

Assignment for

Computer Science Theory for the Information Age

Day 10

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2012 ACM class

June 29, 2013

Exercise 1. Label the points in the plane that are within the circle of radius one as $+1$ and label the points in the annulus of inner radius two and outer radius three as -1 . Find a function ϕ mapping the points to a higher dimensional space where the two sets are linearly separable.

Answer.

Let ϕ be

$$\phi(x, y) = \left(x, y, \sqrt{x^2 + y^2} \right) \quad (1)$$

For

$$\forall (x_1, y_1), (x_2, y_2) ((x_1, y_1) \neq (x_2, y_2)) \Rightarrow (\phi(x_1, y_1) \neq \phi(x_2, y_2)) \quad (2)$$

And the plane $z = 1$ can separate the two sets.

Exercise 2. Prove that the VC-dimension of circles is three.

Proof.

For the three points forming a triangle as they are not in a line. Obviously any set of them can be shattered by a circle. However three points in a line cannot be shattered for any circle containing the external two points, it must contain the internal point.

Consider four points. If any three of them stand in a line, they cannot be shattered by circles as shown above. In another case, if one point is inside the convex hull of the other three points. So let's just consider the four points forming a convex quadrangle. We use two circles each only consisting two diamond points in this quadrangle. This makes the two circles divide the plane into four regions, which is not possible. \square