

# Segmenting images

Anders Kaestner :: Laboratory for Neutron Scattering and Imaging

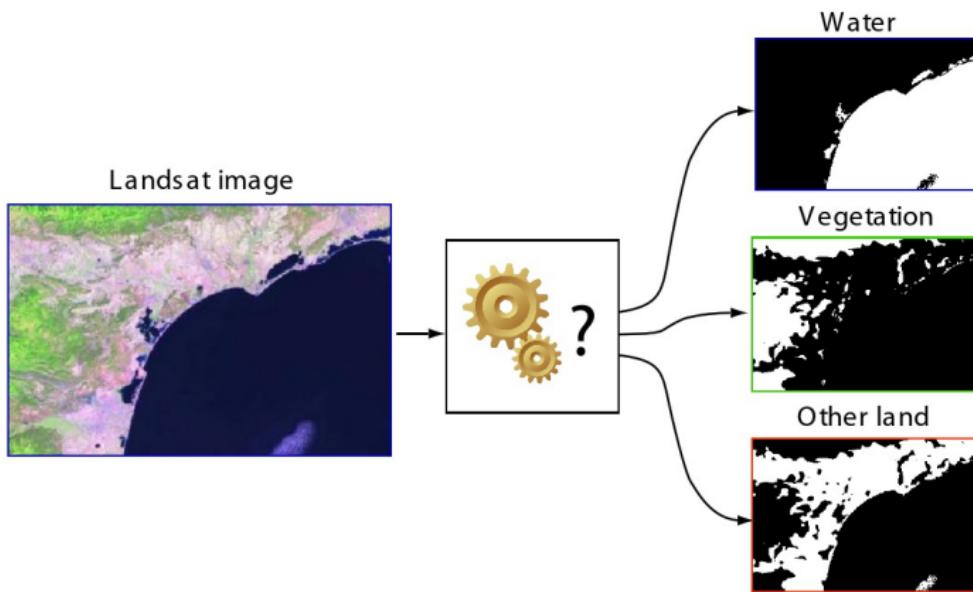


**1 Defining segmentation****2 Neighborhood based segmentation****3 Using probability****4 Multi-class segmentation****5 Bivariate segmentation**

# What is segmentation?

Segmentation is the process to convert the pixels in an image into a limited (small) number of classes depending on:

- The histogram of the image
- A-priori knowledge of the statistics in the image
- Neighborhood information



## Classification

Identify regions based on characteristic properties

- Intensity
- Color
- Texture

## Labelling

Identify individual items, often requires classified image as input

## Unsupervised methods

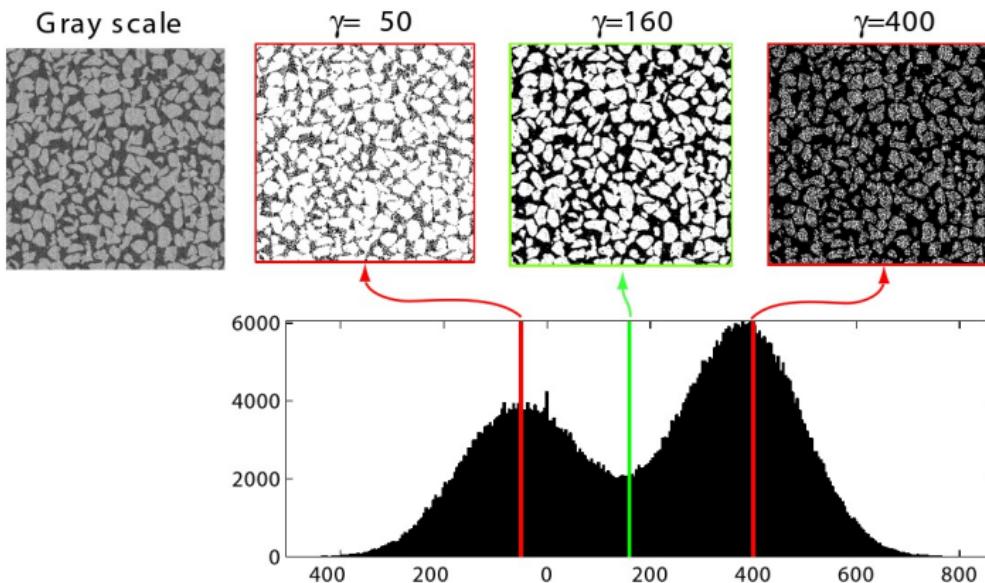
The algorithm can find the regions without human interaction

## Supervised methods

The algorithm needs the input from a human as seed for the processing

Thresholding an image: Compare pixel value against a constant value

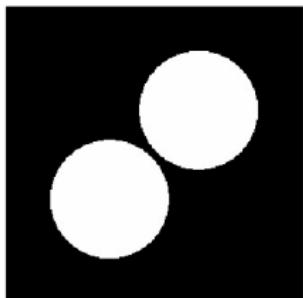
$$g(x) = \begin{cases} 1 & f(x) \geq \gamma \\ 0 & \text{otherwise} \end{cases} \quad \forall x \in \Omega$$



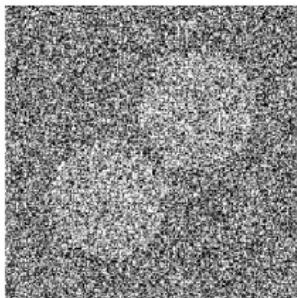
The question now is which threshold value to choose...

Sezgin and Sankur (2004)

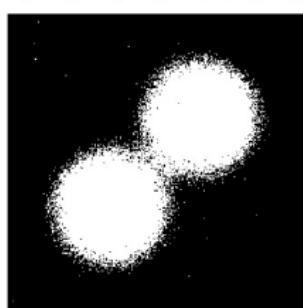
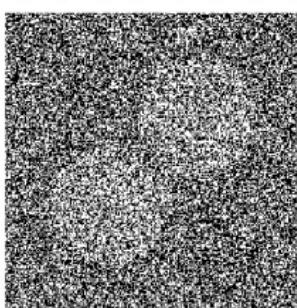
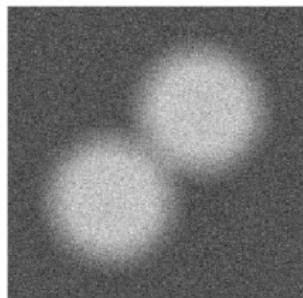
Ideally

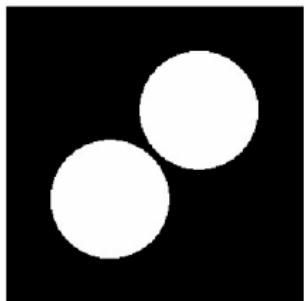
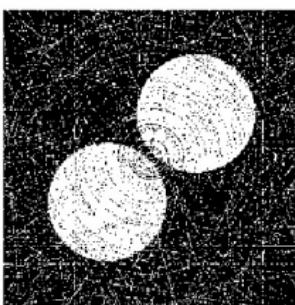
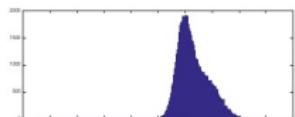
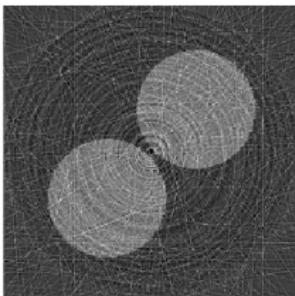
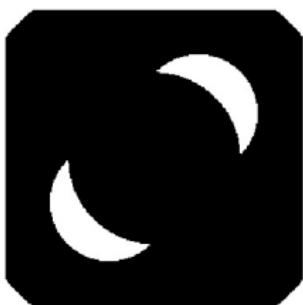
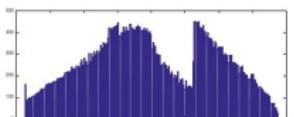
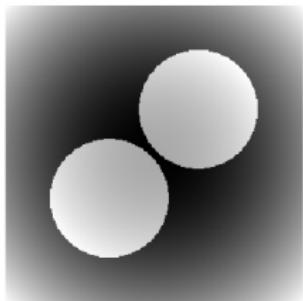


Noise



Unsharpness



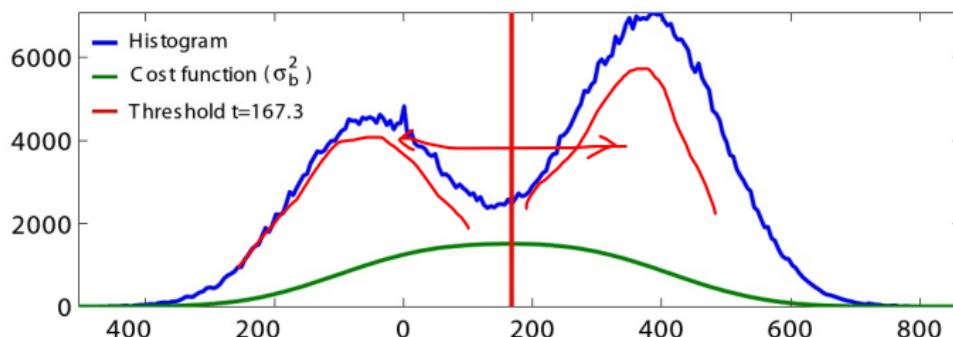
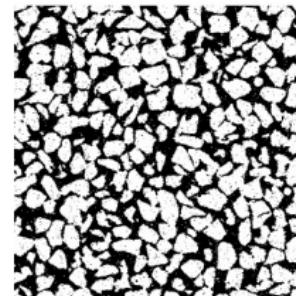
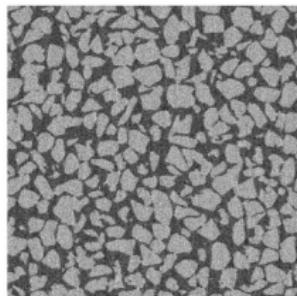
**Ideally****Artifacts****Gradients**

## Histogram-based segmentation

- Use the information provided by the histogram to find a threshold value
- Often work pixelwise → is fast
- Can result in many miss-classified pixels

# Otsu's threshold

A classic algorithm to find a threshold was introduced by Otsu.



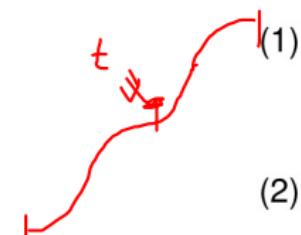
Otsu (1979)

## Algorithm

Find the  $t$  that minimize *in-class* variance

$$\sigma_w^2(t) = q_1(t) \sigma_1^2(t) + q_2(t) \sigma_2^2(t)$$

or that maximize the equivalent *between-class* variance

$$\sigma_b^2(t) = q_1(t)(1 - q_2(t))(\mu_1(t) - \mu_2(t))^2 \quad (2)$$


where  $P$  is a histogram with  $N$  bins

*Class 1*

$$CDF^a \quad q_1(t) = \sum_{i=1}^t P(i)$$

$$Mean \quad \mu_1(t) = \sum_{i=1}^t i \frac{P(i)}{q_1(t)}$$

$$Variance \quad \sigma_1(t) = \sum_{i=1}^t (i - \mu_1(t))^2 \frac{P(i)}{q_1(t)}$$

*Class 2*

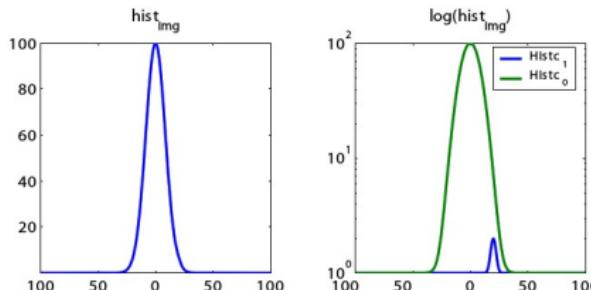
$$q_2(t) = \sum_{i=t+1}^N P(i)$$

$$\mu_2(t) = \sum_{i=t+1}^N i \frac{P(i)}{q_2(t)}$$

$$\sigma_2(t) = \sum_{i=t+1}^N (i - \mu_2(t))^2 \frac{P(i)}{q_2(t)}$$

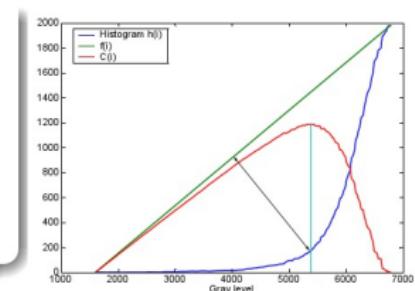
<sup>a</sup>Cumulative Density Function

In cases when the number of pixel in the classes are highly unbalanced (1:100 or even more) could Rosin's method



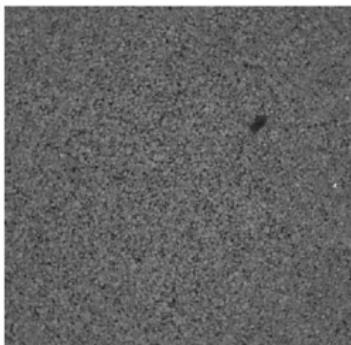
## Algorithm

- Draw a line from histogram max to the end of the tail.
- Compute distance from line to histogram curve.
- Select the threshold at distance max.

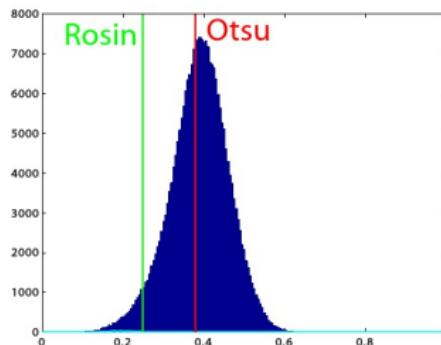


Rosin (2001)

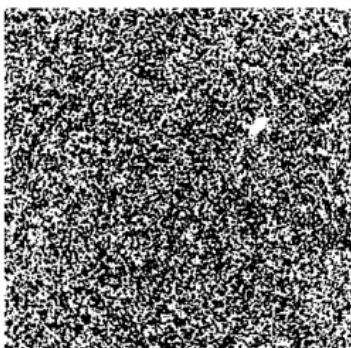
## Example images Rosin



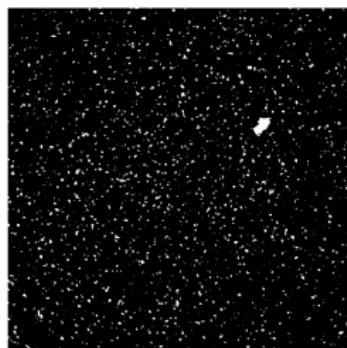
Original image



Histogram



Threshold by Otsu

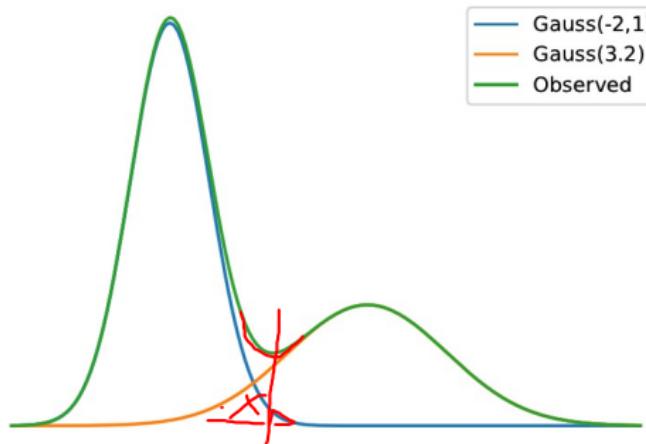


Threshold by Rosin

# Neighborhood based segmentation

A problem with low SNR data is the amount of miss-classified pixels

- The intensity distributions overlap.
- It is difficult to find a single threshold.



## Solution

Introduce region growing in the thresholding process.

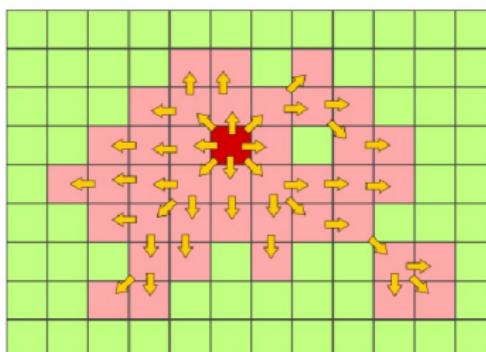
## Principle

Region growing starts from a given pixel or region. The region grows until

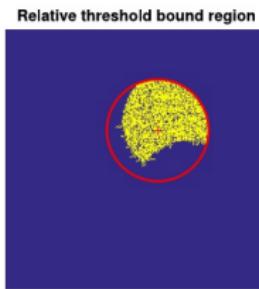
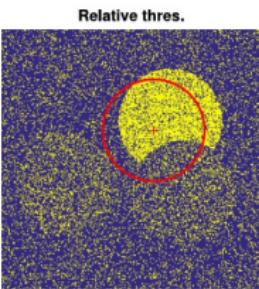
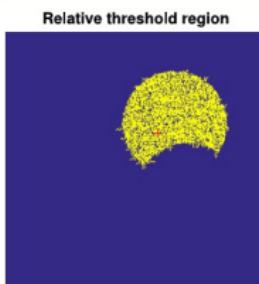
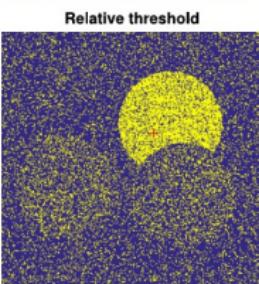
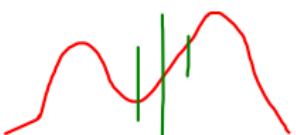
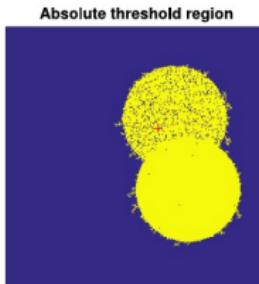
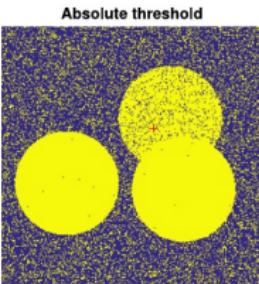
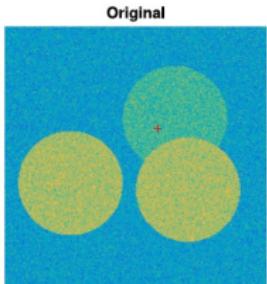
- An absolute threshold intensity is reached.
- The intensity difference reach a threshold value.
- A distance is exceeded, usually combined with intensity thresholds.

This is done by:

- Considering the pixel neighborhood.
- Both unsupervised and supervised.
- Must not necessarily be histogram based.

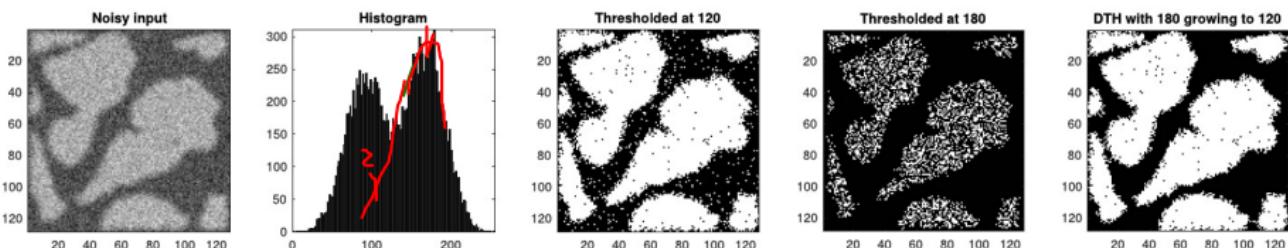


# Variations of region growing



- It is hard to classify data with overlapping class distributions.
- A single threshold either under- or over-segments data
- Combine two thresholds with region growing
  - Set high threshold as seed
  - Perform region growing until lower threshold.

## Example



## Algorithm

This method is defined by the following steps:

- Compute  $\hat{x} = \{x | h(x) = H(\min)\}$ ,  
where  $h(x)$  is a Gaussian fitted to the upper part of the histogram  $H(x)$
- Find thresholds:  $t_{min} = \frac{1}{2}(max_1 + min)$  and  $t_{max} = \frac{1}{2}(min + \hat{x})$
- Threshold in two steps

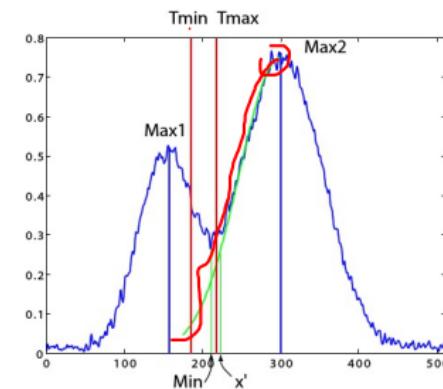
Global pixel-wise threshold

$$f(x) = \begin{cases} 0 & f(x) < t_{min} \\ f(x) & \text{otherwise} \end{cases} \quad (3)$$

Region growing (item edge pixels)

$$f(y) = \begin{cases} 0 & f(y) < t_{max} \\ f(y) & \text{otherwise} \end{cases} \quad y \in NG_x \quad (4)$$

$NG_x$  is the neighborhood of pixel  $x$ .



Vogel and Kretzschmar (1996)

# Using probability

Compute the likelihood ratio

$$\Lambda(R) = \frac{p_{r|\mathcal{H}_1}(\mathbf{R}|\mathcal{H}_1)}{p_{r|\mathcal{H}_0}(\mathbf{R}|\mathcal{H}_0)} \stackrel{\mathcal{H}_1}{\gtrless}_{\mathcal{H}_0} \gamma \quad R \in NG_x, \forall x \in \Omega \quad (5)$$

$\gamma$  is the threshold level

### Advantages

- The threshold can be chosen in terms of detection probability.
- The risk for miss-classification can be computed.

### Disadvantages

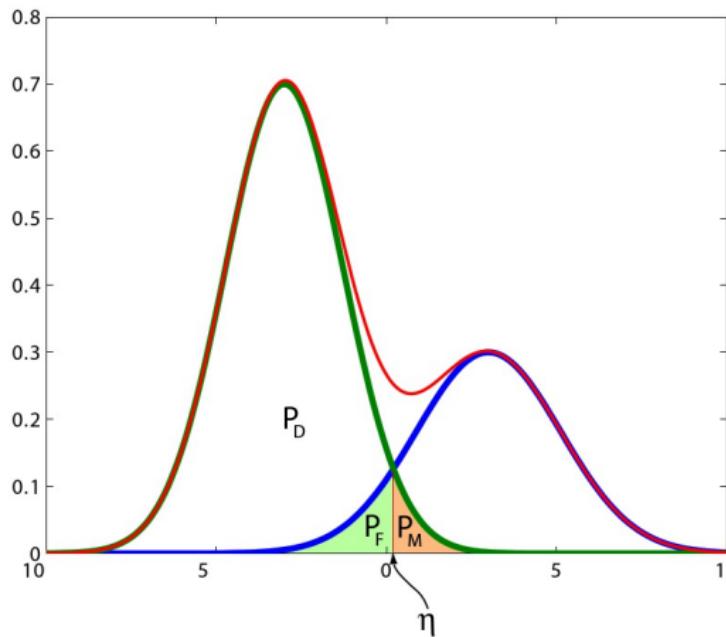
- Require a-priori information on probability distributions of the gray values.
- Might miss small features due local statistics calculations.

For literature on detection theory see Kay (1998) or van Trees (2001)

The threshold  $\eta$  is determined by choosing values for

**Probability of false alarm**  $P_{FA} = P\{t(x) > \gamma' | \mathcal{H}_0\}$

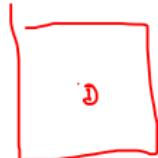
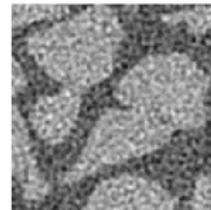
**Probability of detection**  $P_D = P\{t(x) > \gamma' | \mathcal{H}_1\}$



Example – Determine the Likelihood-ratio We have image with gray levels:

$$H_0 : \mathcal{N}(0, \sigma)$$

$$H_1 : \mathcal{N}(\mu, \sigma)$$



⇒ Likelihood ratio for pixel  $x$

$$\Lambda(R_x) = \frac{\prod_{i \in NG_x} \frac{1}{2\pi\sigma} e^{-(r_i - \mu)^2/2\sigma^2}}{\prod_{i \in NG_x} \frac{1}{2\pi\sigma} e^{-(r_i)^2/2\sigma^2}} \stackrel{H_1}{\gtrless} \stackrel{H_0}{\gtrless} \gamma$$

Logarithm and rearrange gives the test variable

$$t(x) = \underbrace{\frac{1}{N} \sum_{i \in NG_x} r_i}_{\text{Neighborhood sum}} \stackrel{H_1}{\gtrless} \underbrace{\frac{\sigma^2}{N\mu} \ln \gamma + \frac{\mu}{2}}_{\text{Constant}} = \gamma'$$

$\mu$  Average of  $H_1$

$\sigma$  Standard deviation of the noise

$r_i$  pixel in neighborhood

$N$  number of pixels in neighborhood

$\gamma$  Threshold level at given confidence level

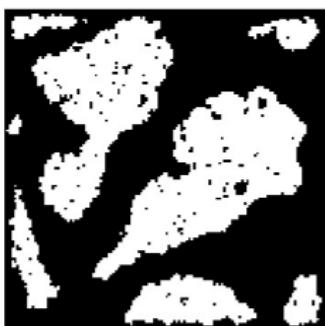
- Compute  $t = h * (h^T * f)$  with  $h = \mathbf{1}_N$

- Determine the threshold level

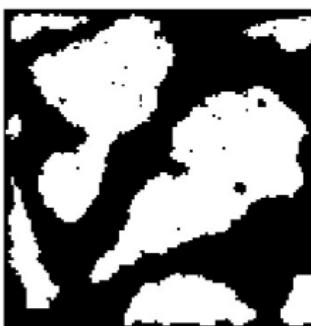
- Neyman–Pearson,

$$P_{FA} = Q\left(\frac{\gamma'}{\sqrt{\sigma^2/N}}\right) \text{ and } P_D = Q\left(\frac{\gamma' - \mu}{\sqrt{\sigma^2/N}}\right)^1$$

- Apply the threshold



$P_D = 95\%$



$P_D = 99\%$



$P_D = 99.9\%$

---

<sup>1</sup>  $Q(x) = \frac{1}{2}\operatorname{erf}(x/\sqrt{2})$  error probability that an observation is in  $[-x, x]$ ,  $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$

The Bayesian decision is based on the cost function

$$\mathcal{R} = \sum_{i=0}^1 \sum_{j=0}^1 C_{ij} P(\mathcal{H}_i | \mathcal{H}_j) P(\mathcal{H}_j) \quad (6)$$

Then the threshold becomes

$$\Lambda(x) = \frac{\underbrace{\frac{p(x|\mathcal{H}_1)}{p(x|\mathcal{H}_0)}}_{\text{likelihood ratio}}}{\frac{(C_{10} - C_{00})P(\mathcal{H}_0)}{(C_{01} - C_{11})P(\mathcal{H}_1)}} > \gamma \quad (7)$$

Under the assumption that  $C_{10} > C_{00}$  and  $C_{01} > C_{11}$

$C_{00}$  Cost for correct assignment to  $\mathcal{H}_0$ .

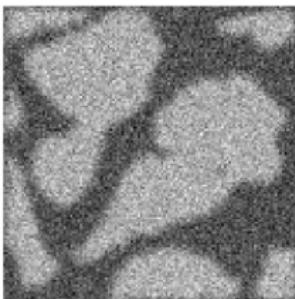
$C_{10}$  Cost for incorrect assignment to  $\mathcal{H}_1$ .

$C_{11}$  Cost for correct assignment to  $\mathcal{H}_1$ .

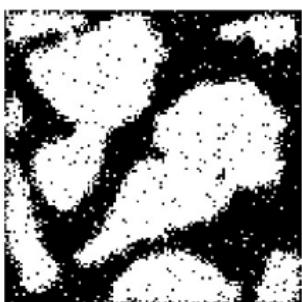
$C_{01}$  Cost for incorrect assignment to  $\mathcal{H}_0$ .



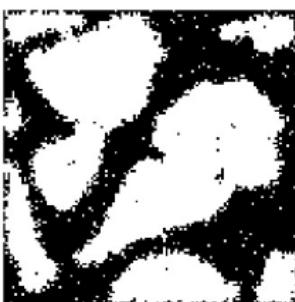
Original



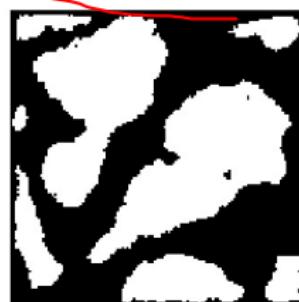
Noisy



Otsu



Hysteresis



Hypothesis testing

**Quiz question:** Why does hypothesis test do misclassify less?

# Multi-class segmentation

In most experiments samples contain more than two phases.

## Example

Porous media	Anatomy	Non-destructive testing
<ul style="list-style-type: none"><li>■ Matrix</li><li>■ Fluids</li><li>■ Void</li></ul>	<ul style="list-style-type: none"><li>■ Bones</li><li>■ Different tissues</li><li>■ Air</li></ul>	<ul style="list-style-type: none"><li>■ Plastic</li><li>■ Metals</li><li>■ Fluids</li><li>■ Air</li></ul>

## Problem

*Each phase should have gray levels that are clearly separated and the interfaces should be sharp.*

*In real data this is rarely true due to noise and unsharpness.*

- Multi-class segmentation requires more thresholds
- More comparisons
- Ambiguous assignments can occur

### Otsu thresholding

Otsu foresaw the segmentation of multiple classes. The handling is described in the paper.

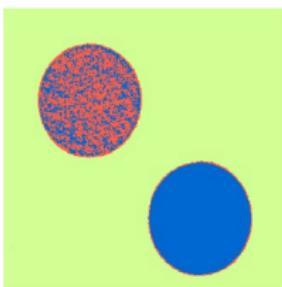
### Likelihood ratio

Adding classes requires

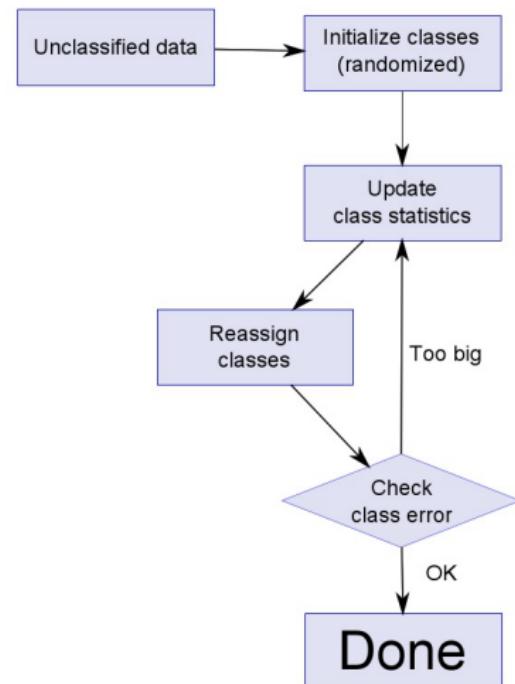
- More class statistics.
- One likelihood test per class pair

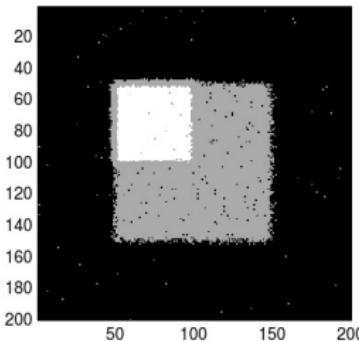
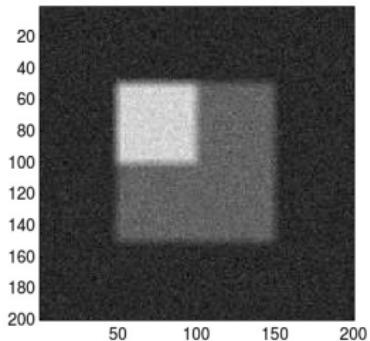
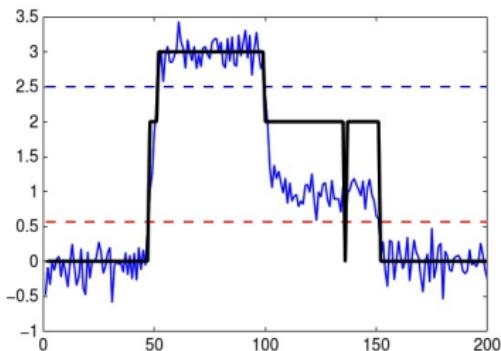
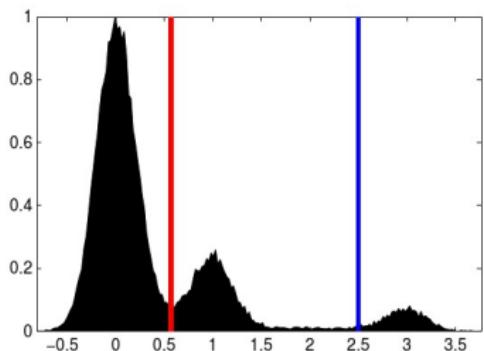
## Some features

- Multi-class data
- Iterative method
- Pixelwise operations
- Global optimization
- Unsupervised
- No a priori information needed

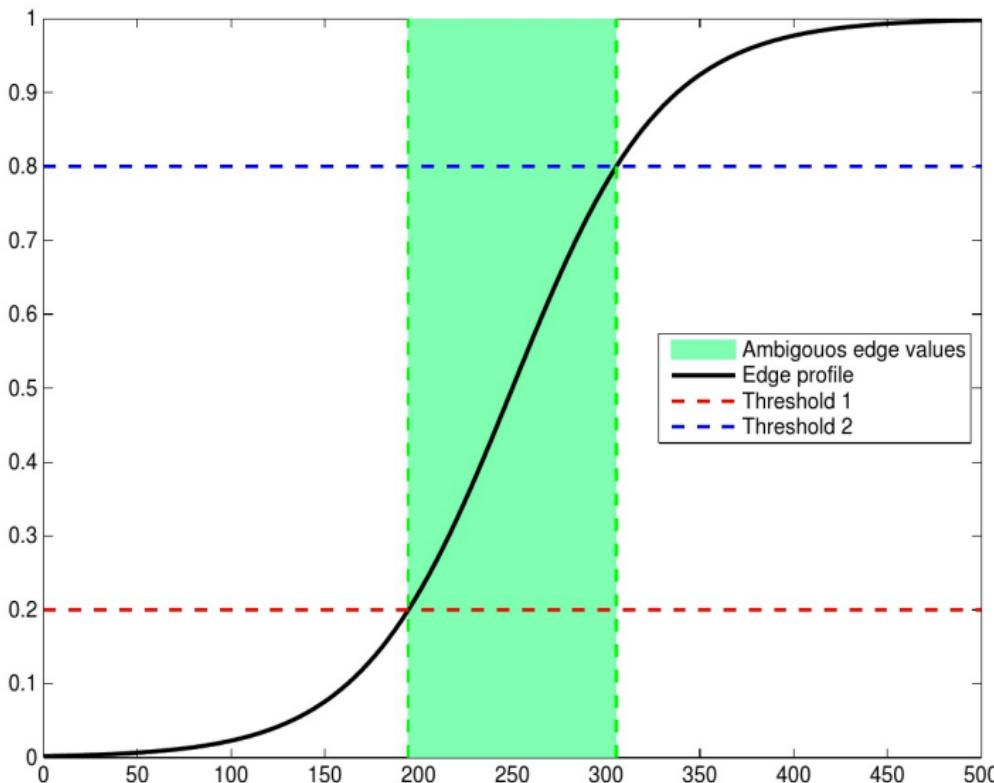


## Algorithm

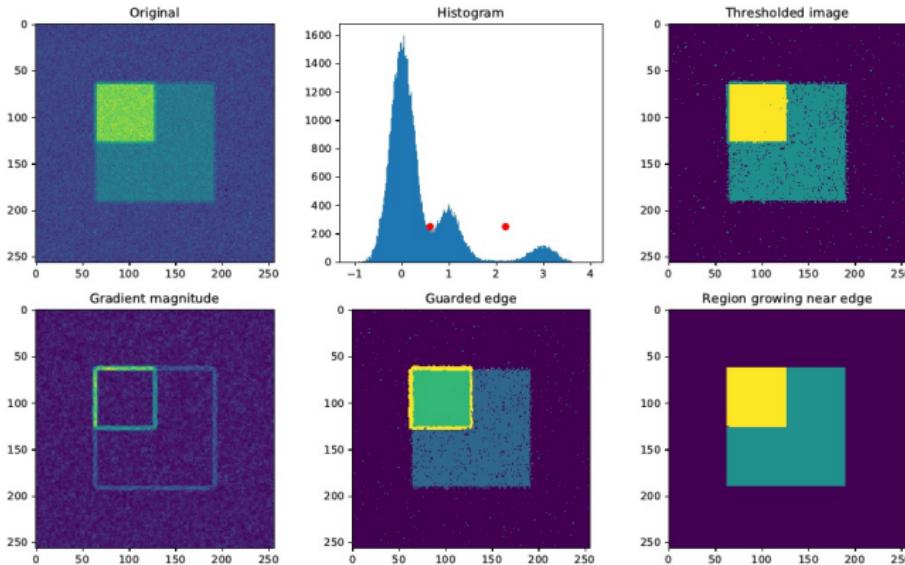




## Looking closer on an edge

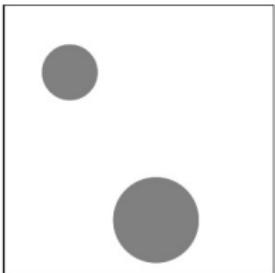


- Use a thresholded LoG image as edge mask  $m$
- Apply segmentation to pixels  $\notin m \rightarrow$  raw segmented image  $s$
- Region growing with class assignment from item edge pixels in  $c$

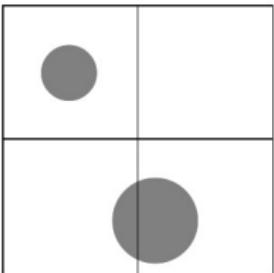


An iterative approach splitting regions when they don't fulfill a some criterion.

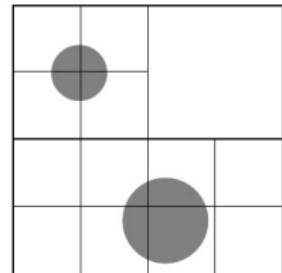
Start



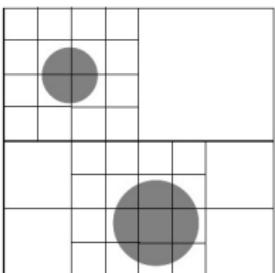
Split 1



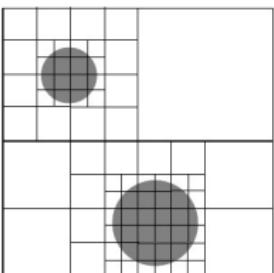
Split 2



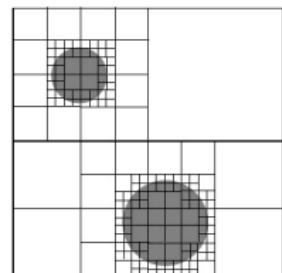
Split 3



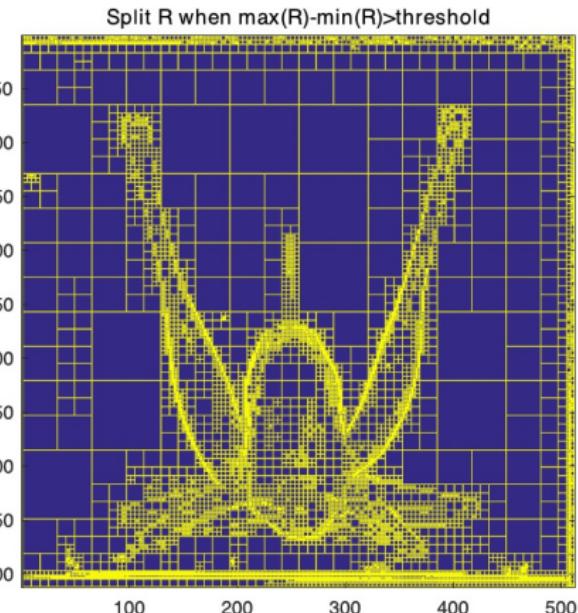
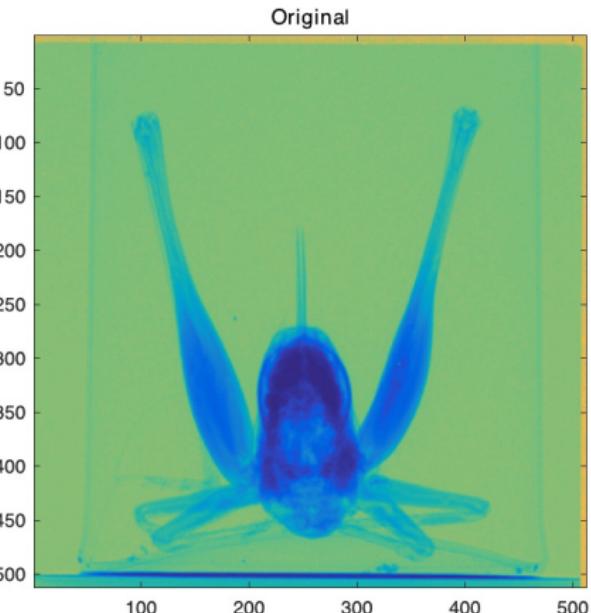
Split 4



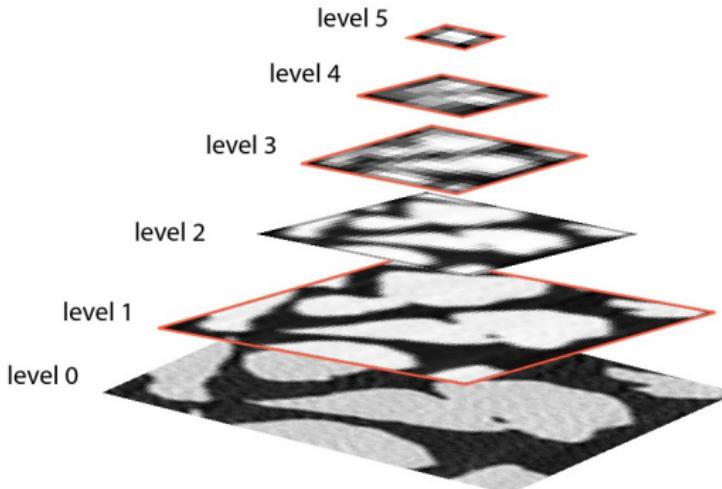
Split 5



## Quad tree decomposition



Work with multiple resolutions



Purpose:

- Less sensitive to noise
- Faster processing

## Burt's pyramid

A region based algorithm working on different scales.

It

- Works iteratively
- Interprets the scale pyramid as a tree
- Re-links the tree edges depending on parent relations
- Automatically determines the number of classes

## Algorithm for Burt's Segmentation

**Initialization** Build the pyramid

### Step 1 Relink

- Bottom-up
- Link child to father with smallest difference

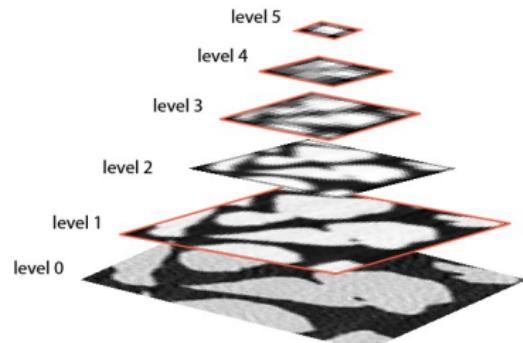
### Step 2 Update levels

- Bottom-up
- Compute average of linked children, assign to father

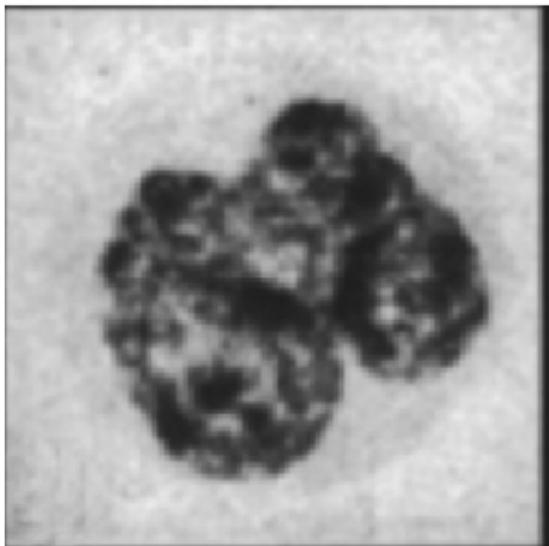
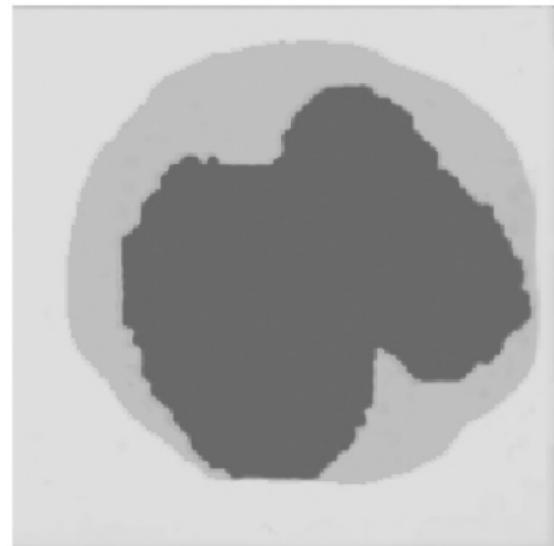
### Step 3 Update segments

- Top-down
- Assign segment of father to all linked children

**Iterate** step 1–3 until stability

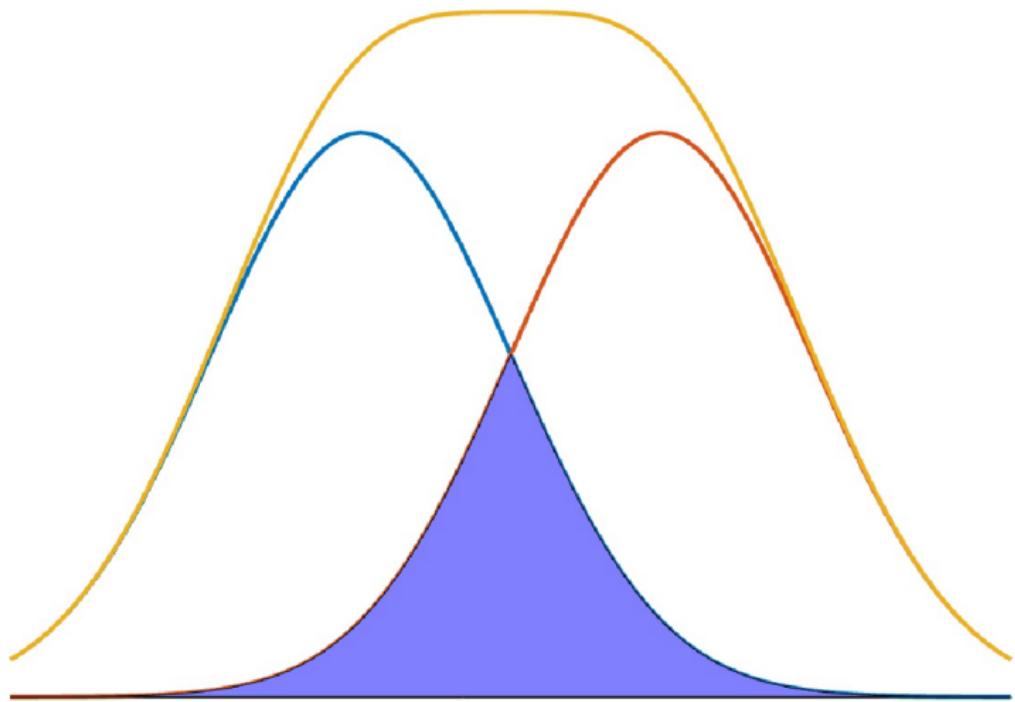


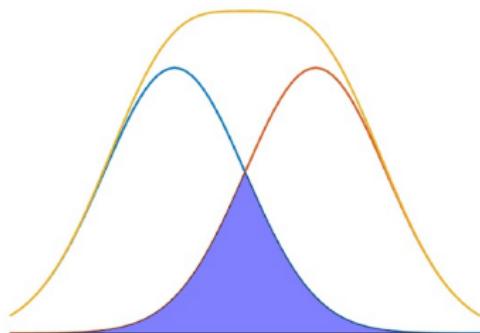
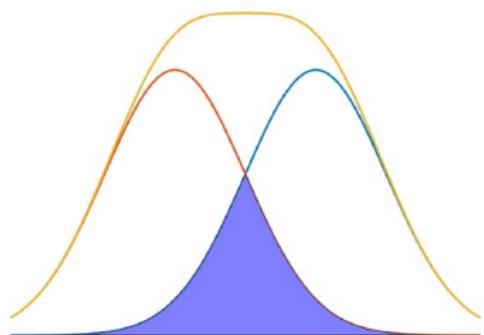
Burt et al. (1981)

**Original****Segmented**

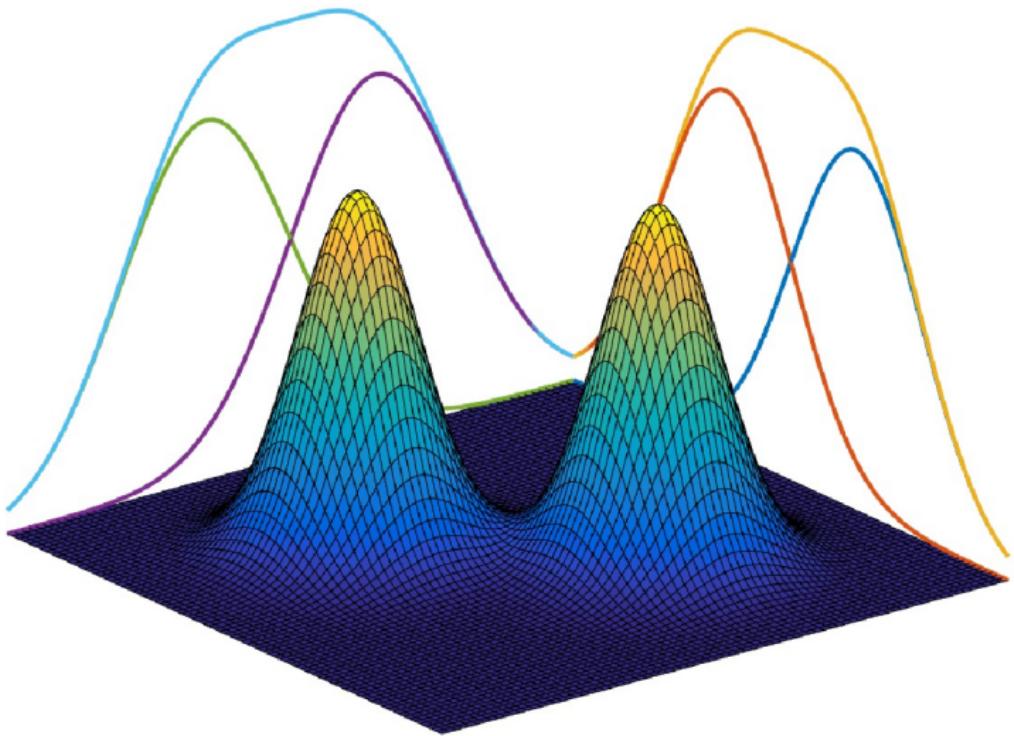
# Bivariate segmentation

# Single modality histogram

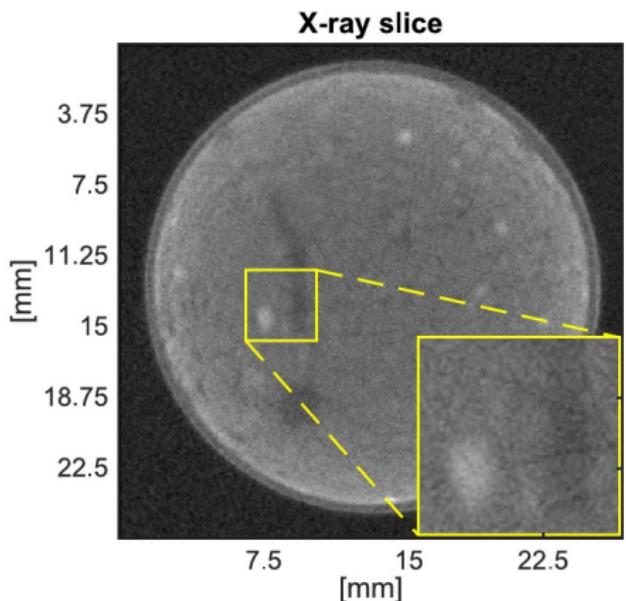
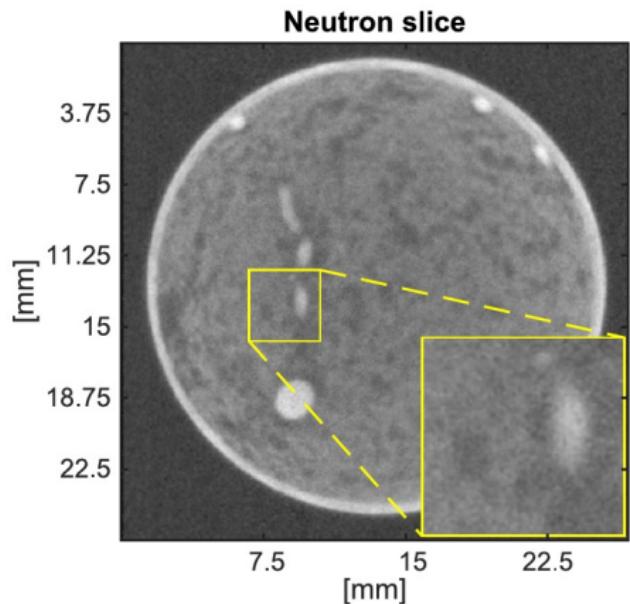


**Modality A****Modality B**

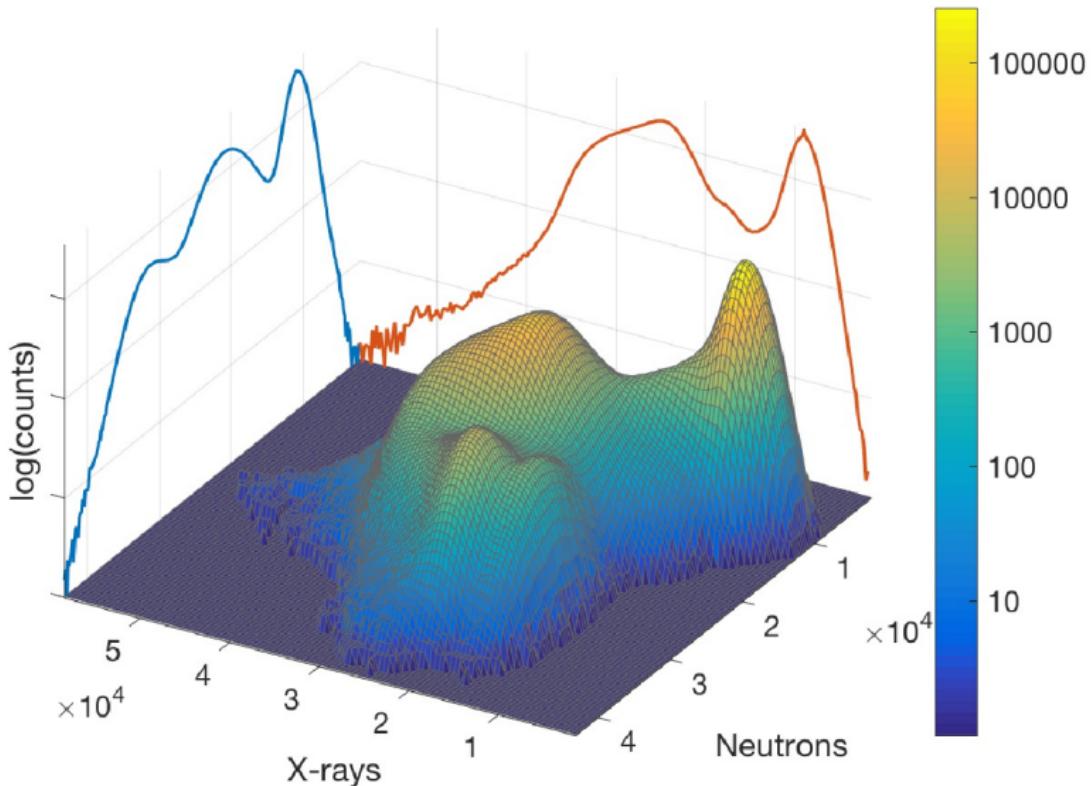
# Bivariate histogram



## Example: Roots in soil



## Bivariate histogram of roots



## Data

- Images from  $M$  modalities  
 $f_1, \dots, f_M$
- Registered

## Classes

The  $N$  classes are described by:

$$\mathcal{H}_1 : p(\mu_1, \sigma_1)$$

$$\mathcal{H}_2 : p(\mu_2, \sigma_2)$$

⋮

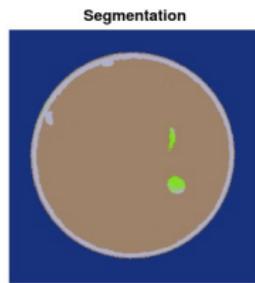
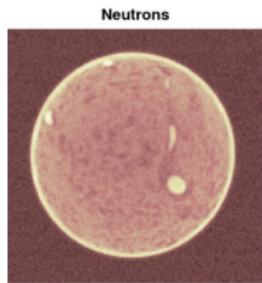
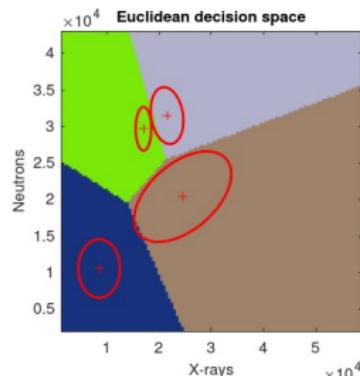
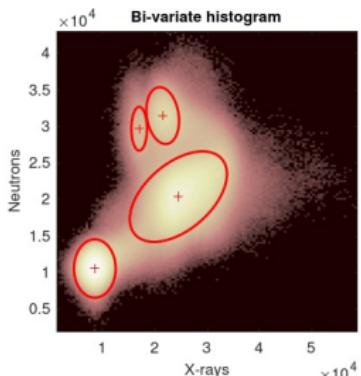
$$\mathcal{H}_N : p(\mu_N, \sigma_N)$$

## Classifier options

- Smallest Euclidean class distance (sample mean)
- Multivariate class distances (sample mean and covariance)
- Non-linear discrimination → Machine learning

Duda et al. (2001)

# Segmentation by Euclidean distance



Kaestner et al. (2017)

We have talked about:

- Thresholds
- Threshold estimation
- Classification methods

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