Morphological operations

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Outline – Morphological operations

What is morphological processing?

Introduction

What is Morphological Image Analysis?

- A branch of image analysis based on set algebra
- Originally introduced by Serra (1990)
- Uses structures and neighborhoods

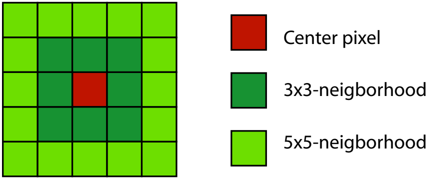
Notation in this lecture is according to Soille (2002)

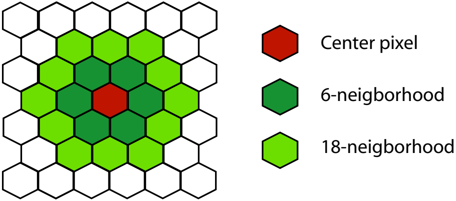
Neighborhoods and connectivity

Common grids in morphological image analysis are

- Rectangular grid → Straight-forward implementation
  - Four connectivity
  - Eight connectivity
  - Can be used for 3D data
- Hexagonal grid → No neighborhood ambiguity
  - Always six connectivity

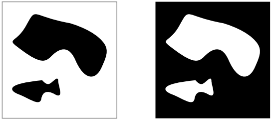
Neighborhoods



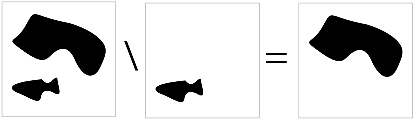


Some useful set operations

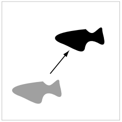
Complement $f^c(\mathbf{x}) = t_{max} - f(\mathbf{x})$



Set difference $X \setminus Y = X \cap Y^c$



Translation $f_{\mathbf{b}}(\mathbf{x}) = f(\mathbf{x} - \mathbf{b})$

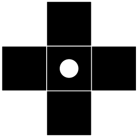


Reflection $\check{B} = \{-\mathbf{b} | \mathbf{b} \in B\}$

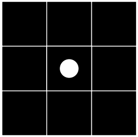
Structure elements

Structure elements are masks for the morphological operator.

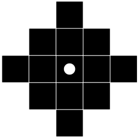
Discrete disc



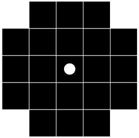
$R = 1$



$R = \sqrt{2}$



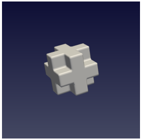
$R = 2$



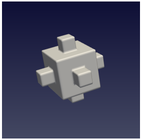
$R = \sqrt{5}$

Discrete sphere

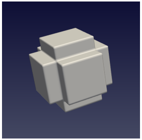
$R = 1$



$R = 1.5$



$R = 2$



$R = 2.5$

Bi-level images

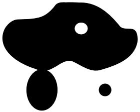
Morphology on black and white images

The basic operators of Black and White morphology act as search and replace operation for at given pixel configuration.

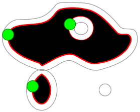
Erosion

Erosion is a region shrinking operation for a given structure element *SE*.

$$\varepsilon_{SE}(f) = \{\mathbf{x} | SE_{\mathbf{x}} \subseteq f\} \quad \Rightarrow \quad \varepsilon_{SE}$$



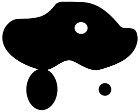
$=$



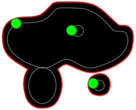
Dilation

Dilation is a region growing operation for a given structure element *SE*.

$$\delta_{SE}(f) = \{\mathbf{x} | SE_{\mathbf{x}} \cap f \neq \varnothing\} \quad \Rightarrow \quad \varepsilon_{SE}$$

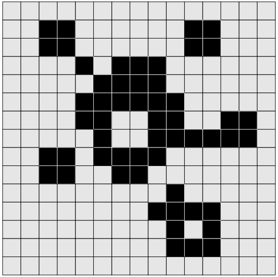


$=$



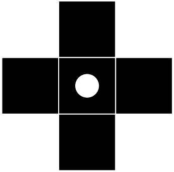
Erosion :: Detailed example

Detailed example

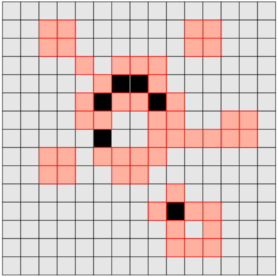


Eroding

SE=

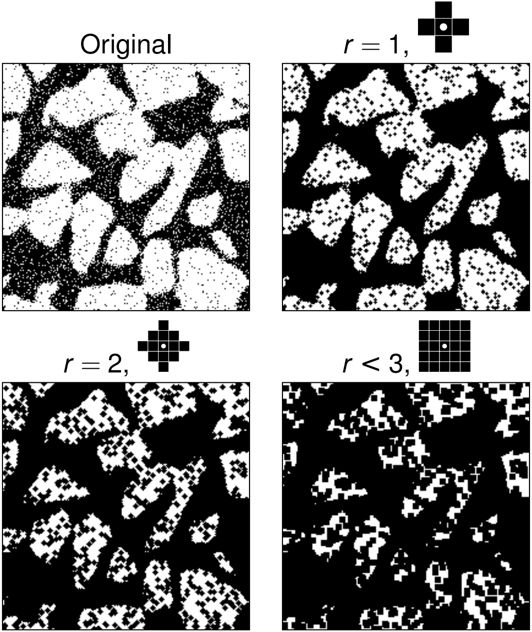


using



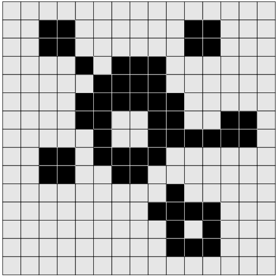
gives

Erosion :: Examples with increasing SE size

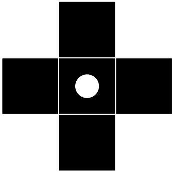


Dilation

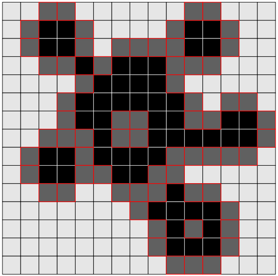
Detailed example



Dilating

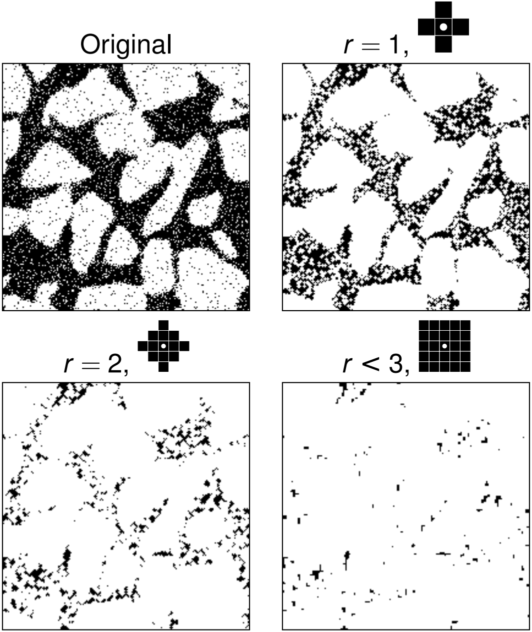
SE= 

using



gives

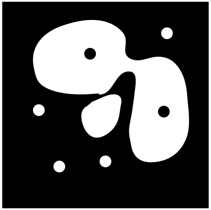
Dilation :: Examples with increasing SE size



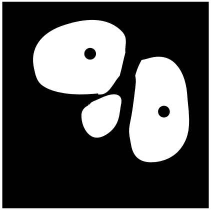
Combining Erosion and Dilation

Undesired image features be removed by combining erosion and dilation.

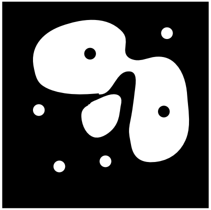
Opening $\delta(\varepsilon(f))$



$\Rightarrow$



Closing $\varepsilon(\delta(f))$



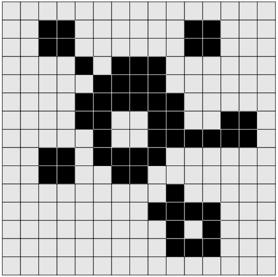
$\Rightarrow$

Open

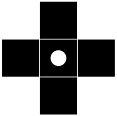
The Open operator γ_B is defined as

$$\gamma_B(f) = \delta_{\breve{B}}\left[\varepsilon_B\right] \quad (1)$$

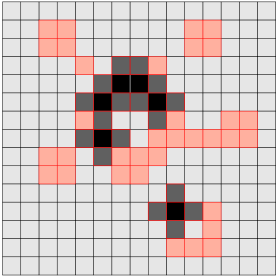
Opening example



Opening

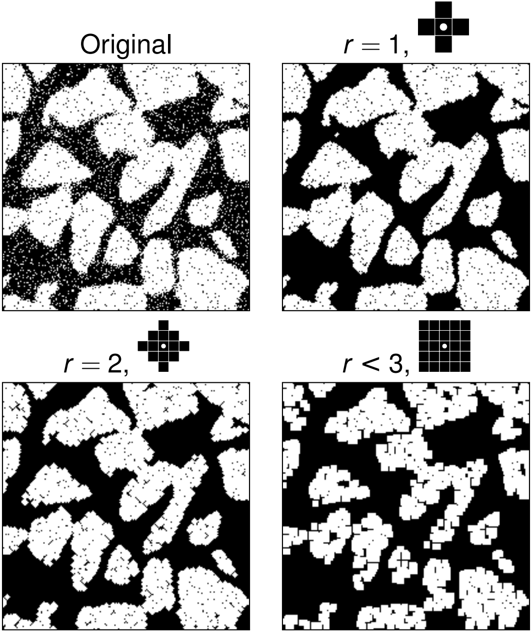
SE= 

using



gives

Open :: Examples with increasing SE size

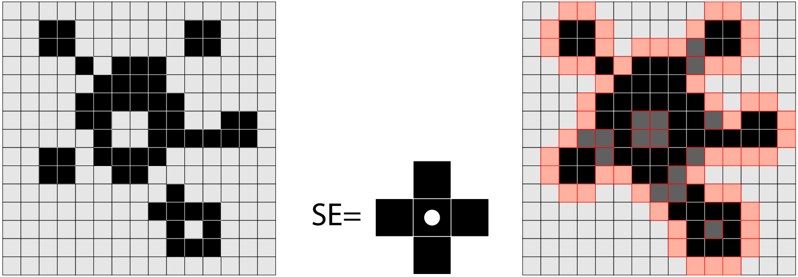


Close

The close operator ϕ_B is defined as

$$\phi_B(f) = \varepsilon_{\check{B}}[\delta_B] \tag{2}$$

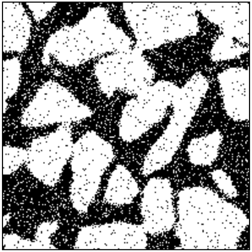
Closing example

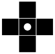


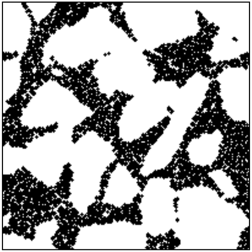
Closing using gives

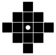
Close :: Examples with increasing SE size

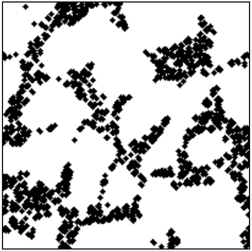
Original

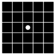


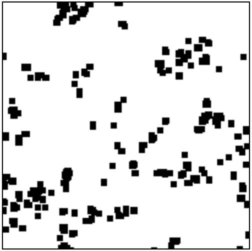
$r = 1$, 



$r = 2$, 



$r < 3$, 



Gray scale morphology

Gray-scale morphology

A generalization of binary morphology is *gray-scale morphology*
To handle the increased number of intensity levels the basic operations

- Erosion
- Dilation

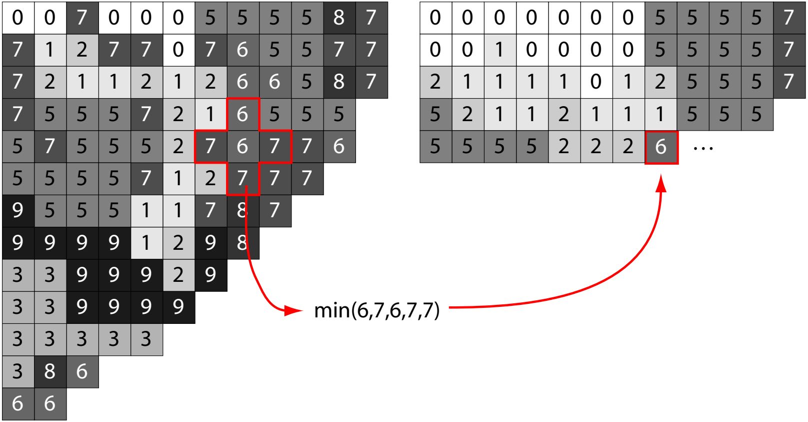
must be redefined...
...while others only act differently based on ε and δ

Gray-scale erosion

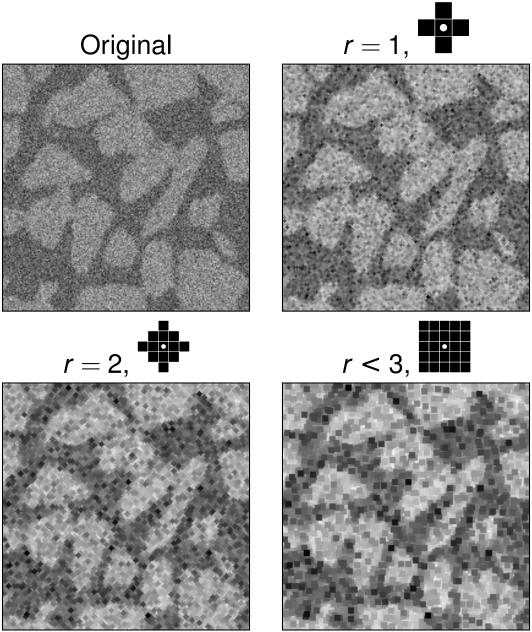
Erosion is defined as

$$[\varepsilon_{SE}(f)](\mathbf{x}) = \min_{\mathbf{b}\in SE} f(\mathbf{x}+\mathbf{b}) \tag{3}$$

Example



Gray-scale erosion increasing SE



Gray-scale dilation

Dilation is defined as

$$[\delta_{SE}(f)](\mathbf{x}) = \max_{\mathbf{b}\in SE} f(\mathbf{x}+\mathbf{b}) \qquad (4)$$

Example

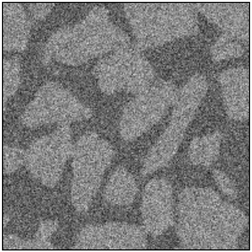
0	0	7	0	0	0	5	5	5	5	8	7
7	1	2	7	7	0	7	6	5	5	7	7
7	2	1	1	2	1	2	6	6	5	8	7
7	5	5	5	7	2	1	6	5	5	5	
5	7	5	5	5	2	7	6	7	7	6	
5	5	5	5	7	1	2	7	7	7		
9	5	5	5	1	1	7	8	7			
9	9	9	9	1	2	9	8				
3	3	9	9	9	2	9					
3	3	9	9	9	9						
3	3	3	3	3							
3	8	6									
6	6										

max(6,7,6,7,7)

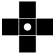
7	7	7	7	7	5	7	6	5	8	8	8
7	7	7	7	7	7	7	7	6	7	8	7
7	7	5	7	7	2	7	6	6	8	8	8
7	7	5	7	7	7	7	6	7	7	8	
7	7	7	5	7	7	7	7	…			

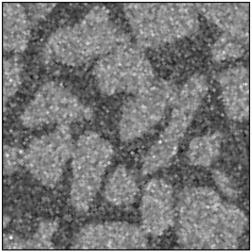
Gray-scale Dilation increasing SE

Original

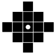


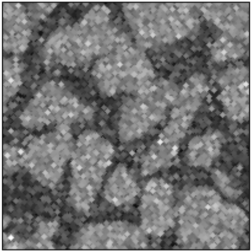
$r = 1$,



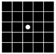


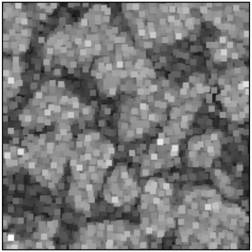
$r = 2$,





$r < 3$,





The rank filter

The erode, dilate, median filters can all be described as *rank*-filters:

Definition

The rank filter sort all pixels in a neighborhood and selects one with index i.

Example

Data: $\{2, 5, 7, 2, 4, 1, 8\}$

Sorted value	Rank
1	0 (Min, Erosion)
2	1
2	2
4	3 (Median)
5	4
7	5
8	6 (Max, Dilation)

Top-hats

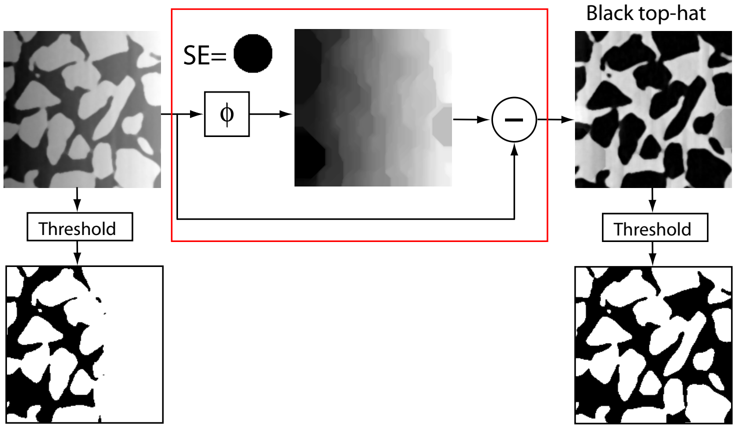
Open and close operations work as well with gray-scale images.
An application of gray-scale open/close is the top-hats defined as

- White top-hat
$$WTH = f - \gamma(f) \tag{5}$$
- Black top-hat
$$BTH = \phi(f) - f \tag{6}$$

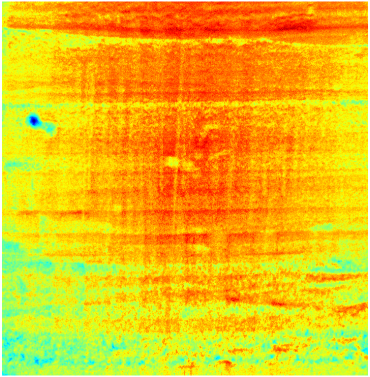
The BTH is related to WTH as $BTH = WTH\complement$

Inhomogeneous illumination correction

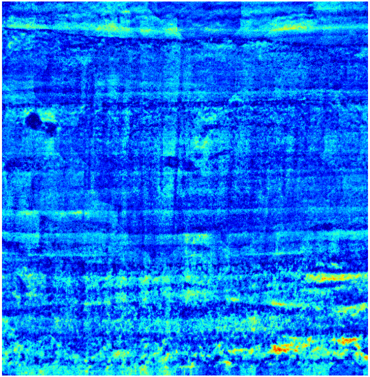
Example with black tophat



Top-hats :: Example - Snow profile



Original



White top-hat (disk r=30)

Image provided by M. Matzl, SLF, Davos

Summary

In this part we talked about:

- Neighborhoods and Connectivity
- Structure elements
- Erosion and Dilation as primitive operations of morphology
- Open and Closing as combinations of ε and δ
- Top-hats
- Bi-level and Gray-scale images

References I

Serra, J. (1990). *Image Analysis and Mathematical Morphology*. Academic press.

Soille, P. (2002). *Morphological image analysis*. Springer Verlag, 2nd edition.