Assignment for

Computer Science Theory for the Information Age

Day 8

June 26, 2013

Exercise 1. Read in a photo and convert it to a matrix. Perform a singular value decomposition of the matrix. Reconstruct the photo using only 10%, 25%, 50% of the singular values.

1. Print the reconstruted photo. How good is the quality of the reconstructed photo?
2. What percent of the Forbenius norm is captured in each case?

Answer.

I'm using one famous photo in image processing: *the Lenna*[1]. But to be more interesting, I XiWenLeJianly use the full version of this image:

Figure 1. *The Lenna*, origin

1. ***The Lenna Story***, http://www.cs.cmu.edu/~chuck/lennapg/

It's a color image formed in RGB. First I'm gonne convert RGB color space into **Lab** color space. As I already know what **S.V.D.** will perform to this image, the **Lab color system**[2] is much closer to human's true vision, and does well in photography, color identification and skin-tones adjustment.

The Lab color space consists of three channels: $L*$ for lightness, $a*$ for green-magenta and $b*$ for blue-yellow, each with a matrix. As only the $L*$ channel influence the sharpness and defination of the image, while the $a*$ and $b*$ channels do colors, I'll mostly process only the $L*$ channel matrix, and show a example for dealing with the color channels.

Figure 2. % singular values, almost 100.00% Frobenius norm captured

2. CIELAB, www.hunterlab.com/appnotes/an07_96a.pdf

Figure 3. 25% singular values, 99.99% Frobenius norm captured

Figure 4. 10% singular values, 99.92% Frobenius norm captured

The image start to be awesome. If we go a bit further, say leave only one singular value, we have

Figure 5. only one sigular value left

Extremely awesome. However the colors remine perfect. As the results shows, the L^* channel keeps the sharpness and defination of the image.

Now Let's see into what if we do something to the color channel:

Figure 6. 10% sigular values of a^* channel

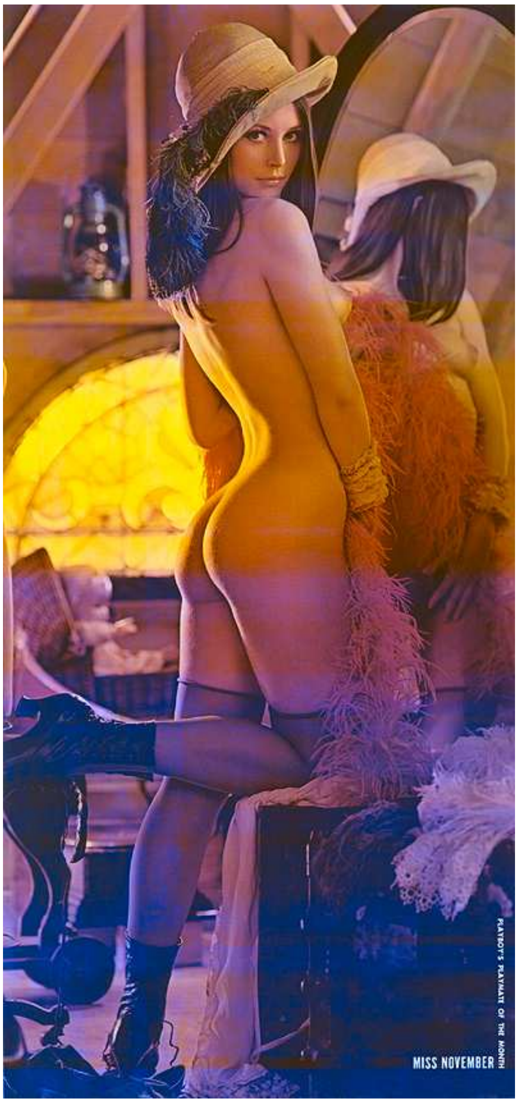

Figure 7. 10% singular values for the $b*$ channel

It seems like the quality of color channels has a rather bigger influence on the quality of the image, something like humans are more sensible to the change of colors.

Exercise 2.

1. Consider the pairwise distance matrix for twenty US cities given below. use the algorithm of Exercise 4.30 to place the cities on a map of the US.

2. Suppose you had airline distances for 50 cities around the world. Could you use these distances to construct a world map?

Answer.

Let $\boldsymbol{X}$ be a 20×20 matrix denoting distances between each pair of cities. From Ex 4.30 we have

$$(\boldsymbol{X}\boldsymbol{X}^T)_{ij} = -\frac{1}{2}[d_{ij}^2 - \frac{1}{n}\sum_{j=1}^{n} d_{ij}^2 - \frac{1}{n}\sum_{i=1}^{n} d_{ij}^2 + \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}^2] \tag{1}$$

perform singular value decomposition on $(\boldsymbol{X}\,\boldsymbol{X}^T)$, we have

$$\boldsymbol{X}\,\boldsymbol{X}^T = \boldsymbol{U}\,\boldsymbol{D}\,\boldsymbol{V}^T \tag{2}$$

notice that

$$\begin{aligned}(\boldsymbol{U}\,\boldsymbol{D}\,\boldsymbol{V}^T)^T &= \boldsymbol{V}\boldsymbol{D}\boldsymbol{U}^T\\ (\boldsymbol{X}\,\boldsymbol{X}^T)^T &= \boldsymbol{X}\,\boldsymbol{X}^T\end{aligned}$$

so we have

$$\boldsymbol{U} = \boldsymbol{V} \tag{3}$$

so that

$$\boldsymbol{X} = \sqrt{\boldsymbol{D}}\,\boldsymbol{V} \tag{4}$$

where $\sqrt{\boldsymbol{D}}$ means taking the square root of each element in the diagonal of $\boldsymbol{D}$. The row vectors of $\boldsymbol{V}$ are the coordinates of the cities.

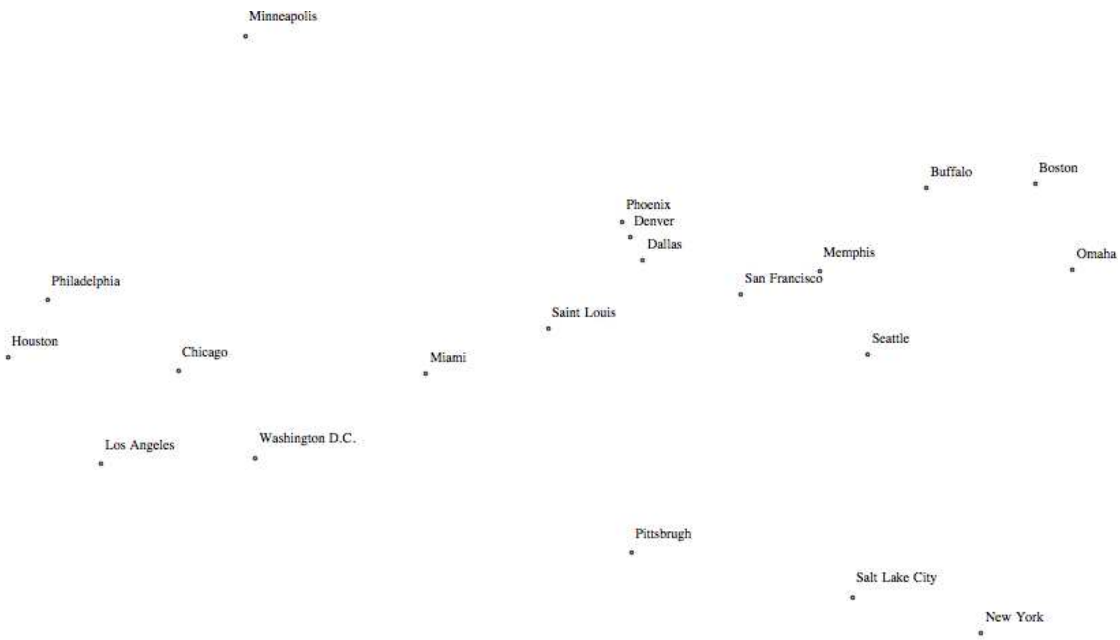

Figure 8. The map generated

However we may not simply use this linear algorithm for airlines, for the earth is not plant.