

Morphological operations

Anders Kaestner :: Laboratory for Neutron Scattering and Imaging



1 Distances**2 Skeletons****3 Geodesic operations****4 Labeling and morphological segmentation**

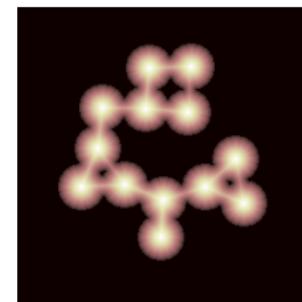
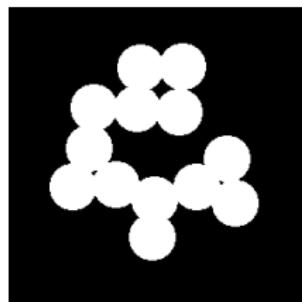
Distances

Definition

A *distance map* shows the shortest distance from an object pixel to the object edge.

- A distance map is computed using a distance transform.
- The result of the transform depends on the choice of distance metric

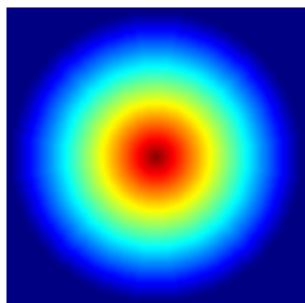
Example



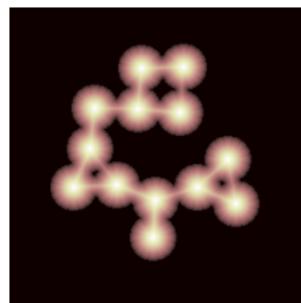
The Euclidean distance is defined as

$$d_{\mathcal{E}}[(x_1, y_1), (x_2, y_2)] = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad (1)$$

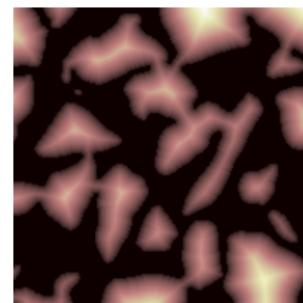
Example



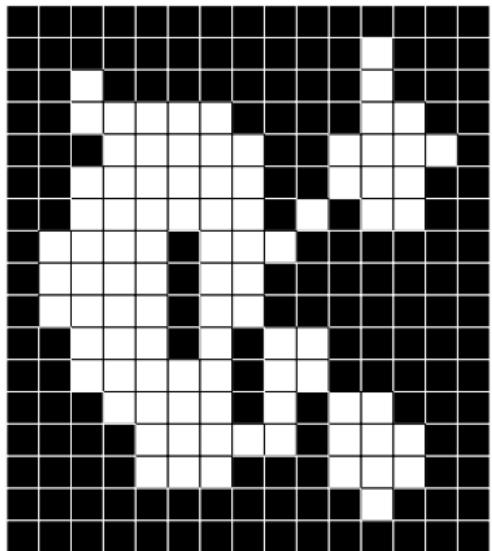
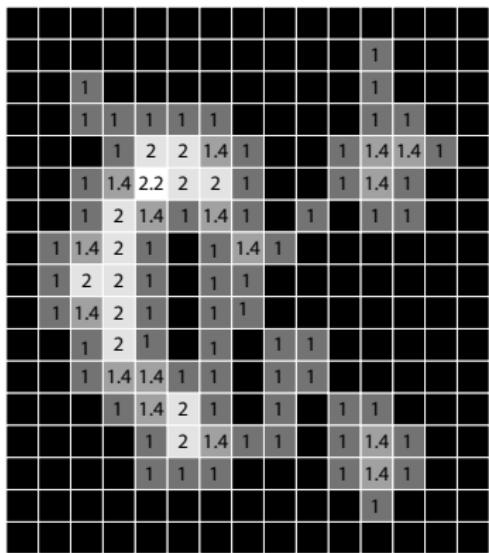
Disk



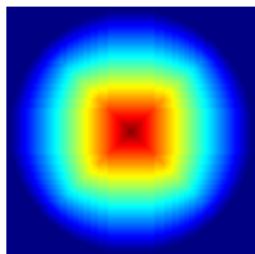
Circles



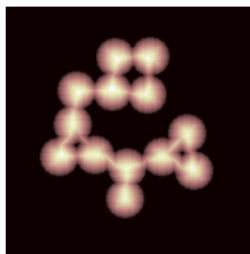
Sand

Bilevel image**Euclidean distance map**

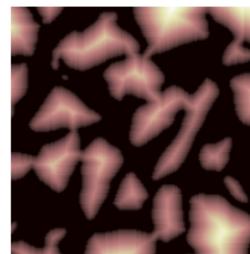
City-block metric $d_4[(x_1, y_1), (x_2, y_2)] = |x_2 - x_1| + |y_2 - y_1|$



Disk

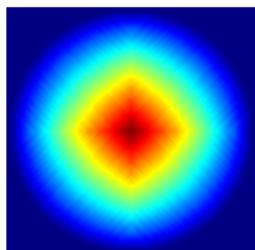


Circles

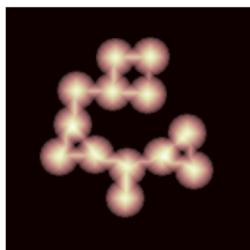


Sand

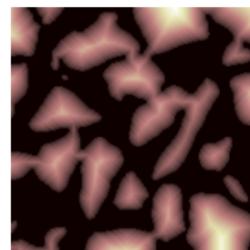
Chessboard metric $d_8[(x_1, y_1), (x_2, y_2)] = \max\{|x_2 - x_1|, |y_2 - y_1|\}$



Disk



Circles

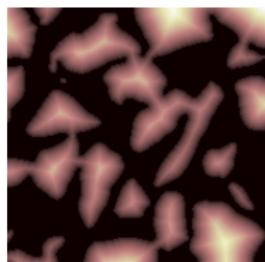


Sand

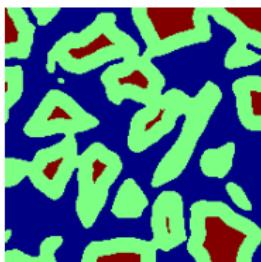
Procedure

- Compute the distance map $d = D(f)$
- $dR = d > R$
- Label $dR \Rightarrow ldr$
- $\#region > R = \max ldr$

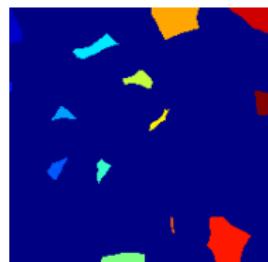
Example



$Dist > R$
 \Rightarrow

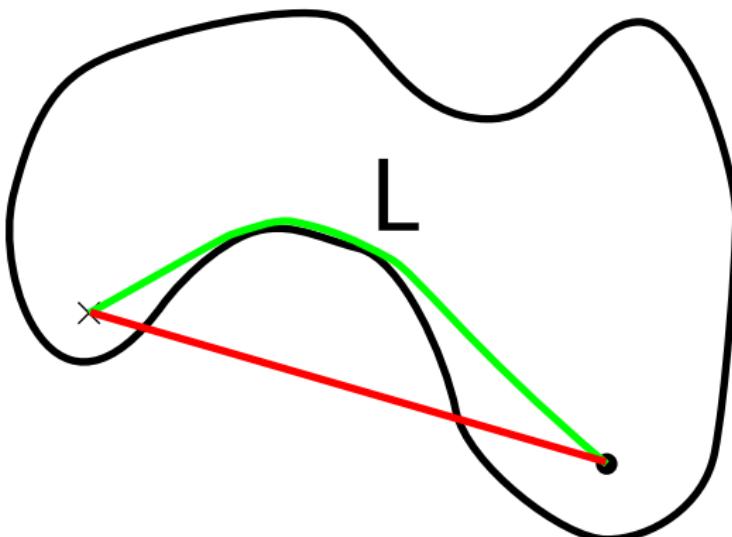


$Label$
 \Rightarrow



Definition

The shortest distance, L , within the mask from a marker to any connected pixel in the mask.



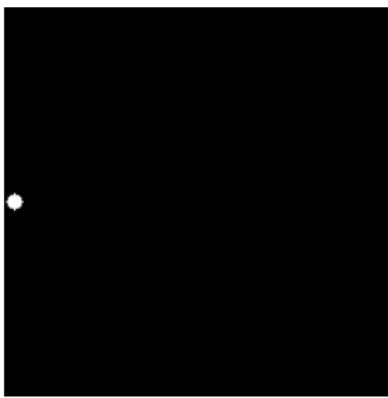
The geodesic distance is constrained by the shape of the object

Example

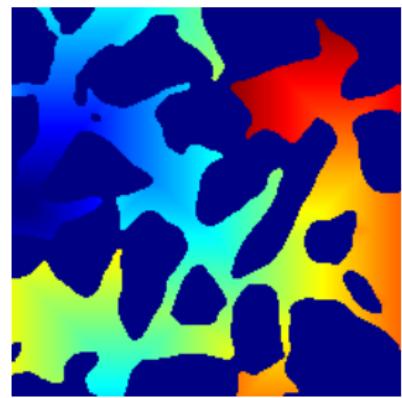
The geodesic distance can be used to find the greatest pore distance.



Porespace mask



Origin of measurement



8 connected geodesic distance

Skeletons

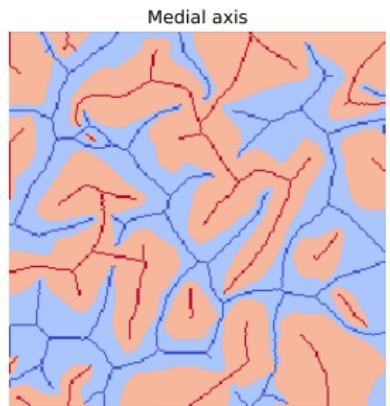
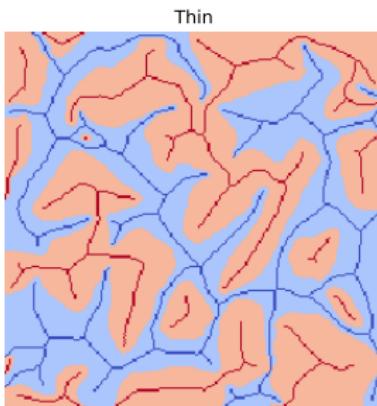
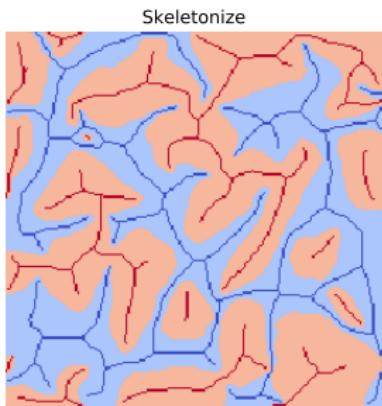
Definition

A skeleton represents the medial axis of the objects in the image. It shall essentially span the convex hull of the object.

The shape of the skeleton depends on the method to extract it

- Thinning until stability
- Extracting the ridge of a distance transform
- many others, see Soille (2002) for more

Testing three algorithms provided by SciKit Image



Geodesic operations

Morphologic operations that are constrained only to act in regions supported by a mask image

Example

Typical operations:

- Reconstruction
- Regional extrema
- Distance maps

Connectivity operations on image f using mask image g

- Reconstruction by Dilation.

$$[R_g^\delta(f)](x) = \min \left\{ g(x), \underbrace{\max \{f(y) | y \in NG(x)\}}_{\text{Dilation of } f} \right\}$$

- Reconstruction by Erosion.

$$[R_g^\varepsilon(f)](x) = \max \left\{ g(x), \underbrace{\min \{f(y) | y \in NG(x)\}}_{\text{Erosion of } f} \right\}$$

- Self-dual Reconstruction

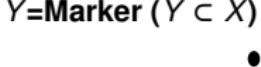
$$[R_g^\nu(f)](x) = \begin{cases} [R_g^\delta(f)](x), & f(x) \leq g(x) \\ R_g^\varepsilon(f)(x), & \text{otherwise} \end{cases}$$

$R_X^\delta Y$ is a connectivity algorithm for B/W images – seeded region growing

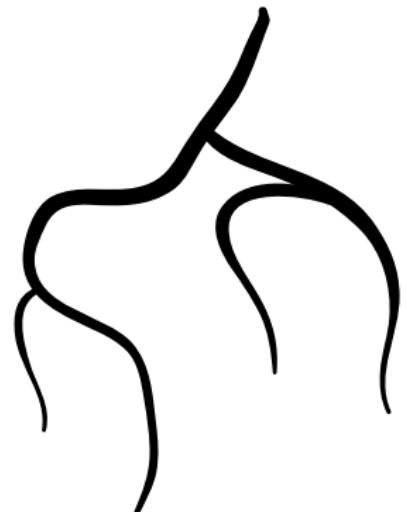
Roots and speckle



$Y = \text{Marker } (Y \subset X)$

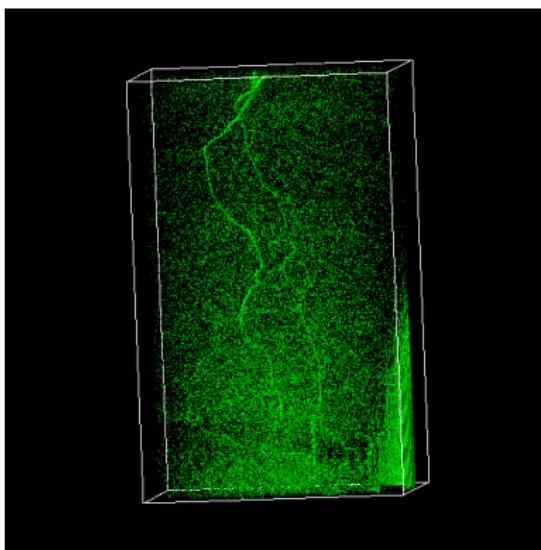


Reconstructed roots

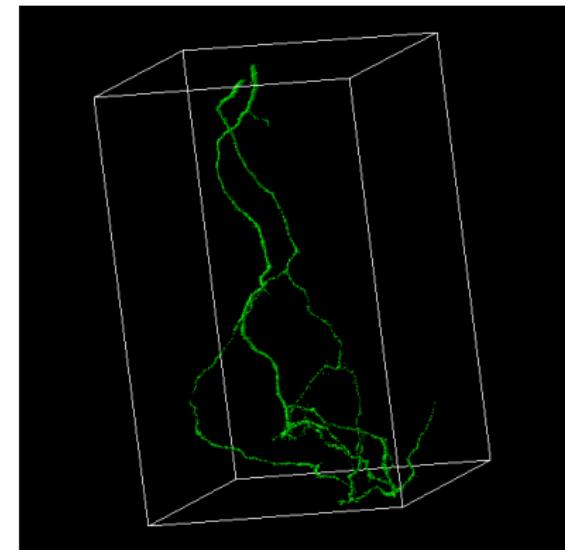


The operator also works in 3D – Tomography of roots in sand

Thresholded image



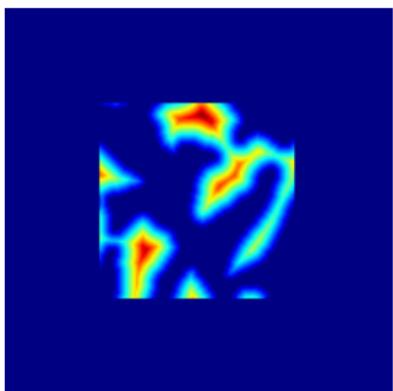
Reconstructed roots



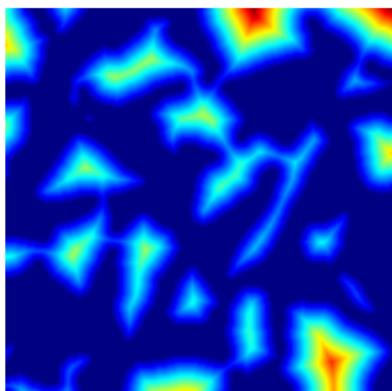
Root network reconstructed using a single voxel per root system

Kaestner et al. (2006)

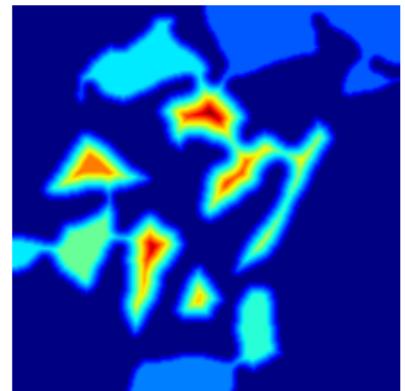
Marker (Y)



Mask (X)



Result



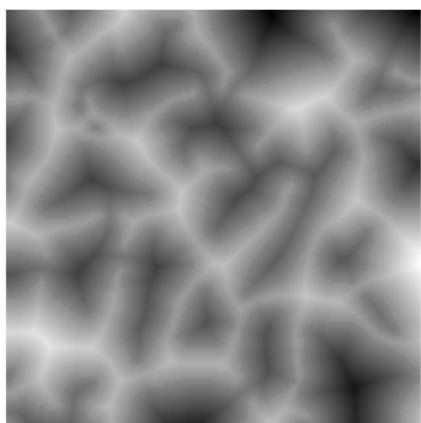
Regional extrema can be found with Reconstruction operators

- Regional min, $RMIN(f) = R_f^\varepsilon(f+1) - f$
- Regional max, $RMAX(f) = f - R_f^\delta(f-1)$
- H min, $HMIN_h(f) = R_f^\varepsilon(f+h)$
- H max $HMAX_h(f) = R_f^\delta(f-h)$
- Extended min $EMIN_h(f) = RMIN(HMIN_h(f))$
- Extended max $EMAX_h(f) = RMAX(HMAX_h(f))$

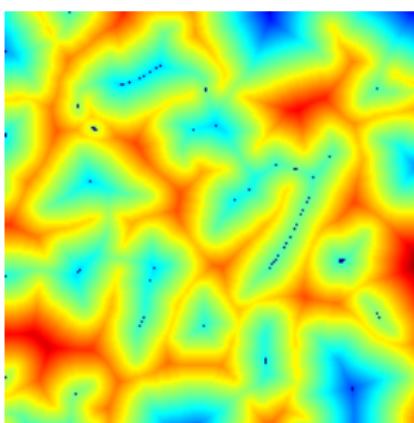
Regional min $RMIN(f) = R_f^\varepsilon(f+1) - f$

Regional max $RMAX(f) = f - R_f^\delta(f-1)$

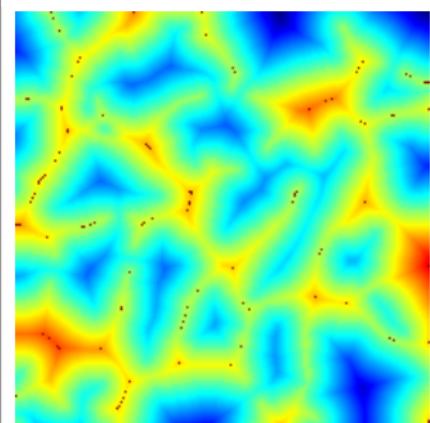
Input



RMIN(f)



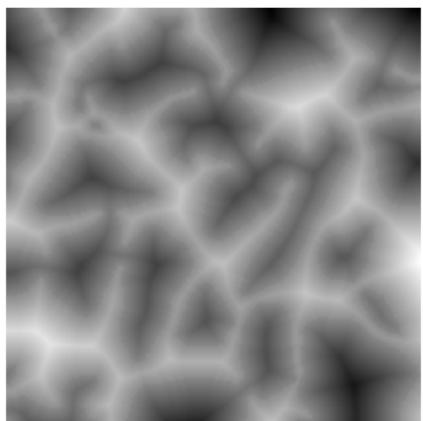
RMAX(f)



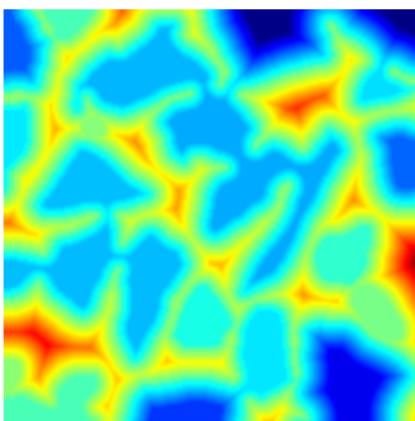
H min $HMIN_h(f) = R_f^\varepsilon(f + h)$

H max $HMAX_h(f) = R_f^\delta(f - h)$

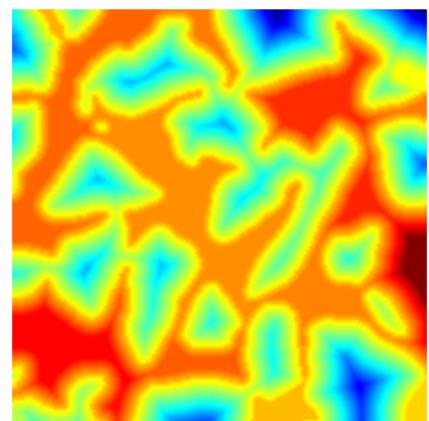
Input



$HMIN_{10}(f)$



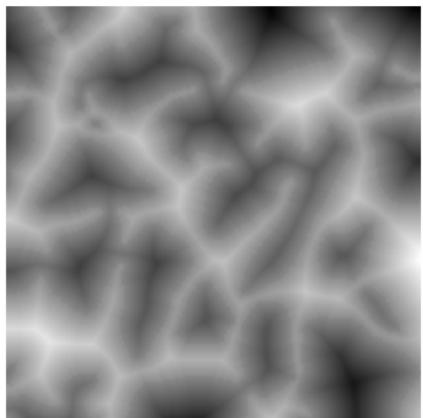
$HMAX_{10}(f)$



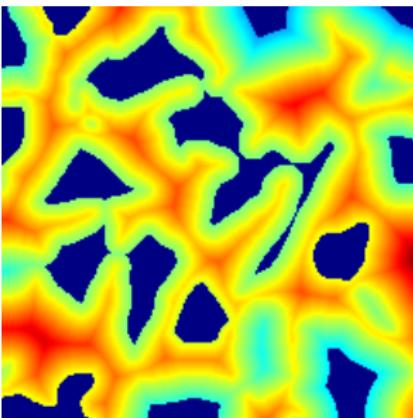
Extended min $EMIN_h(f) = RMIN(HMIN_h(f))$

Extended max $EMAX_h(f) = RMAX(HMAX_h(f))$

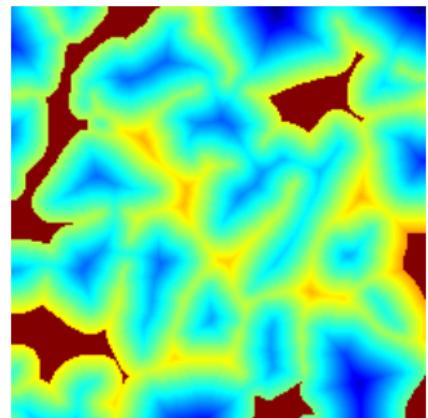
Input



$EMIN_{10}(f)$



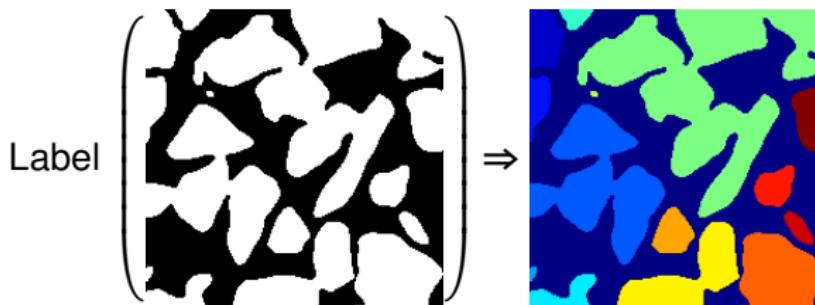
$EMAX_{10}(f)$



Labeling and Morphological segmentation

Definition

Labeling an image corresponds to assigning a label (value) to each disconnected region in the image.



A region growing operation, i.e. the connectivity has an impact on the outcome.

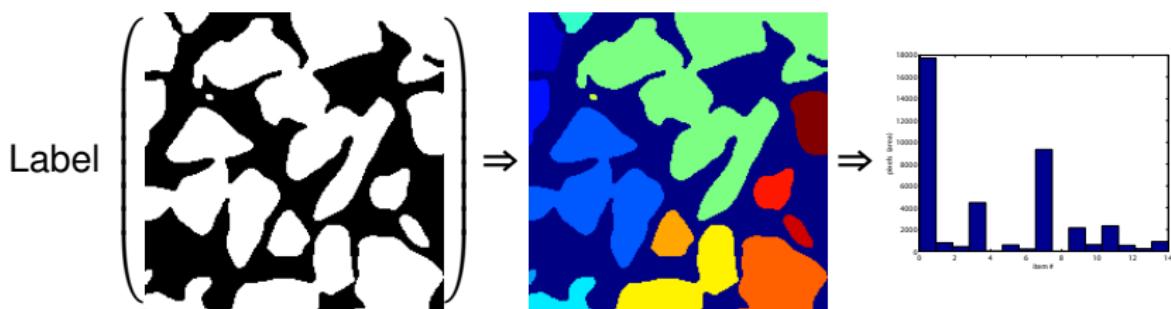
Applications

- Item counting
- Area measurement
- Item extraction, remove small items

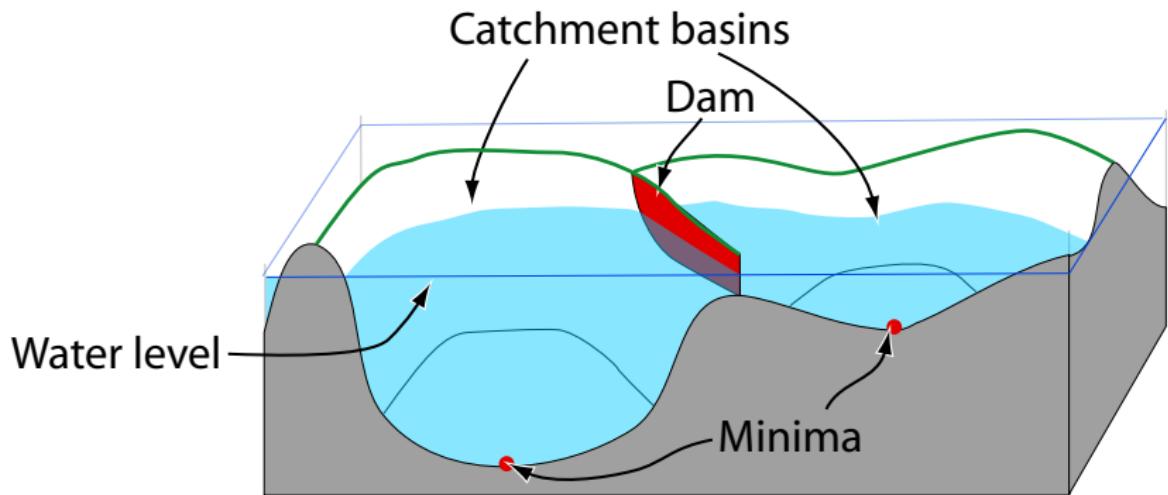
Procedure

- 1 Label the regions in the image
- 2 Compute the histogram of the labeled image
- 3 Each bin in the histogram corresponds to the region areas

Example

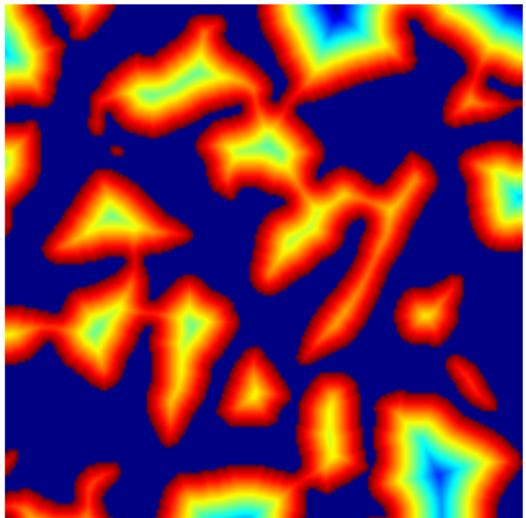


- The purpose of morphological segmentation is to identify objects
- The labeling has the drawback that it cannot identify overlapping objects.
- Watershed segmentation can do this...

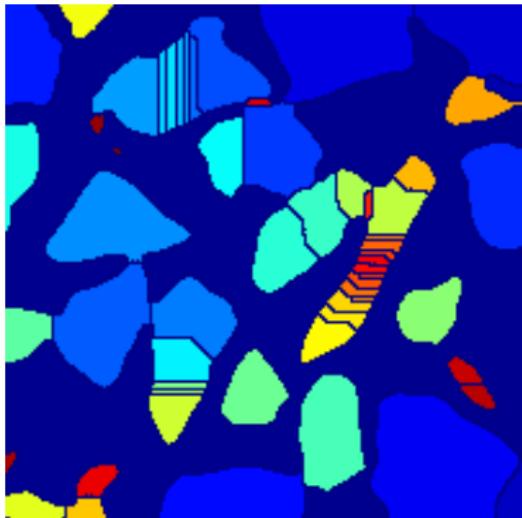


A review of algorithms can be found in Roerdink and Meijster (2001)

Releasing the WS-algorithm on an elevation map...



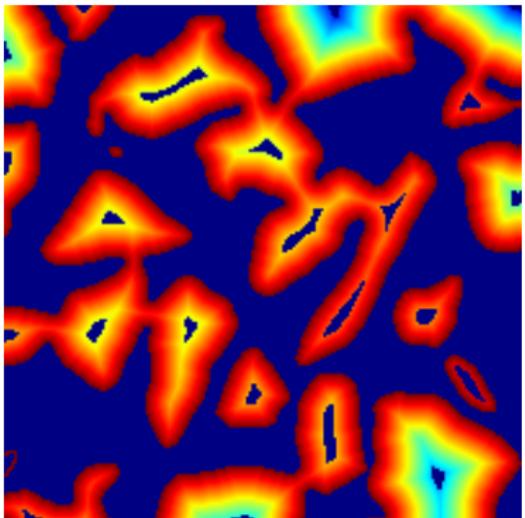
D='Elevation map'



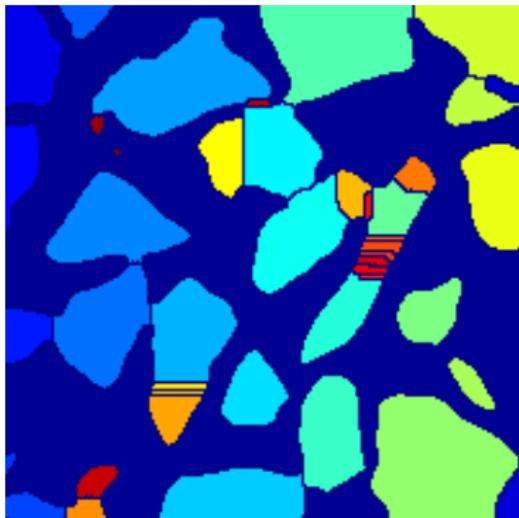
Segmented sand

... mostly result in a large over-segmentation

It is possible to reduce the amount of over-segmentation by imposing starting points



D='Elevation map'



Segmented sand

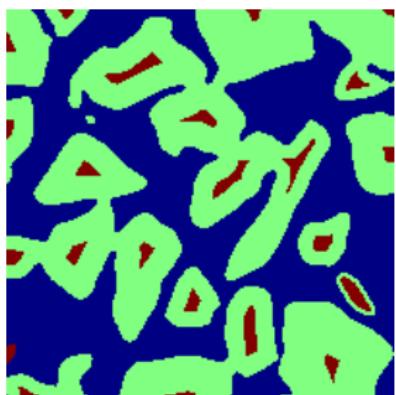
Here was $EMIN_2(D)$ used.

Minima can be imposed using

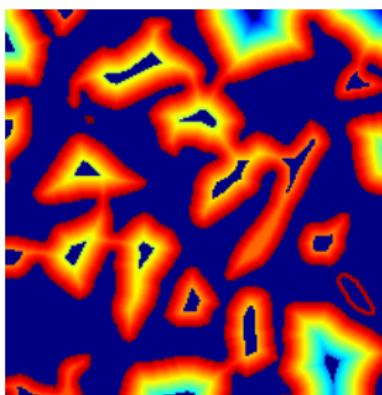
$$R_{(f+1) \wedge f_m}^{\varepsilon}(f_m)$$

with

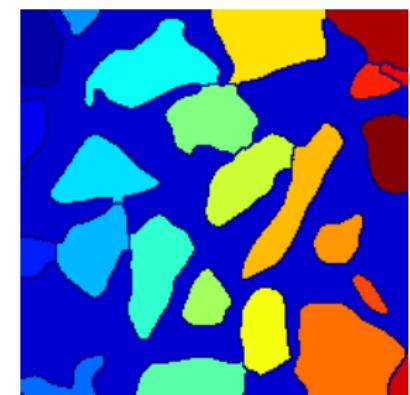
$$f_m(p) = \begin{cases} t_{min} & \text{if } p \text{ belongs to a marker} \\ t_{max} & \text{otherwise} \end{cases}$$



Markers (red)



$R_{(f+1) \wedge f_m}^{\varepsilon}(f_m)$



Segmented

We have talked about:

- Distance maps
- Skeletons
- Geodesic operations
 - Reconstruction
 - Local extrema
 - Geodesic distances
- Labeling and Watershed segmentation

Bibliography

- Kaestner, A., Schneebeli, M., and Graf, F. (2006). Visualizing three-dimensional root networks using computed tomography. *Geoderma*, 136(1–2):459–469.
- Roerdink, J. and Meijster, A. (2001). The watershed transform: Definitions, algorithms and parallelization strategies. *Fundamenta Informaticae*, 41:187–228.
- Soille, P. (2002). *Morphological image analysis*. Springer Verlag, 2nd edition.