# Inequality of any two real number sequences and its study

**Abstract:** Inelastic collision is very common natural phenomenon. So it is very important to study the quantity relation of inelastic collision. The motion of common velocity is a typical inelastic collision. This article first discusses the energy loss problem of the common motion and to get a energy loss equation for the motion of the common velocity. From the point of view that themotion is inelastic collision, the inequality based on natural number field is derived. Then the inequality of any two real numbers sequences based on real number field is extended. Finally, strict mathematical proofs are given and discuss some special cases. At the end of this paper, the inequality is specially extended. A new type of sequence is defined and its properties are discussed.This inequality can provide a powerful mathematical tool for solving related problems and optimizing calculation process.

**Keywords**: Inelastic collision; Motion of common velocity; Real number sequence Inequality; Vector.

## 1. Inequalities based on the field of natural numbers

Inelastic collision is a very common natural phenomenon. In case of relevant problems we usually use the conservation of momentum law. The motion of common velocity is a typical inelastic collision. Therefore, this paper studies the energy loss of the common motion and uncover the quantitative relationship.

Under ideal conditions, let's say I have $n(n \in \mathbb{N}^*)$ objects on the horizontal plane. At some point a collision occurs. After the collision, the motion of common velocity occurs. In the collision process there is the following relationship: Let's say the total kinetic energy of the system before the collision is $E_1$,the total kinetic energy of the system after the collision is $E_2$,the common velocity of the system is $v_0$.There are

$$\mathbf{E_1 = \frac{1}{2}\sum_{i=1}^{n} m_i v_i^2}$$

$$\mathbf{E_2 = \frac{1}{2} v_0^2 \sum_{i=1}^{n} m_i}$$

Momentum conservation during collision, so:

$$\mathbf{\sum_{i=1}^{n} m_i\, v_i = v_0 \sum_{i=1}^{n} m_i}$$

$$\mathbf{v_0 = \frac{\sum_{i=1}^{n} m_i\, v_i}{\sum_{i=1}^{n} m_i}}$$

$$\mathbf{E_2 = \frac{(\sum_{i=1}^{n} m_i\, v_i)^2}{2\sum_{i=1}^{n} m_i}}$$

Let's say that $\Delta E$ is the kinetic energy that the system loses. There are $\Delta E = E_1 - E_2$.

To sum up there are

**(1.1)** $$\mathbf{\Delta E = \frac{1}{2}\left[\sum_{i=1}^{n} m_i v_i^2 - \frac{(\sum_{i=1}^{n} m_i v_i)^2}{\sum_{i=1}^{n} m_i}\right] \qquad (n \in \mathbb{N}^*; i = 1, 2, 3, \cdots, n)}$$

Because the motion of common velocity is an inelastic collision, so $\Delta E > 0$.And because the value range of masses and velocities is the set of natural numbers, so when the mass or velocity of the system or both is $0$ easy to know $\Delta E = 0$.

To sum up there are inequality based on set of natural numbers: For the sequence of natural numbers $\{m_n\}$and $\{v_n\}$ there are

**(1.2)** $$\sum_{i=1}^{n} \mathbf{m_i} \sum_{i=1}^{n} \mathbf{m_i v_i^2} \geq \left(\sum_{i=1}^{n} \mathbf{m_i v_i}\right)^2 \qquad (\mathbf{m_i}, \mathbf{v_i} \in \mathbb{N}; \mathbf{n} \in \mathbb{N}^*; \mathbf{i} = \mathbf{1}, \mathbf{2}, \mathbf{3}, \cdots, \mathbf{n})$$

## 2. Extension and certification

Then can reasonably guess: Is it possible to generalize (1.2) to the real number domain?

Namely for $n(n \in N^*)$ the same any two real numbers sequences $\{a_n\}$ and $\{b_n\}$ all have:

**(2.1)** $$\sum_{i=1}^{n} \mathbf{a_i} \sum_{i=1}^{n} \mathbf{a_i b_i^2} \geq \left(\sum_{i=1}^{n} \mathbf{a_i b_i}\right)^2 \qquad (\mathbf{a_i}, \mathbf{b_i} \in \mathbb{R}; \mathbf{n} \in \mathbb{N}^*; \mathbf{i} = \mathbf{1}, \mathbf{2}, \mathbf{3}, \cdots, \mathbf{n})$$

So here's the proof:

We take (2.1) apart and we get

① $(a_1 + a_2 + \cdots + a_n)(a_1 b_1^2 + a_2 b_2^2 + \cdots + a_n b_n^2) \geq (a_1 b_1 + a_2 b_2 + \cdots + a_n b_n)^2$

② $\sum_{i=1}^{n} a_i^2 b_i^2 + a_1\left(\sum_{i=1}^{n} a_i b_i^2 - a_1 b_1^2\right) + a_2\left(\sum_{i=1}^{n} a_i b_i^2 - a_2 b_2^2\right) + \cdots + a_n\left(\sum_{i=1}^{n} a_i b_i^2 - a_n b_n^2\right) \geq \sum_{i=1}^{n} a_i^2 b_i^2 + a_1 b_1\left(\sum_{i=1}^{n} a_i b_i - a_1 b_1\right) + a_2 b_2\left(\sum_{i=1}^{n} a_i b_i - a_2 b_2\right) + \cdots + a_n b_n\left(\sum_{i=1}^{n} a_i b_i - a_n b_n\right)$

③ $a_1 a_2(b_1^2 + b_2^2) + a_1 a_3(b_1^2 + b_3^2) + \cdots + a_1 a_n(b_1^2 + b_n^2) + a_2 a_3(b_2^2 + b_3^2) + a_2 a_4(b_2^2 + b_4^2) + \cdots + a_2 a_n(b_2^2 + b_n^2) + \cdots + a_{n-1} a_n(b_{n-1}^2 + b_n^2) \geq 2(a_1 a_2 b_1 b_2 + a_1 a_3 b_1 b_3 + \cdots + a_1 a_n b_1 b_n + a_2 a_3 b_2 b_3 + a_2 a_4 b_2 b_4 + \cdots + a_2 a_n b_2 b_n + \cdots + a_{n-1} a_n b_{n-1} b_n)$

Because $a^2+b^2 \geq 2ab \quad (a,b \in \mathbb{R})$

So ③ is tenable.

So (2.1) get proven.

Namely:**(2.1)** $\sum_{i=1}^{n} \mathbf{a_i} \sum_{i=1}^{n} \mathbf{a_i}\, \mathbf{b_i^2} \geq (\sum_{i=1}^{n} \mathbf{a_i}\, \mathbf{b_i})^2 \quad (\mathbf{a_i}, \mathbf{b_i} \in \mathbb{R}; \mathbf{n} \in \mathbb{N}^*; i=1,2,3,\cdots,n)$ tenable.

Easy to know,(2.1) the two-dimensional form is

**(2.2)** $(\mathbf{a}+\mathbf{b})(\mathbf{ac^2}+\mathbf{bd^2}) \geq (\mathbf{ac}+\mathbf{bd})^2 \quad (\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d} \in \mathbb{R})$

To observe the (2.2) known when $a \neq 0, b \neq 0$ and $a+b \neq 0$, only in the condition of $c = d$ is (2.2) *take the equal sign.*

In particular, in one dimensional form namely when $n = 1$ the (2.1) perpetually take the equal sign.

In the $n(n \geq 1)$ dimensional form the condition for the equality of (2.1) is as follows:

(1)All the items in $\{b_n\}$ are equal.

(2)All of the items in $\{b_n\}$ are 0.

(3)All of the items in $\{a_n\}$ are 0.

(4)There are $(n-1)$ items in $\{a_n\}$ that are all 0.

## 3. Inequality promotion

Because ③ is true and we combine it with our algorithm for vectors,

So you get the vector form of the inequality.

Because $\vec{\mathbf{a}}^2 + \vec{\mathbf{b}}^2 \geq 2\vec{\mathbf{a}}\vec{\mathbf{b}}$

So the vector form of the inequality is

**(3.1)** $\sum_{i=1}^{n} \vec{\mathbf{a}}_i \sum_{i=1}^{n} \vec{\mathbf{a}}_i \vec{\mathbf{b}}_i^2 \geq \left(\sum_{i=1}^{n} \vec{\mathbf{a}}_i\, \vec{\mathbf{b}}_i\right)^2 \quad (\mathbf{n} \in \mathbb{N}^*; \mathbf{i} = \mathbf{1}, \mathbf{2}, \mathbf{3}, \cdots, \mathbf{n})$

So it's a two dimensional vector form of inequality is

**(3.2)** $$(\vec{\mathbf{a}}+\vec{\mathbf{b}})(\vec{\mathbf{a}}\vec{\mathbf{c}}^{\mathbf{2}}+\vec{\mathbf{b}}\vec{\mathbf{d}}^{\mathbf{2}}) \geq \left(\vec{\mathbf{a}}\vec{\mathbf{c}}+\vec{\mathbf{b}}\vec{\mathbf{d}}\right)^{\mathbf{2}}$$

Other forms of inequality are also summarized. There is at least one inequality is tenable in (3.3) for a real number sequence that meets the criteria.

**(3.3)** $$\sqrt{\sum_{\mathbf{i=1}}^{\mathbf{n}}\mathbf{a_i}}+\sqrt{\sum_{i=1}^{n}a_i\,b_i^2} \geq \sqrt{\sum_{\mathbf{i=1}}^{\mathbf{n}}\mathbf{a_i}(b_i+1)^2} \quad \textbf{or} \quad \sqrt{\sum_{\mathbf{i=1}}^{\mathbf{n}}\mathbf{a_i}}+\sqrt{\sum_{i=1}^{n}a_i\,b_i^2} \geq \sqrt{\sum_{\mathbf{i=1}}^{\mathbf{n}}\mathbf{a_i}(b_i-1)^2}$$

$(a_i \in \mathbb{N}; b_i \in \mathbb{R}; n \in \mathbb{N}^*; i = 1,2,3,\cdots,n)$

Here are some of the conditions that make the (3.3) equal sign true:

(1)All of the items in $\{\mathrm{a_n}\}$ **are 0.**

(2)All of the items in $\{\mathrm{b_n}\}$ **are 0.**

Prove: $\left(\sqrt{\sum_{i=1}^{n}a_i}+\sqrt{\sum_{i=1}^{n}a_i\,b_i^2}\right)^2 = \sum_{\mathbf{i=1}}^{\mathbf{n}}\mathbf{a_i}+\sum_{i=1}^{n}a_i\,b_i^2+2\sqrt{\sum_{\mathbf{i=1}}^{\mathbf{n}}\mathbf{a_i}}\sqrt{\sum_{i=1}^{n}a_i\,b_i^2} \geq \sum_{\mathbf{i=1}}^{\mathbf{n}}\mathbf{a_i}+\sum_{i=1}^{n}a_i\,b_i^2+2\left|\sum_{\mathbf{i=1}}^{\mathbf{n}}\mathbf{a_i}\,\mathbf{b_i}\right| = \sum_{\mathbf{i=1}}^{\mathbf{n}}\mathbf{a_i}(b_i+1)^{\mathbf{2}}$ **or** $\sum_{\mathbf{i=1}}^{\mathbf{n}}\mathbf{a_i}+\sum_{i=1}^{n}a_i\,b_i^2+2\left|\sum_{\mathbf{i=1}}^{\mathbf{n}}\mathbf{a_i}\,\mathbf{b_i}\right| = \sum_{\mathbf{i=1}}^{\mathbf{n}}\mathbf{a_i}(b_i-1)^{\mathbf{2}}$

Sqrt(square root) get: $\sqrt{\sum_{\mathbf{i=1}}^{\mathbf{n}}\mathbf{a_i}}+\sqrt{\sum_{i=1}^{n}a_i\,b_i^2} \geq \sqrt{\sum_{\mathbf{i=1}}^{\mathbf{n}}\mathbf{a_i}(b_i+1)^{\mathbf{2}}}$ **or** $\sqrt{\sum_{\mathbf{i=1}}^{\mathbf{n}}\mathbf{a_i}}+\sqrt{\sum_{i=1}^{n}a_i\,b_i^2} \geq \sqrt{\sum_{\mathbf{i=1}}^{\mathbf{n}}\mathbf{a_i}(b_i-1)^{\mathbf{2}}}$

So **(3.3)** is proved.

Easy to know, the two-dimensional form of the **(3.3)** is

**(3.4)** $$\sqrt{\mathbf{a}+\mathbf{b}}+\sqrt{\mathbf{ac^2}+\mathbf{bd^2}} \geq \sqrt{\mathbf{a}(\mathbf{c}+\mathbf{1})^{\mathbf{2}}+\mathbf{b}(\mathbf{d}+\mathbf{1})^{\mathbf{2}}} \ \textbf{or}\ \sqrt{\mathbf{a}+\mathbf{b}}+\sqrt{\mathbf{ac^2}+\mathbf{bd^2}} \geq \sqrt{\mathbf{a}(\mathbf{c}-\mathbf{1})^{\mathbf{2}}+\mathbf{b}(\mathbf{d}-\mathbf{1})^{\mathbf{2}}} \qquad (\mathbf{a},\mathbf{b} \in \mathbb{N}; \mathbf{c},\mathbf{d} \in \mathbb{R})$$

When $\mathrm{a} = \mathrm{b} = 0$, (3.4) equal sign holds.

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