

Morphological operations

Anders Kaestner :: Laboratory for Neutron Scattering and Imaging



1 What is morphological processing?

2 Morphology on black and white images

3 Gray-scale morphology

What is morphological processing?

What is Morphological Image Analysis?

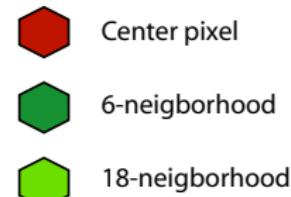
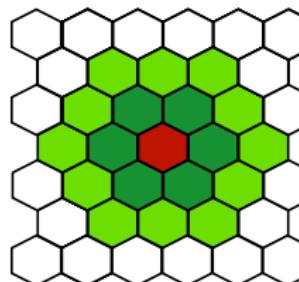
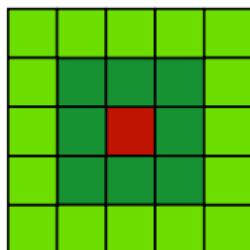
- A branch of image analysis based on set algebra
- Originally introduced by Serra (1990)
- Uses structures and neighborhoods

Notation in this lecture is according to Soille (2002)

Common grids in morphological image analysis are

- Rectangular grid → Straight-forward implementation
 - Four connectivity
 - Eight connectivity
 - Can be used for 3D data
- Hexagonal grid → No neighborhood ambiguity
 - Always six connectivity

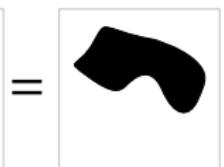
Neighborhoods



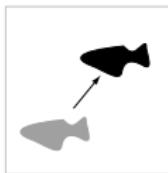
Complement $f^c(x) = t_{max} - f(x)$



Set difference $X \setminus Y = X \cap Y^c$



Translation $f_b(x) = f(x - b)$

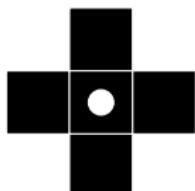


Reflection $\check{B} = \{-b | b \in B\}$

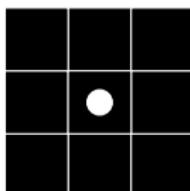


Structure elements are masks for the morphological operator.

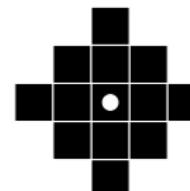
Discrete disc



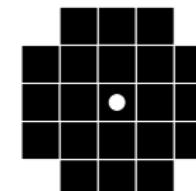
$$R = 1$$



$$R = \sqrt{2}$$



$$R = 2$$



$$R = \sqrt{5}$$

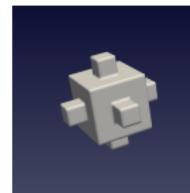
Discrete sphere



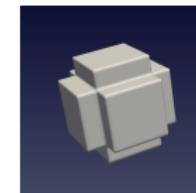
$$R = 1$$



$$R = 1.5$$



$$R = 2$$



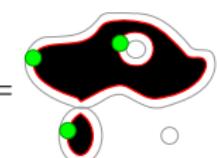
$$R = 2.5$$

Bi-level images

The basic operators of Black and White morphology act as search and replace operation for a given pixel configuration.

Erosion

Erosion is a region shrinking operation for a given structure element SE .

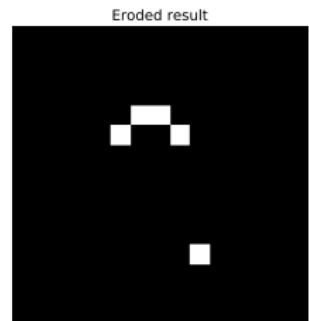
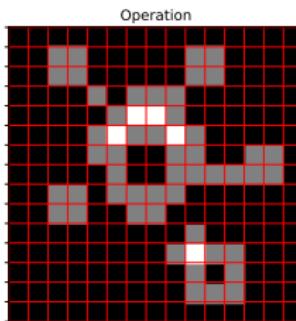
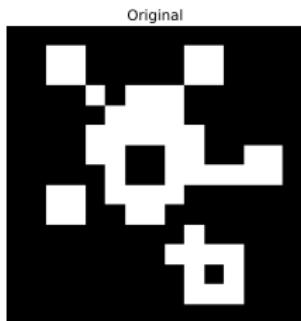
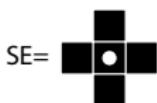
$$\varepsilon_{SE}(f) = \{x | SE_x \subseteq f\} \Rightarrow \varepsilon_{SE} \left(\begin{array}{c} \text{Black blob} \\ \text{with internal hole} \\ \text{and two small dots} \end{array} \right) = \text{Red outline of the blob}$$


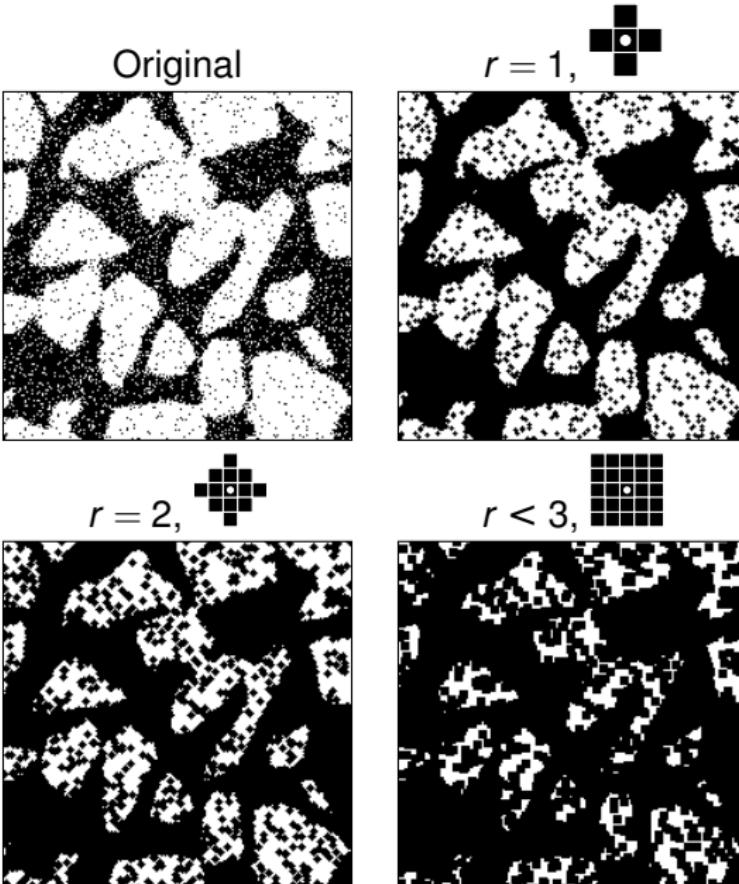
Dilation

Dilation is a region growing operation for a given structure element SE .

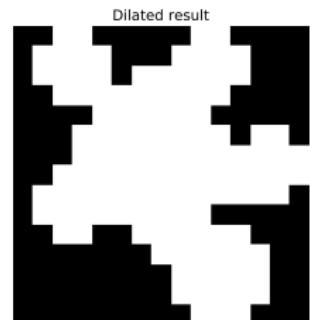
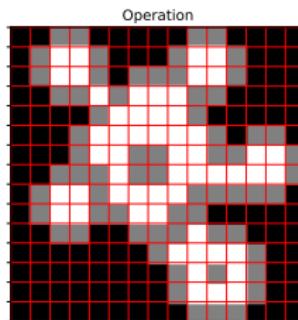
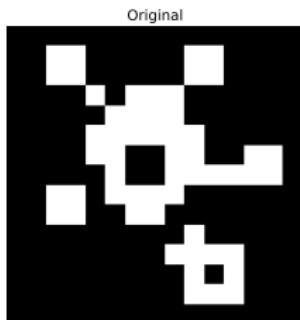
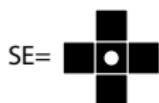
$$\delta_{SE}(f) = \{x | SE_x \cap f \neq \emptyset\} \Rightarrow \delta_{SE} \left(\begin{array}{c} \text{Black blob} \\ \text{with internal hole} \\ \text{and two small dots} \end{array} \right) = \text{Red outline of the blob with green dots at the boundary}$$

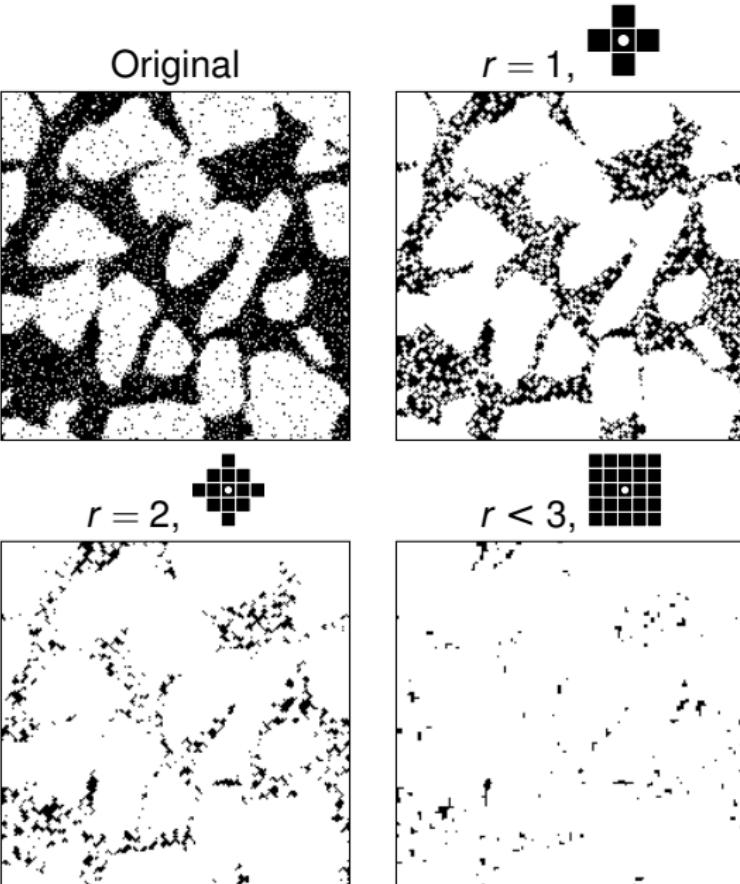

Detailed example





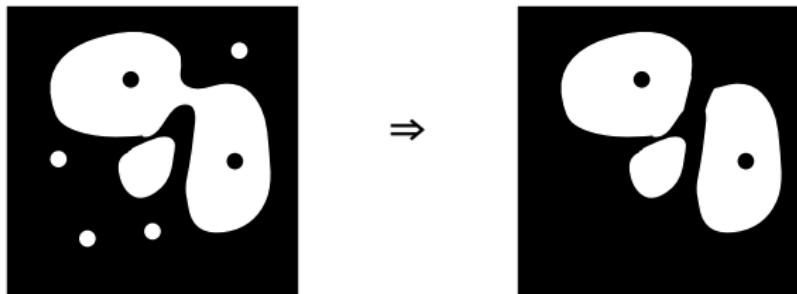
Detailed example



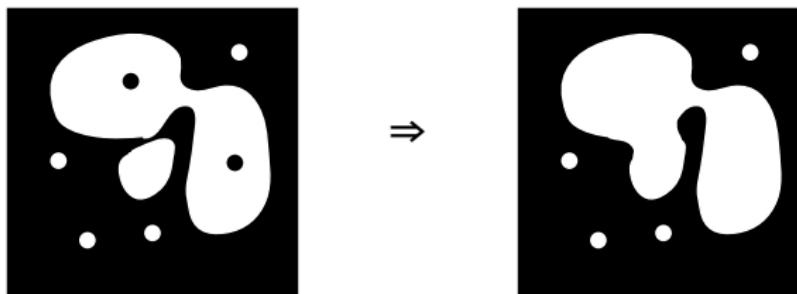


Undesired image features be removed by combining erosion and dilation.

Opening $\delta(\varepsilon(f))$



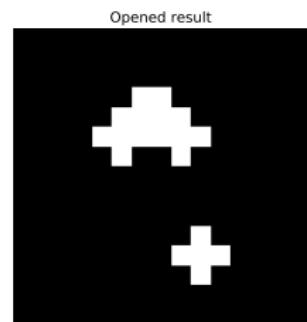
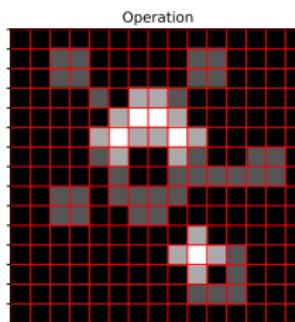
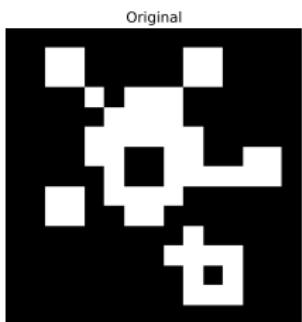
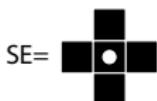
Closing $\varepsilon(\delta(f))$

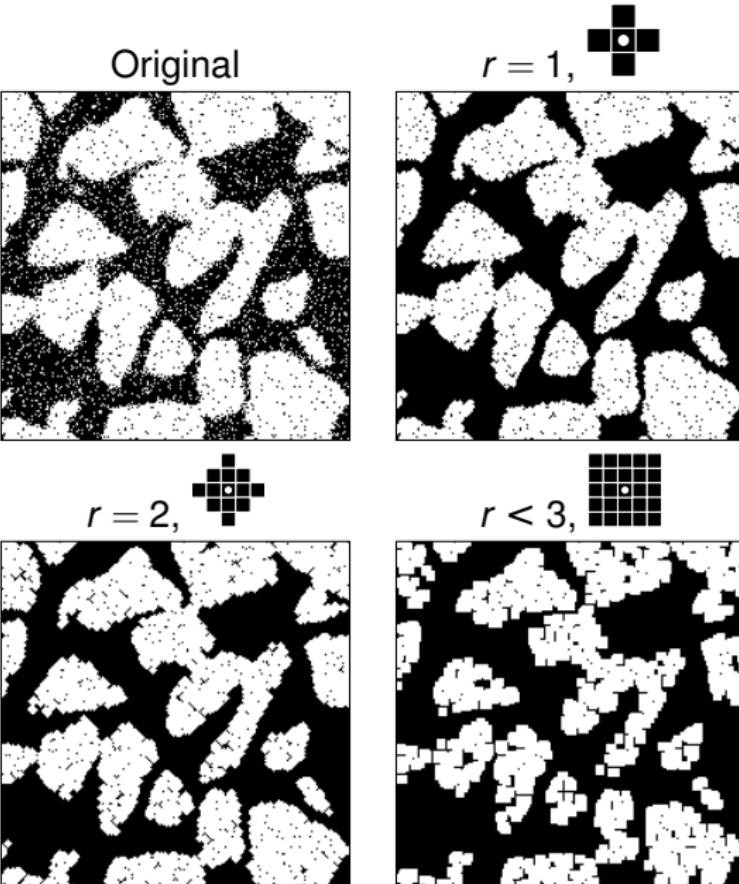


The Open operator γ_B is defined as

$$\gamma_B(f) = \delta_B[\varepsilon_B] \quad (1)$$

Opening example

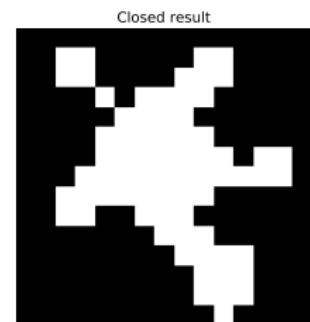
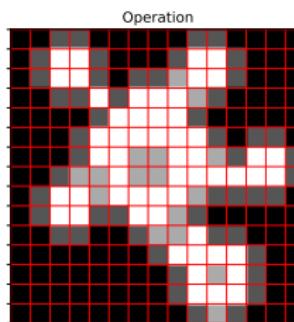
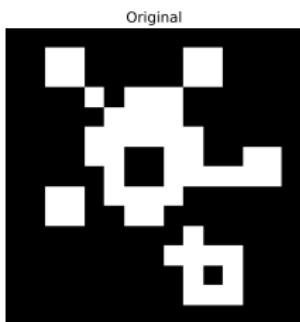
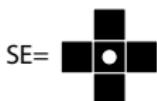




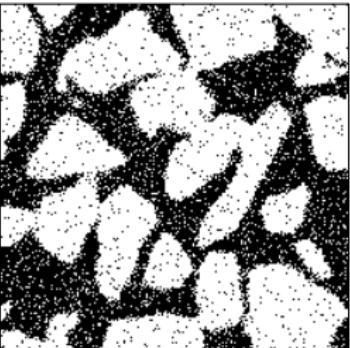
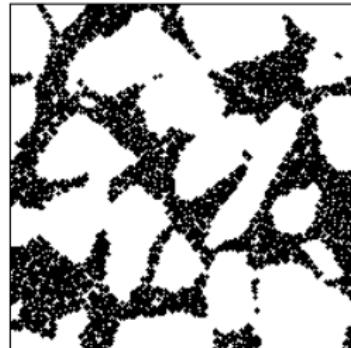
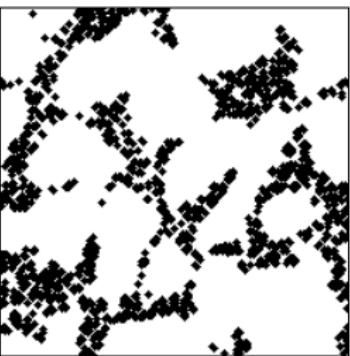
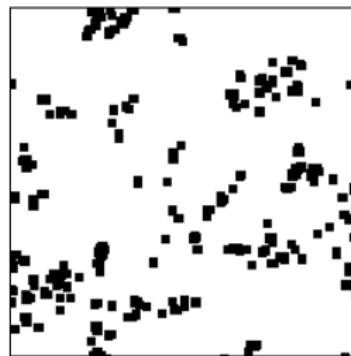
The close operator ϕ_B is defined as

$$\phi_B(f) = \varepsilon_B [\delta_B] \quad (2)$$

Closing example



Original

 $r = 1,$  $r = 2,$  $r < 3,$ 

Gray scale morphology

A generalization of binary morphology is *gray-scale morphology*
To handle the increased number of intensity levels the basic operations

- Erosion
- Dilation

must be redefined...

...while others only act differently based on ε and δ

Erosion is defined as

$$[\varepsilon_{SE}(f)](x) = \min_{\mathbf{b} \in SE} f(x + \mathbf{b}) \quad (3)$$

Example

0	0	7	0	0	0	5	5	5	5	8	7
7	1	2	7	7	0	7	6	5	5	7	7
7	2	1	1	2	1	2	6	6	5	8	7
7	5	5	5	7	2	1	6	5	5	5	
5	7	5	5	5	2	7	6	7	7	6	
5	5	5	5	7	1	2	7	7	7		
9	5	5	5	1	1	7	8	7			
9	9	9	9	1	2	9	8				
3	3	9	9	9	2	9					
3	3	9	9	9	9						
3	3	3	3	3	3						
3	8	6									
6	6										

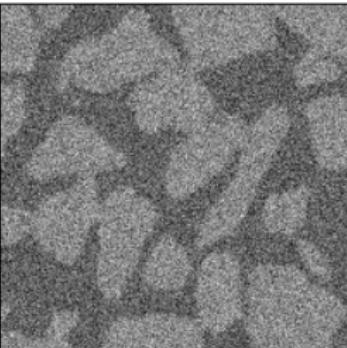
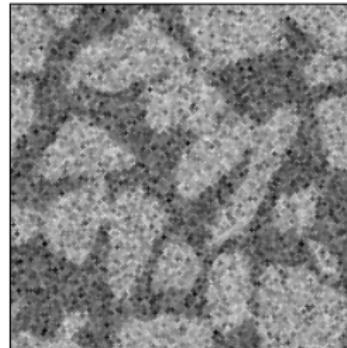
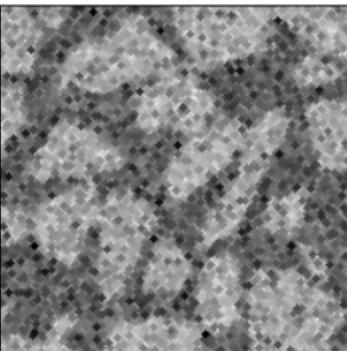
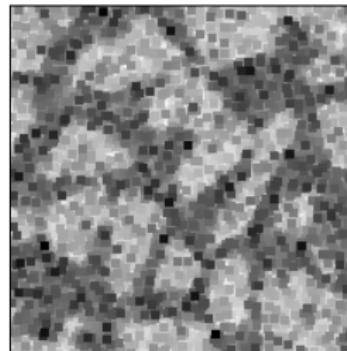
0	0	0	0	0	0	0	0	5	5	5	7
0	0	1	0	0	0	0	0	5	5	5	7
2	1	1	1	1	0	1	2	5	5	5	7
5	2	1	1	2	1	1	1	5	5	5	
5	5	5	5	2	2	2	6				...

$$\min(6,7,6,7,7)$$



Gray-scale erosion increasing SE

Original

 $r = 1,$  $r = 2,$  $r < 3,$ 

Dilation is defined as

$$[\delta_{SE}(f)](x) = \max_{\mathbf{b} \in SE} f(x + \mathbf{b}) \quad (4)$$

Example

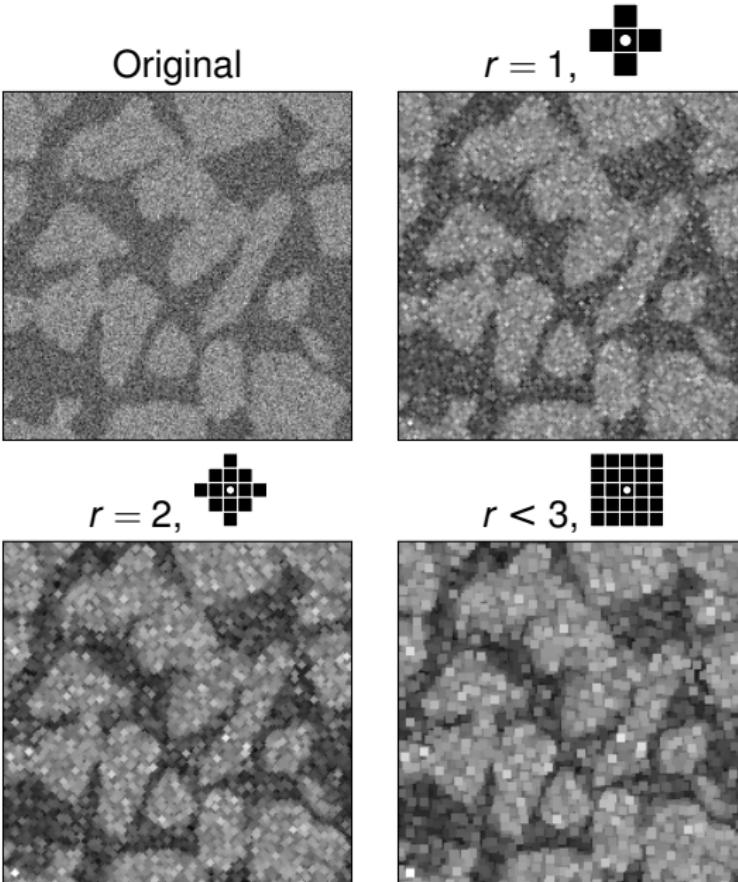
0	0	7	0	0	0	5	5	5	5	8	7
7	1	2	7	7	0	7	6	5	5	7	7
7	2	1	1	2	1	2	6	6	5	8	7
7	5	5	5	7	2	1	6	5	5	5	
5	7	5	5	5	2	7	6	7	7	6	
5	5	5	5	7	1	2	7	7	7		
9	5	5	5	1	1	7	8	7			
9	9	9	9	1	2	9	8				
3	3	9	9	9	2	9					
3	3	9	9	9	9						
3	3	3	3	3	3						
3	8	6									
6	6										

7	7	7	7	7	5	7	6	5	8	8	8
7	7	7	7	7	7	7	7	6	7	8	7
7	7	5	7	7	2	7	6	6	8	8	8
7	7	5	7	7	7	7	6	7	7	8	
7	7	7	5	7	7	7	7	7	7	7	...

$$\max(6,7,6,7,7)$$



Gray-scale Dilation increasing SE



The erode, dilate, median filters can all be described as *rank*-filters:

Definition

The rank filter sort all pixels in a neighborhood and selects one with index i .

Example

Data: $\{2, 5, 7, 2, 4, 1, 8\}$

Sorted value	Rank
1	0 (Min, Erosion)
2	1
2	2
4	3 (Median)
5	4
7	5
8	6 (Max, Dilation)

Open and close operations work as well with gray-scale images.
An application of gray-scale open/close are the top-hats defined as

- White top-hat

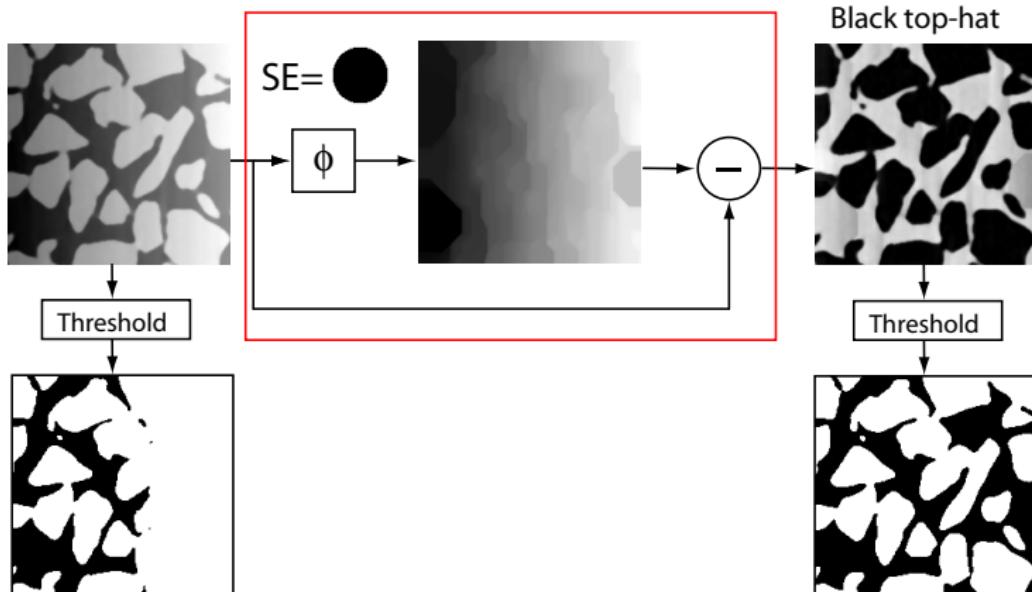
$$WTH = f - \gamma(f) \quad (5)$$

- Black top-hat

$$BTH = \phi(f) - f \quad (6)$$

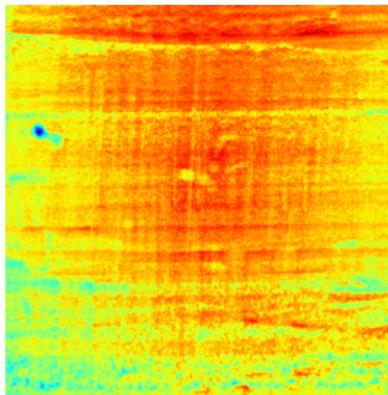
The BTH is related to WTH as $BTH = WTH \complement$

Example with black top-hat

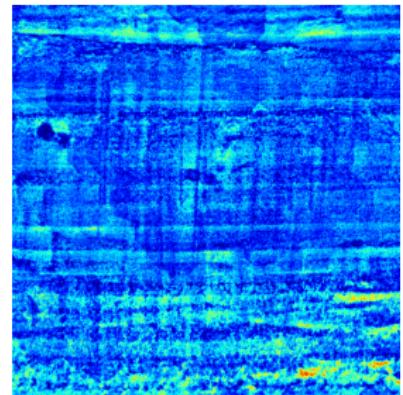


Snow profile

M. Schneebeli, SLF, Davos

Original picture

M. Matzl, SLF, Davos

White top-hat

SE=Disk R=30

In this part we talked about:

- Neighborhoods and Connectivity
- Structure elements
- Erosion and Dilation as primitive operations of morphology
- Open and Closing as combinations of ε and δ
- Top-hats
- Bi-level and Gray-scale images

- Serra, J. (1990). *Image Analysis and Mathematical Morphology*. Academic press.
- Soille, P. (2002). *Morphological image analysis*. Springer Verlag, 2nd edition.