In this example the only way the data appear in the likelihood ratio test is in a sum. This is an example of a *sufficient statistic*, which we denote by $l(\mathbf{R})$ (or simply l when the argument is obvious). It is just a function of the received data which has the property that $\Lambda(\mathbf{R})$ can be written as a function of l. In other words, when making a decision, knowing the value of the sufficient statistic is just as good as knowing $\mathbf{R}$. In Example 1, l is a linear function of the R_i. A case in which this is not true is illustrated in Example 2.

Example 2. Several different physical situations lead to the mathematical model of interest in this example. The observations consist of a set of N values: $r_1, r_2, r_3, \ldots, r_N$. Under both hypotheses, the r_i are independent, identically distributed, zero-mean Gaussian random variables. Under H_1 each r_i has a variance σ_1^2. Under H_0 each r_i has a variance σ_0^2. Because the variables are independent, the joint density is simply the product of the individual densities. Therefore

$$p_{\mathbf{r}|H_1}(\mathbf{R}|H_1) = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi}\,\sigma_1} \exp\left(-\frac{R_i^2}{2\sigma_1^2}\right) \tag{27}$$

and

$$p_{\mathbf{r}|H_0}(\mathbf{R}|H_0) = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi}\,\sigma_0} \exp\left(-\frac{R_i^2}{2\sigma_0^2}\right). \tag{28}$$

Substituting (27) and (28) into (13) and taking the logarithm, we have

$$\frac{1}{2}\left(\frac{1}{\sigma_0^2} - \frac{1}{\sigma_1^2}\right) \sum_{i=1}^{N} R_i^2 + N \ln\frac{\sigma_0}{\sigma_1} \underset{H_0}{\overset{H_1}{\gtrless}} \ln \eta. \tag{29}$$

In this case the sufficient statistic is the sum of the squares of the observations

$$l(\mathbf{R}) = \sum_{i=1}^{N} R_i^2, \tag{30}$$

and an equivalent test for $\sigma_1^2 > \sigma_0^2$ is

$$l(\mathbf{R}) \underset{H_0}{\overset{H_1}{\gtrless}} \frac{2\sigma_0^2\sigma_1^2}{\sigma_1^2 - \sigma_0^2}\left(\ln \eta - N \ln\frac{\sigma_0}{\sigma_1}\right) \triangleq \gamma. \tag{31}$$

For $\sigma_1^2 < \sigma_0^2$ the inequality is reversed because we are multiplying by a negative number:

$$l(\mathbf{R}) \underset{H_1}{\overset{H_0}{\gtrless}} \frac{2\sigma_0^2\sigma_1^2}{\sigma_0^2 - \sigma_1^2}\left(N \ln\frac{\sigma_0}{\sigma_1} - \ln \eta\right) \triangleq \gamma'; \qquad (\sigma_1^2 < \sigma_0^2). \tag{32}$$

These two examples have emphasized Gaussian variables. In the next example we consider a different type of distribution.

Example 3. The Poisson distribution of events is encountered frequently as a model of shot noise and other diverse phenomena (e.g., [1] or [2]). Each time the experiment is conducted a certain number of events occur. Our observation is just this number which ranges from 0 to ∞ and obeys a Poisson distribution on both hypotheses; that is,

$$\Pr(n \text{ events}) = \frac{(m_i)^n}{n!} e^{-m_i}, \qquad n = 0, 1, 2 \ldots, i = 0, 1, \tag{33}$$

where m_i is the parameter that specifies the average number of events:

$$E(n) = m_i. \tag{34}$$