

Class Note for

Computer Science Theory for the Information Age

Day 4

Homework:

3.11

search www for an undirected graph or a data base that can be converted to a graph. Find the connected components and count the number of each size.

Proposition 1.

another stress! no matter how large the graphy $G\left(n, \frac{d}{n}\right)$ is, the expectation of triangles in each is independent from n .

a review for first moment method and second moment argument.

Proposition 2. *see into threshold for hamilton circle*

There's $\frac{1}{2}(n-1)!$ possible circles (without direction) in a n -graphy.

$$E(x) = \frac{1}{2}(n-1)! \left(\frac{d}{n}\right)^n \approx \left(\frac{n}{e}\right)^n \left(\frac{d}{n}\right)^n = \left(\frac{d}{e}\right)^n$$

with a threshold in $d=e$.

however usually $d = \ln n$ is a threshold,

Next for giant component

Proposition 3. *ask how much giant component a graph has*

breadth first search

generate the graph with when search the graph! NOT break the independence of edges.

probability that a vertex is not discovered in first i steps is $\left(1 - \frac{d}{n}\right)^n$

Let Z_i be the number of vertices discovered in i steps.

$$Z_i = \text{binomial}\left(n-1, 1 - \left(1 - \frac{d}{n}\right)^n\right).$$

Proposition 4. *if there would be two giant component?*

answer is no.

next for Branching process

Proposition 5. *denote P_i as a root has i children.*

make a generating function of $f(x) = \sum_{i=0}^{\infty} p_i x^i$!see into generating function

to prove $f_{j+1}(x) = f_j(f(x))$

the generating function for $x_1 + x_2$ where x_1 and x_2 are independent random variable with generate function $f(x)$ is $f^2(x)$.for:

$$f(x) = \sum_{i=0}^{\infty} p_i x^i$$

$$f^2(x) = p_0^2 + (p_0 p_1 + p_1 p_0) x + (p_0 p_2 + p_1 p_1 + p_2 p_0) x^2 + \dots \text{ obviously.}$$

Let z_j be number of children in j th generation.