MS205 Automata Theory	Dec 12, 2013

Problem Set 1

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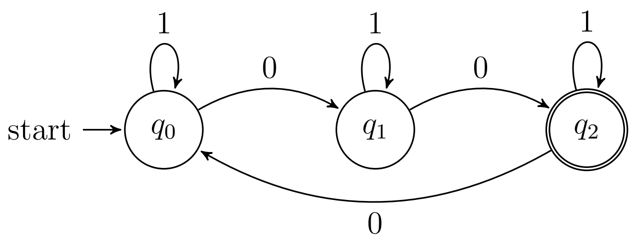
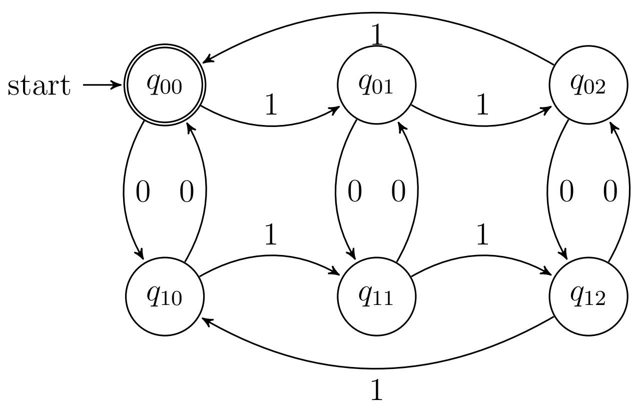
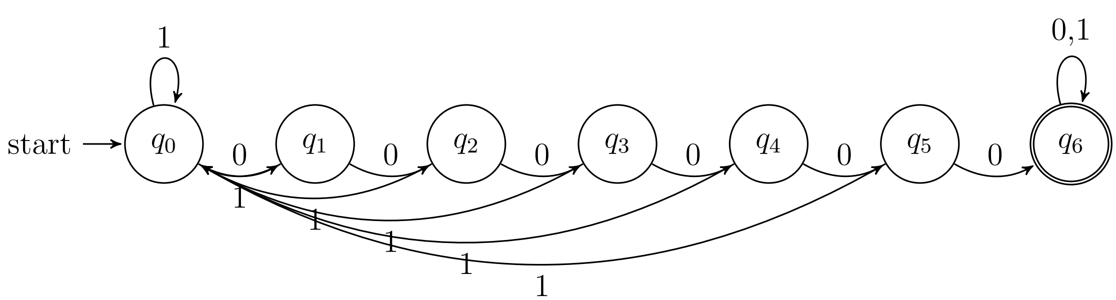
Problem 1

1. 

2. 

3. Since the problem isn't clear enough, take it as search for 6 consecutive 0s.



Problem 2

Proof. We show it by contradiction.

Suppose that the number of subsets of the set of all finite length strings, which is equivalent to the number of functions from elements to $\{0,1\}$ is countably infinite, so that we can label these functions $\{f_1, f_2, ..., f_n, ...\}$ with distinct integers of $\mathbb{N}$. We can also label all finite length strings $\{s_1, s_2, ..., s_n, ...\}$, such like first by length and second by dictionary order, since we already know their number is countably infinite. Given one such labeling, we form a function f_p as following:

$$f_p(s_i) \neq f_i(s_i)$$

saying equivalently we construct a subset containing the ith string while the ith subset not, or to the contrary. We can easily see f_p is not in any f_i since it differs from any f_i at s_i, this is contradict to our assumption. So the number of subsets of the set of all finite length strings is not countably infinite. $\square$

Problem 3

1. Countably infinite. Number of strings of every fixed length is finite. And we can order strings first by length second by dictionary order.

2. Countably infinite. Clearly it's at least countably infinite, and any finite automata can be expressed by certain string of finite length.

3. Countably infinite. The reason is the same as for finite automata.

4. Not countably infinite. The proof is similar to Problem 2.

5. Not countably infinite. The proof is similar to Problem 2.