ETH zürich

PAUL SCHERRER INSTITUT

Segmenting images

Anders Kaestner :: Laboratory for Neutron Scattering and Imaging

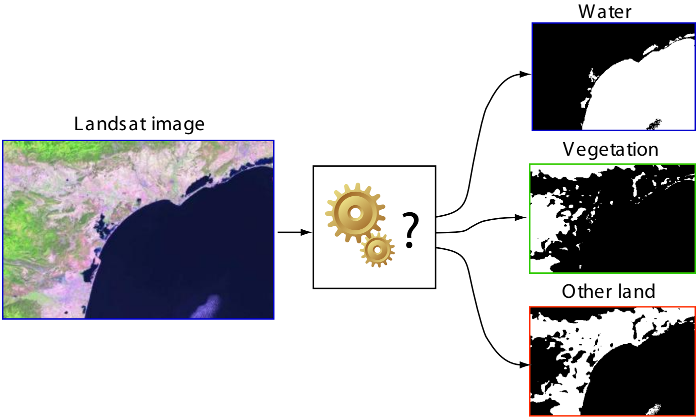
Outline :: Segmentation

1. Defining segmentation
2. Neighborhood based segmentation
3. Using probability
4. Multi-class segmentation
5. Bivariate segmentation

What is segmentation?

Segmentation is the process to convert the pixels in an image into a limited (small) number of classes depending on:

- The histogram of the image
- A-priori knowledge of the statistics in the image
- Neighborhood information



Some vocabulary

Classification

Identify regions based on characteristic properties

- Intensity
- Color
- Texture

Labelling

Identify individual items, often requires classified image as input

Unsupervised methods

The algorithm can find the regions without human interaction

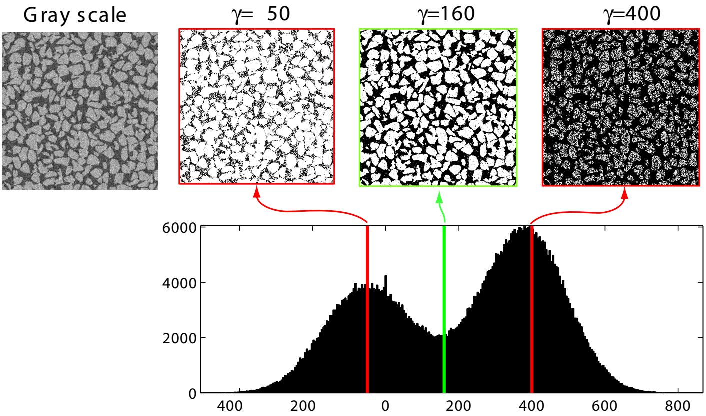
Supervised methods

The algorithm needs the input from a human as seed for the processing

Thresholding

Thresholding an image: Compare pixel value against a constant value

$$g(\mathbf{x}) = \begin{cases} 1 & f(\mathbf{x}) \geq \gamma \\ 0 & \text{otherwise} \end{cases} \qquad \forall \mathbf{x} \in \Omega$$

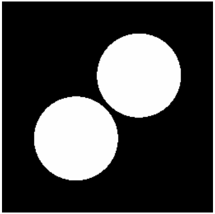


The question now is which threshold value to choose. . .

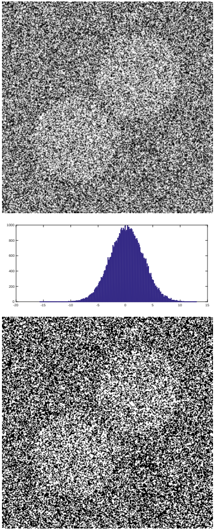
?

Difficulties with segmentation I

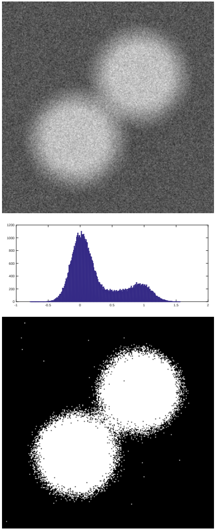
Ideally



Noise

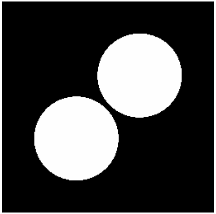


Unsharpness

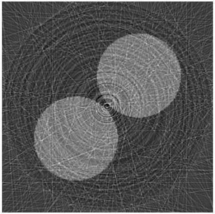


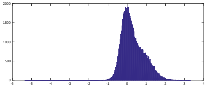
Difficulties with segmentation II

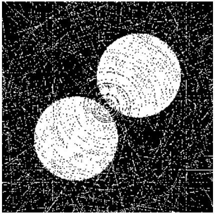
Ideally



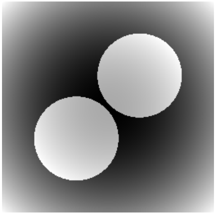
Artifacts

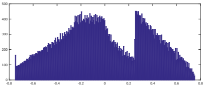


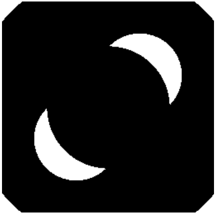




Gradients







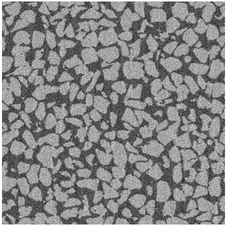
Histogram-based segmentation

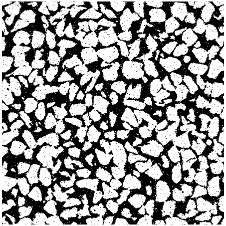
Histogram-based segmentation

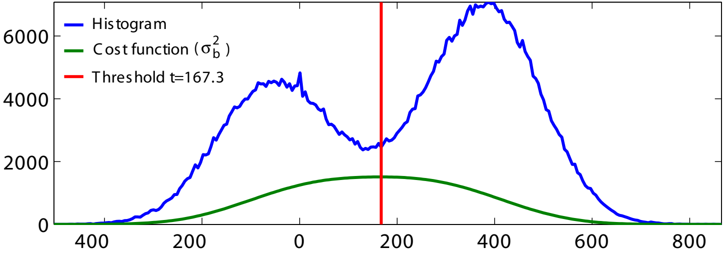
- Use the information provided by the histogram to find a threshold value
- Often work pixelwise $\rightarrow$ is fast
- Can result in many miss-classified pixels

Otsu's threshold

A classic algorithm to find a threshold was introduced by Otsu (1979).







Otsu's threshold algorithm

Algorithm

Find the t that minimize *in-class* variance

$$\sigma_w^2(t) = q_1(t)\,\sigma_1^2(t) + q_2(t)\,\sigma_2^2(t) \tag{1}$$

or that maximize the equivalent *between-class* variance

$$\sigma_b^2(t) = q_1(t)(1 - q_1(t))(\mu_1(t) - \mu_2(t))^2 \tag{2}$$

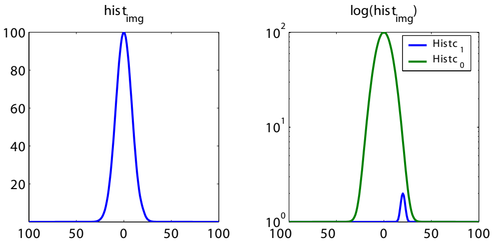
where

	Class 1	*Class 2*
CDF[a]	$q_1(t) = \sum_{i=1}^{t} P(i)$	$q_2(t) = \sum_{i=t+1}^{I} P(i)$
Mean	$\mu_1(t) = \sum_{i=1}^{t} i\,\frac{P(i)}{q_1(t)}$	$\mu_2(t) = \sum_{i=t+1}^{I} i\,\frac{P(i)}{q_2(t)}$
Variance	$\sigma_1(t) = \sum_{i=1}^{t} (i - \mu_1(t))^2\,\frac{P(i)}{q_1(t)}$	$\sigma_2(t) = \sum_{i=t+1}^{I} (i - \mu_2(t))^2\,\frac{P(i)}{q_2(t)}$

[a] Cumulative Density Function

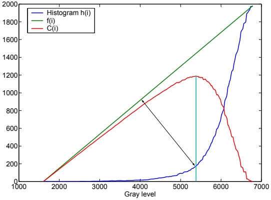
Rosin's method

In cases when the number of pixel in the classes are highly unbalanced (1:100 or even more) could Rosin's method Rosin (2001)

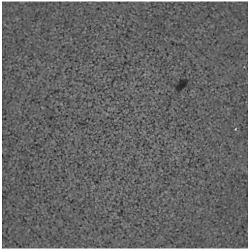


Algorithm

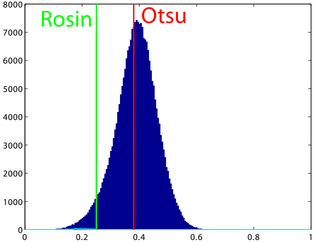
- Draw a line from histogram max to the end of the tail.
- Compute distance from line to histogram curve.
- Select the threshold at distance max.



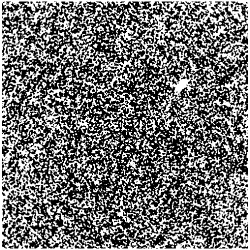
Example images Rosin



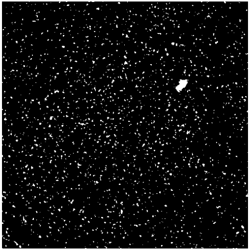
Original image



Histogram



Threshold by Otsu



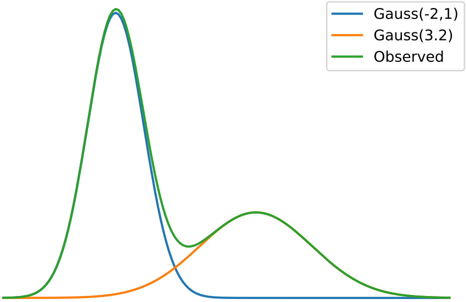
Threshold by Rosin

Neighborhood based segmentation

Motivation

A problem with low SNR data is the amount of miss-classified pixels

- The intensity distributions overlap.
- It is difficult to find a threshold.



Solution: Introduce region growing.

Region growing

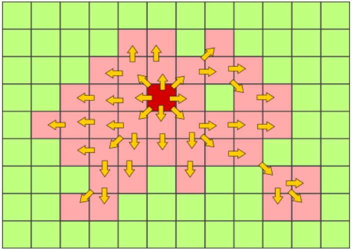
Principle

Region growing starts from a given pixel or region. The region grows until

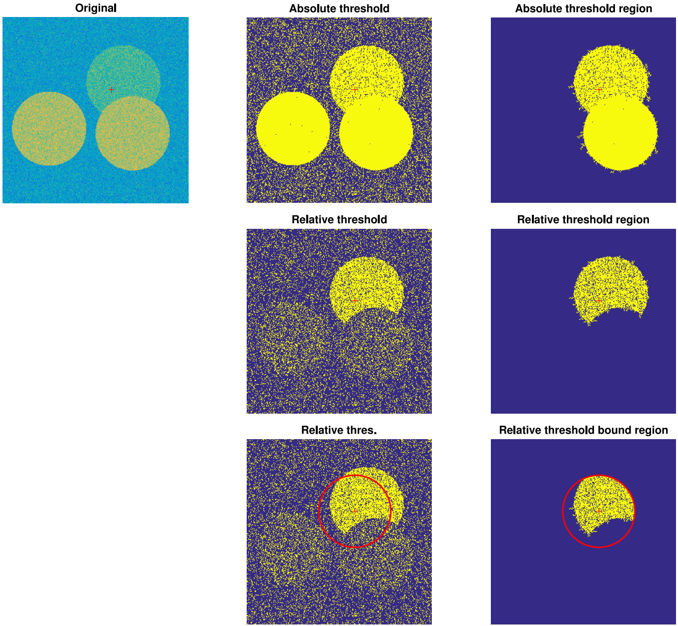
- An absolute threshold intensity is reached.
- The intensity difference reach a threshold value.
- A distance is exceeded, usually combined with intensity thresholds.

This is done by:

- Considering the pixel neighborhood.
- Both unsupervised and supervised.
- Must not necessarily be histogram based.



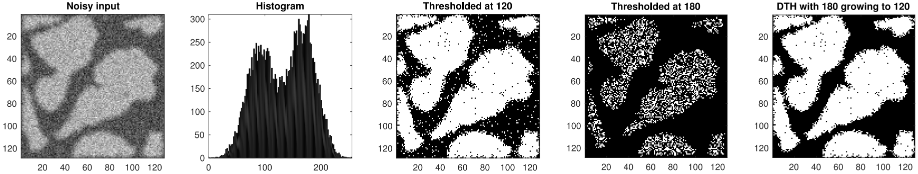
Variations of region growing



Threshold with hysteresis

- It is hard to classify data with overlapping class distributions.
- A single threshold either under- or over-segments data
- Combine two thresholds with region growing
  - Set high threshold as seed
  - Perform region growing until lower threshold.

Example



Algorithm

This method is defined by the following steps:

- Compute $\hat{x} = \{x|h(x) = H(min)\}$,
  where $h(x)$ is a Gaussian fitted to the upper part of the histogram $H(x)$
- Find thresholds: $t_{min} = \frac{1}{2}(max_1 + min)$ and $t_{min} = \frac{1}{2}(min + \hat{x})$
- Threshold in two steps

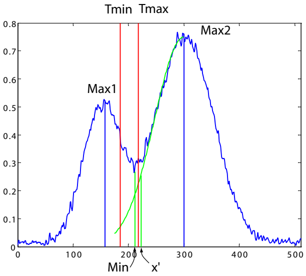
Global pixel-wise threshold

$$f(\mathbf{x}) = \begin{cases} 0 & f(\mathbf{x}) < t_{min} \\ f(\mathbf{x}) & \text{otherwise} \end{cases} \tag{3}$$

Region growing (item edge pixels)

$$f(\mathbf{y}) = \begin{cases} 0 & f(\mathbf{y}) < t_{max} \\ f(\mathbf{y}) & \text{otherwise} \end{cases} \quad \mathbf{y} \in NG_{\mathbf{x}} \tag{4}$$

NG_x is the pixel neighborhood of x



Vogel and Kretzschmar (1996)

Using probability

Likelihood-tests

Compute the likelihood ratio

$$\Lambda(R) = \frac{p_{r|\mathcal{H}_1}(\mathbf{R}|\mathcal{H}_1)}{p_{r|\mathcal{H}_0}(\mathbf{R}|\mathcal{H}_0)} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma \qquad R \in NG_{\mathbf{x}}, \ \forall \mathbf{x} \in \Omega \tag{5}$$

γ is the threshold level

Advantages

- The threshold can be chosen in terms of detection probability.
- The risk for miss-classification can be computed.

Disadvantages

- Require a-priori information on probability distributions of the gray values.
- Might miss small features due local statistics calculations.

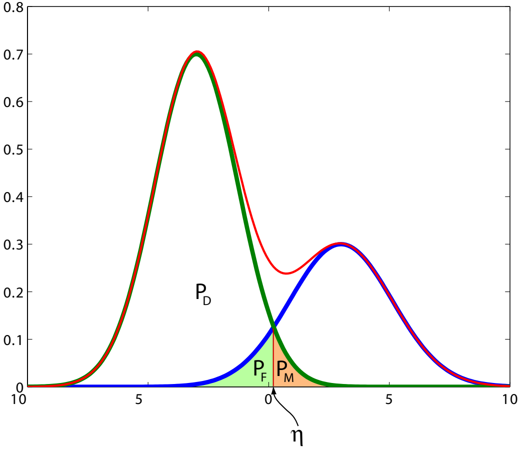
For literature on detection theory see Kay (1998) or van Trees (2001)

The Neyman–Pearson threshold

The threshold η is determined by choosing values for

Probability of false alarm $P_{FA} = P\{t(x) > \gamma' | \mathcal{H}_0\}$

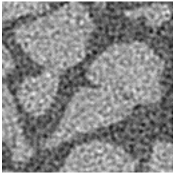
Probability of detection $P_D = P\{t(x) > \gamma' | \mathcal{H}_1\}$



Likelihood ratio

Example – Determine the Likelihood-ratio We have image with gray levels:

$$
\begin{aligned}
H_0 &: \mathcal{N}(0, \sigma) \\
H_1 &: \mathcal{N}(\mu, \sigma)
\end{aligned}
$$



$\Rightarrow$ Likelihood ratio for pixel $\mathbf{x}$

$$\Lambda(R_{\mathbf{x}}) = \frac{\prod_{i \in NG_x} \frac{1}{2\pi\sigma} e^{-(r_i-\mu)^2/2\sigma^2}}{\prod_{i \in NG_x} \frac{1}{2\pi\sigma} e^{-(r_i)^2/2\sigma^2}} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma$$

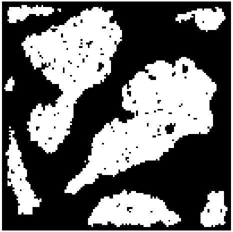
Logarithm and rearrange gives the test variable

$$t(\mathbf{x}) = \underbrace{\frac{1}{N} \sum_{i \in NG_{\mathbf{x}}} r_i}_{\text{Neighborhood sum}} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \underbrace{\frac{\sigma^2}{N\mu} \ln \gamma + \frac{\mu}{2}}_{\text{Constant}} = \gamma'$$

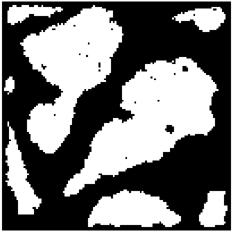
- μ Average of H_1
- σ Standard deviation of the noise
- r_i pixel in neighborhood
- N number of pixels in neighborhood
- γ Threshold level at given confidence level

Likelihood ratio :: Implementation

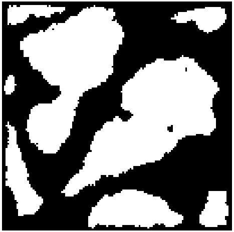
- Compute $t = h * (h^T * f)$ with $h = \mathbf{1}_N$
- Determine the threshold level
  - Neyman–Pearson,
    $P_{FA} = Q\left(\frac{\gamma'}{\sqrt{\sigma^2/N}}\right)$ and $P_D = Q\left(\frac{\gamma'-\mu}{\sqrt{\sigma^2/N}}\right)$ [1]
- Apply the threshold



$P_D = 95\%$



$P_D = 99\%$



$P_D = 99.9\%$

[1] $Q(x) = \frac{1}{2}\mathrm{erf}(x/\sqrt{2})$ computes the error probability for a single measurement lies in $[-x, x)$ with $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$

Bayes threshold

The Bayesian decision is based on the cost function

$$\mathcal{R} = \sum_{i=0}^{1}\sum_{j=0}^{1} C_{ij}\, P(\mathcal{H}_i|\mathcal{H}_j)\, P(\mathcal{H}_j) \tag{6}$$

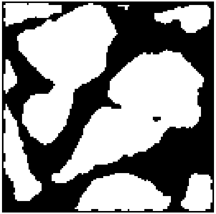
Then the threshold becomes

$$\Lambda(\mathbf{x}) = \underbrace{\frac{p(\mathbf{x}|\mathcal{H}_1)}{p(\mathbf{x}|\mathcal{H}_0)}}_{\text{likelihood ratio}} > \frac{(C_{10}-C_{00})\,P(\mathcal{H}_0)}{(C_{01}-C_{11})\,P(\mathcal{H}_1)} = \gamma \tag{7}$$

Under the assumption that $C_{10} > C_{00}$ and $C_{01} > C_{11}$

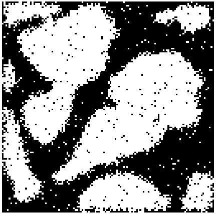
- C_{00} Cost for correct assignment to $\mathcal{H}_0$.
- C_{10} Cost for incorrect assignment to $\mathcal{H}_1$.
- C_{11} Cost for correct assignment to $\mathcal{H}_0$.
- C_{01} Cost for incorrect assignment to $\mathcal{H}_1$.

Comparing Segmentation Methods

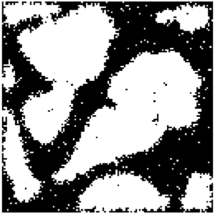


Original

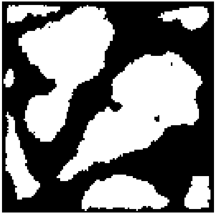
Noisy



Otsu



Hysteresis



Hypothesis testing

Quiz question: Why does hypothesis test do misclassify less?

Multi-class segmentation

Multi-class segmentation

In most experiments samples contain more than two phases.

Example

Porous media

- Matrix
- Fluids
- Void

Anatomy

- Bones
- Different tissues
- Air

Non-destructive testing

- Plastic
- Metals
- Fluids
- Air

Problem

Each phase should have gray levels that are clearly separated and the interfaces should be sharp.
In real data this is rarely true due to noise and unsharness.

Extending known methods to multi-class

- Multi-class segmentation requires more thresholds
- More comparisons
- Ambiguous assignments can occur

Otsu thresholding

Otsu foresaw the segmentation of multiple classes. The handling is described in the paper.

Likelihood ratio

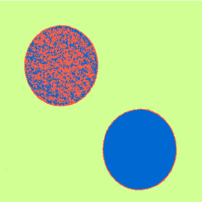
Adding classes requires

- More class statistics.
- One likelihood test per class pair

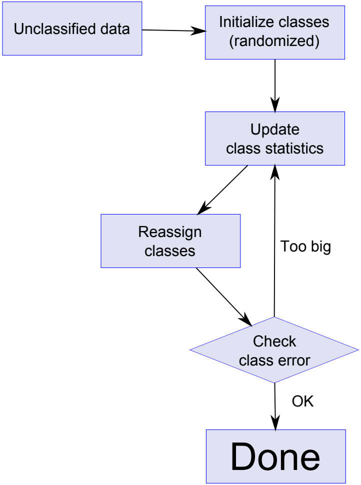
Unsuperwised classification :: Fuzzy C Means

Some features

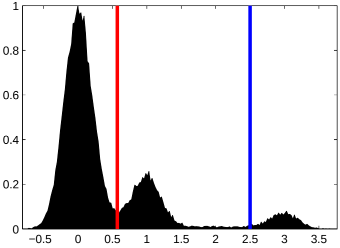
- Multi-class data
- Iterative method
- Pixelwise operations
- Global optimization
- Unsupervised
- No a priori information needed

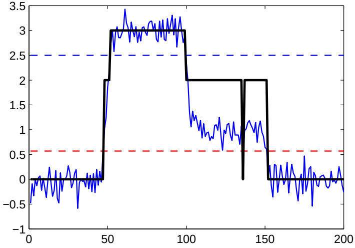


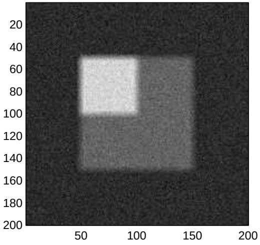
Algorithm

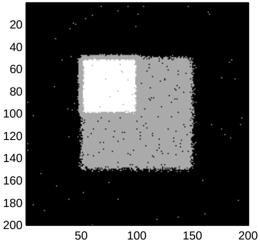


Problems with multi-phase segmentation

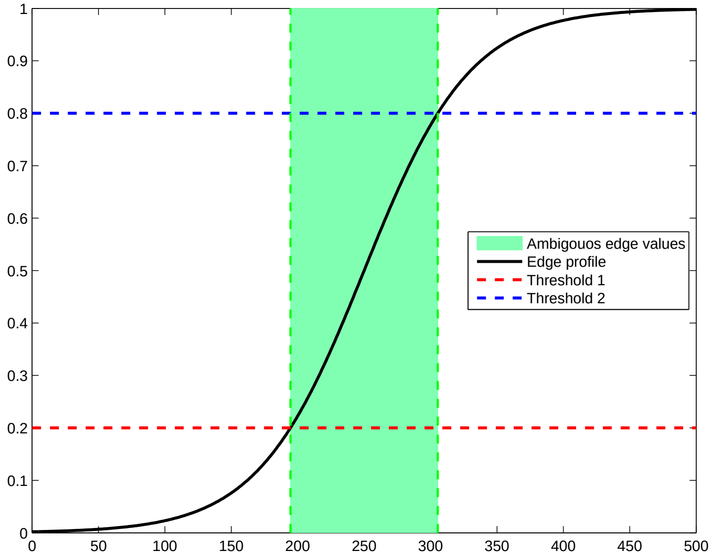








Looking closer on an edge

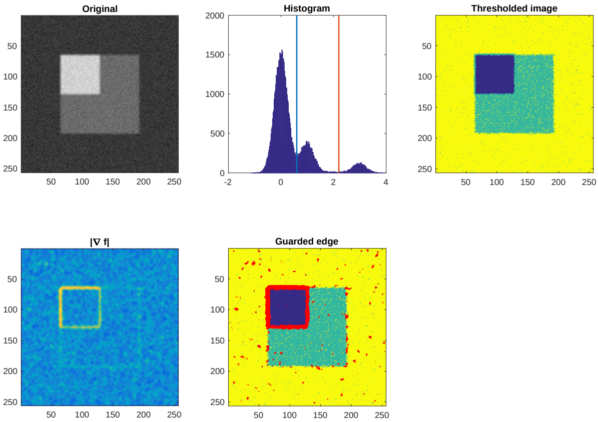


ETH zürich

Segmentation with edge guard

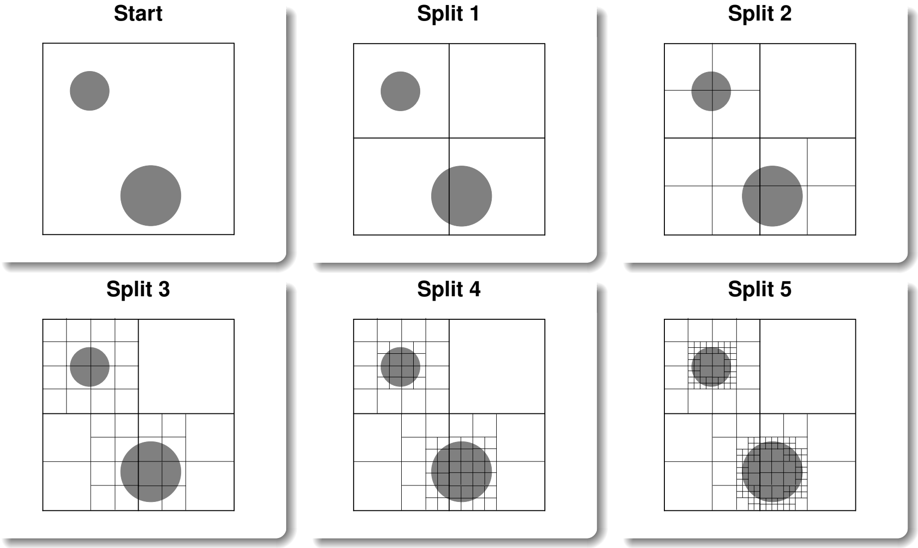
PAUL SCHERRER INSTITUT PSI

- Use a thresholded LoG image as edge mask m
- Apply segmentation to pixels $\notin m \rightarrow$ raw segmented image s
- Region growing with class assignment from item edge pixels in c

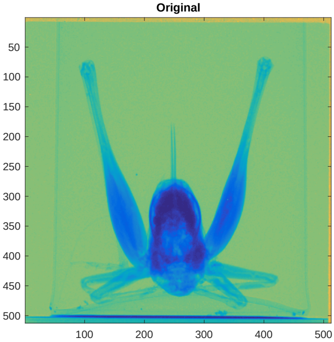


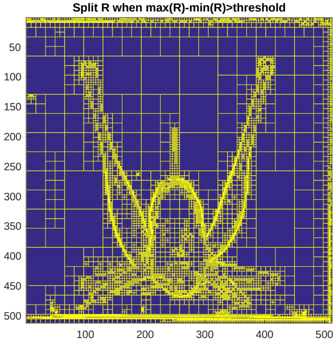
Quad trees

An iterative approach splitting regions when they don’t fulfill a some criterion.



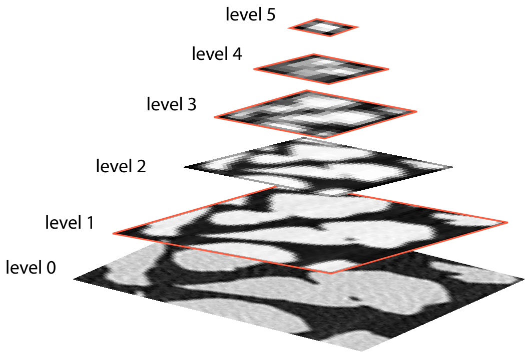
Quad tree decomposition





Scales

Work with multiple resolutions



Purpose:

- Less sensitive to noise
- Faster processing

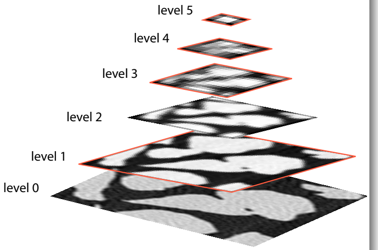
Burt's pyramid

A region based algorithm working on different scales.
It

- Works iteratively
- Interprets the scale pyramid as a tree
- Re-links the tree edges depending on parent relations
- Automatically determines the number of classes

Pyramid Based Segmentation

Algorithm for Burt's Segmentation



Initialization Build the pyramid

Step 1 Relink

- Bottom-up
- Link child to father with smallest difference

Step 2 Update levels

- Bottom-up
- Compute average of linked children, assign to father

Step 3 Update segments

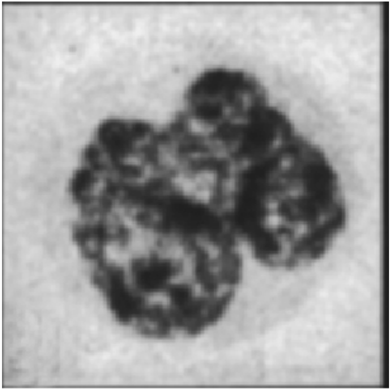
- Top-down
- Assign segment of father to all linked children

Iterate step 1–3 until stability

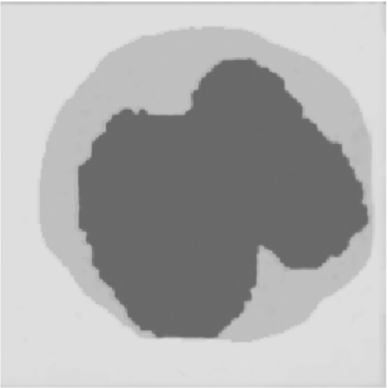
Burt et al. (1981)

Example: Pyramid Based Segmentation (2D)

Original

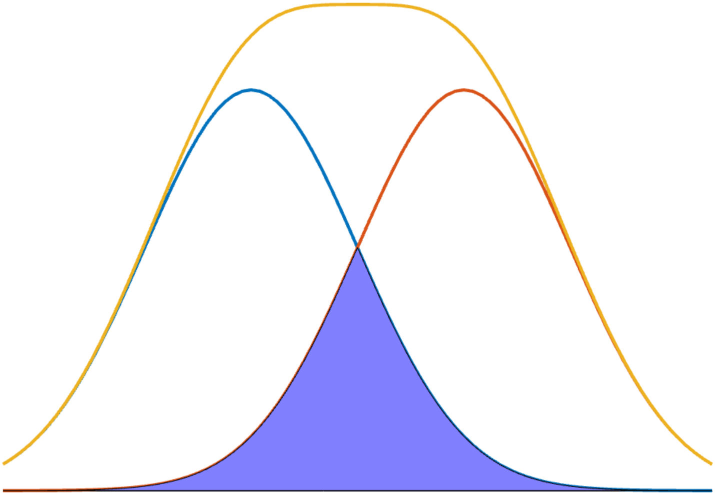


Segmented



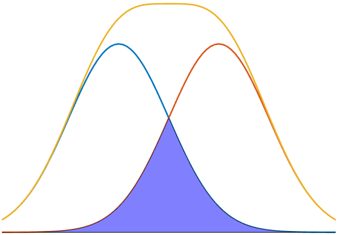
Bivariate segmentation

Single modality histogram

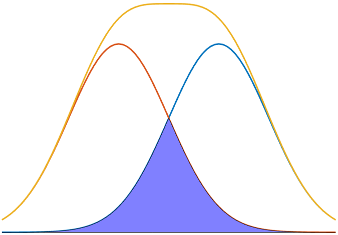


Two modalities separate

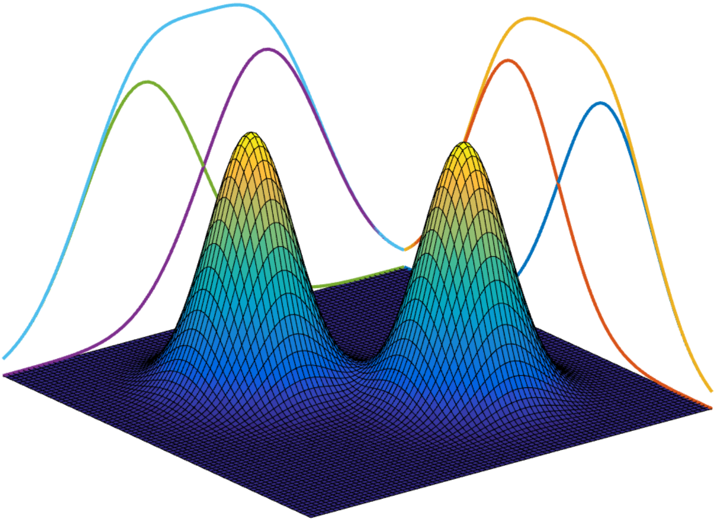
Modality A



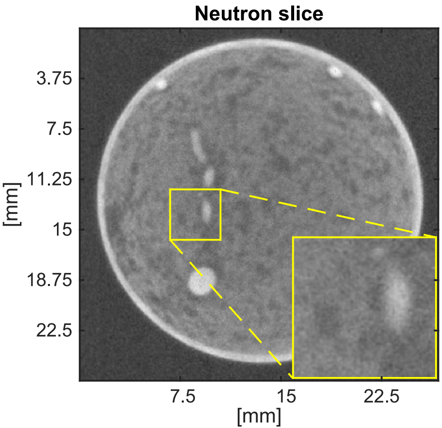
Modality B

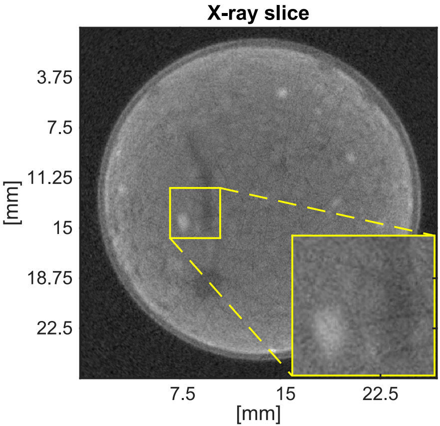


Bivariate histogram

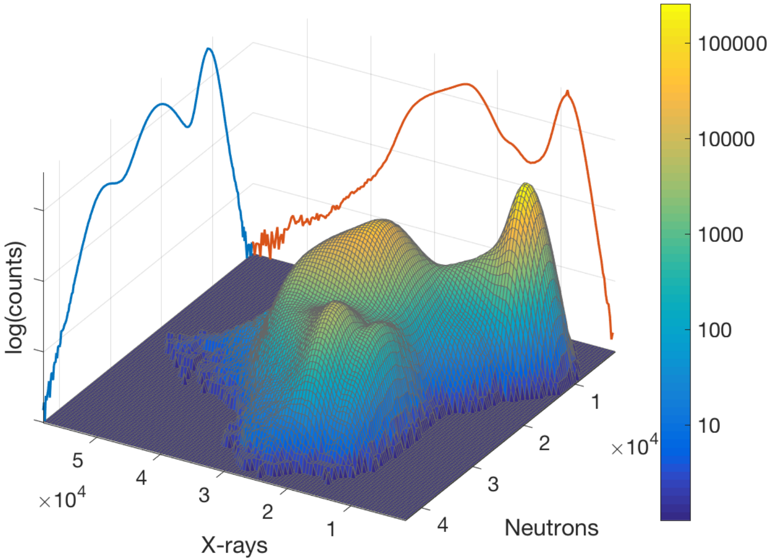


Example: Roots in soil





Bivariate histogram of roots



Classification methods

Data

- Images from M modalities $f_1, \ldots, f_M$
- Registered

Classes

The N classes are described by:

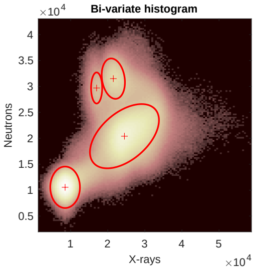
$$
\begin{aligned}
\mathcal{H}_1 &: p(\boldsymbol{\mu_1}, \boldsymbol{\sigma_1}) \\
\mathcal{H}_2 &: p(\boldsymbol{\mu_2}, \boldsymbol{\sigma_2}) \\
&\vdots \\
\mathcal{H}_N &: p(\boldsymbol{\mu_N}, \boldsymbol{\sigma_N})
\end{aligned}
$$

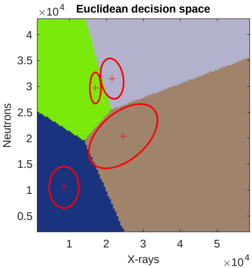
Classifier options

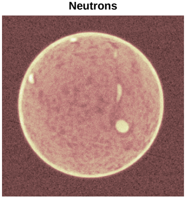
- Smallest Euclidean class distance (sample mean)
- Multivariate class distances (sample mean and covariance)
- Non-linear discrimination → Machine learning

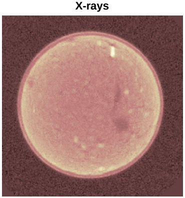
Duda et al. (2001)

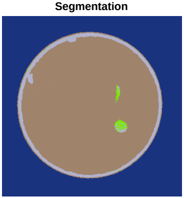
Segmentation by Euclidean distance











?

Summary

We have talked about:

- Thresholds
- Threshold estimation
- Classification methods

References I

Burt, P. J., Hong, T.-H., and Rosenfeld, A. (1981). Segmentation and estimation of image region properties through cooperative hierarchial computation. *IEEE transactions on Systems, Man, and Cybernetics*, SMC-11(12):802–809.

Duda, R., Hart, P., and Stork, D. (2001). *Pattern classification*. John Wiley & Sons, New York, 2 edition.

Kay, S. (1998). *Fundamentals of statistical signal processing: Detection theory*, volume 2. Prentice Hall.

Otsu, N. (1979). A threshold selection method from gray-level histograms. *IEEE Trans. Syst., Man, Cybern.*, 9(1):62–66.

Rosin, P. L. (2001). Unimodal thresholding. *Pattern Recognition*, 34(11):2083–2096.

van Trees, H. (2001). *Detection, estimation and modulation theory, part I*. Academic press.

Vogel, H. and Kretzschmar, A. (1996). Topological characterization of pore space in soil – sample preparation and digital image-processing. *Geoderma*, 73(1–2):23–38.