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Problem Set 2

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Problem 1 Let $B = \{x \in \Sigma^* || x| < maxlength(h(\Sigma))\}$. We construct an NFA $M_h = (Q_h, \Sigma, \delta_h, q_{0h}, F_h)$ such that

$$\begin{cases} Q_h = Q \times B \\ \delta_h((q,b),a) = \begin{cases} (q,ba) \text{if } ba \in B \\ (\delta(q,c),\epsilon) \forall h(c) = ba \text{ if } ba \in h(\Sigma) \end{cases} \\ F_h = (q,\epsilon) | q \in F \end{cases}$$

Problem 2

$$h^{-1}(L) = (ba)^*$$

Proof. It's not hard to see $h((ba)^*) = (1001)^* \subset L$. We claim $\forall x \in (a+b)^* - (ba)^*, h(x) \notin L$. Such x must satisfy at least one of the following properties:

- A prefix a.
- A sufix b.
- An infix aa or bb.

Any of such letters would lead to a consecutive individual 0, which can't be in L. □

Problem 3 Suppose set of all strings of 0's and 1's of length k, with an odd number of 0's and an even number of 1's end at state C, with an even number of 0's and an odd number of 1's end at state B, with an even number of 0's and an even number of 1's end at state A, and with an odd number of 0's and an odd number of 1's is accepted by the machine.

Problem 4 For each question, suppose the language is regular, then by Pumping Lemma $\exists n \in \mathbb{N}, \forall x \in L, |x| > n, \exists u, v \neq \epsilon, w, |uv| < n, s.t. uv^i w \in L$

1. Let $x = scs^R \in L$ and $|x| = 2n + 1$. By Pumping Lemma there exists $x = uvw$ satisfying. Clearly since $|uv| < n$, v is an infix of w. We have $y = uw = (s/v)cs^R \in L$, which is a contradiction since letter c is no longer at the middle of the string.

2. Let $x = a^n b^n c^n$, $x \in L$ and hence $x = uvw$ satisfies pumping lemma. Clearly since $|uv| < n$, $v \in a^+$, by Pumping Lemma we have $y = uw = (a^n - v)b^n c^n \in L$, this contradicts $i_a \geq j_b \geq k_c$.

3. Let $x = 0^p$, $p > n$ and p is a prime. Hence there exists some $x = uvw$ satisfies pumping lemma. Clearly $v = 0^t$, $0 < t < n$. By Pumping Lemma we have $y = uv^{p-t}w = 0^{(t+1)(p-t)} \in L$, which is a contradiction since $(t + 1)(p - t)$ is not prime.