Morphological operations

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Outline :: Morphological operations

Distances

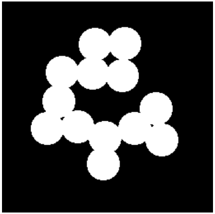
Distance map

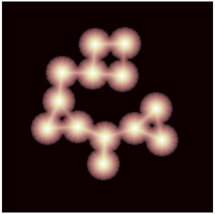
Definition

A *distance map* shows the shortest distance from an object pixel to the object edge.

- A distance map is computed using a distance transform.
- The result of the transform depends on the choice of distance metric

Example



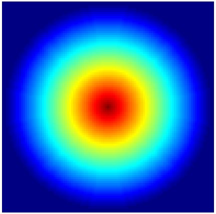


Euclidean distance metrics

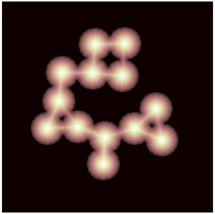
The Euclidean distance is defined as

$$d_{\mathcal{E}}[(x_1, y_1), (x_2, y_2)] = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad (1)$$

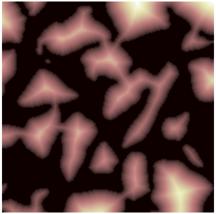
Example



Disk



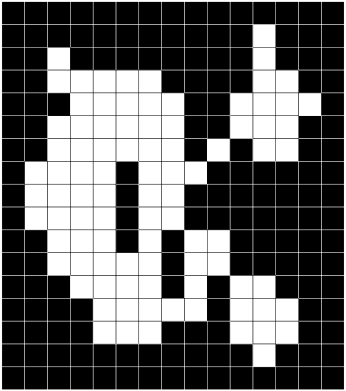
Circles



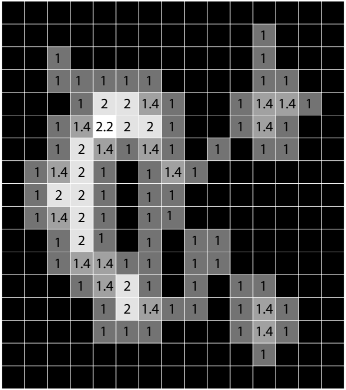
Sand

Detailed example with Euclidean distances

Bilevel image

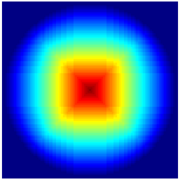


Euclidean distance map

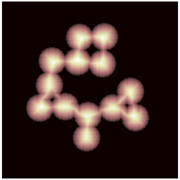


Other distance metrics

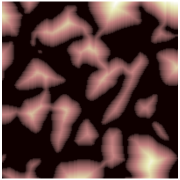
City-block metric $d_4[(x_1, y_1), (x_2, y_2)] = |x_2 - x_1| + |y_2 - y_1|$



Disk

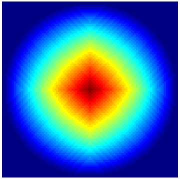


Circles

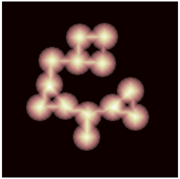


Sand

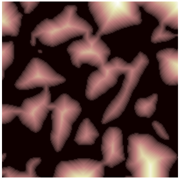
Chessboard metric $d_8[(x_1, y_1), (x_2, y_2)] = \max\{|x_2 - x_1|, |y_2 - y_1|\}$



Disk



Circles



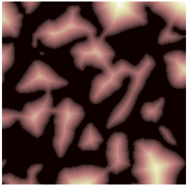
Sand

Count regions with radius>R

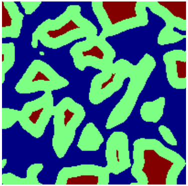
Procedure

- Compute the distance map $d = D(f)$
- $dR = d > R$
- Label $dR \Rightarrow ldr$
- $\#region > R = \max ldr$

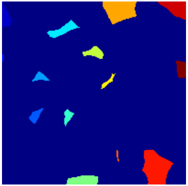
Example



Dist>R $\Rightarrow$



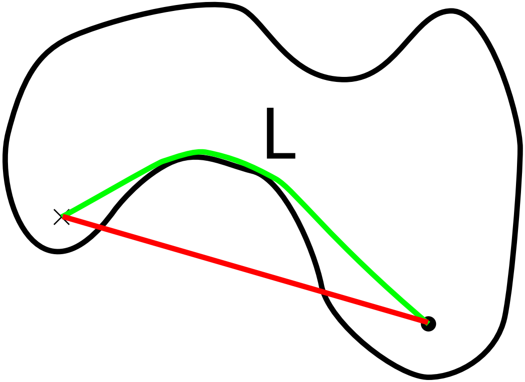
Label $\Rightarrow$



Geodesic Distances

Definition

The shortest distance, L, within the mask from a marker to any connected pixel in the mask.



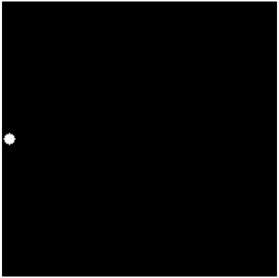
The geodesic distance is constrained by the shape of the object

Constrained length of the pore space

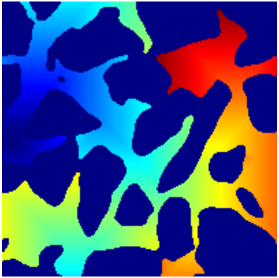
Example

The geodesic distance can be used to find the greatest pore distance.

Porespace mask



Origin of measurement



8 connected geodesic distance

Skeletons

Skeletons

Definition

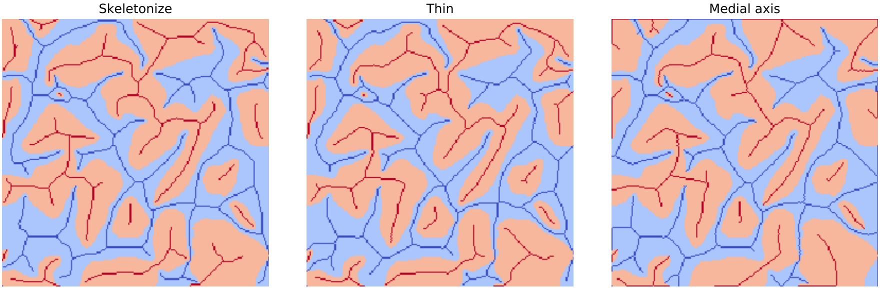
A skeleton represents the medial axis of the objects in the image. It shall essentially span the convex hull of the object.

The shape of the skeleton depends on the method to extract it

- Thinning until stability
- Extracting the ridge of a distance transform
- many others, see Soille (2002) for more

Skeletons using different algorithms

Testing three algorithms provided by SciKit Image



Geodesic operations

Geodesic Operations

Morphologic operations that are constrained only to act in regions supported by a mask image

Example

Typical operations:

- Reconstruction
- Regional extrema
- Distance maps

Geodesic Reconstruction

Connectivity operations on image f using mask image g

- Reconstruction by Dilation.

$$[R_g^{\delta}(f)](\mathbf{x}) = \min\left\{ g(\mathbf{x}), \underbrace{\max\left\{f(\mathbf{y}) | \mathbf{y} \in NG(\mathbf{x})\right\}}_{\text{Dilation of } f} \right\}$$

- Reconstruction by Erosion.

$$[R_g^{\varepsilon}(f)](\mathbf{x}) = \max\left\{ g(\mathbf{x}), \underbrace{\min\left\{f(\mathbf{y}) | \mathbf{y} \in NG(\mathbf{x})\right\}}_{\text{Erosion of } f} \right\}$$

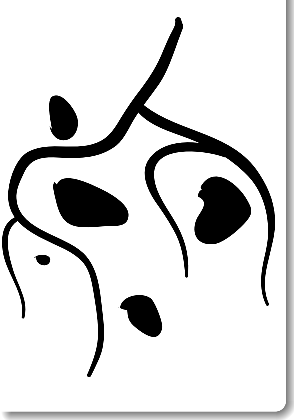
- Self-dual Reconstruction

$$[R_g^{\nu}(f)](\mathbf{x}) = \begin{cases} [R_g^{\delta}(f)](\mathbf{x}), & f(\mathbf{x}) \leq g(\mathbf{x}) \\ R_g^{\varepsilon}(f)(\mathbf{x}), & \textit{otherwise} \end{cases}$$

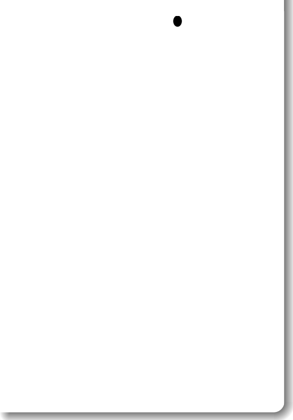
Reconstruction by dilation – Bilevel images

$R^{\delta}_{X} Y$ s a connectivity algorithm for B/W images – seeded region growing

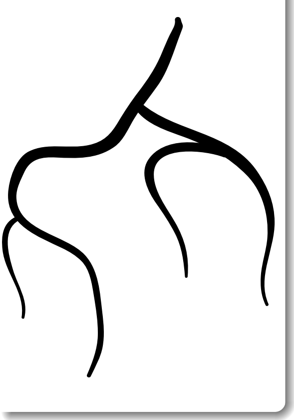
Roots and speckle



Y**=Marker (**$Y \subset X$**)**



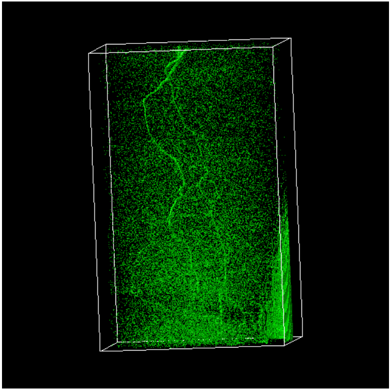
Reconstructed roots



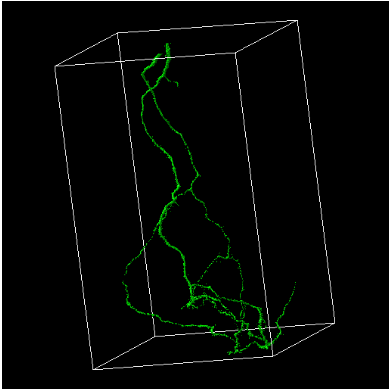
3D example with $R^{\delta}_{X}(Y)$

The operator also works in 3D – Tomography of roots in sand

Thresholded image



Reconstructed roots

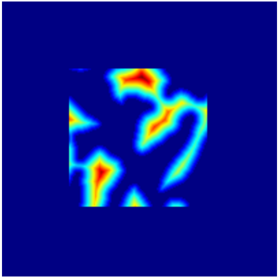


Root network reconstructed using a single voxel per root system

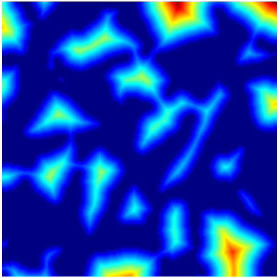
Kaestner et al. (2006)

Reconstruction by dilation – Gray-scale images

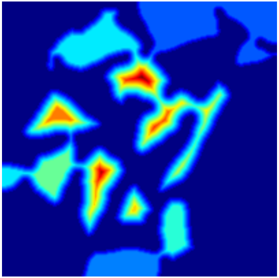
Marker (Y)



Mask (X)



Result



Regional extrema

Regional extrema can be found with Reconstruction operators

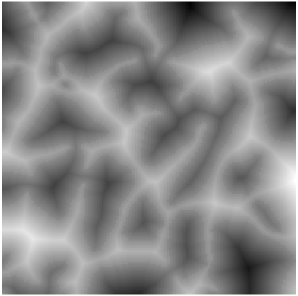
- Regional min, $RMIN(f) = R_f^{\varepsilon}(f+1) - f$
- Regional max, $RMAX(f) = f - R_f^{\delta}(f-1)$
- H min, $HMIN_h(f) = R_f^{\varepsilon}(f+h)$
- H max $HMAX_h(f) = R_f^{\delta}(f-h)$
- Extended min $EMIN_h(f) = RMIN(HMIN_h(f))$
- Extended max $EMAX_h(f) = RMAX(HMAX_h(f)$

Regional extrema – RMIN and RMAX

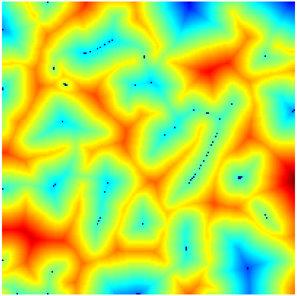
Regional min $RMIN(f) = R_f^{\varepsilon}(f+1) - f$

Regional max $RMAX(f) = f - R_f^{\delta}(f-1)$

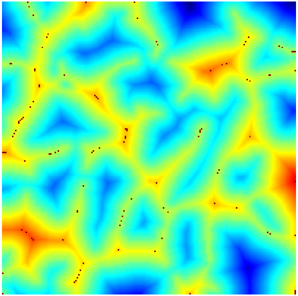
Input



RMIN(f)



RMAX(f)

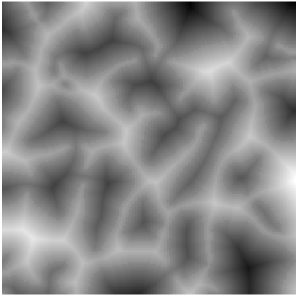


Regional extrema – HMIN and HMAX

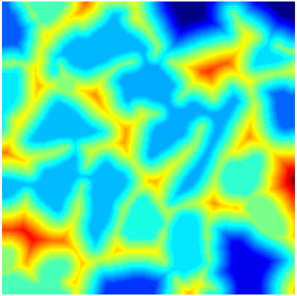
H min $HMIN_h(f) = R_f^{\varepsilon}(f+h)$

H max $HMAX_h(f) = R_f^{\delta}(f-h)$

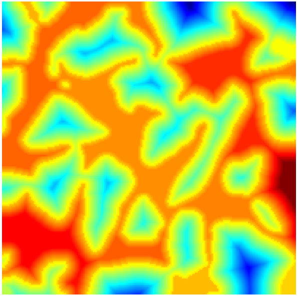
Input



$HMIN_{10}(f)$



$HMAX_{10}(f)$

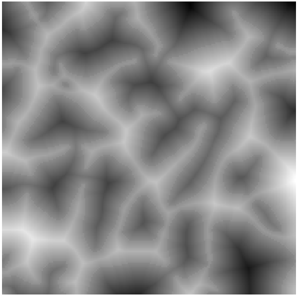


Regional extrema – EMIN and EMAX

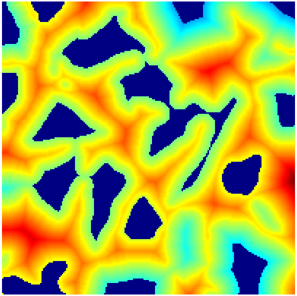
Extended min $EMIN_h(f) = RMIN(HMIN_h(f))$

Extended max $EMAX_h(f) = RMAX(HMAX_h(f))$

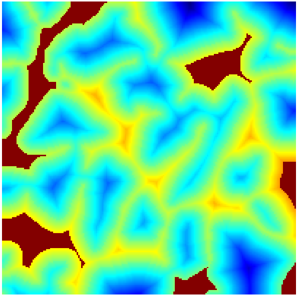
Input



EMIN$_{10}$(f)



EMAX$_{10}$(f)



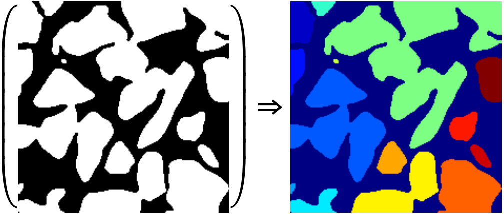
Labeling and Morphological segmentation

Labeling

Definition

Labeling an image corresponds to assigning a label (value) to each disconnected region in the image.

Label



A region growing operation, i.e. the connectivity has an impact on the outcome.

Applications

- Item counting
- Area measurement
- Item extraction, remove small items

Labeling – Measure approximate object area

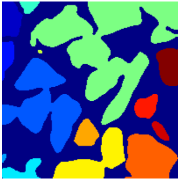
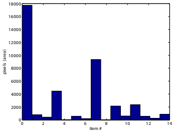
Area of an item

The item area is computed as the sum of all pixels belonging to the item.

Procedure for all items

1. Label the regions in the image
2. Compute the histogram of the labeled image (#bins = #labels)
3. Each bin in the histogram corresponds to the region areas

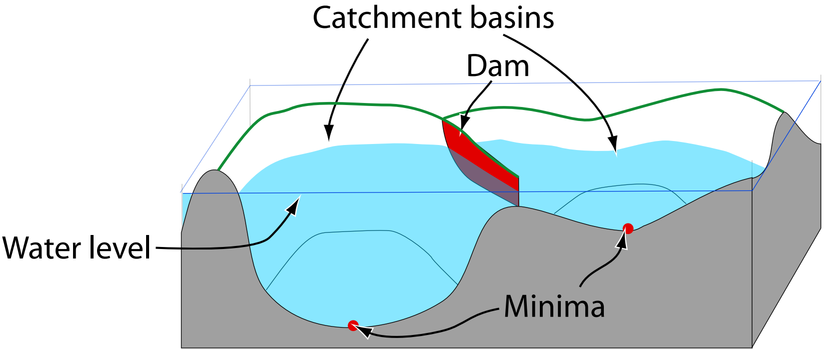
Example

Label () ⇒  ⇒ 

Morphological Segmentation

- The purpose of morphological segmentation is to identify objects
- The labeling has the drawback that it cannot identify overlapping objects.
- Watershed segmentation can do this...

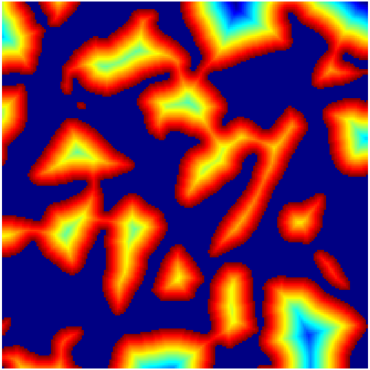
Watershed segmentation – How does it work?



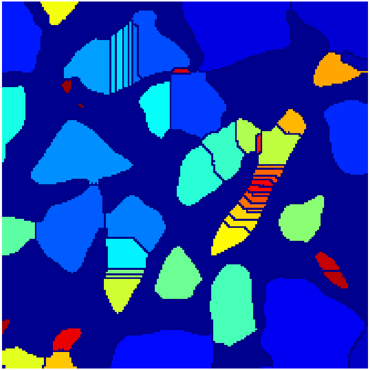
A review of algorithms can be found in Roerdink and Meijster (2001)

Watershed segmentation – over-segmentation

Releasing the WS-algorithm on an elevation map. . .



D='Elevation map'

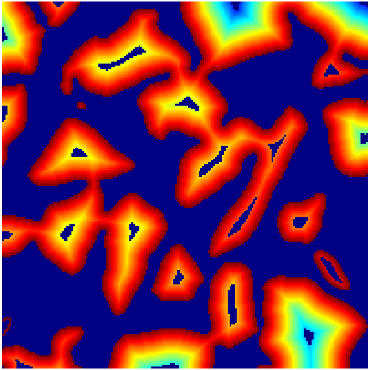


Segmented sand

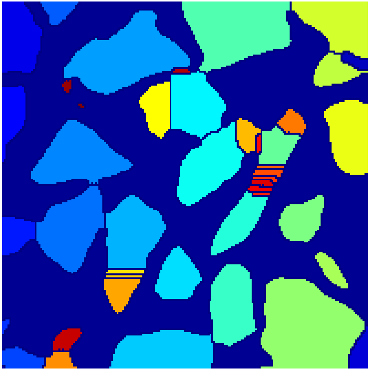
. . . mostly result in a large over-segmentation

Watershed – Reducing over-segmentation

It is possible to reduce the amount of over-segmentation by imposing starting points



D='Elevation map'



Segmented sand

Here was $EMIN_2(D)$ used.

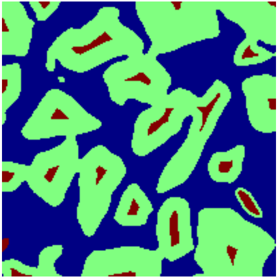
Watershed segmentation – Minima imposition

Minima can be imposed using

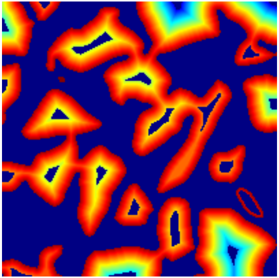
$$R^{\varepsilon}_{(f+1)\wedge f_m}(f_m)$$

with

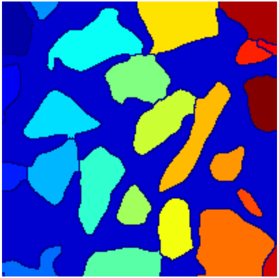
$$f_m(p) = \begin{cases} t_{min} & \text{if } p \text{ belongs to a marker} \\ t_{max} & \text{otherwise} \end{cases}$$



Markers (red)



$R^{\varepsilon}_{(f+1)\wedge f_m}(f_m)$



Segmented

Summary

We have talked about:

- Distance maps
- Skeletons
- Geodesic operations
  - Reconstruction
  - Local extrema
  - Geodesic distances
- Labeling and Watershed segmentation

Bibliography

Kaestner, A., Schneebeli, M., and Graf, F. (2006). Visualizing three-dimensional root networks using computed tomography. *Geoderma*, 136(1–2):459–469.

Roerdink, J. and Meijster, A. (2001). The watershed transform: Definitions, algorithms and parallelization strategies. *Fundamenta Informaticae*, 41:187–228.

Soille, P. (2002). *Morphological image analysis*. Springer Verlag, 2nd edition.