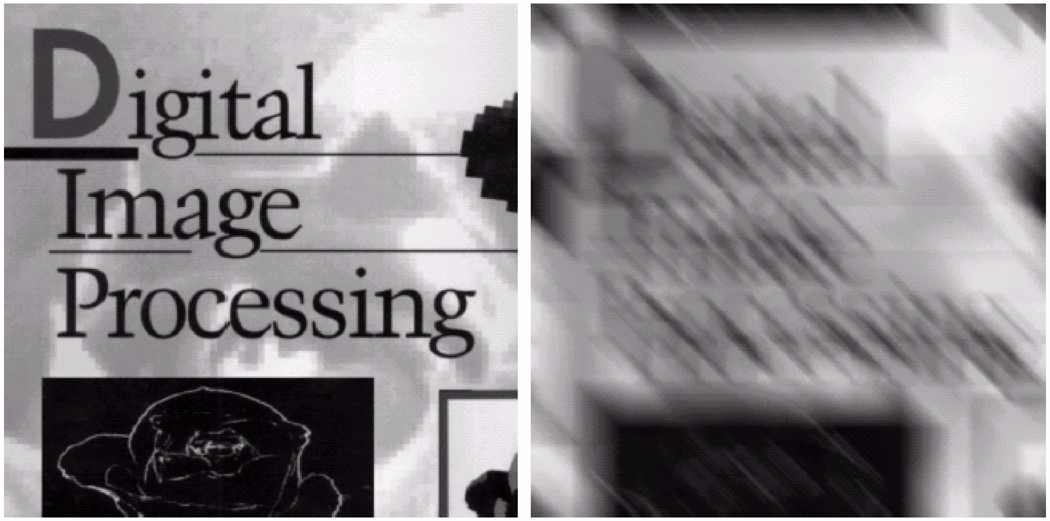
FIGURE 3.29 Enhancement by image subtraction. (a) Mask image. (b) An image (taken after injection of a contrast medium into the bloodstream) with mask subtracted out.

Taking an image without injecting a contrast agent first. Then take the image again after the organ is injected some special contrast agent (which go into the bloodstreams only). Then subtract the two images --- A popular technique in medical imaging

Image Deblurring

- Noise removal considered thus far assumes the image is corrupted by additive noise
  - Each pixel is corrupted by a noise value, independent of neighboring pixels
- Image blurring
  - When the camera moves while taking a picture
  - Or when the object moves
  - Each pixel value is the sum of surrounding pixels → The blurred image is a filtered version of the original
- Deblurring methods:
    - Inverse filter: can adversely amplify noise
    - Wiener filter = generalized inverse filter
    - Many advanced adaptive techniques

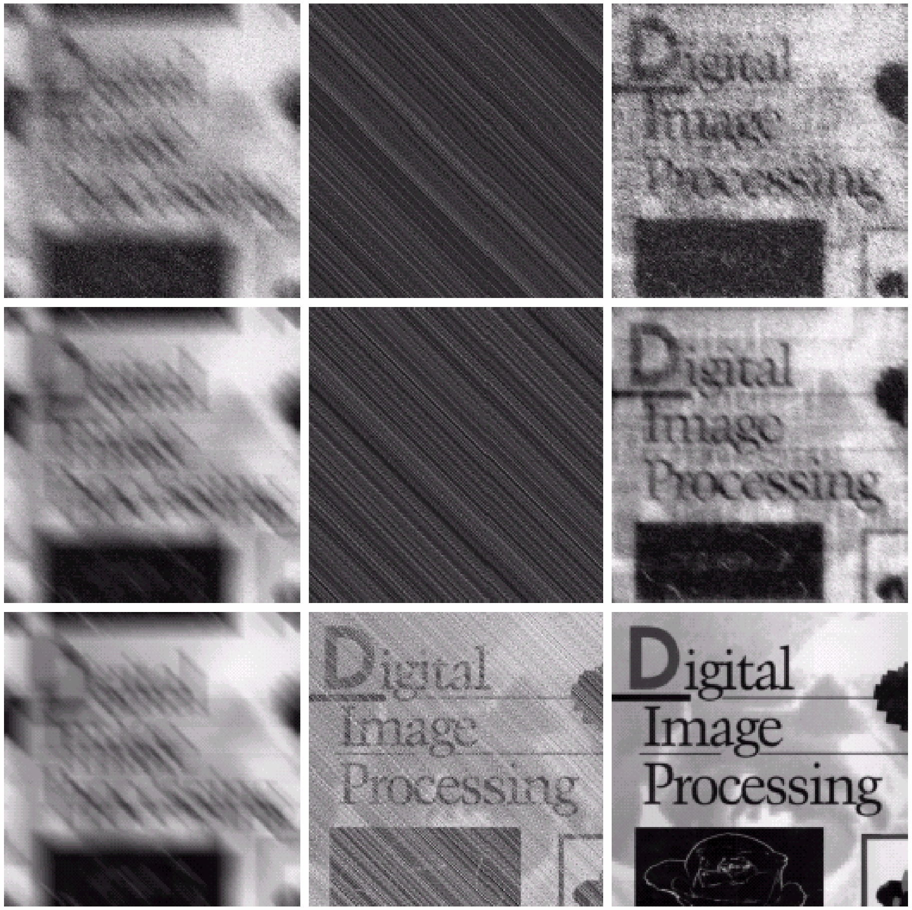
Example of Motion Blur



a b

FIGURE 5.26 (a) Original image. (b) Result of blurring using the function in Eq. (5.6-11) with $a = b = 0.1$ and $T = 1$.

Inverse Filtering vs. Wiener Filtering



a b c
d e f
g h i

FIGURE 5.29 (a) Image corrupted by motion blur and additive noise. (b) Result of inverse filtering. (c) Result of Wiener filtering. (d)–(f) Same sequence, but with noise variance one order of magnitude less. (g)–(i) Same sequence, but noise variance reduced by five orders of magnitude from (a). Note in (h) how the deblurred image is quite visible through a "curtain" of noise.

Constrained Least Squares Filtering

a b c

FIGURE 5.30 Results of constrained least squares filtering. Compare (a), (b), and (c) with the Wiener filtering results in Figs. 5.29(c), (f), and (i), respectively.

What Should You Know

- How does averaging and weighted averaging filter works?
- How does median filter works?
- What method is better for additive Gaussian noise?
- What method is better for salt-and-pepper noise?
- How does high-pass filtering and sharpening work?
- For smoothing and sharpening:
  - Can perform spatial filtering using given filters
  - Deriving frequency response is not required, but should know the desired shape for the frequency responses in different applications
- What is the challenge in noise removal and sharpening?
- What causes blurring?
- Principle of deblurring: technical details not required

References

Gonzalez and Woods, *Digital image processing*, 2nd edition, Prentice Hall, 2002. Chap 4 Sec 4.3, 4.4; Chap 5 Sec 5.1 – 5.3 (pages 167-184 and 220-243)