Basics of images

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Outline :: Images

Defining images

What is an image?

Full Definition of image

1 a reproduction or imitation of the form of a person or thing

2 **a** the optical counterpart of an object produced by an optical device (as a lens or mirror) or an electronic device
b a visual representation of something: as (1) : a likeness of an object produced on a photographic material (2) : a picture produced on an electronic display (as a television or computer screen)

3 exact likeness : semblance <God created man in his own image — Genesis 1:27(Revised Standard Version)>

4 a tangible or visible representation : incarnation

5 (1) a mental picture or impression of something (2) : a mental conception held in common by members of a group and symbolic of a basic attitude and orientation

6 a vivid or graphic representation or description

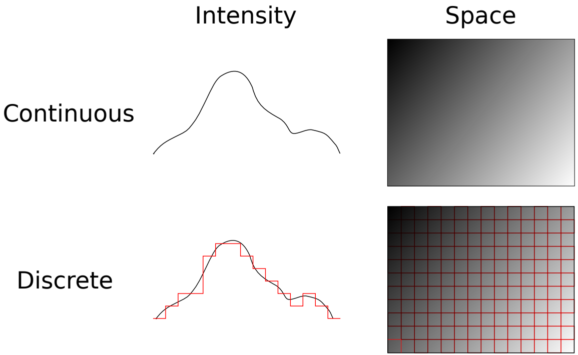
7 a popular conception (as of a person, institution, or nation) projected especially through the mass media

8 a set of values given by a mathematical function (as a homomorphism) that corresponds to a particular subset of the domain

Digital images

Digital images are characterized by

- Intensity information on a discrete grid
- Intensity is represented by numbers with different precision
- Finite geometric extents
- Can be stored and processed by computers



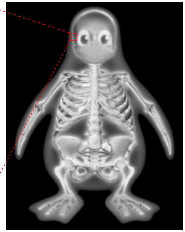
The smallest elements of a digital image

Definition

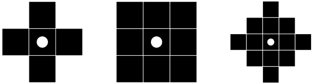
A pixel is the smallest element of an image. It has

- intensity and color
- a position in the image
- in 3D it is called voxel

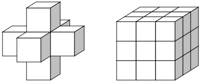
83	104	124	120	94	67	79	83	92	100
84	104	124	123	98	69	78	84	92	100
85	104	124	126	103	72	77	84	91	102
86	102	123	129	107	76	78	84	92	109
86	101	122	130	112	81	80	86	96	118
86	102	121	132	116	84	82	88	99	125
84	101	124	134	123	91	88	87	103	132
84	100	123	133	124	94	90	90	106	136
84	99	122	133	128	100	93	93	109	140
83	98	120	132	133	105	95	95	110	142



2D – Pixel neighborhoods



3D – Voxels neighborhoods



Pixel size and resolution

Pixel size

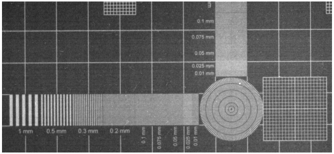
- Step size of sampling grid
- Elements stored

Resolution

- Effect of optical transfer function
- Should be greater than pixel size
- Defines the smallest pixel size

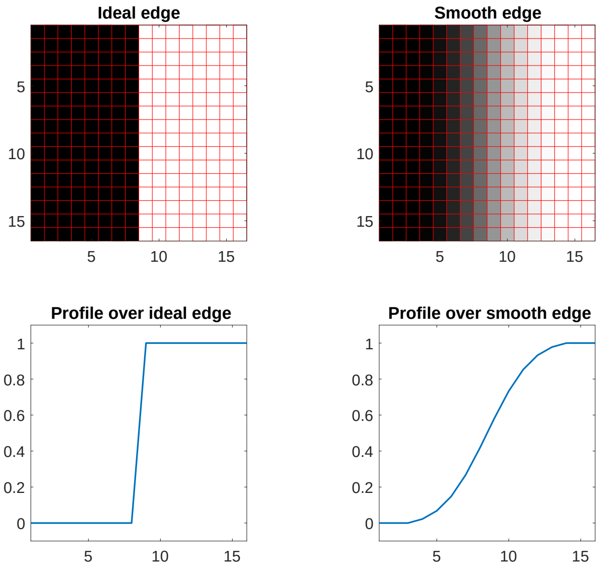
Pixel sampling

Sampling according to Nyquist: The pixel size must be less than half the highest spatial frequency in the image.



Pixel size: 46 μm

Edges at different resolutions and pixel sizes



Different types of images

2D

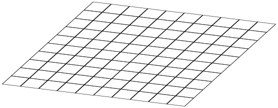
- Pictures
- Radiographs
- CT slices

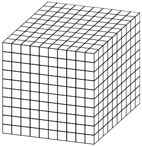
3D

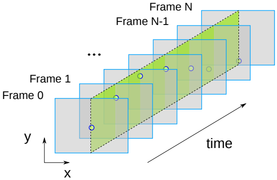
- Volumes
  x, y, z
- Movies
  x, y, t

4D

- Volume movie
  x, y, z, t







Intensities and Data types

Intensity dynamics

- Digital images can only represent a limited number of intensities
- Contrast
- Saturation
- Starvation

Integers

Bi-level	1 bit - 2 levels
Byte	8 bits – 256 levels
Short	16 bits – 65536 levels
Int	32 bits – $4.3 \cdot 10^9$ levels
Long	64 bits – $1.8 \cdot 10^{19}$ levels

Floating point

single	32 bits
double	64 bits

How many bits are needed?

- Contrast difference
- Separate many different sample features
- Sensitivity to rounding errors

Few bits

- High contrast
- Clean images
- Segmented data

Many bits

- Low contrast
- Noisy images
- Gradual changes

Floating point

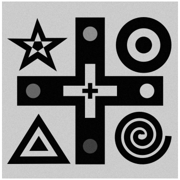
- High intensity dynamics
- Quantification to physical properties
- In algorithms

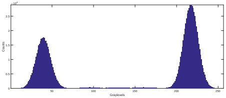
The histogram

Definition

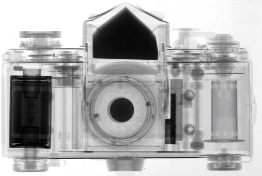
A histogram is a function that shows the distribution of the gray levels in an image

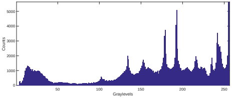
Simple image



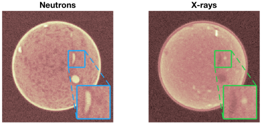


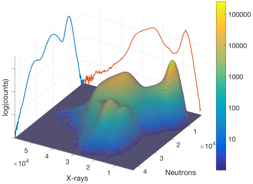
Complex image





Bivariate images



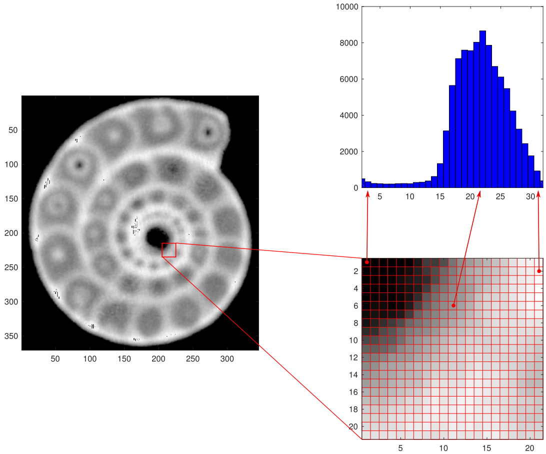


How to measure a histogram

A histogram H for image f is computed as

$$H[idx(f(\mathbf{x}))] = H[idx(f(\mathbf{x}))] + 1, \quad \forall \mathbf{x} \in \Omega \tag{1}$$

$$idx(y) = \left\lfloor N_{bins} \frac{y - y_{Low}}{y_{High} - y_{Low}} \right\rfloor \tag{2}$$



Visualizing gray levels

Human perception

- The human eye can't handle more than 8–30 shades of gray
- It is easier to see steep than slow changes

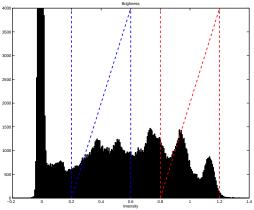
Reduce dynamics

Contrast Slope of the transfer function from intensity to displayed gray levels.

Brightness Intensity position of the displayed contrast interval.

Effects of brightness and contrast

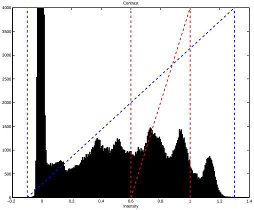
Brightness



Low

High

Contrast



Narrow

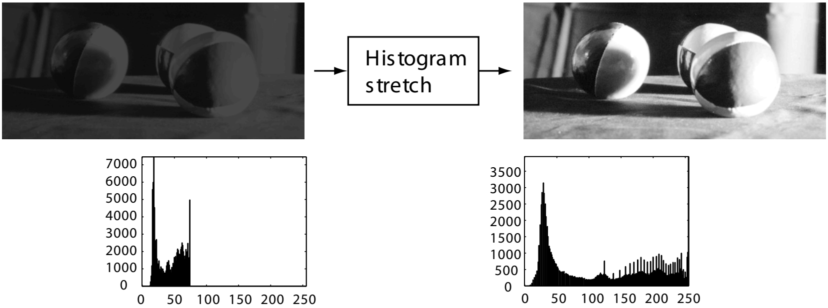
Wide

Histogram stretching

For images with small dynamics can the dynamics be stretched using

$$g(\mathbf{x}) = K \frac{f(\mathbf{x}) - \min(f)}{\max(f) - \min(f)}$$

i.e a pixelwise operation



Note: This is a visibility enhancing operation that redistributes the intensities.

Pixel-wise operations

Pixel-wise operations operate on each pixel independently

- Using one or more images.
- Results in a new image with the same dimensions.

Arithmetic operations

- Addition/Subtraction
- Multiplication/Division

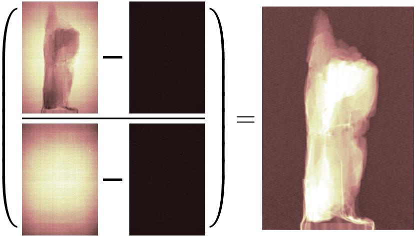
Functions

- Exponents/Roots
- Logarithms
- Trigonometric functions
- etc

Pixel-wise arithmetics: Beer-Lamberts law

From Beer-Lamberts law, $I = I_0 e^{-\int_L \mu(x)\,dx}$, we get

$$p = -\log\left(\frac{I - I_{DC}}{I_0 - I_{DC}}\right) = -\log$$



- p Processed projection, $\int_L \mu(x)\,dx$
- I Measured radiogram
- I_{DC} Dark current image (removes noise floor)
- I_{OB} Open beam image

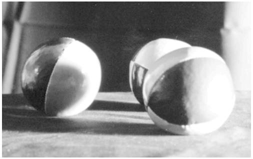
Example functions: Gamma correction

Gamma correction is a non-linear way to change the distribution of the gray levels

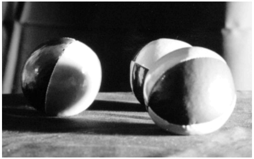
$$g = f^{\gamma}$$

where γ is a constant

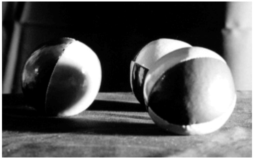
γ=0.5



γ=1



γ=1.5



Warning

This will change the gray value of the pixels non-linearly
→ use only for visualization unless it is physically motivated.

Image noise

What is noise?

General definition

Noise is unintended additional information that harms the quality of an image.

- Introduces uncertainty in the interpretation.
- Often has a statistical nature.
- Always present in experiment data.

Bev Doolittle, *Woodland encounter*

Noise sources and types

Noise sources

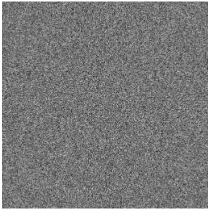
Counting noise Physical origin (Poison)

Thermal noise Electronics (Gaussian)

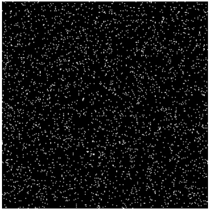
Conversion noise A/D conversion (Binomial)

Noise types

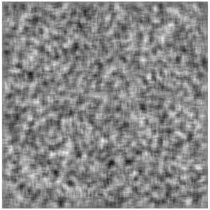
Gaussian



Salt ’n pepper



Structured



Noise models – Distributions

Gaussian noise

- Additive
- Easy to model
- Other distributions obtain Gaussian shape at large numbers

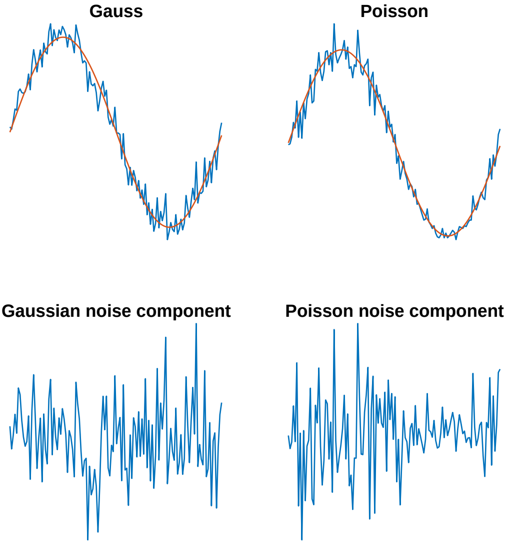
$$N(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\left(\frac{x-\mu}{2\sigma}\right)^2}$$

Poisson noise

- Multiplicative
- Physically correct for event counting

$$P_\lambda(k) = \frac{\lambda^k}{k!} e^{-\lambda}$$

Noise examples



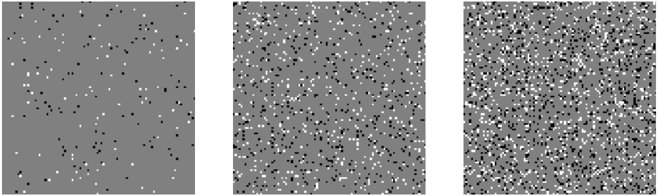
Noise models – Salt'n'Pepper noise

- A type of outlier noise
- Noise strength give as the probability of an outlier
- Additive, multiplicative, independent replacement

Example

$$sp(x) = \begin{cases} -1 & x \leq \lambda_1 \\ 0 & \lambda_1 < x \leq \lambda_2 \\ 1 & \lambda_2 < x \end{cases} \qquad \begin{array}{l} x \in \mathcal{U}(0,1) \\ \lambda_1 < \lambda_2 \\ \lambda_1 + \lambda_2 = \text{noise fraction} \end{array}$$

λ_1=1% and λ_2=99% | λ_1=5% and λ_2=95% | λ_1=10% and λ_2=90%



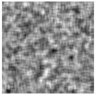
Noise models – Structured noise

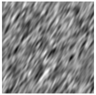
- Spatially correlated
- Example: Detector structure

Example random field models

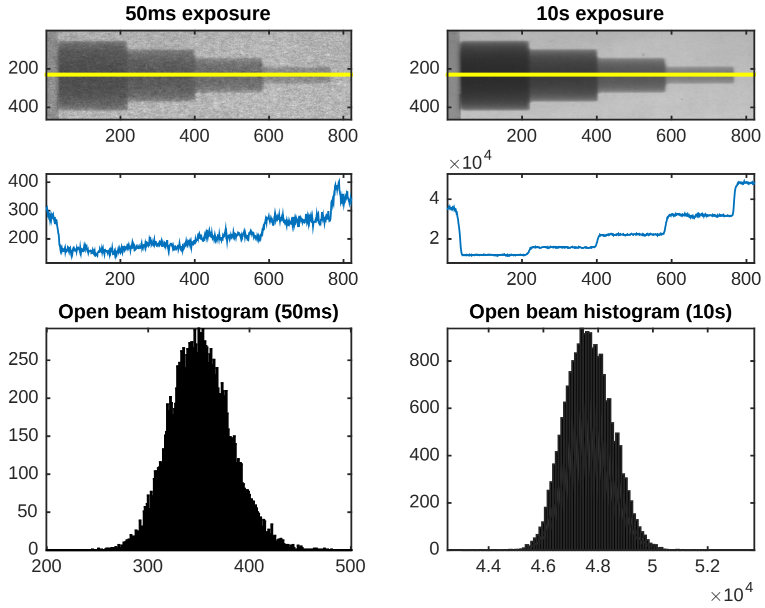
Averaging process is implemented using convolution (details in next lecture).

$$
\begin{aligned}
n(x,y) &\in \mathcal{N}(\mu,\sigma) \\
ns &= K * n \qquad K = \text{convolution kernel}
\end{aligned}
$$

u_{5x5} $*$  $=$ 

 $*$  $=$ 

Noisy profiles



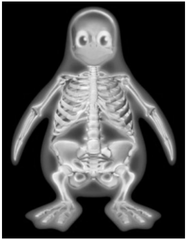
Signal to noise ratio

A metric to describe noise strength

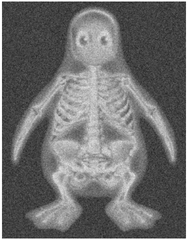
$$SNR = \frac{\mu_{image}}{\sigma_{image}} \tag{3}$$

$$SNR_{db} = 20 \log \frac{\mu_{image}}{\sigma_{image}} \tag{4}$$

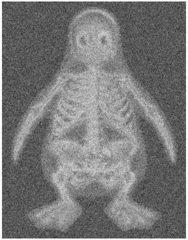
- Select a region
- Compute average intensity
- Compute std deviation
- Apply eqns 3 or 4



SNR = ∞



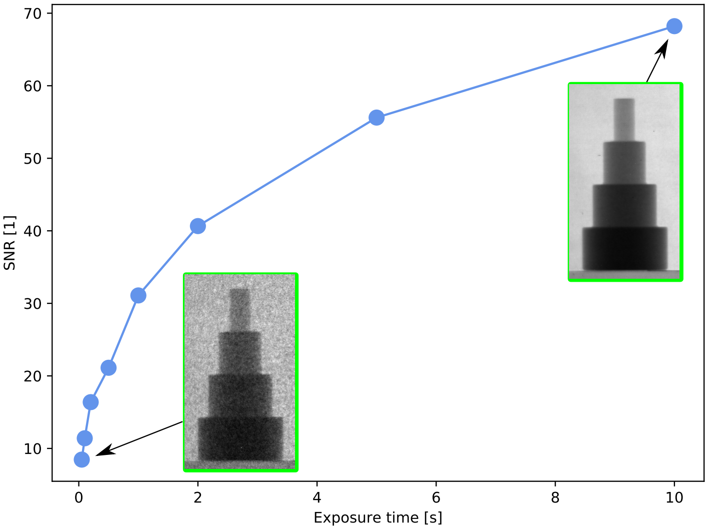
SNR = 5



SNR = 2

SNR = 1

SNR for different exposure times

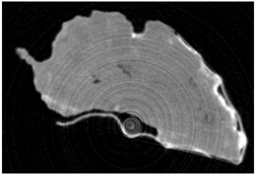


SNR for Poisson noise: $SNR = \frac{\mu}{\sigma} = \frac{\lambda}{\sqrt{\lambda}} = \sqrt{\lambda} \sim \sqrt{t}$

Artifacts from the acquisition

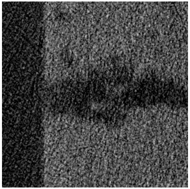
Rings

- Appear in most CT acquisitions
- Caused by stuck pixels in the projection data
- Can mostly be supressed during reconstruction



Lines

- Frequent in neutron CT slices
- Caused by spots on single projections
- Can mostly be supressed during reconstruction



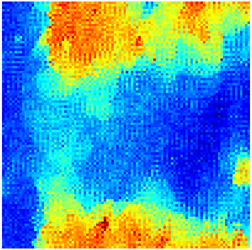
Numerical artifacts

Rounding errors

- May appear with sum operations on large data sets.
- At some point the new term is smaller than the precision.

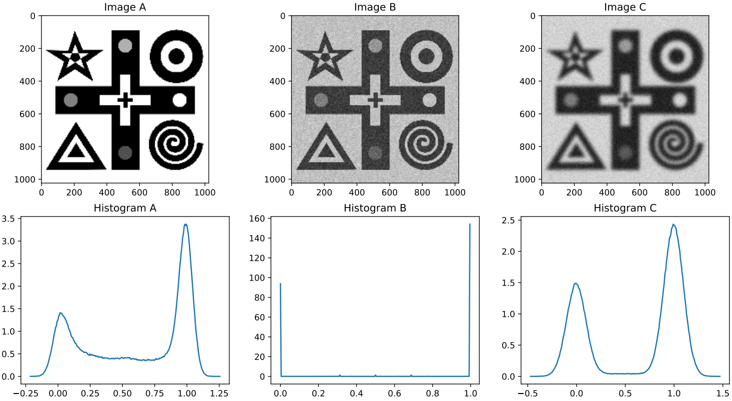
Instable processing

- Due to incorrect regularization
- Wrong parameterization
- Incorrect implementation. . . bugs *etc*



Quiz – Histograms and Noise

Which image is has which histogram? . . . why?



Affine transformations

Typical affine transformations

Motivation

Affine tranformations are needed when images are

- Acquired at different times
- Acquired using different equipment
- Distorted by acquisition system

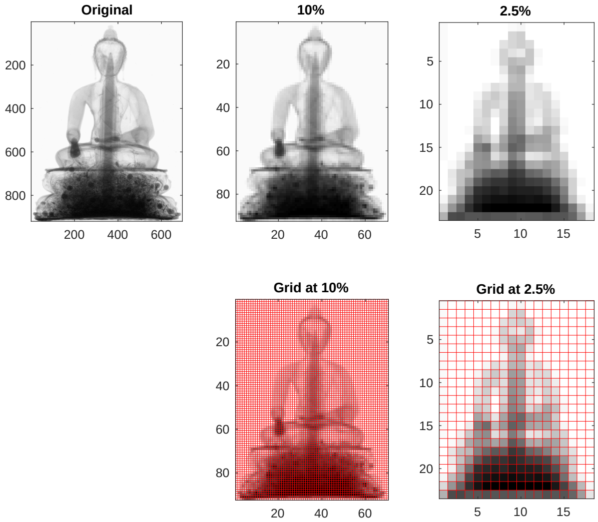
to align data sets

Concept

- Basic concept
- Transform operator

Scale

Scaling is resampling of the data on a grid with different pixel size



Translation

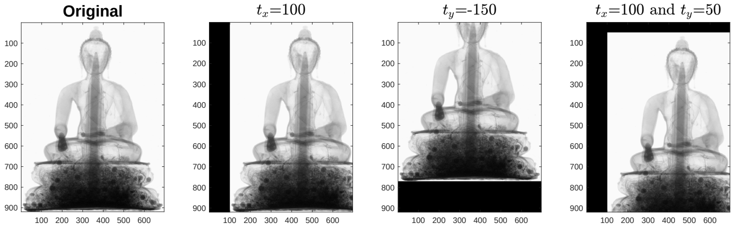
The translation operator

Each new pixel position is computed as

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \tag{5}$$

Note: No interpolation is needed if t_x and t_y are integers.

Translation examples



Rotation

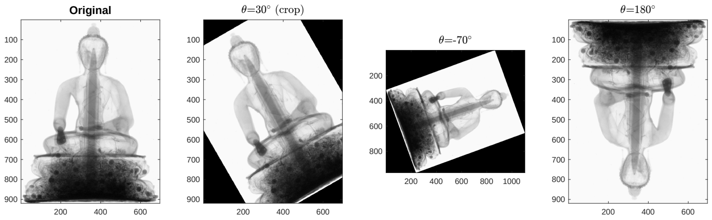
The rotation operator

Each new pixel position is computed as

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \tag{6}$$

Note: Interpolation is always needed.

Rotation examples



Skew

The skew operator

Each new pixel position is computed as

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & \tan\alpha & 0 \\ \tan\beta & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \tag{7}$$

Skew examples

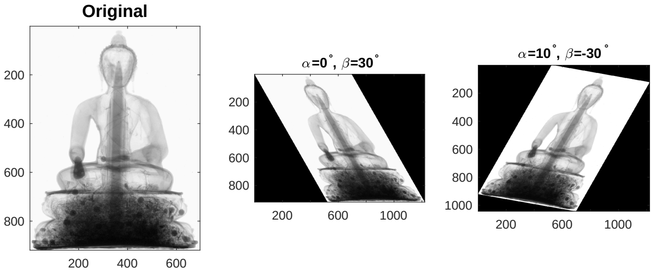


Image registration: Put it all together

An important application of affine transformation is image registration.

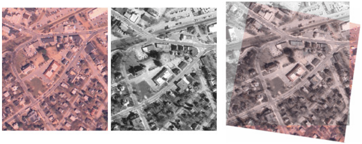
Definition

The process to align two or more images in space.
It requires

- An iterative process to minimize some metric (MSE, Correlation, etc.)
- Different affine transforms (translation, rotation, scale) are applied.
- The process benefits from a priori information

Applications

- Time series acquisitions with unstable samples.
- Image fusion of data from different modalities.



Goshtasby (2012)

Image storage

Some words about file formats

- Image data must be stored on some media
- Data type must serve the purpose of the next user
- Storage size
- Many file formats are available
- Save series of 2D files or as single 3D block

'Wellknown' image formats formats

JPEG

- Lossy compression
- 2D
- Color and Grey
- 8-bit

PNG

- ZIP compression
- 2D, multi-frame
- Color and grey (transparency)
- 1- to 16-bits

TIFF

- ZIP compression
- 2D, multi-frame
- Color and grey
- 1- to 32-bits
- Supports single prec. floating point

Meta data

These formats only have limited support for meta data
→ Must be saved separately.

'Scientific' formats

These formats are designed to support meta data and multi dimensional data

FITS

- Simple format
- Any data type
- Multidimensional

DICOM

- Standard for medical imaging.
- Patient meta data.

HDF4/5

- Very flexible
- Structure must be defined
- Maddison et al. (1997)
- De Carlo et al. (2014)

Summary

- Defining images (Russ (2016), chapters 1, 2)
- Noise (Russ (2016), chapter 1, Buzug (2008), section 2.6)
- Affine transformations
- File formats (Russ (2016), chapter 2)

References I

Buzug, T. (2008). *Introduction to Computed Tomography: From photon statistics to modern cone-beam CT*. Springer.

De Carlo, F., Gürsoy, D., Marone, F., Rivers, M., Parkinson, D. Y., Khan, F., Schwarz, N., Vine, D. J., Vogt, S., Gleber, S.-C., Narayanan, S., Newville, M., Lanzirotti, T., Sun, Y., Hong, Y. P., and Jacobsen, C. (2014). Scientific data exchange: a schema for HDF5-based storage of raw and analyzed data. *Journal of Synchrotron Radiation*, 21(6):1224–1230.

Goshtasby, A. (2012). *Image registration*. Springer London.

Maddison, D., Swofford, D., and Maddison, W. (1997). Nexus: An extensible file format for systematic information. *Systematic Biology*, 46(7):590–621.

Russ, J. (2016). *Image processing handbook*. CRC Press.