H2.Hamlet

- The assignment is due at Gradescope on Tuesday, January 26 at 12 noon.
- You can either type your homework using LaTex or scan your handwritten work. We will provide a LaTex template for each homework. If you writing by hand, please fill in the solutions in this template, inserting additional sheets as necessary. This will facilitate the grading.
- You are permitted to study and discuss the problems with 2 other students (per problem; any section). However, *you must write up your own solutions, in your own words*. Do not submit anything you cannot explain. If you do collaborate with any of the other students on any problem, please do list all your collaborators in your submission for each problem.
- Similarly, please list any other source you have used for each problem, including other textbooks or websites.
- *Show your work.* Answers without justification will be given little credit.

PROBLEM 1 (25 POINTS) *Solve exercise 3 in Chapter 4 in the Kleinberg-Tardos textbook. (trucking)*

Solution: Collaborated with: Yael Sulkin, Bryan Lee

Algorithm The greedy algorithm this proof will be proving is the following:

Let all trucks have maximum weight, W
Let the number of trucks sent to Boston, $numTrucks$.
Let the truckweight at kth iteration be $truckWeight$.
Let i be the package that most recently arrived, woth weight of i be w_i
while packages arrive **do**
  if $truckWeight + w_i < W$ **then**
    update $truckweight = truckweight + w_i$
  else
    increase $numTrucks$ by 1
    reset $truckWeight$ to 0
  end if
end while

Proof We will prove this algorithm inductively

Extra Space for your solution

PROBLEM 2 (25 POINTS) *Solve exercise 5 in Chapter 4 in the Kleinberg-Tardos textbook. (cell phone towers)*

Please note! Here and elsewhere, when the authors (or your instructors) say "give an algorithm" without further instructions, they mean "give an algorithm AND prove that it is correct and runs in polynomial time". This should be assumed henceforth.

Solution: Collaborated with: Yael Sulkin

Starting at west-most point $R = 0$, let's assume we travel such that when we stop R = easternmost point.
while R **do**
  if house is found **then**
    move 4 meters and place a cell tower
    move 4 meters forward again such that $R = R + 8$
  end if
end while

Proof Let set A be the set of bases placed by our algorithm and set O be the set of bases placed by the optimal algorithm. If our set A is not optimal and we are looking for the minimal amount of bases then we know that if $|A| = n$ and $|O| = m$ then $m < n$. If $m < n$ then we know there exists some base, b in A such that it either covers 0 houses or there also exists some b' such that b' can be moved to cover the houses within b. We will continue this proof using case analysis.

In the case where b in A covers 0 houses, this, in fact, contradicts the algorithm for A which states "if a house is found, move 4 meters and place a cell tower". This case is impossible for our proposed greedy algorithm.

In the case where b' exists in A such that b' can be moved, let b be located at distance x and b' be located at distance x' such that $x \neq x'$ and $x, x' \geq 0$. According to our case statement, there exists h in $x - 4, x + 4$ such that b' can be moved to cover $x - 4, x + 4 \cup x' - 4, x' + 4$ or $x - 4, x + 4, x' - 4, x' + 4$. This is in fact impossible because b' will have to at most cover $|(x - 4) - (x' + 4)| = |x - x' - 8|$. Since x and x' cannot both be 0, the distance b' has to cover will always be greater than 8 unless x and x' are exactly 8 meters apart. If —x - x' - 8— is greater than 8, then the distance b must cover contradicts our givens. If —x - x' - 8— is exactly 0 then x and x' must be exactly 8 meters apart which satisfies our greedy algorithm.

Extra Space for your solution

PROBLEM 3 (25 POINTS) *The Running Sums problem is defined as follows:*

Input: *a sequence $(a_1, \dots, a_n)$, where each a_i is either 1 or -1.*

Desired output: *a sequence $(b_1, \dots, b_n)$, where each b_i is either 0 or 1.*
*Your goal is to **minimize** $\sum_{1 \le i \le n} b_i$, subject to the **constraint** that*

$$\sum_{1 \le i \le j} (a_i + b_i) \ \geq \ 0 \,, \qquad \text{for all} \quad j \in \{1, 2, \dots, n\} \,.$$

Give an algorithm for this problem that, for full credit, should run in $O(n)$ steps.

Solution: Your solution goes here.

Extra Space for your solution

PROBLEM 4 (25 POINTS) *In the Hopping Game, there is a sequence of n spaces. You begin at space 0 and at each step, you can hop 1,2,3, or 4 spaces forward. However, some of the spaces have obstacles and if you land on an obstacle, you lose.*

Give a greedy algorithm which, given an array $A[1, \ldots, n-1]$ of Boolean values with $A[i]$ indicating the presence/absence of obstacle at position $i \in [1, n-1]$, find the minimum number of hops needed to reach space n without losing, if it is possible to do so. (We assume spaces 0 and n are obstacle-free, and are not part of the input.) Prove that your algorithm is correct. For full credit, your algorithm should run in time $O(n)$.

Extra Space for your solution