

Segmenting images

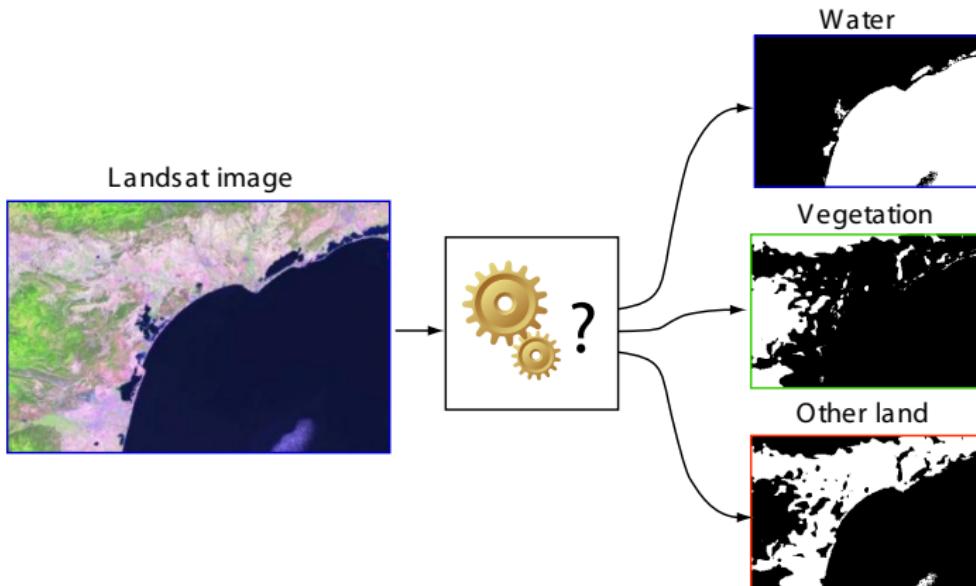
Anders Kaestner :: Laboratory for Neutron Scattering and Imaging



1 Defining segmentation**2 Neighborhood based segmentation****3 Using probability****4 Multi-class segmentation****5 Bivariate segmentation**

Segmentation is the process to convert the pixels in an image into a limited (small) number of classes depending on:

- The histogram of the image
- A-priori knowledge of the statistics in the image
- Neighborhood information



Classification

Identify regions based on characteristic properties

- Intensity
- Color
- Texture

Labelling

Identify individual items, often requires classified image as input

Unsupervised methods

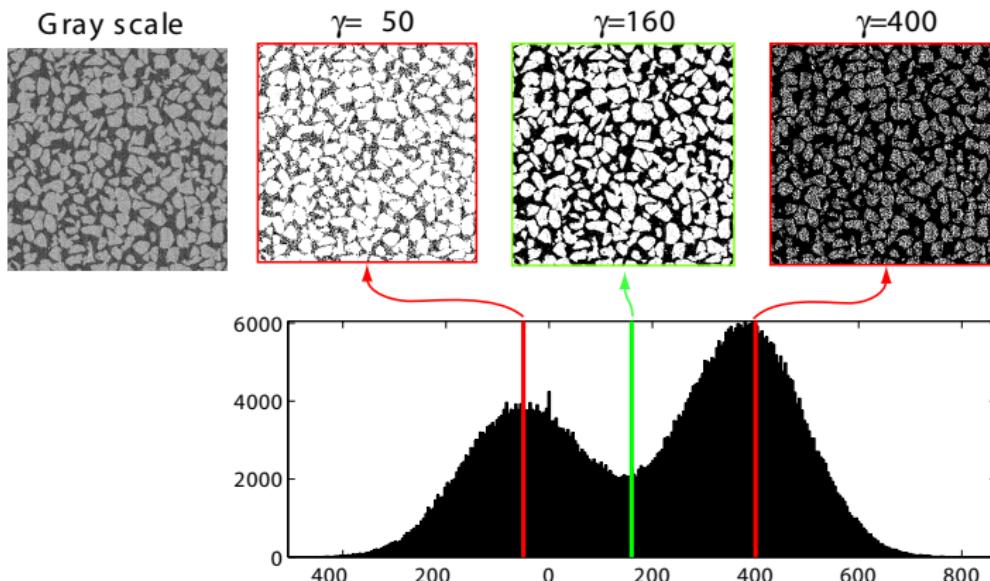
The algorithm can find the regions without human interaction

Supervised methods

The algorithm needs the input from a human as seed for the processing

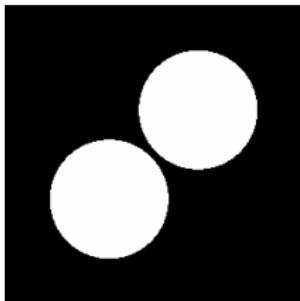
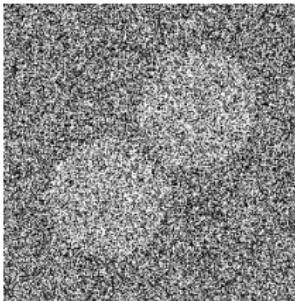
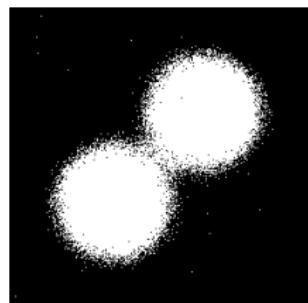
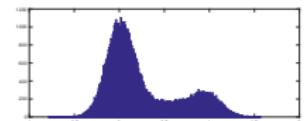
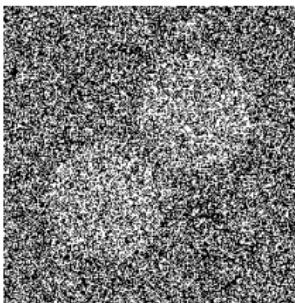
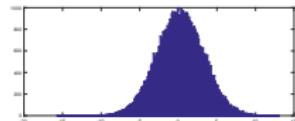
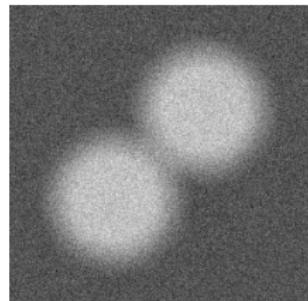
Thresholding an image: Compare pixel value against a constant value

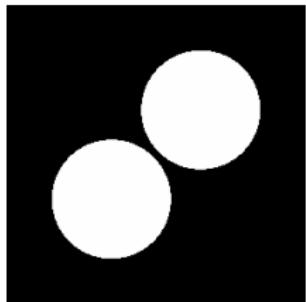
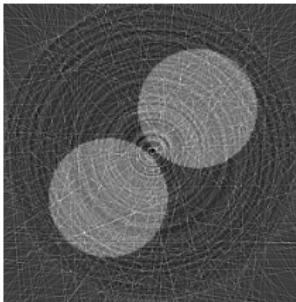
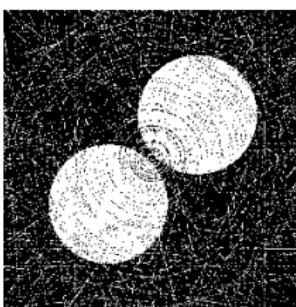
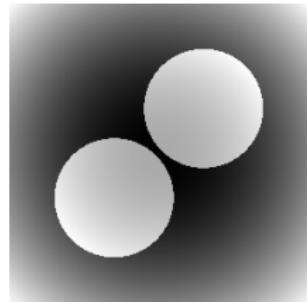
$$g(x) = \begin{cases} 1 & f(x) \geq \gamma \\ 0 & \text{otherwise} \end{cases} \quad \forall x \in \Omega$$



The question now is which threshold value to choose...

Sezgin and Sankur (2004)

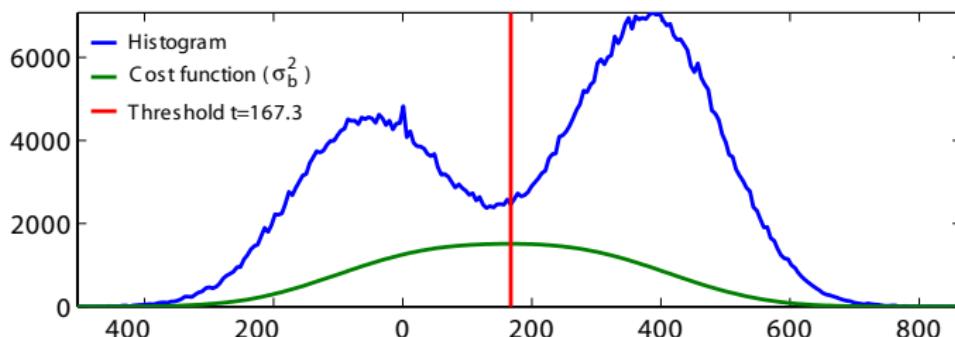
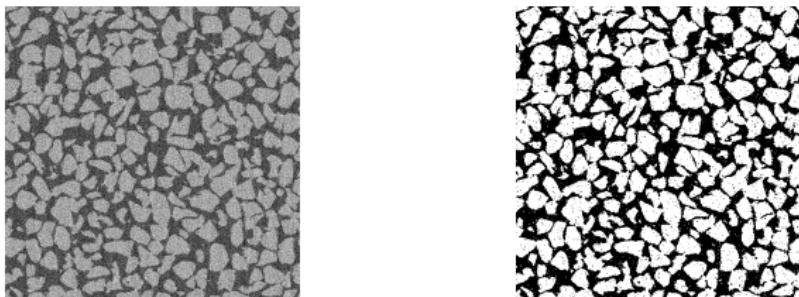
Ideally**Noise****Unsharpness**

Ideally**Artifacts****Gradients**

Histogram-based segmentation

- Use the information provided by the histogram to find a threshold value
- Often work pixelwise → is fast
- Can result in many miss-classified pixels

A classic algorithm to find a threshold was introduced by Otsu.



Otsu (1979)

Algorithm

Find the t that minimize *in-class* variance

$$\sigma_w^2(t) = q_1(t) \sigma_1^2(t) + q_2(t) \sigma_2^2(t) \quad (1)$$

or that maximize the equivalent *between-class* variance

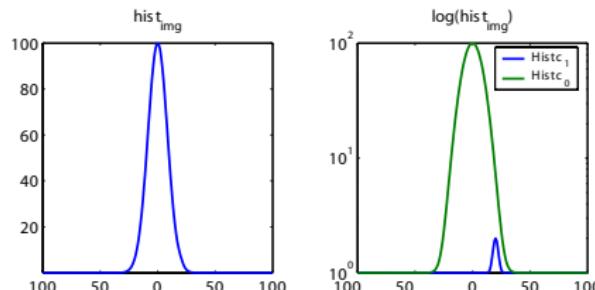
$$\sigma_b^2(t) = q_1(t)(1 - q_2(t))(\mu_1(t) - \mu_2(t))^2 \quad (2)$$

where P is a histogram with N bins

	<i>Class 1</i>	<i>Class 2</i>
CDF^a	$q_1(t) = \sum_{i=1}^t P(i)$	$q_2(t) = \sum_{i=t+1}^N P(i)$
$Mean$	$\mu_1(t) = \sum_{i=1}^t i \frac{P(i)}{q_1(t)}$	$\mu_2(t) = \sum_{i=t+1}^N i \frac{P(i)}{q_2(t)}$
$Variance$	$\sigma_1(t) = \sum_{i=1}^t (i - \mu_1(t))^2 \frac{P(i)}{q_1(t)}$	$\sigma_2(t) = \sum_{i=t+1}^N (i - \mu_2(t))^2 \frac{P(i)}{q_2(t)}$

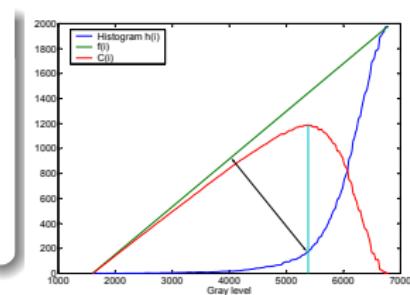
^aCumulative Density Function

In cases when the number of pixel in the classes are highly unbalanced (1:100 or even more) could Rosin's method



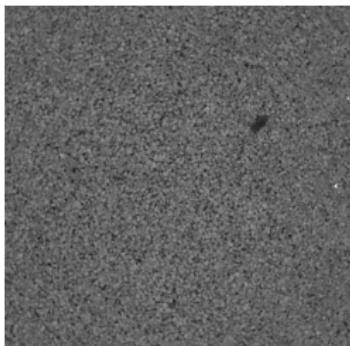
Algorithm

- Draw a line from histogram max to the end of the tail.
- Compute distance from line to histogram curve.
- Select the threshold at distance max.

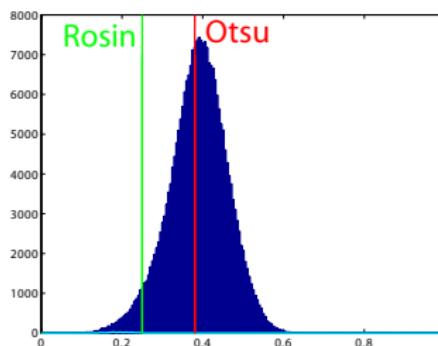


Rosin (2001)

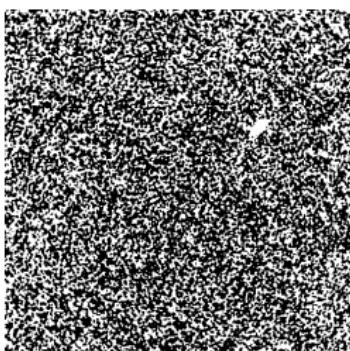
Example images Rosin



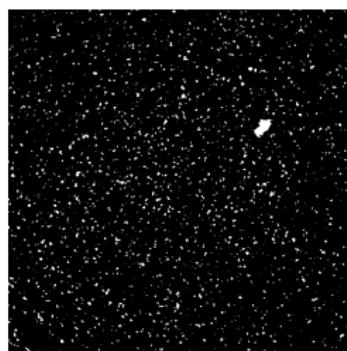
Original image



Histogram



Threshold by Otsu

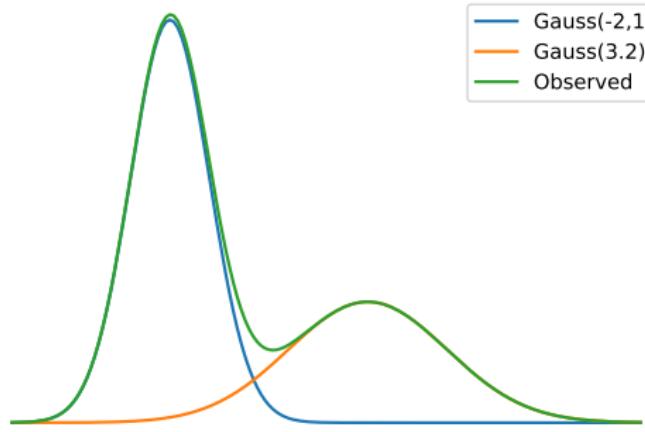


Threshold by Rosin

Neighborhood based segmentation

A problem with low SNR data is the amount of miss-classified pixels

- The intensity distributions overlap.
- It is difficult to find a single threshold.



Solution

Introduce region growing in the thresholding process.

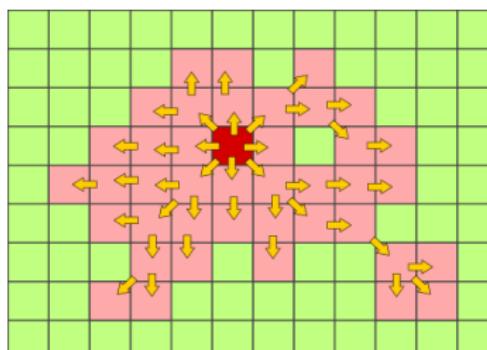
Principle

Region growing starts from a given pixel or region. The region grows until

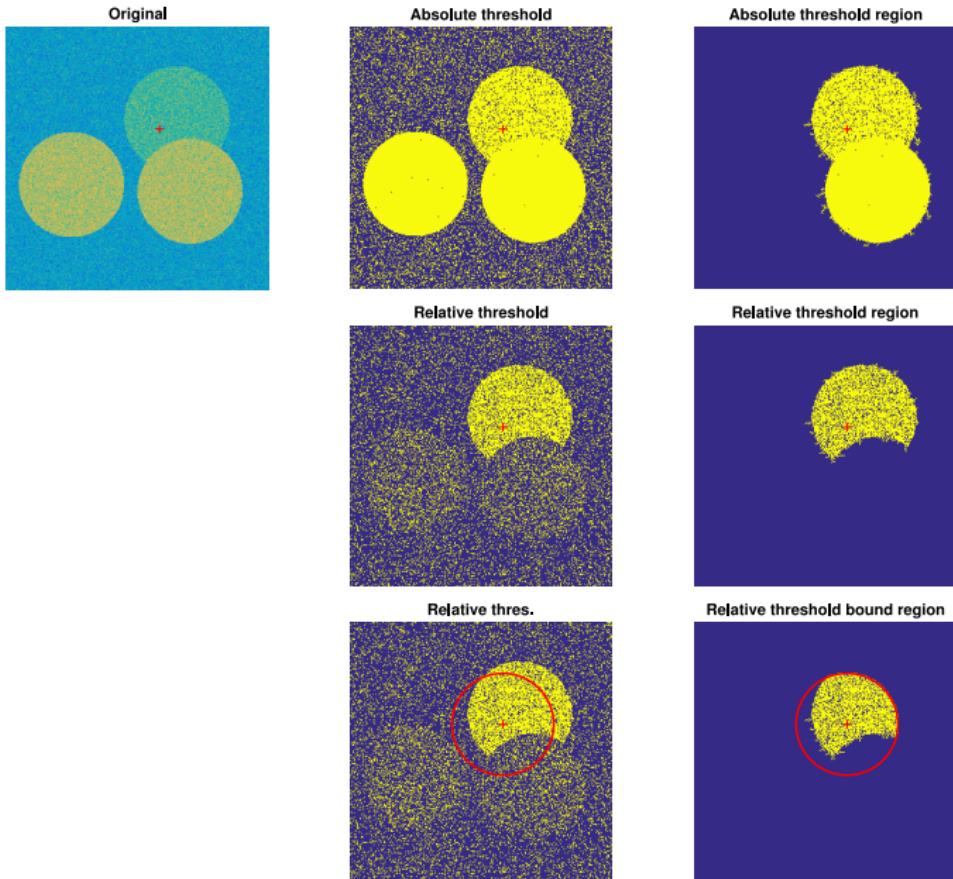
- An absolute threshold intensity is reached.
- The intensity difference reach a threshold value.
- A distance is exceeded, usually combined with intensity thresholds.

This is done by:

- Considering the pixel neighborhood.
- Both unsupervised and supervised.
- Must not necessarily be histogram based.

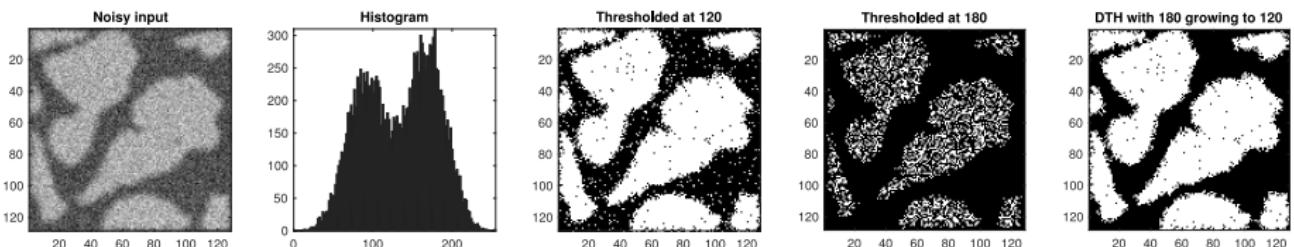


Variations of region growing



- It is hard to classify data with overlapping class distributions.
- A single threshold either under- or over-segments data
- Combine two thresholds with region growing
 - Set high threshold as seed
 - Perform region growing until lower threshold.

Example



Algorithm

This method is defined by the following steps:

- Compute $\hat{x} = \{x | h(x) = H(\min)\}$,
where $h(x)$ is a Gaussian fitted to the upper part of the histogram $H(x)$
- Find thresholds: $t_{min} = \frac{1}{2}(max_1 + min)$ and $t_{max} = \frac{1}{2}(min + \hat{x})$
- Threshold in two steps

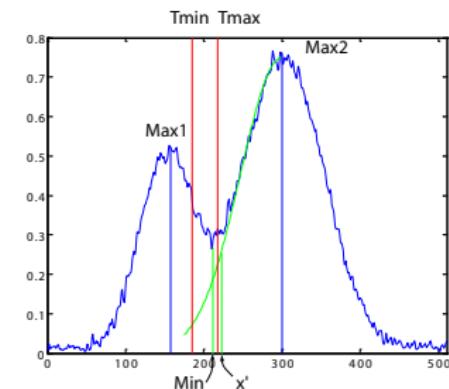
Global pixel-wise threshold

$$f(x) = \begin{cases} 0 & f(x) < t_{min} \\ f(x) & \text{otherwise} \end{cases} \quad (3)$$

Region growing (item edge pixels)

$$f(y) = \begin{cases} 0 & f(y) < t_{max} \\ f(y) & \text{otherwise} \end{cases} \quad y \in NG_x \quad (4)$$

NG_x is the neighborhood of pixel x .



Vogel and Kretzschmar (1996)

Using probability

Compute the likelihood ratio

$$\Lambda(R) = \frac{p_{r|\mathcal{H}_1}(\mathbf{R}|\mathcal{H}_1)}{p_{r|\mathcal{H}_0}(\mathbf{R}|\mathcal{H}_0)} \stackrel{\mathcal{H}_1}{\underset{\mathcal{H}_0}{\gtrless}} \gamma \quad R \in NG_x, \forall x \in \Omega \quad (5)$$

γ is the threshold level

Advantages

- The threshold can be chosen in terms of detection probability.
- The risk for miss-classification can be computed.

Disadvantages

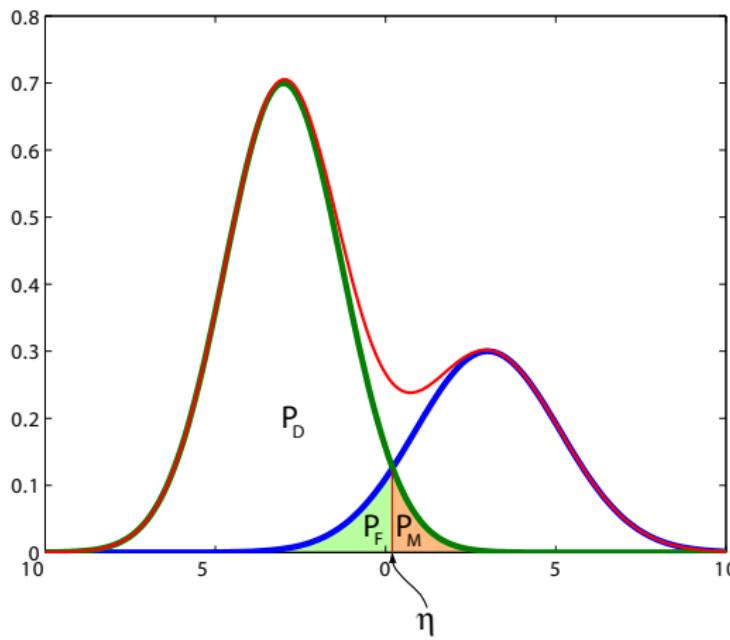
- Require a-priori information on probability distributions of the gray values.
- Might miss small features due local statistics calculations.

For literature on detection theory see Kay (1998) or van Trees (2001)

The threshold η is determined by choosing values for

Probability of false alarm $P_{FA} = P\{t(x) > \gamma' | \mathcal{H}_0\}$

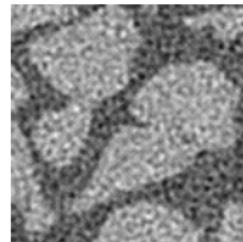
Probability of detection $P_D = P\{t(x) > \gamma' | \mathcal{H}_1\}$



Example – Determine the Likelihood-ratio We have image with gray levels:

$$H_0 : \mathcal{N}(0, \sigma)$$

$$H_1 : \mathcal{N}(\mu, \sigma)$$



⇒ Likelihood ratio for pixel x

$$\Lambda(R_x) = \frac{\prod_{i \in NG_x} \frac{1}{2\pi\sigma} e^{-(r_i - \mu)^2/2\sigma^2}}{\prod_{i \in NG_x} \frac{1}{2\pi\sigma} e^{-(r_i)^2/2\sigma^2}} \stackrel{H_1}{\gtrless} \stackrel{H_0}{\gtrless} \gamma$$

Logarithm and rearrange gives the test variable

$$t(x) = \underbrace{\frac{1}{N} \sum_{i \in NG_x} r_i}_{\text{Neighborhood sum}} \stackrel{H_1}{\gtrless} \underbrace{\frac{\sigma^2}{N\mu} \ln \gamma + \frac{\mu}{2}}_{\text{Constant}} = \gamma'$$

μ Average of H_1

σ Standard deviation of the noise

r_i pixel in neighborhood

N number of pixels in neighborhood

γ Threshold level at given confidence level

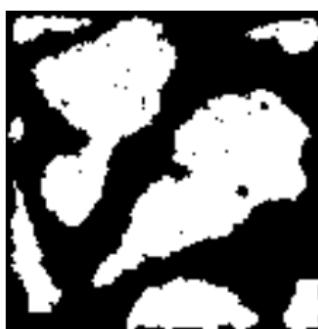
- Compute $t = h * (h^T * f)$ with $h = \mathbf{1}_N$

- Determine the threshold level

- Neyman–Pearson,

$$P_{FA} = Q\left(\frac{\gamma'}{\sqrt{\sigma^2/N}}\right) \text{ and } P_D = Q\left(\frac{\gamma' - \mu}{\sqrt{\sigma^2/N}}\right)^1$$

- Apply the threshold

 $P_D = 95\%$  $P_D = 99\%$  $P_D = 99.9\%$

¹ $Q(x) = \frac{1}{2}\operatorname{erf}(x/\sqrt{2})$ error probability that an observation is in $[-x, x]$, $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$

The Bayesian decision is based on the cost function

$$\mathcal{R} = \sum_{i=0}^1 \sum_{j=0}^1 C_{ij} P(\mathcal{H}_i | \mathcal{H}_j) P(\mathcal{H}_j) \quad (6)$$

Then the threshold becomes

$$\Lambda(x) = \frac{\underbrace{p(x|\mathcal{H}_1)}_{\text{likelihood ratio}}}{p(x|\mathcal{H}_0)} > \frac{(C_{10} - C_{00}) P(\mathcal{H}_0)}{(C_{01} - C_{11}) P(\mathcal{H}_1)} = \gamma \quad (7)$$

Under the assumption that $C_{10} > C_{00}$ and $C_{01} > C_{11}$

C_{00} Cost for correct assignment to \mathcal{H}_0 .

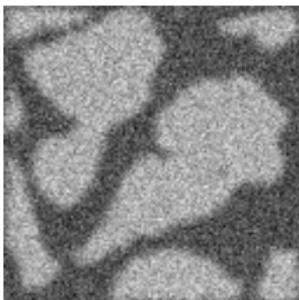
C_{10} Cost for incorrect assignment to \mathcal{H}_1 .

C_{11} Cost for correct assignment to \mathcal{H}_1 .

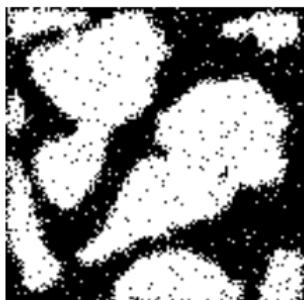
C_{01} Cost for incorrect assignment to \mathcal{H}_0 .



Original



Noisy



Otsu



Hysteresis



Hypothesis testing

Quiz question: Why does hypothesis test do misclassify less?

Multi-class segmentation

In most experiments samples contain more than two phases.

Example

Porous media	Anatomy	Non-destructive testing
<ul style="list-style-type: none">■ Matrix■ Fluids■ Void	<ul style="list-style-type: none">■ Bones■ Different tissues■ Air	<ul style="list-style-type: none">■ Plastic■ Metals■ Fluids■ Air

Problem

Each phase should have gray levels that are clearly separated and the interfaces should be sharp.

In real data this is rarely true due to noise and unsharness.

- Multi-class segmentation requires more thresholds
- More comparisons
- Ambiguous assignments can occur

Otsu thresholding

Otsu foresaw the segmentation of multiple classes. The handling is described in the paper.

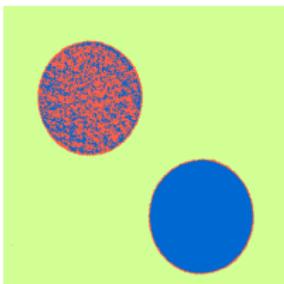
Likelihood ratio

Adding classes requires

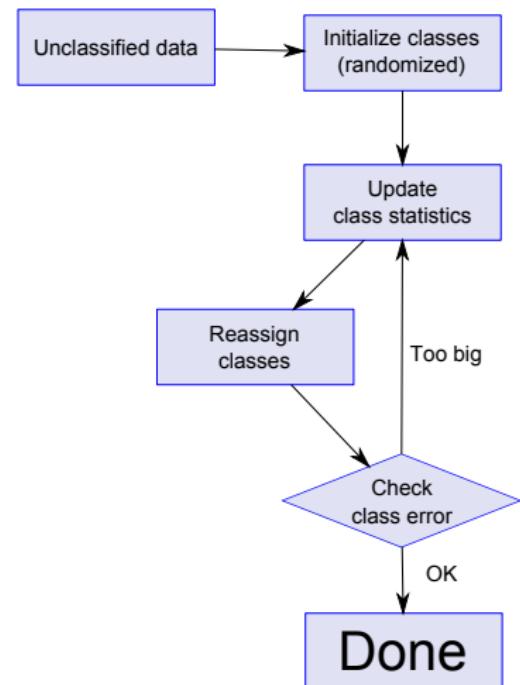
- More class statistics.
- One likelihood test per class pair

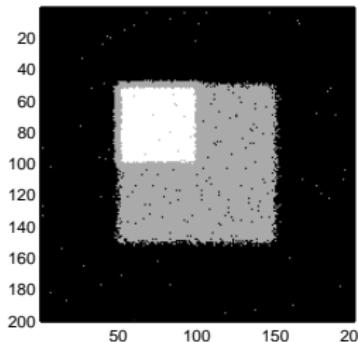
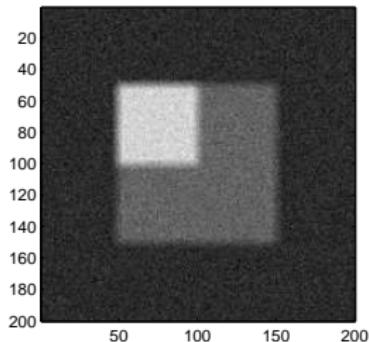
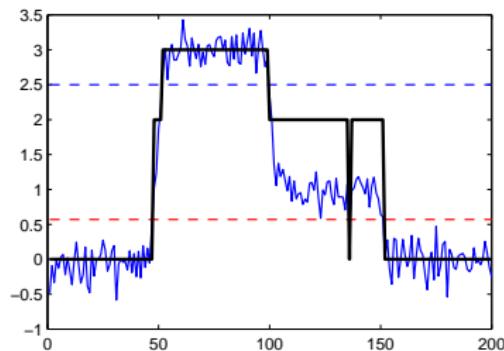
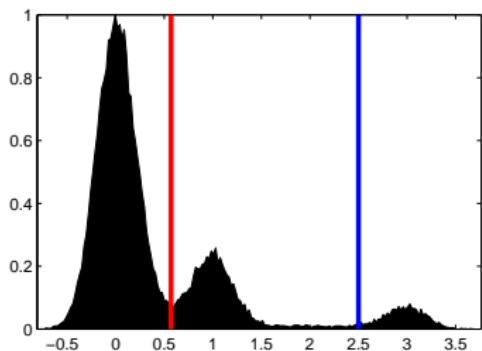
Some features

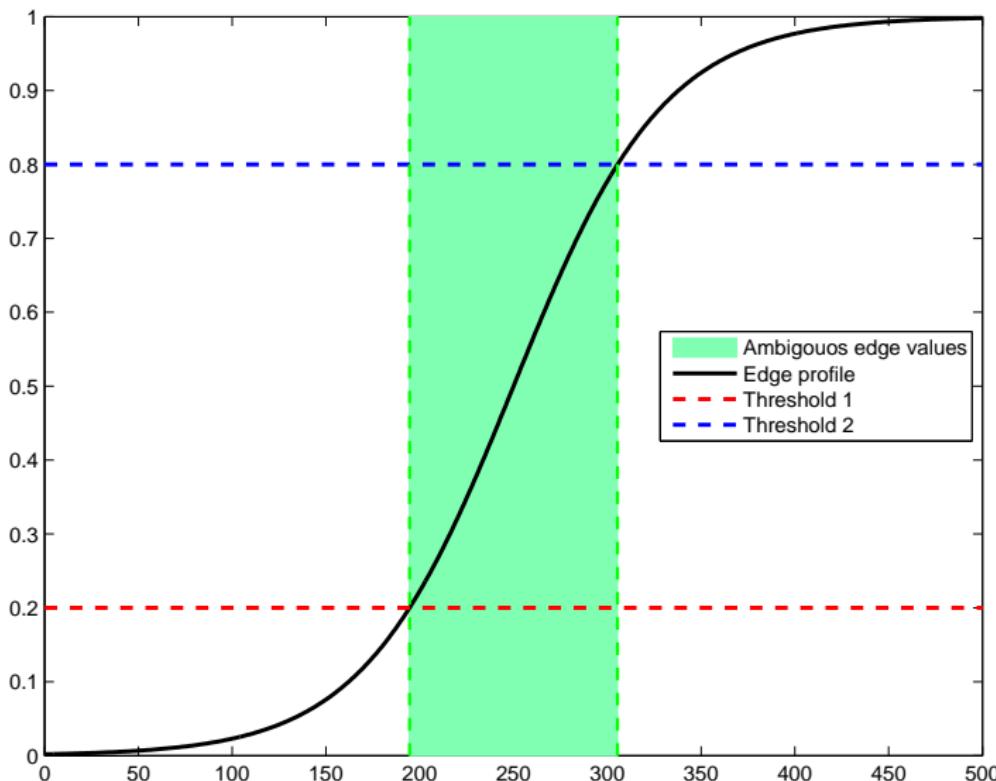
- Multi-class data
- Iterative method
- Pixelwise operations
- Global optimization
- Unsupervised
- No a priori information needed



Algorithm

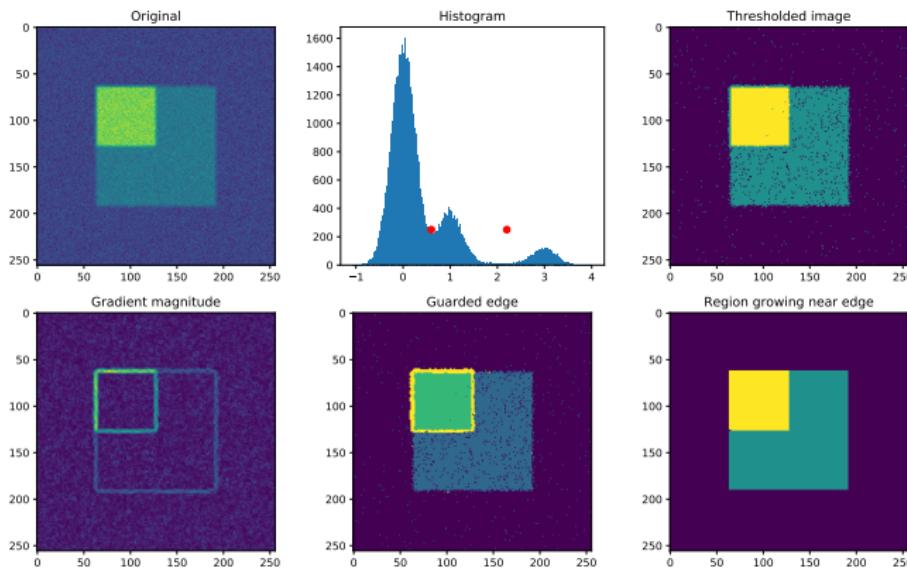






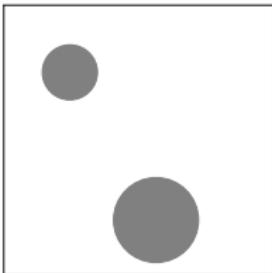
Segmentation with edge guard

- Use a thresholded LoG image as edge mask m
- Apply segmentation to pixels $\notin m \rightarrow$ raw segmented image s
- Region growing with class assignment from item edge pixels in c

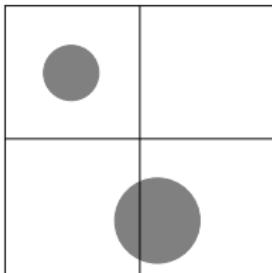


An iterative approach splitting regions when they don't fulfill a some criterion.

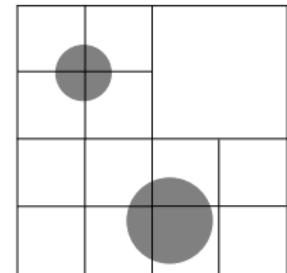
Start



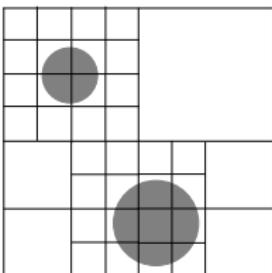
Split 1



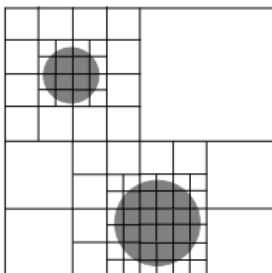
Split 2



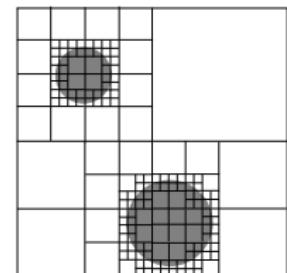
Split 3



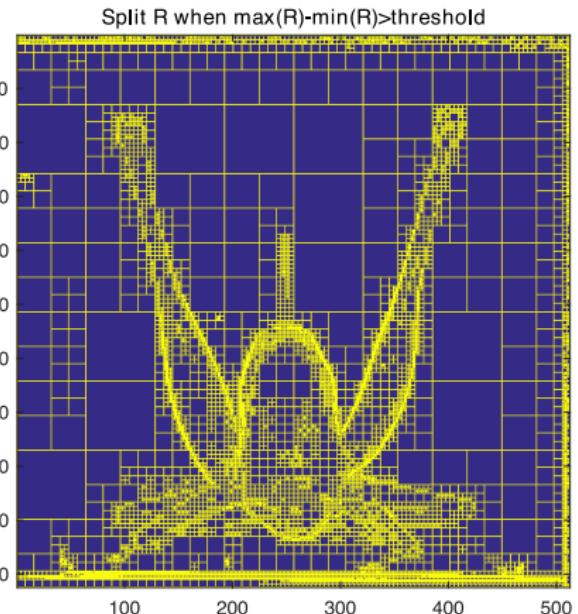
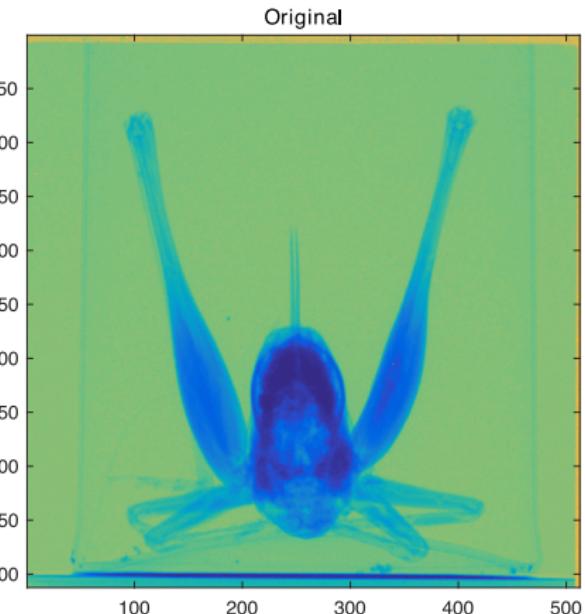
Split 4



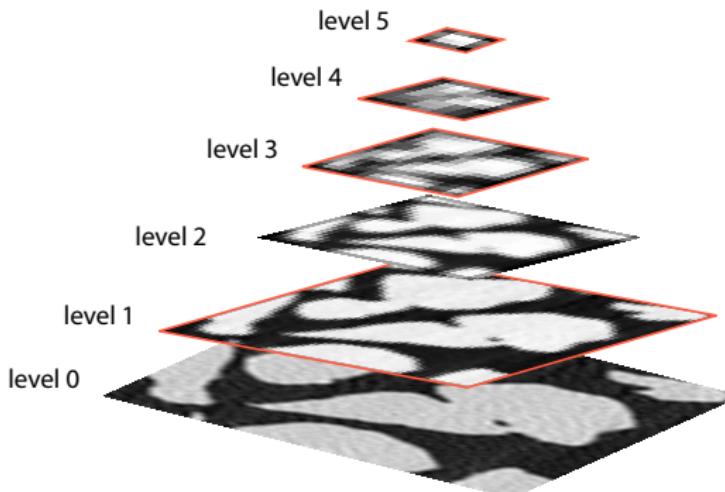
Split 5



Quad tree decomposition



Work with multiple resolutions



Purpose:

- Less sensitive to noise
- Faster processing

Burt's pyramid

A region based algorithm working on different scales.

It

- Works iteratively
- Interprets the scale pyramid as a tree
- Re-links the tree edges depending on parent relations
- Automatically determines the number of classes

Algorithm for Burt's Segmentation

Initialization Build the pyramid

Step 1 Relink

- Bottom-up
- Link child to father with smallest difference

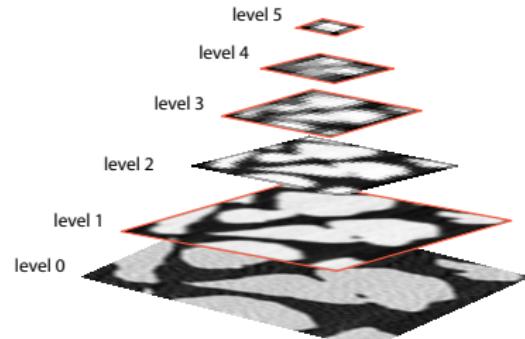
Step 2 Update levels

- Bottom-up
- Compute average of linked children, assign to father

Step 3 Update segments

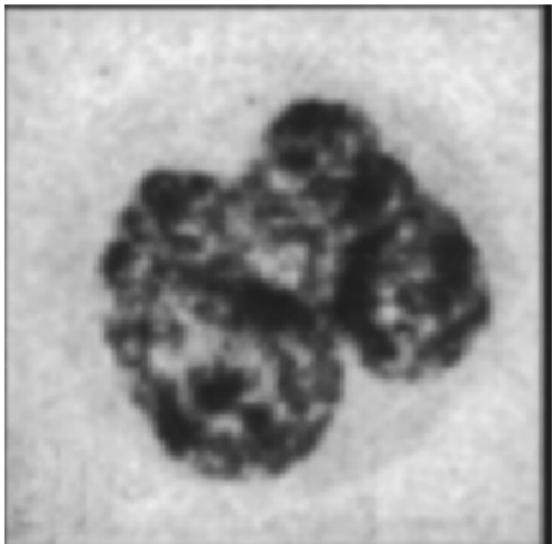
- Top-down
- Assign segment of father to all linked children

Iterate step 1–3 until stability

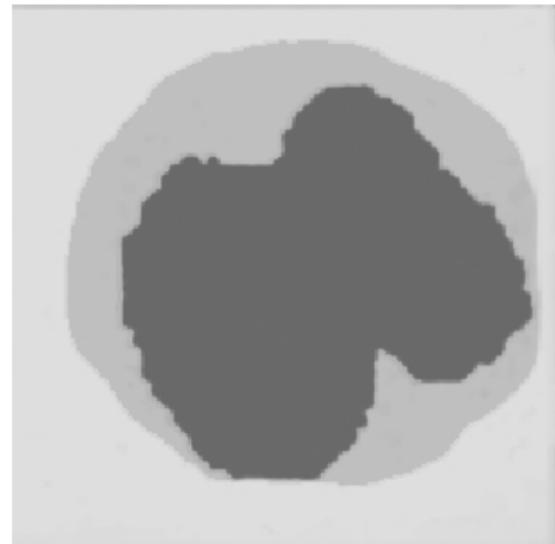


Burt et al. (1981)

Original

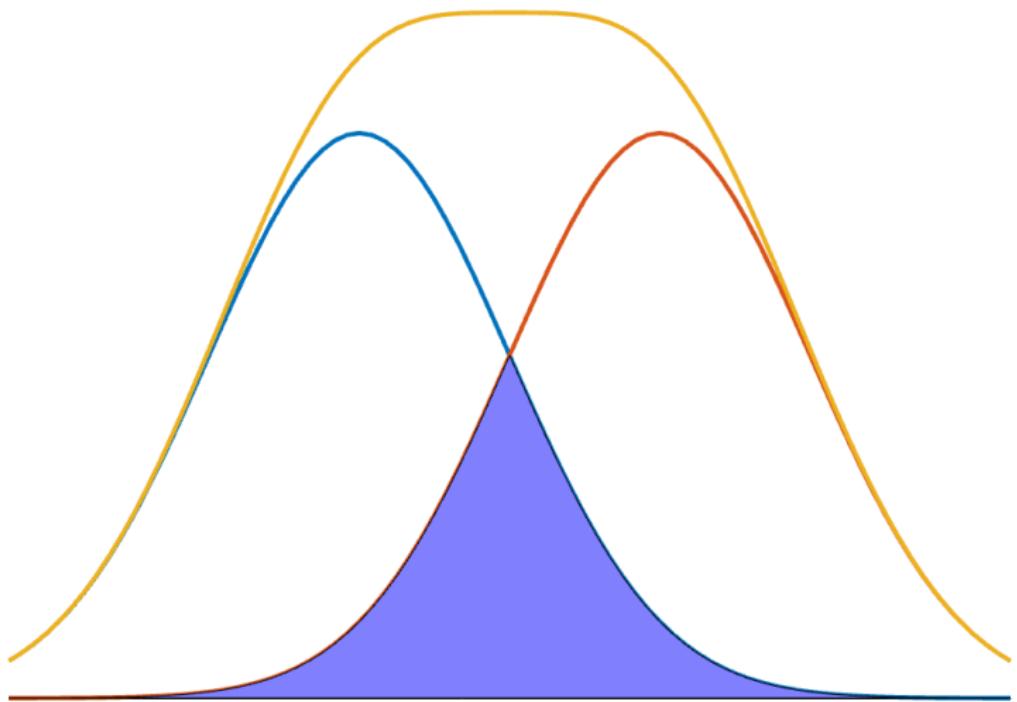


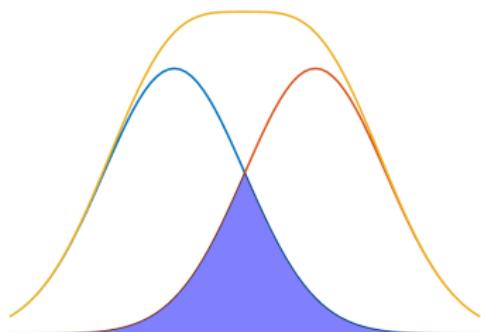
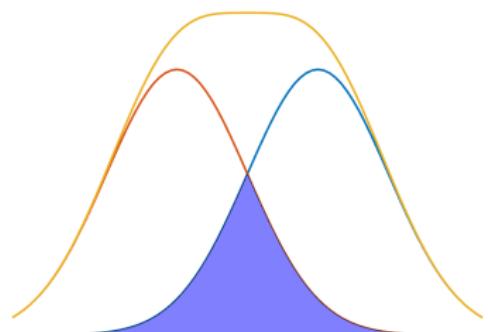
Segmented



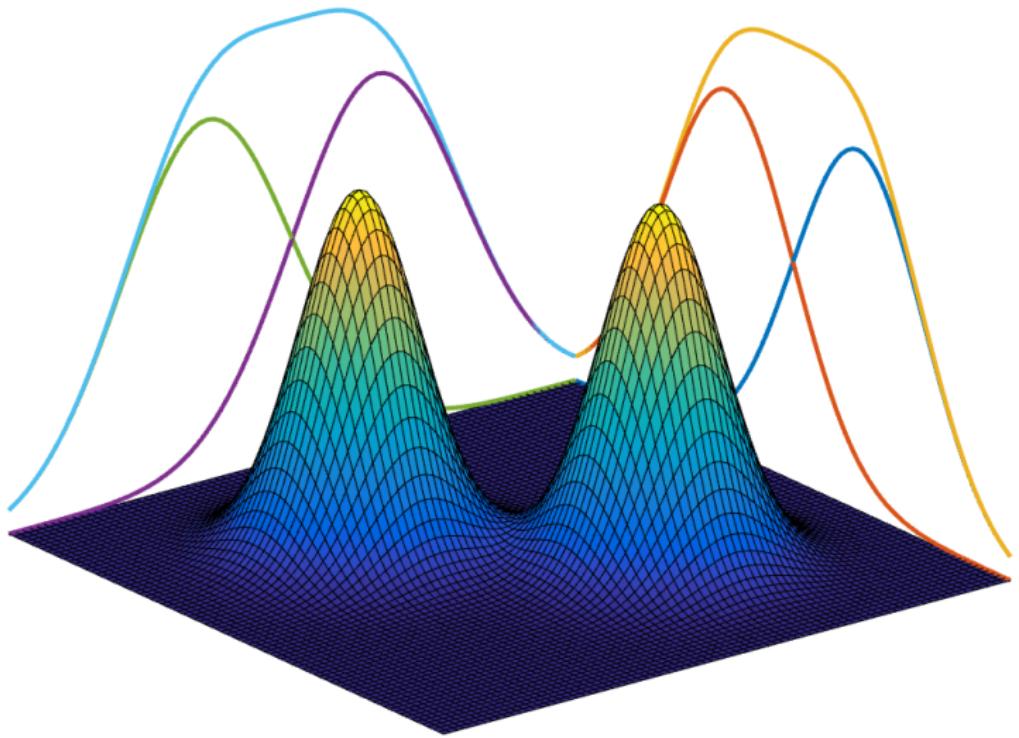
Bivariate segmentation

Single modality histogram

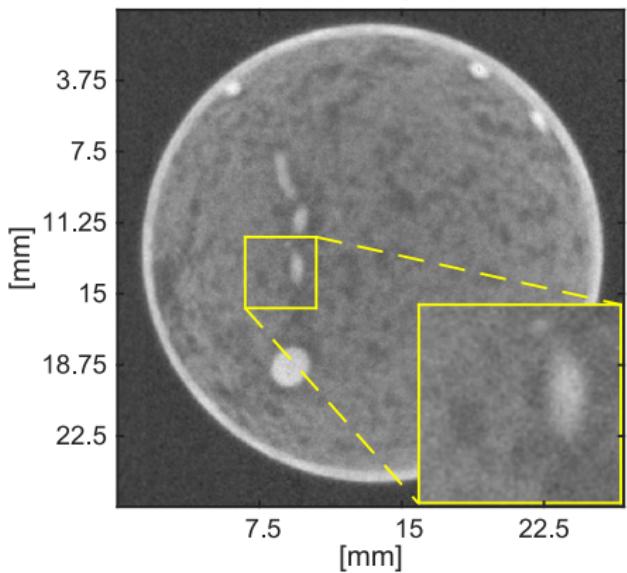
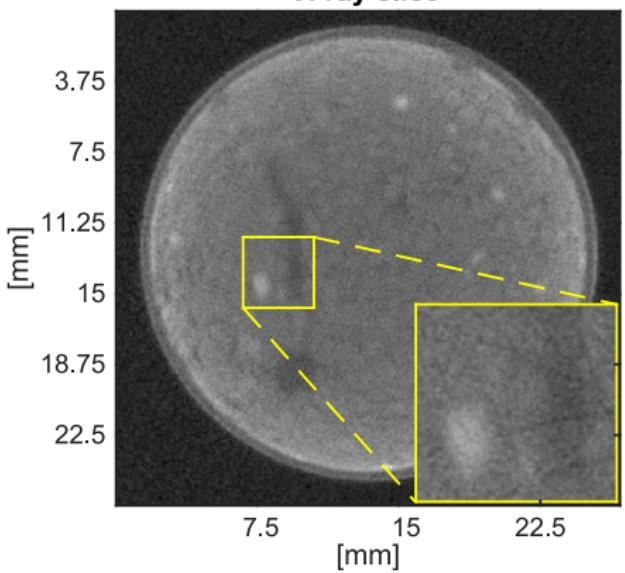


Modality A**Modality B**

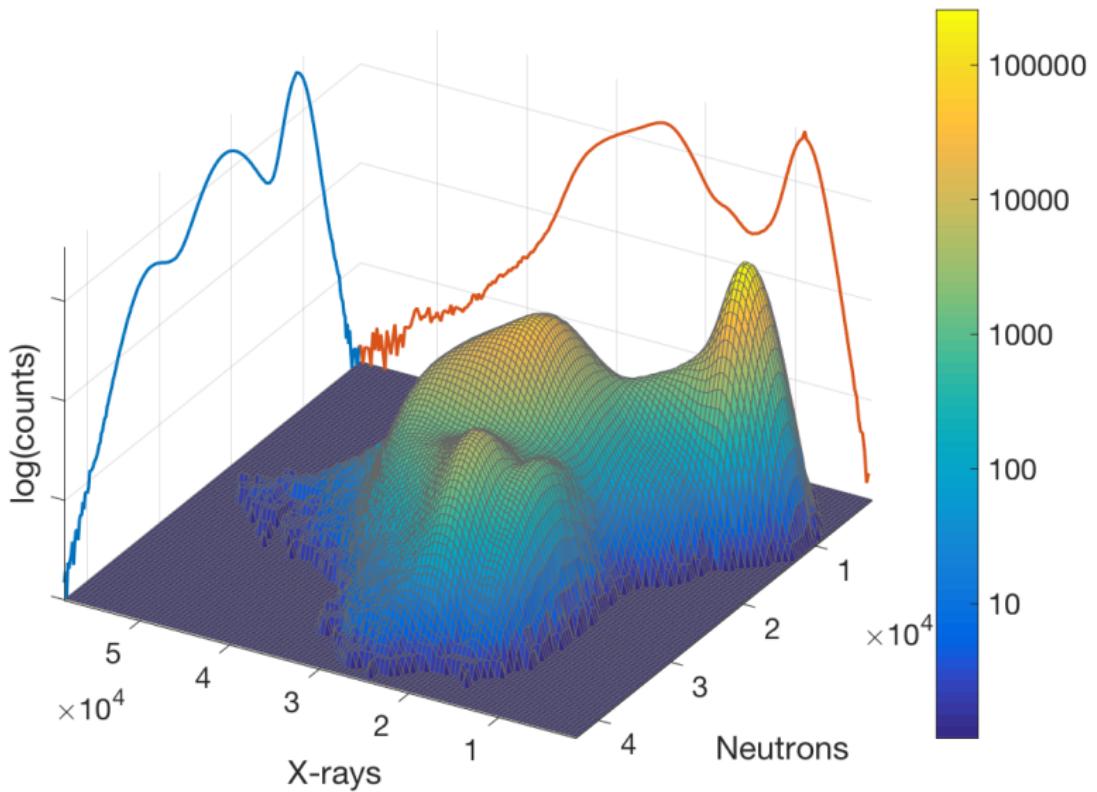
Bivariate histogram



Example: Roots in soil

Neutron slice**X-ray slice**

Bivariate histogram of roots



Data

- Images from M modalities
 f_1, \dots, f_M
- Registered

Classes

The N classes are described by:

$$\mathcal{H}_1 : p(\mu_1, \sigma_1)$$

$$\mathcal{H}_2 : p(\mu_2, \sigma_2)$$

⋮

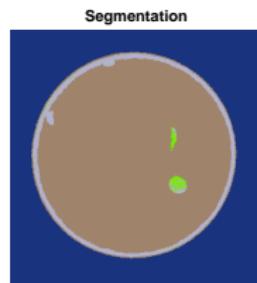
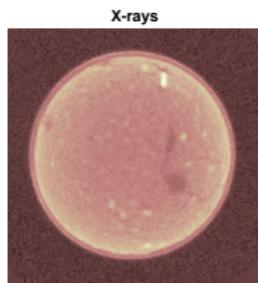
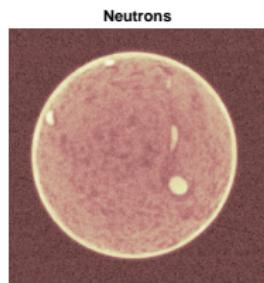
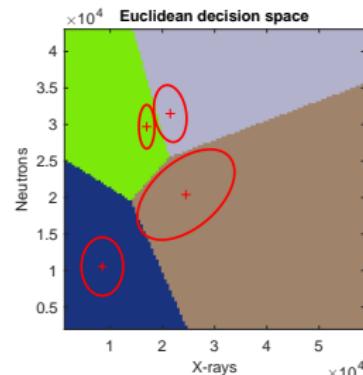
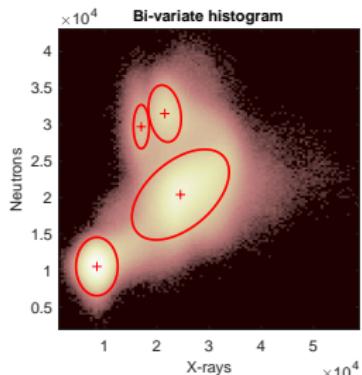
$$\mathcal{H}_N : p(\mu_N, \sigma_N)$$

Classifier options

- Smallest Euclidean class distance (sample mean)
- Multivariate class distances (sample mean and covariance)
- Non-linear discrimination → Machine learning

Duda et al. (2001)

Segmentation by Euclidean distance



Kaestner et al. (2017)

We have talked about:

- Thresholds
- Threshold estimation
- Classification methods

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