

Analyzing images

Anders Kaestner :: Laboratory for Neutron Scattering and Imaging



1 Introduction**2 Basic image metrics****3 Characterizing shapes****4 Displacement analysis****5 Classification****6 Summary**

Introduction

The previous lectures we looked into methods to prepare images.

Filtering

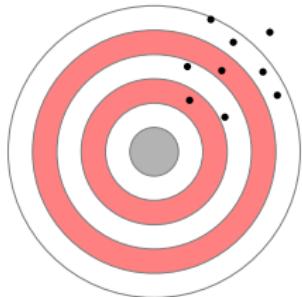
- Intensity adjustments
- Denoising
- Edge detection

Segmentation

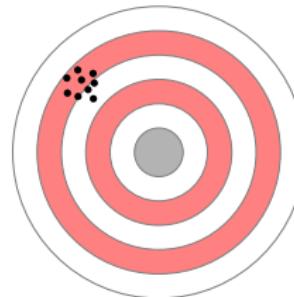
- Classification
- Labeling
- Post segmentation cleaning

These steps are needed to provide information for the analysis.

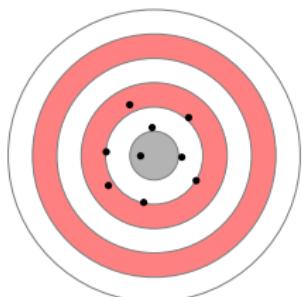
Low Precision and Accuracy



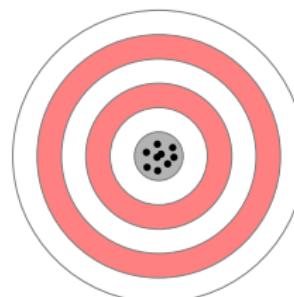
High Precision and Low Accuracy



Low Precision, High Accuracy



High precision and accuracy



Basic image metrics

Counting pixels is the most simple measurement on an image:

Area and volume

- 2D** The number of pixels corresponds to the area.
Each pixel has the unit area (pixel size)²

$$Area_i = L_{pixel}^2 \cdot \#(\text{pixels} \in \text{label}_i)$$

- 3D** The number of voxels corresponds to the volume.
Each pixel has the unit volume (voxel size)³

$$Volume_i = L_{voxel}^3 \cdot \#(\text{voxels} \in \text{label}_i)$$

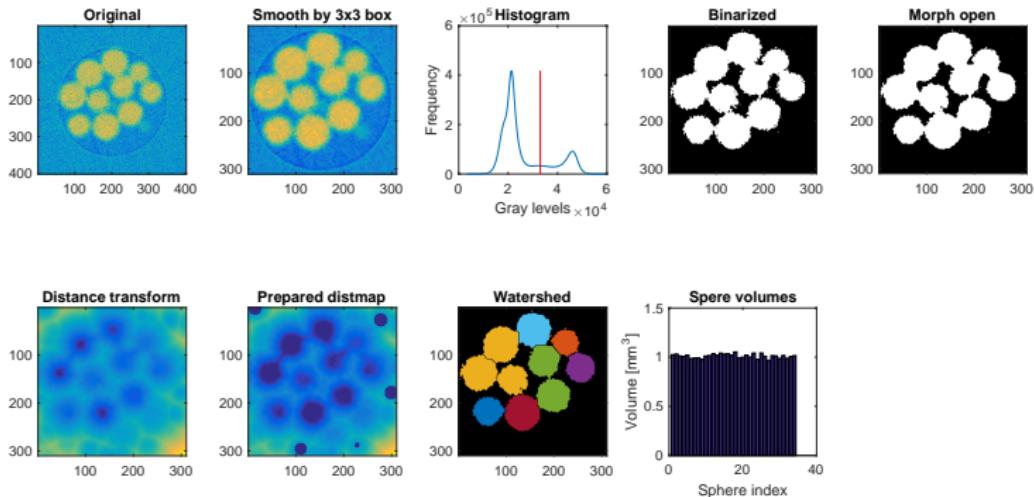
Labeled images

Area/volume per item in a labeled image: Using a histogram with N_{Labels} bins.

Measure volume per item

Example

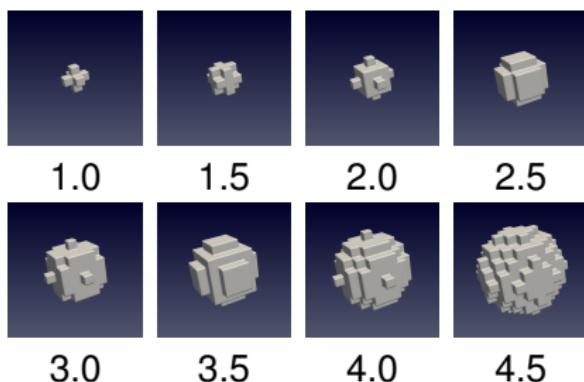
We want to measure the volume of items in a packing of spheres



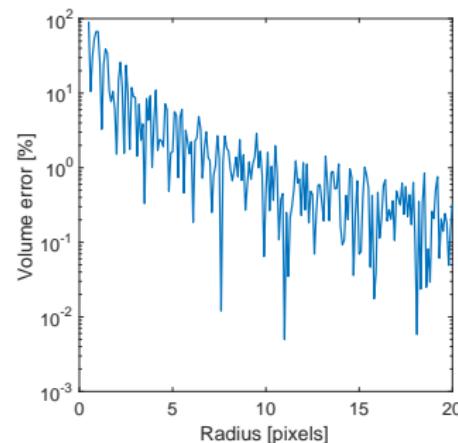
Problem with a discrete grid

- The accuracy decrease when the item size approach the voxel size.
- It is difficult reproduce fine details.

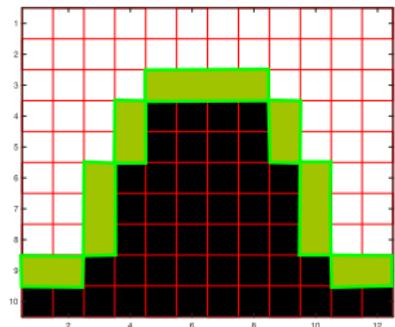
Discrete spheres



Volume error

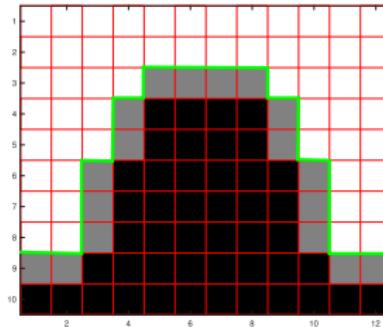


Count pixels



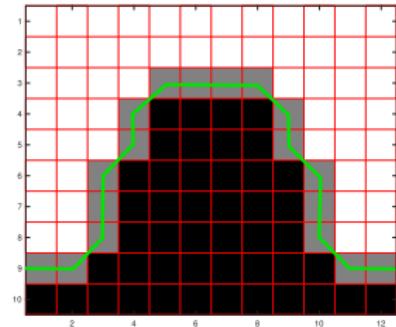
Simple processing.

Pixel outline



Requires neighborhood.

Pixel midline



Requires neighborhood.

Some words about accuracy and precision

- Boundary position depends on threshold.
- The precision depends on noise and metric.

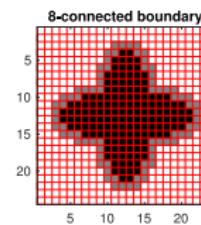
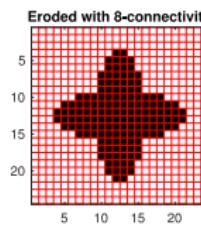
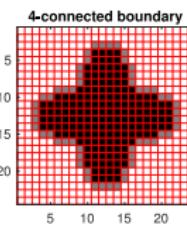
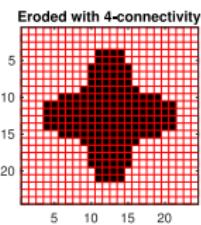
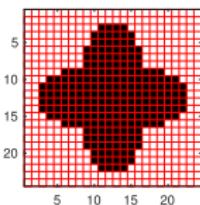
Identify boundary pixels

The boundary pixels of a bilevel image are identified by

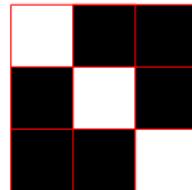
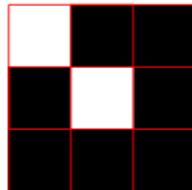
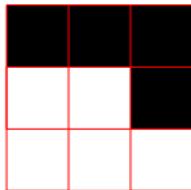
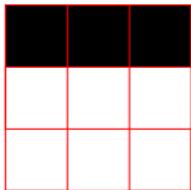
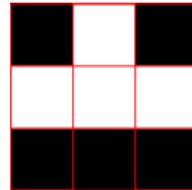
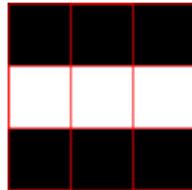
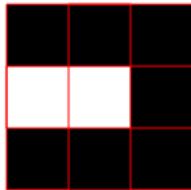
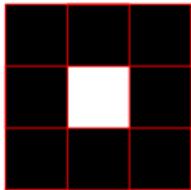
$$b(f) = f - \varepsilon_{SE}(f)$$

with structure element SE representing the direct neighbours.

Example



Each edge pixel type must be identified as a rotation of



In 3D more patterns are added.

Interpolate edge shape

Sub-pixel accuracy can be achieved with edge interpolation.

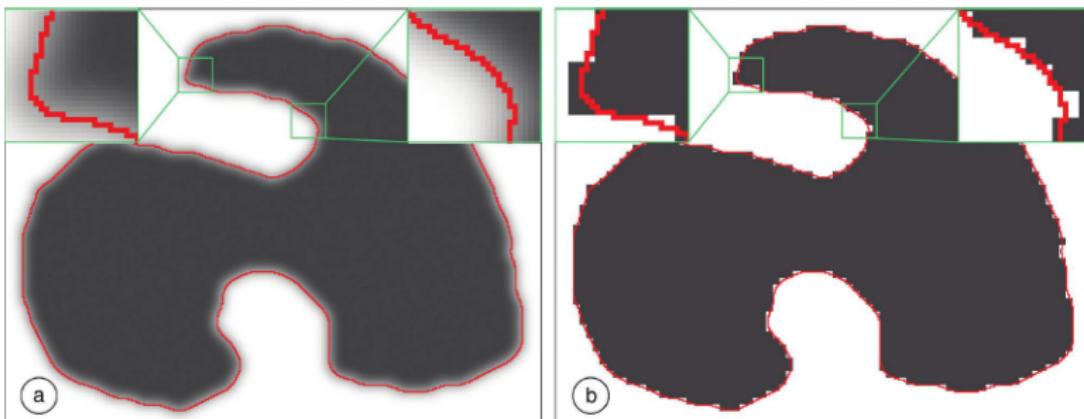


Figure 10.72 Fitting a smooth boundary for perimeter measurement: (a) contour line on a smoothed (antialiased) feature; (b) the contour line superimposed on the original pixels.

Russ (2016)

The surface area and volume can be used to characterize the particle shape using a sphere as reference shape:

Sphericity is a measure how round a particle p is using the ratio between surface area and volume

$$\Phi = \frac{\pi^{1/3} (6 V_p)^{2/3}}{A_p}$$

The sphericity is one for a sphere other shapes have less than one.

Equivalent radius $R = \left(V \frac{3}{4\pi} \right)^{1/3}$

Motivation

Some applications require the knowledge how the intensity depends on the distance.

- Physical quantities like density or water content.
- Edge sharpness

Segmentation

Identify features.

$$f \rightarrow b$$

Distance map

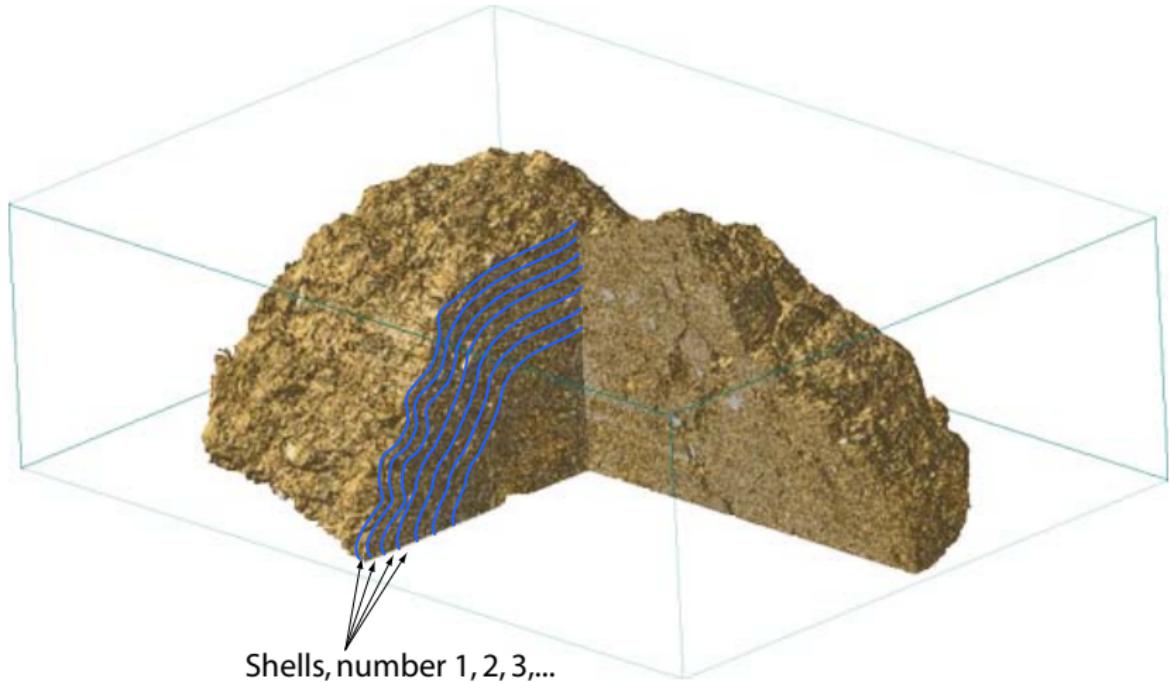
Apply distance transform.

$$D = \mathcal{D}(b)$$

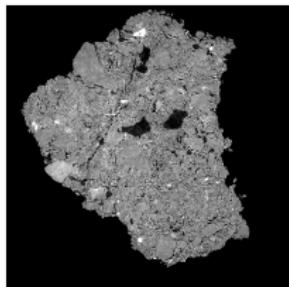
Collect profile

- $s(d) = \{p | D(p) = d\}$.
- Compute average intensity $E[s(d)]$, $\forall d \in D$.

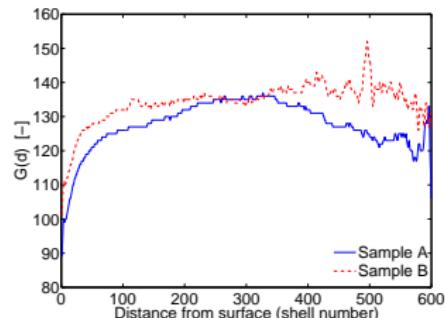
Shell peeling



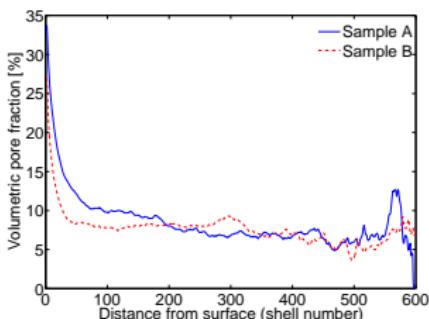
CT slice



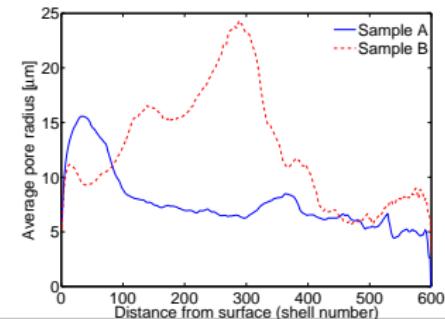
Density



Porosity



Pore radius



Characterizing shapes

Center of box

- Identify bounding box (smallest box including entire item)
- Using box coordinate to find center point

Simple but may be misleading.

Center of gravity

The mid point of item f is

$$\text{CoG}_x = \frac{\sum_{i \in f} x_i}{\text{Area}(f)}$$
$$\text{CoG}_y = \frac{\sum_{i \in f} y_i}{\text{Area}(f)}$$

For 3D a third equation is added, normalization by volume.

van Assen et al. (2002)

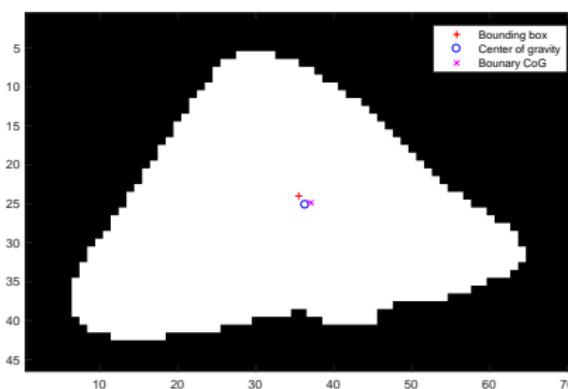
Density weighted

$$CoG_x = \frac{\sum_{i \in f} f(i) x_i}{\sum_{i \in f} f(i)}$$

Boundary only

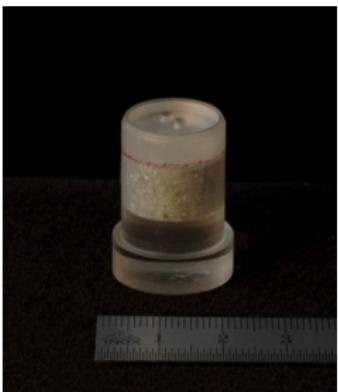
$$b(f) = f - \varepsilon_{SE}(f)$$

$$CoG_x = \frac{\sum_{i \in b(f)} x_i}{\sum b(f)}$$

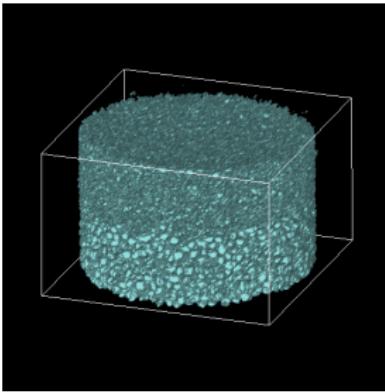
Example

Segmenting two grain size fractions

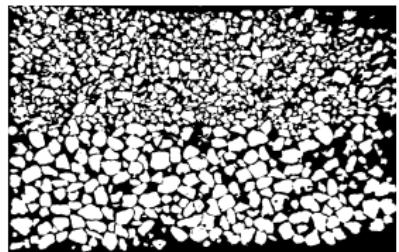
Sample



Volume



Vertical slice



Kaestner et al. (2005)

Part I

Segmenting sand grains

- 1 Create elevation map

$$s_d = d_{\mathcal{E}}(s) - d_{\mathcal{E}}(s^c)$$

- 2 Use Watershed segmentation



- 3 Compute

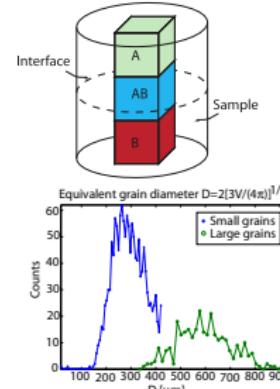
- CoG
- Equivalent grain radius

for each item.

Part II

Classifying sand grains

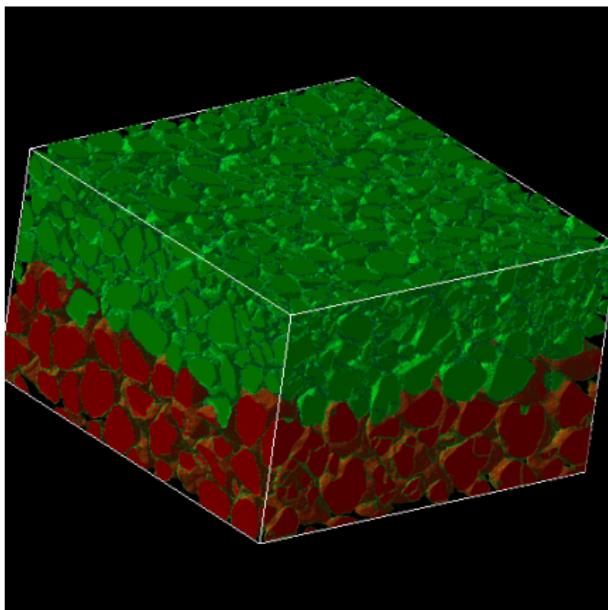
- 1 Collect class statistics



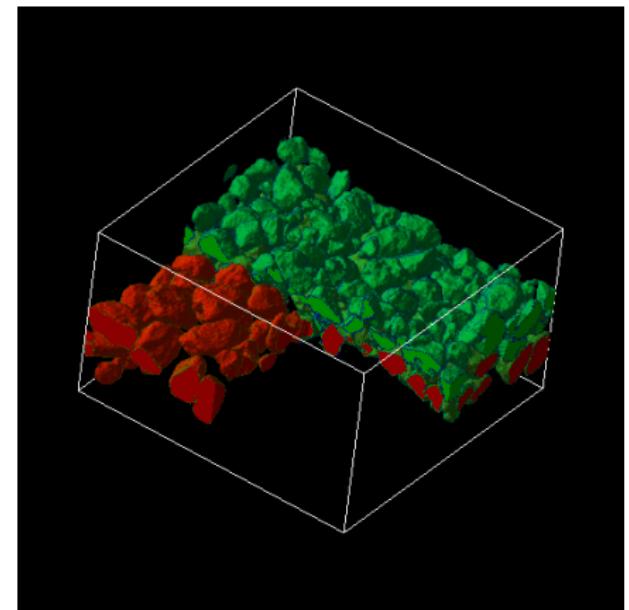
- 2 Use a spatial Bayesian threshold

$$\mathcal{R}(d) = \frac{1}{1 + e^{-d/\sigma}} \frac{P_B}{P_A}$$

Segmented sand fractions



Sand grains in the interface



Centroid distances

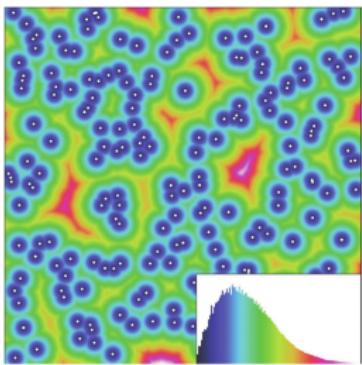


Figure 10.20 Random distribution of points (white) and the Poisson distribution of distances from centroid

Different particle distributions

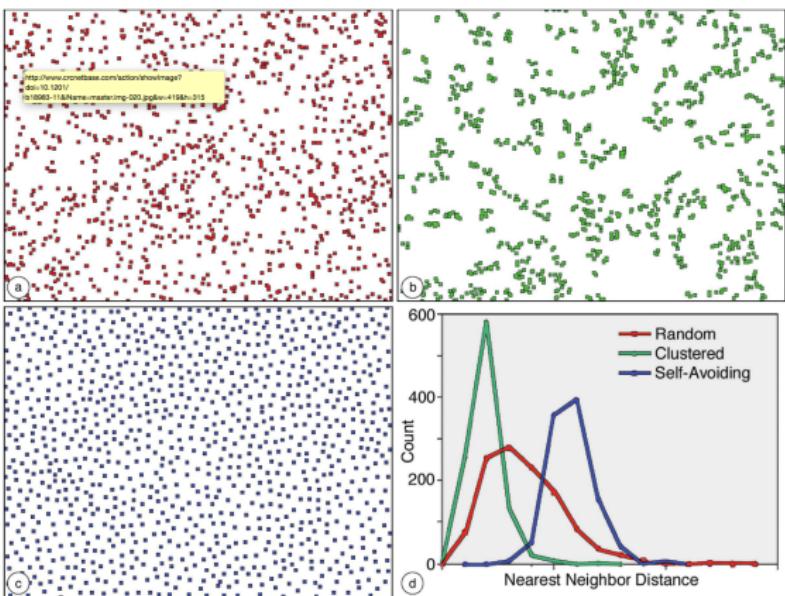


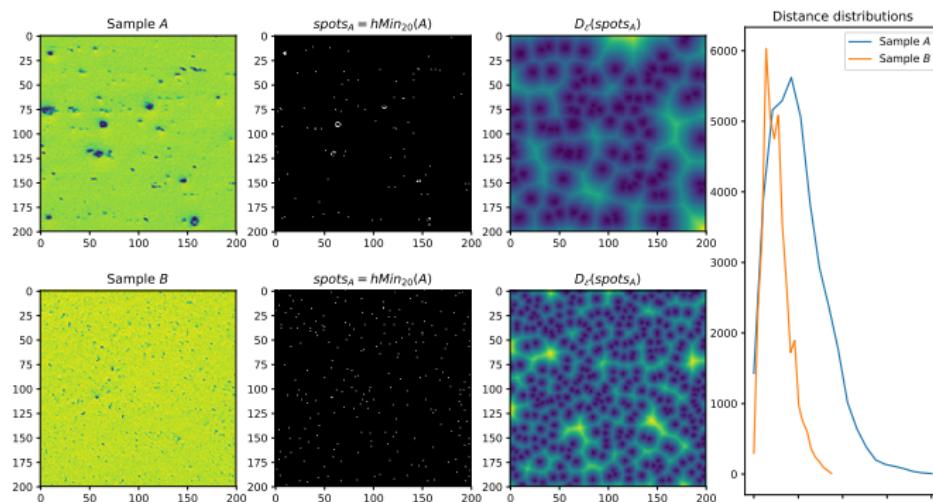
Figure 10.21 Feature distributions illustrating (a) random, (b) clustered, and (c) spaced or self-avoiding arrangements; and (d) histograms of nearest-neighbor distances for each of the point distributions. The mean value of the clustered distribution is less than, and that for the self-avoiding distribution is greater than, the mean for the random one.

Russ (2016), p. 576

Procedure

- 1 Microscope images of samples with defects
- 2 Find spots using hMin operation
- 3 Measure the distance between the defects using an Euclidean distance transform

Results



Moments

$$\mu_{m,n} = \sum_{i,j} x_i^m y_j^n \cdot v(i,j)$$

$\mu_{0,0}$ Area

$\mu_{1,0}, \mu_{0,1}$ Corresponds to centroid

x, y Positions in item

v Optional weight,
e.g. the density.

Normalized moment

$$\eta_{n,m} = \frac{\mu_{m,n}}{\mu_{0,0}^{(m+n+2)/2}}$$

Orientation

$$\theta = \frac{1}{2} \tan^{-1} \left(\frac{2\mu_{1,1}}{\mu_{2,0} - \mu_{0,2}} \right)$$

Grain orientation



Figure 10.17 Measurement of feature orientation: (a) rice grains; (b) individually color-coded according to moment angle; (c) distribution of grains according to angle.

Russ (2016), p. 574

Invariant moments

The following normalized moments are invariant to translation, orientation, and size.

$$m_1 = \eta_{20} + \eta_{02}$$

$$m_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2$$

$$m_3 = (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \eta_{03})^2$$

$$m_4 = (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2$$

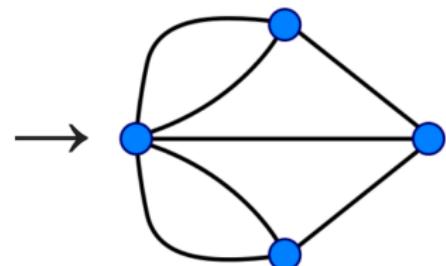
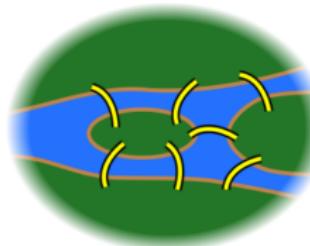
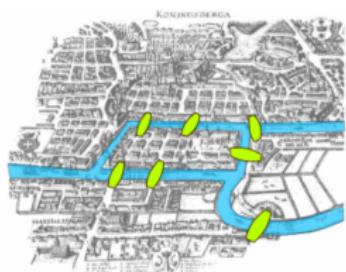
$$m_5 = (\eta_{30} - 3\eta_{12})^2(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 + 3(\eta_{21} + \eta_{03})^2] \\ + (3\eta_{21} - \eta_{03})^2(\eta_{21} + \eta_{03})[(\eta_{30} + \eta_{12})^2 + 3(\eta_{21} + \eta_{03})^2]$$

$$m_6 = (\eta_{20} - \eta_{02})[(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] \\ + 4\eta_{11}(\eta_{30} + \eta_{12})(\eta_{21} + \eta_{03})$$

$$m_7 = (3\eta_{21} - \eta_{03})^2(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 + 3(\eta_{21} + \eta_{03})^2] \\ + (3\eta_{12} - \eta_{30})^2(\eta_{21} + \eta_{03})[(\eta_{30} + \eta_{12})^2 + 3(\eta_{21} + \eta_{03})^2]$$

Königsberg and graph theory

Question: Is it possible to walk each bridges only once?



Wikipedia

Definition

The Euler-Poincare characteristic $\chi = V - E + F$ combines the number of

V vertices/nodes

E edges/connections

F faces/loops

to quantify the topology and connectivity of a shape.

$$\chi_4 = \mathcal{N}\left(\begin{array}{|c|c|} \hline \text{black} & \text{white} \\ \hline \text{white} & \text{white} \\ \hline \end{array}\right) + \mathcal{N}\left(\begin{array}{|c|c|} \hline \text{black} & \text{white} \\ \hline \text{white} & \text{black} \\ \hline \end{array}\right) - \mathcal{N}\left(\begin{array}{|c|c|} \hline \text{black} & \text{black} \\ \hline \text{black} & \text{white} \\ \hline \end{array}\right)$$

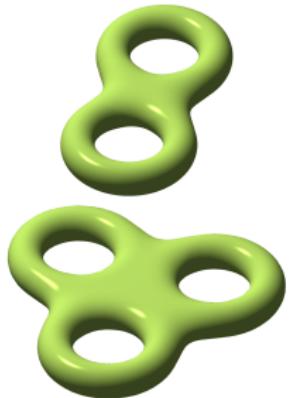
$$\chi_8 = \mathcal{N}\left(\begin{array}{|c|c|} \hline \text{black} & \text{white} \\ \hline \text{white} & \text{white} \\ \hline \end{array}\right) - \mathcal{N}\left(\begin{array}{|c|c|} \hline \text{white} & \text{black} \\ \hline \text{black} & \text{white} \\ \hline \end{array}\right)$$

The configurations are identified using a hit and miss transform (a morphological operation).

It tell us about the complexity of porous structure.

Soille (2002), Vogel (2002)

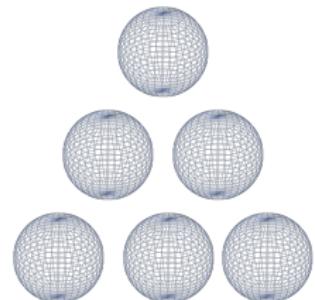
Negative



Zero



Positive



Displacement analysis

- Digital image correlation is a method to measure local image displacements.
 - Absolute displacement
 - Direction
- Requires two images for the measurement
- It uses correlation for the analysis.
- Can also be applied to volume data.
- Does not need segmentation to work.

Definition

Correlation measures how similar two signals X and Y are, using

$$c(k) = \frac{\text{cov}(X, Y)}{\sigma_X \sigma_Y} = \frac{\sum_{i=1}^N (x_{k-i} - \bar{x})(y_i - \bar{y})}{(N-1) s_x s_y}$$

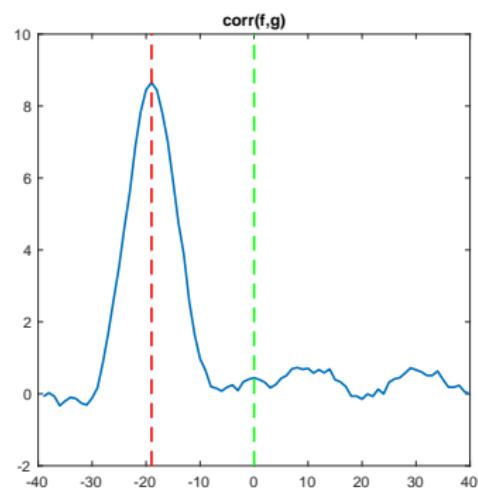
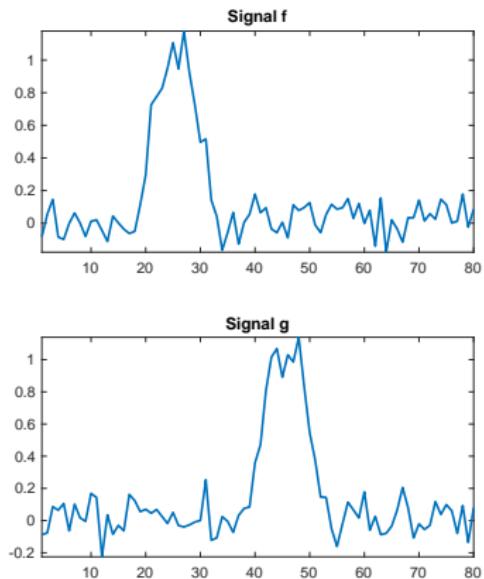
This process can be implemented more efficiently using the FFT

$$c(k) = \mathcal{F}^{-1}\{\mathcal{F}\{x\} \cdot \mathcal{F}\{y\}^*\}$$

The relative displacement distance is given as the k where $c(k)$ is at maximum.

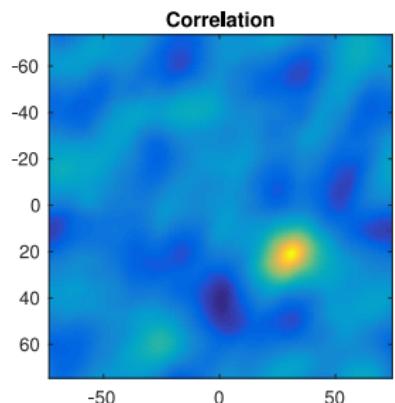
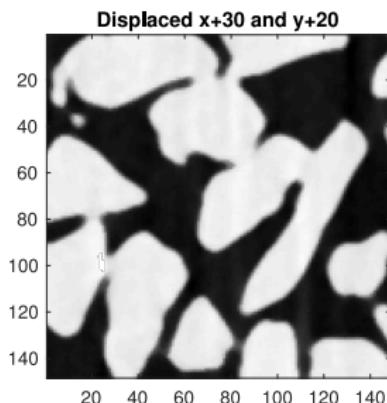
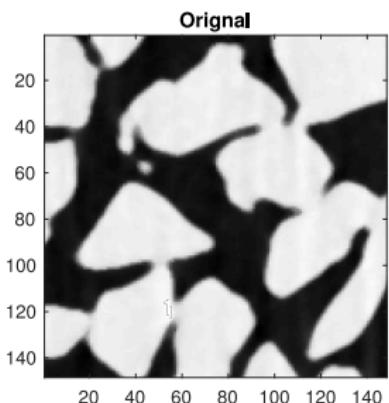
Example

Two phase shifted signals f and g



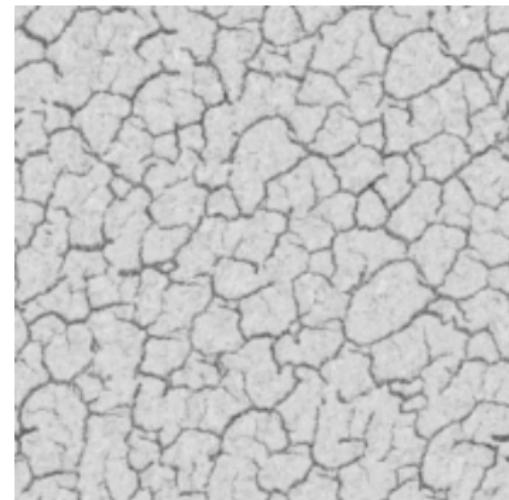
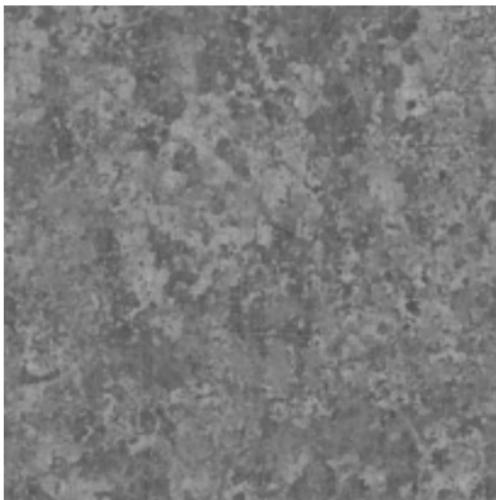
The correlation function computed with FFT gives the displacement distance.

Global correlation



Textures and stress

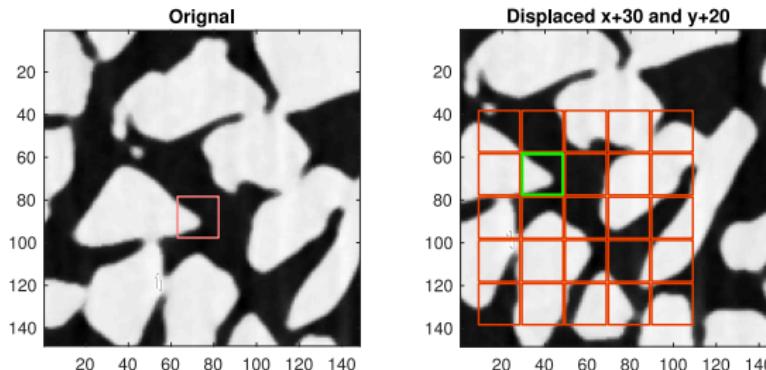
- A texture is an image pattern often with a noise like appearance.
- It can represent barely resolved sample features.



Samples under stress have local texture displacements.

Principle

- The global correlation can only detect if the sample was displaced.
- We want to measure local displacements/changes relative to a reference.
- A displacement field can be computed as correlation with gauge regions.



- The gauge region with best correlation is chosen as local displacement
- A vector field can be generated this way.
- Amplitudes or directions

Example: Water flow in shear band

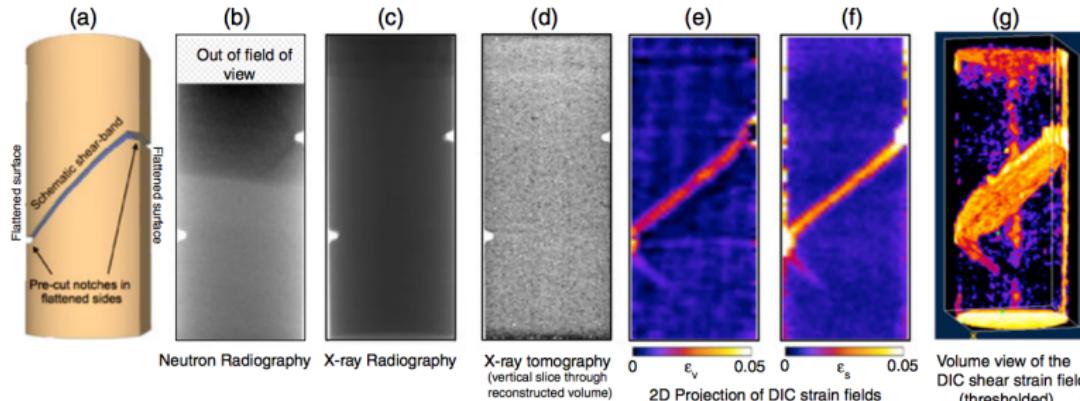


Figure 1. (a) Schematic of the sample geometry (the distance between the flattened faces and the width of the following images are 35 mm). (b) Neutron radiograph of the sample prior to water imbibition; note dark region (higher attenuation) to the top where oil had invaded the sample. (c) X-ray radiography of the sample before water imbibition (note that the slightly darker area at the top is the additional attenuation due to the holder for the sample and not the effect of the oil). (d) Vertical slice though the middle of the X-ray tomography volume (before water imbibition). (e, f) Volumetric and shear strain fields from the volumetric DIC presented as a 2-D median image across the sample parallel to the flattened surfaces. Note that positive volumetric strain indicates compaction and also that some artifacts of the DIC procedure can be seen in the volumetric strain field in the form of subvertical stripes and also subhorizontal stripes emanating from the two notches. (g) Volume rendering of the shear-strain field after thresholding to remove the lower strain values.

Hall (2013)

Classification

- Last week we used classification methods on gray levels to identify regions.
 - Classification methods can also be used on more abstract information:
 - Size
 - Shape
 - Position
- to learn more about the sample.

Our first task is to find metrics carrying most information.

Tools in this quest are:

- Single and multivariate histograms
- Statistical methods
 - Correlation $\text{corr}(X, Y)$
 - Mutual information

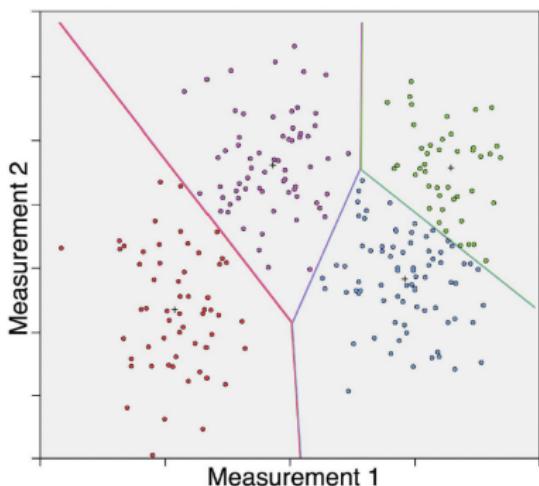
$$I(X; Y) = \sum_{y \in Y} \sum_{x \in X} p(x, y) \log\left(\frac{p(x, y)}{p(x)p(y)}\right)$$

$I(X; Y) = 0$ if X and Y are independent.

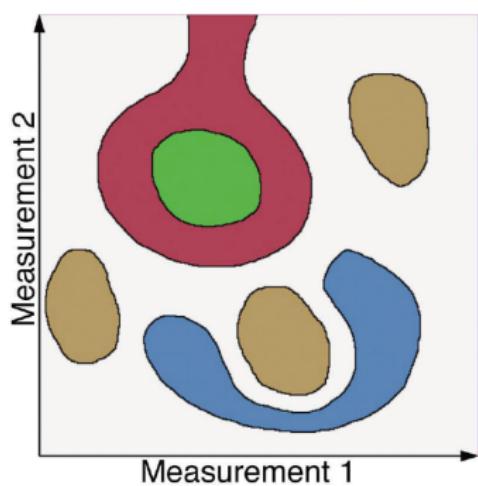
Wikipedia

- Principal component analysis.

Linear separation



Non-linear separation



Principle

Given a training set of observations, select the class for a new observation as the one that have the local majority. The value k is the number of voters.

Example

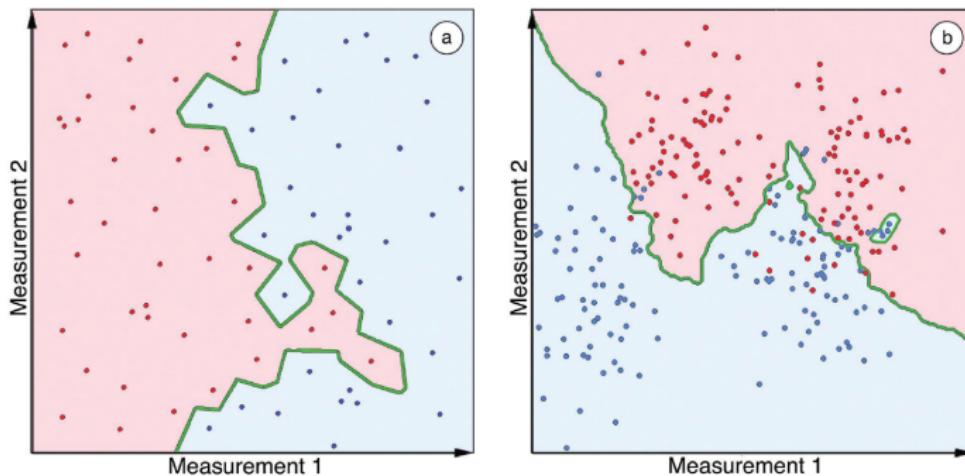


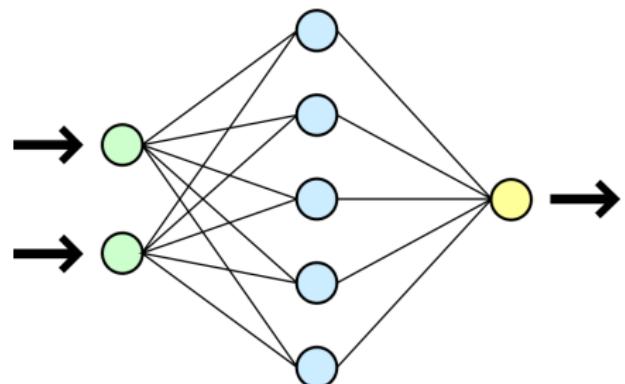
Figure 12.58 The irregular boundary between two classes using k-nearest-neighbor classification:
(a) $k = 1$; (b) $k = 9$.

Methods

- Artificial neural networks (ANN)
- Support vector machines
- Self-organizing maps

Require a learning phase to set the weights.

ANN



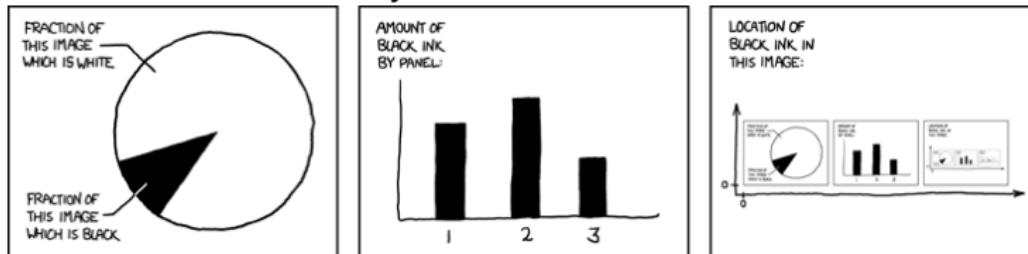
Haykin (1994), Duda et al. (2001)

Summary

We have looked at different ways to quantify information from images

- Size, volume, surface area (Ch 10).
- Shape features (Ch 11).
- Shape guided measurements
 - Distance guided
 - Digital image correlation
- It was not possible to cover all metrics... chapter 10-11 provides more.
- Classification of features (Ch 12).

In the end you want to make statistics



xkcd.com

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