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CSE 461

Introduction to Robotics

Introduction to Control System Theory

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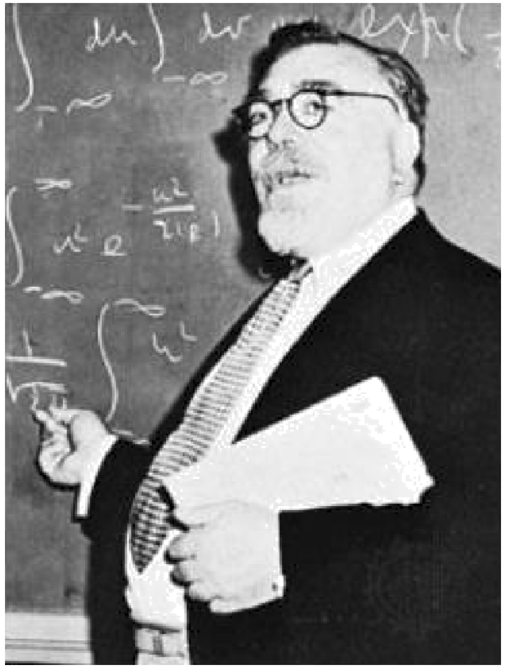
Control Theory

- Roots in another science: *Cybernetics*

  Cybernetics is the study of feedback and derived concepts such as communication and control in living organisms, machines and organizations

- Expression was coined by Norbert Weiner in 1948.



Control Systems Example

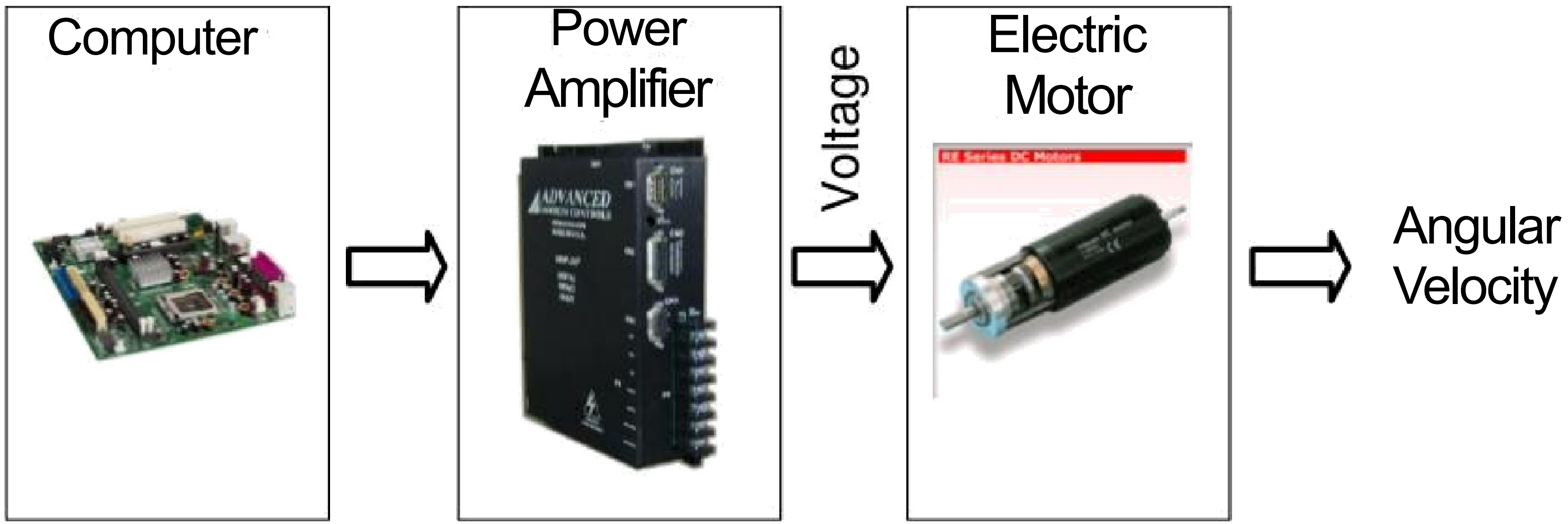
- Body temperature regulation
  - If cold, shiver (muscles produce heat)
  - If hot, sweat (evaporation takes away heat)
- Maintaining social peace
  - If a crime is found (sensor), the guilty party is punished (actuator).
- Cruise control in cars
  - You set a speed, Cruise control will increase fuel intake uphill, and decrease it downhill.
- Etc...

Why Control Theory

- Systematic approach to analysis and design
  - Transient response
  - Consider sampling times, control frequency Taxonomy of basic controllers (PID, open-loop, Model-based, Feedforward...)
  - Select controller based on desired characteristics
- Predict system response to some input
  - Speed of response (e.g., adjust to workload changes)
  - Oscillations (variability)
- Assessing stability of system

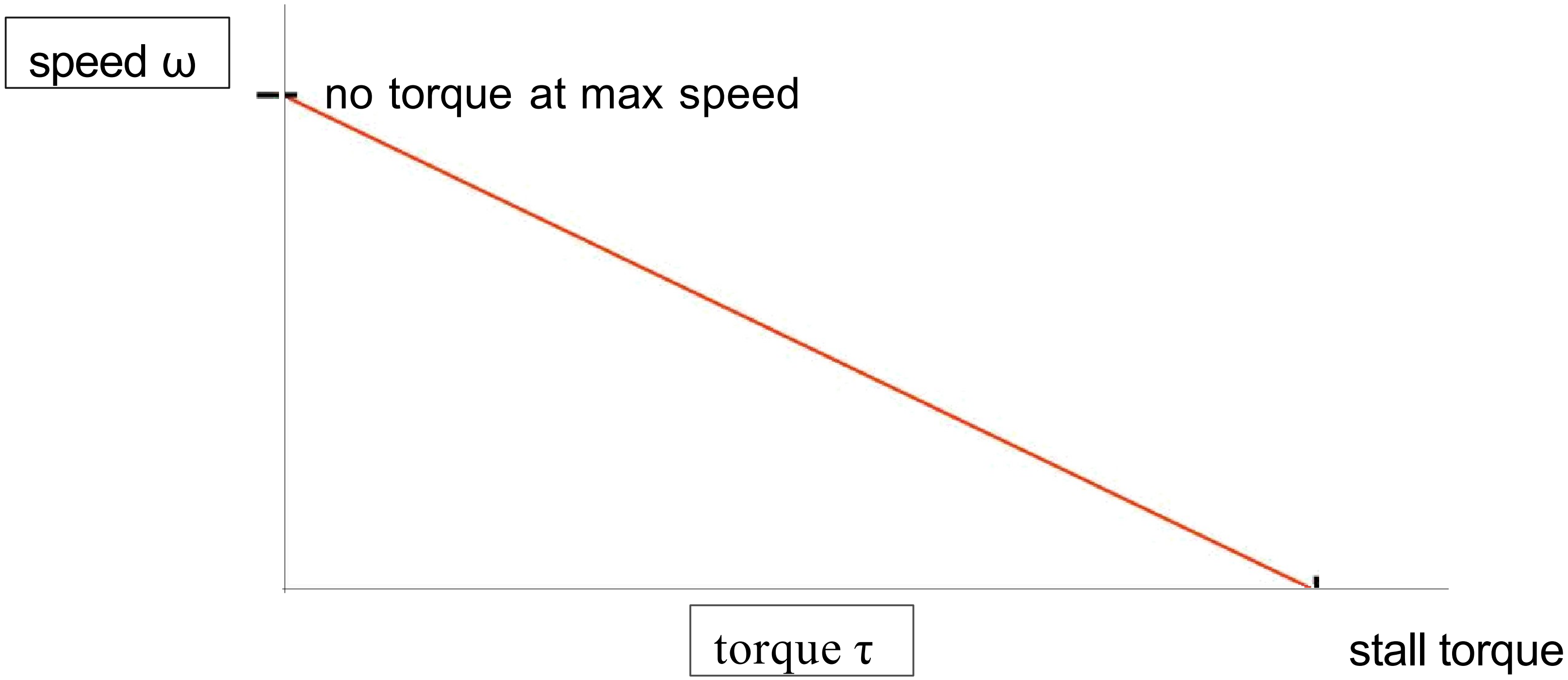
Open Loop

- We want to spin a motor at a given angular velocity. We can apply a fixed voltage to it, and <u>never</u> check to see if it is rotating properly.
- Called open loop.



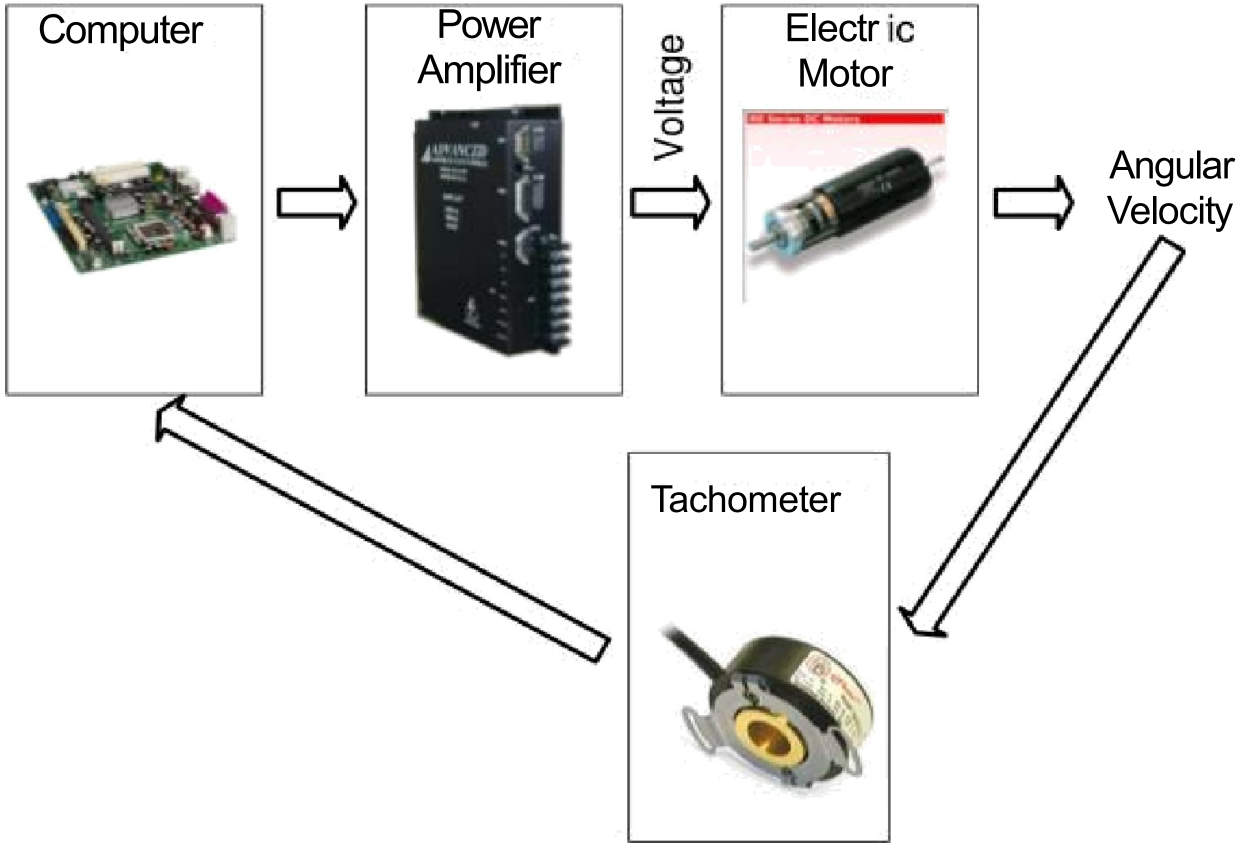
Open Loop

- What if there is a changing load on the motor?
  - —Our output velocity will change!



Closing the Loop

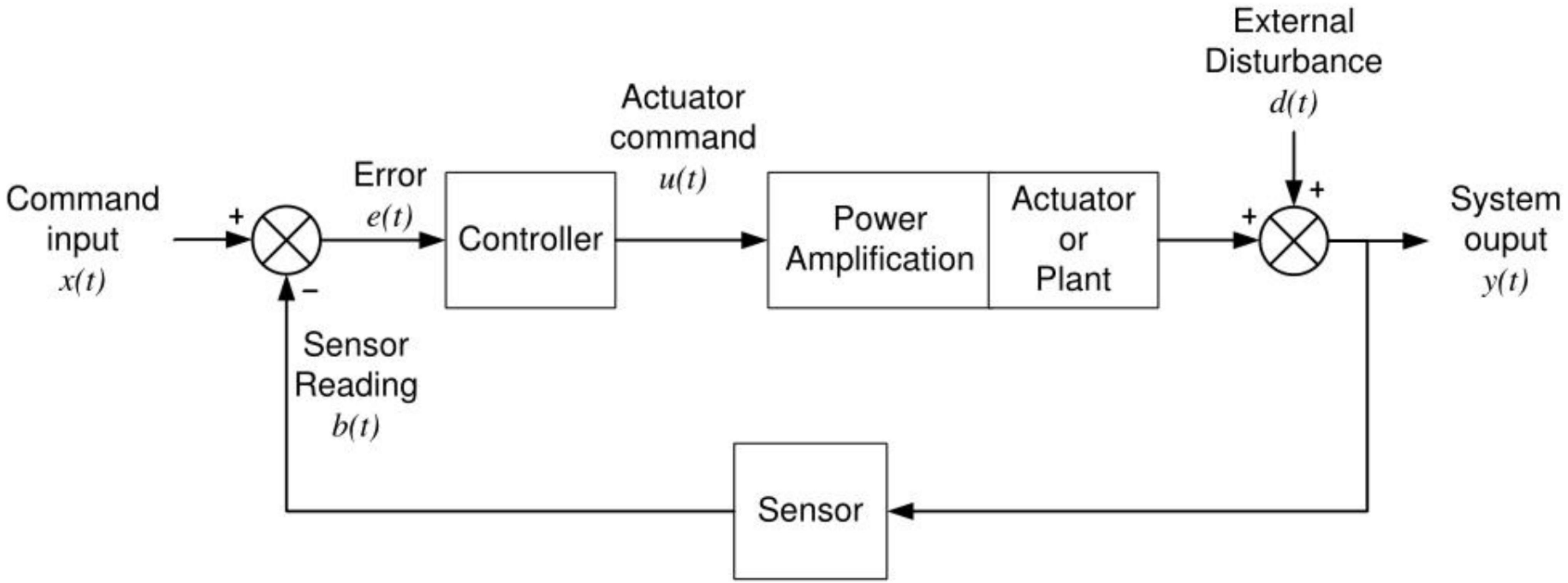
- Let's measure the actual angular velocities.
- Now we can compensate for changes in load by *feeding back* some information.



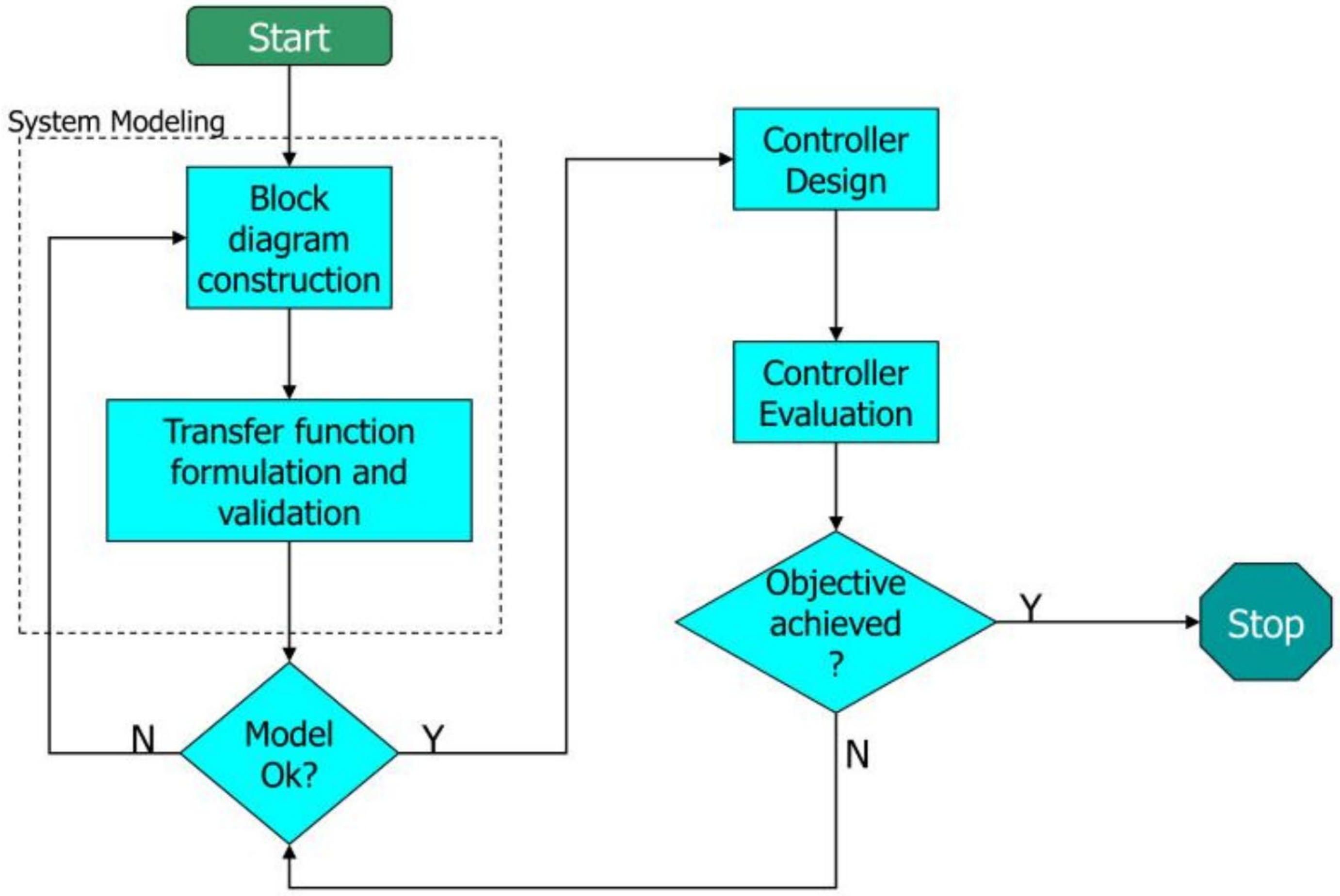
Characteristics of Feedback System

- Power amplification
- Actuator
- Feedback
  - measurement (sensor)
- Error signal
- Controller

Classic Feedback Diagram



Controller Design Methodology



Control System Goals

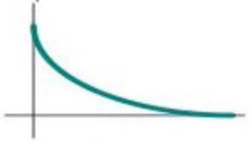
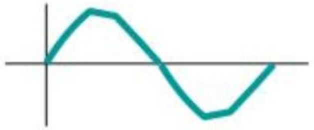
- Regulation
  - Thermostat
- Tracking
  - robot movement, adjust TCP window to network bandwidth
- Optimization
  - best mix of chemicals, minimize response times

Approaches to System Modeling

- First Principles
  - Based on known laws.
  - Physics, Queueing theory.
  - Difficult to do for complex systems.

- Experimental (System Identification)
  - Requires collecting data
  - Is there a good “training set"?

Common Laplace Transform

Name	$f(t)$		$F(s)$
Impulse δ	$f(t)=\begin{cases}1 & t=0\\ 0 & t>0\end{cases}$		1
Step	$f(t)=1$		$\frac{1}{s}$
Ramp	$f(t)=t$		$\frac{1}{s^2}$
Exponential	$f(t)=e^{-at}$		$\frac{1}{s+a}$
Sine	$f(t)=\sin(\omega t)$		$\frac{\omega}{s^2+\omega^2}$
Damped Sine	$f(t)=e^{-at}\sin(\omega t)$		$\frac{\omega}{(s+a)^2+\omega^2}$

Properties of Laplace Transform

1. Linearity	$\sum_{n=1}^{N} \alpha_n x_n(t)$	$\sum_{n=1}^{N} \alpha_n X_n(s)$
2. Time shift	$x(t-t_0)u(t-t_0)$	$X(s)\exp(-st_0)$
3. Frequency shift	$\exp(s_0 t)x(t)$	$X(s-s_0)$
4. Time scaling	$x(\alpha t), \alpha > 0$	$1/\alpha\, X(s/\alpha)$
5. Differentiation	$dx(t)/dt$	$s\,X(s) - x(0^-)$
6. Integration	$\int_0^t x(\tau)\,d\tau$	$\frac{1}{s} X(s)$
7. Multiplication by t	$t\,x(t)$	$-\frac{dX(s)}{ds}$
8. Modulation	$x(t)\cos\omega_0 t$	$\frac{1}{2}[X(s-j\omega_0) + X(s+j\omega_0)]$
	$x(t)\sin\omega_0 t$	$\frac{1}{2j}[X(s-j\omega_0) - X(s+j\omega_0)]$
9. Convolution	$x(t) * h(t)$	$X(s)H(s)$
10. Initial value	$x(0^+)$	$\lim_{s\to\infty} s\,X(s)$
11. Final value	$\lim_{t\to\infty} x(t)$	$\lim_{s\to 0} s\,X(s)$

Transfer Function

- Definition: Transfer function is defined as the ratio of LT of output to the L.T of input. When all the initial condition assume to be zero.

$$H(s) = Y(s) / X(s)$$

- Relates the output of a linear system to its input.
- Describes how a linear system responds to an impulse, called impulse response
- All linear operations allowed
  - Scaling, addition, multiplication

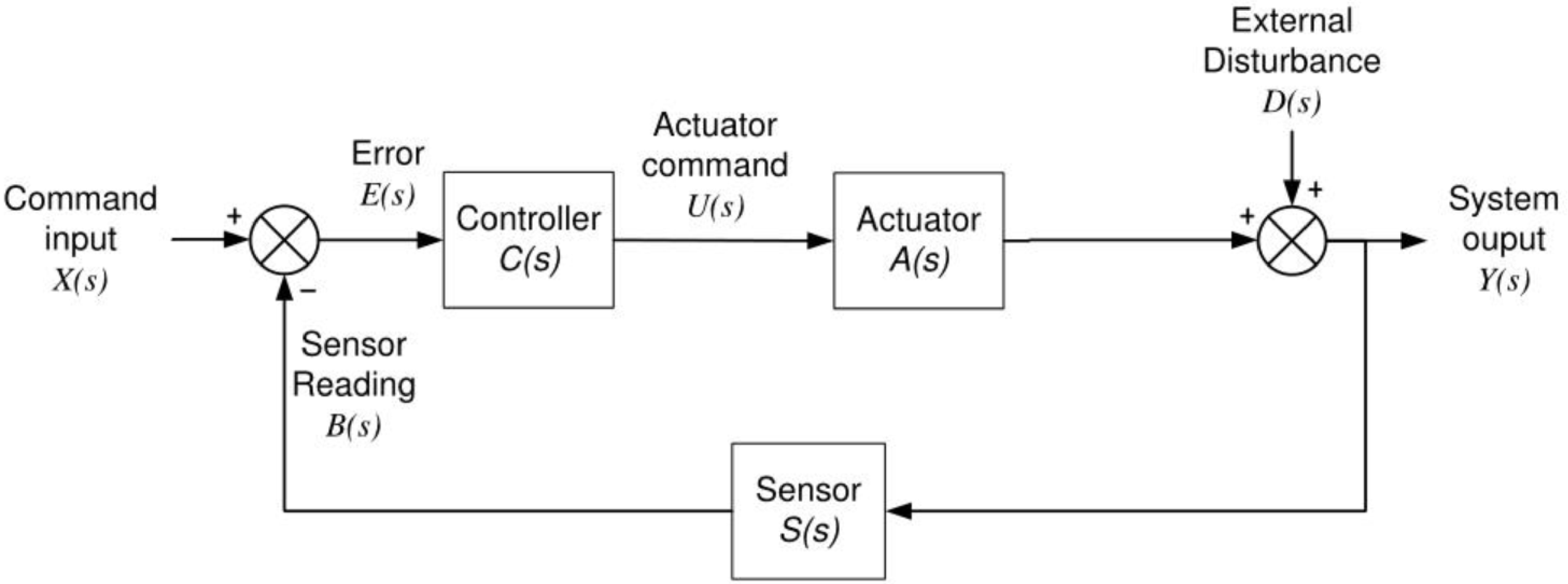
Block Diagram

- Expresses flows and relationships between elements in system.
- Blocks may recursively be systems.
- Rules
  - Cascaded elements: convolution

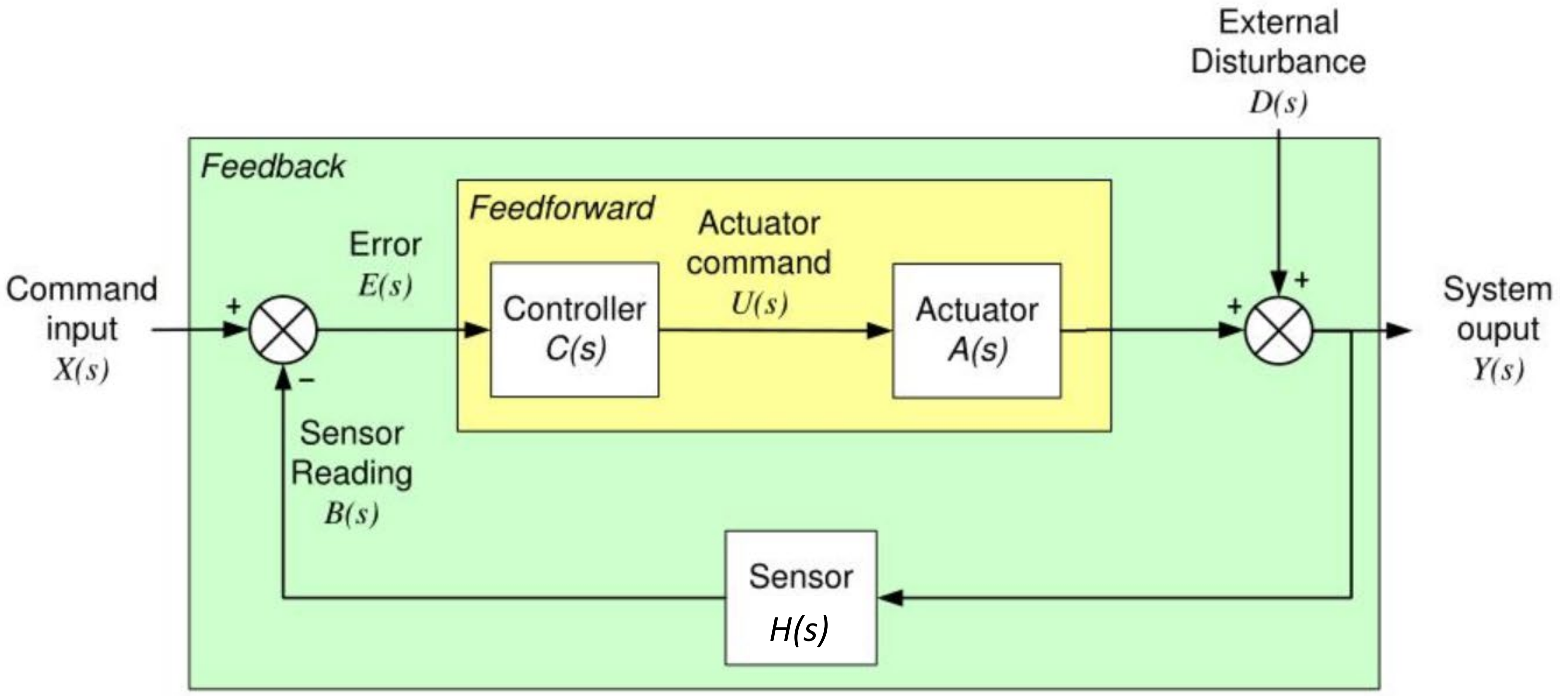
$$\boxed{a(t)} \longrightarrow \boxed{b(t)} = a(t) * b(t)$$

  - Summation and deference elements

Laplace Transform of Classic Feedback System



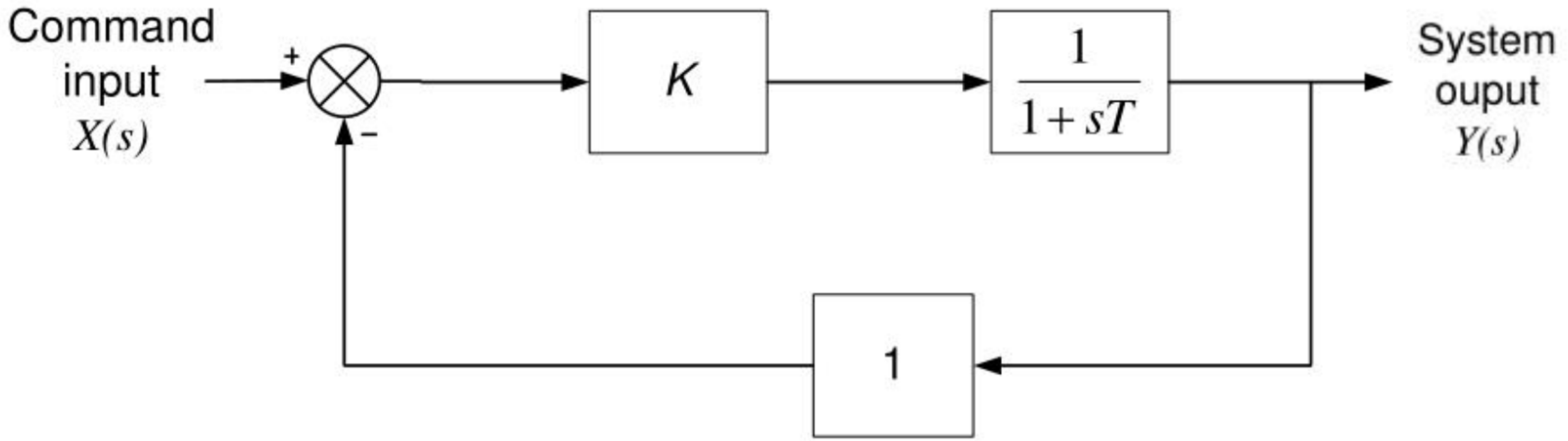
Key Transfer Function



$$\text{Feedforward}: \frac{Y(s)}{E(s)} = C(s)A(s)$$

$$\text{Feedback}: \frac{Y(s)}{X(s)} = \frac{C(s)A(s)}{1 + C(s)A(s)H(s)}$$

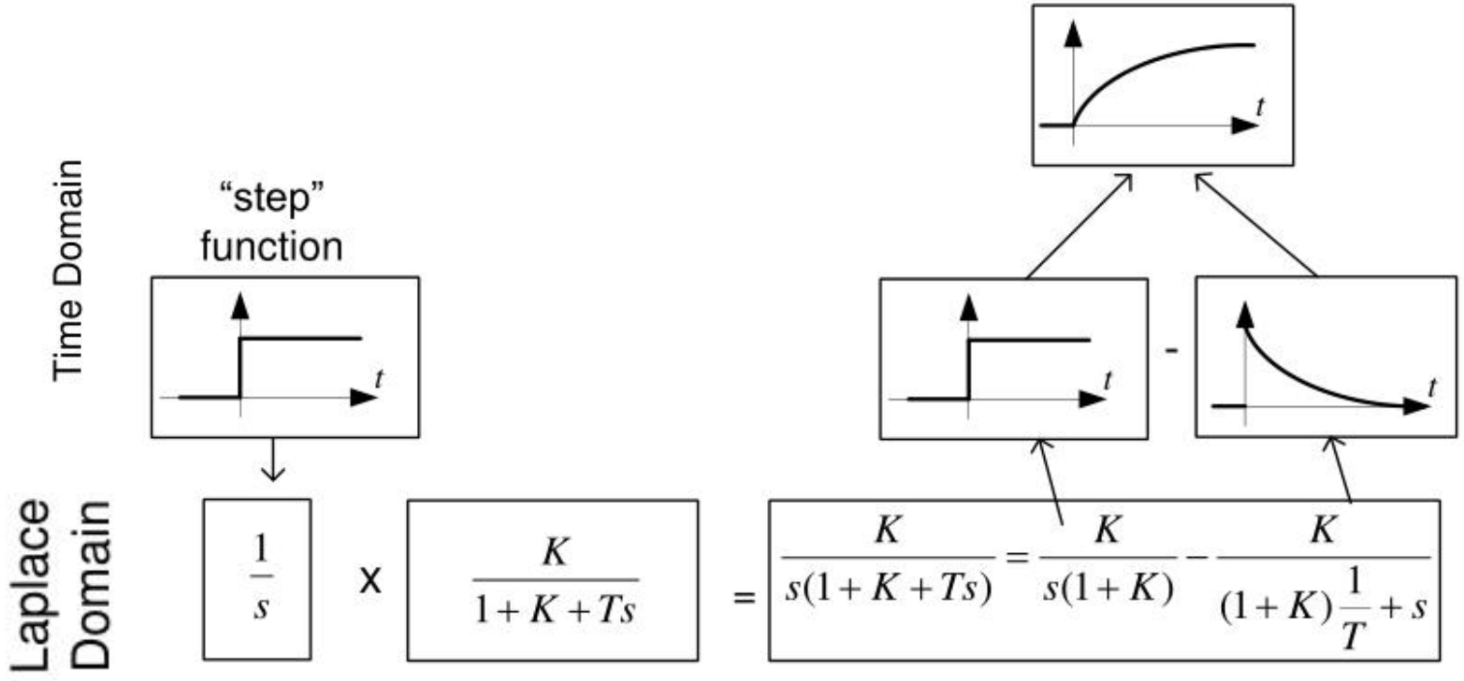
First Order System



$$\frac{Y(s)}{X(s)} = \frac{K}{1+K+sT}$$

Response of the System

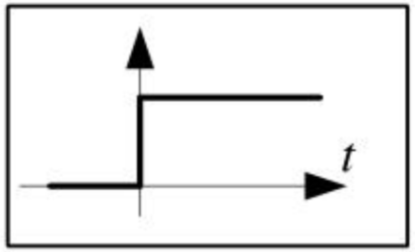
- Impulse Response $$\frac{K}{1+K+sT}$$



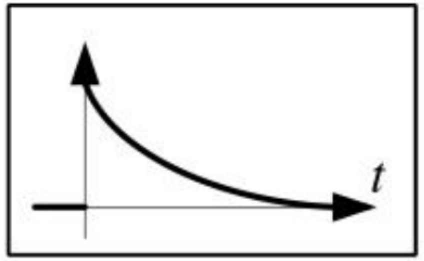
Steady-State Vs Transient

- Step Response illustrates how a system response can be decomposed into two components

- Steady-state part:



- Transient



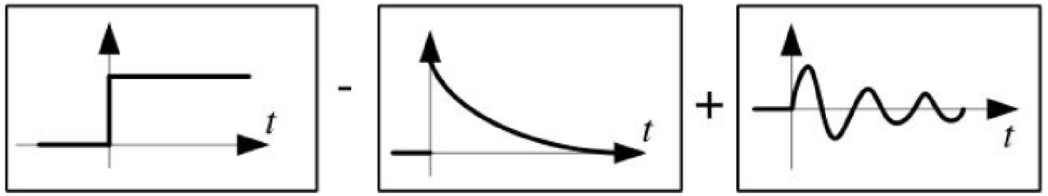
Second Order System

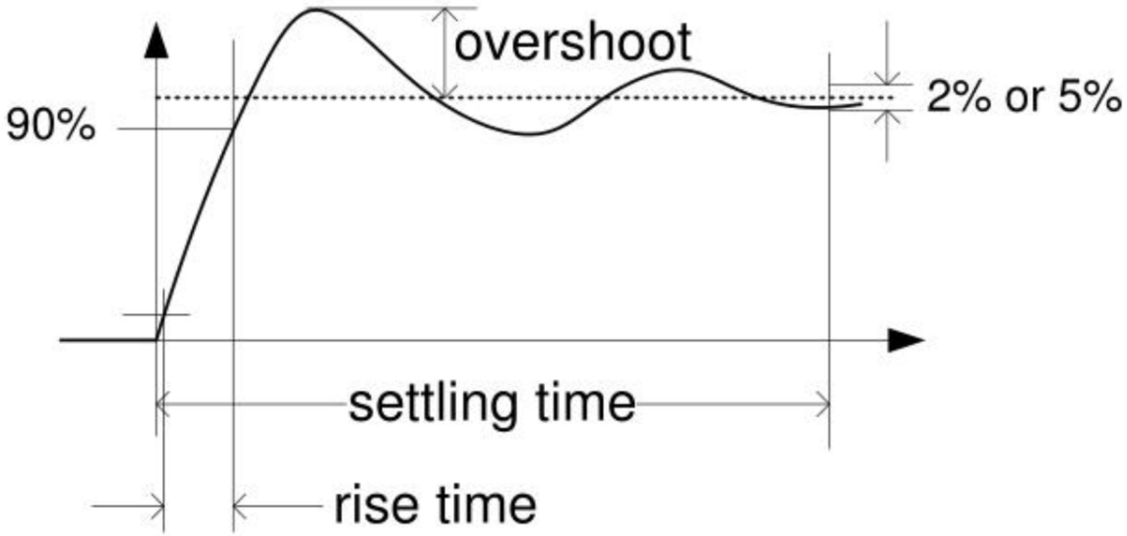
- Impulse response

$$\frac{Y(s)}{X(s)} = \frac{K}{Js^2 + Bs + K} = \frac{\omega_N^2}{s^2 + 2\xi\omega_N s + \omega_N^2}$$

Second Order Response

- Typical response to step input is:





overshoot -- % of final value exceeded at first *oscillation*

rise time -- time to span from 10% to 90% of the final value

settling time -- time to reach within 2% or 5% of the final value

PID Controller

- It produces an output, which is the combination of the outputs of proportional , integral & derivative controllers

$$u(t) \propto e(\mathrm{t}) + \int e(t) + \frac{\mathrm{d}}{\mathrm{d}t} e(t)$$

$$\gg u(t) = K_P e(\mathrm{t}) + K_I \int e(t) + K_D \frac{\mathrm{d}}{\mathrm{d}t} e(t)$$

Laplace transform in both side

$$U(S) = K_P E(\mathrm{S}) + \frac{K_I}{S} E(\mathrm{S}) + K_D S E(\mathrm{S})$$

$$U(S) = E(\mathrm{S}) \left(K_P + \frac{K_I}{S} + K_D S \right)$$

$$\frac{U(S)}{E(\mathrm{S})} = \left(K_P + \frac{K_I}{S} + K_D S \right) = \frac{K_P S + K_I + K_D S^2}{S}$$

Basic PID Controller Function

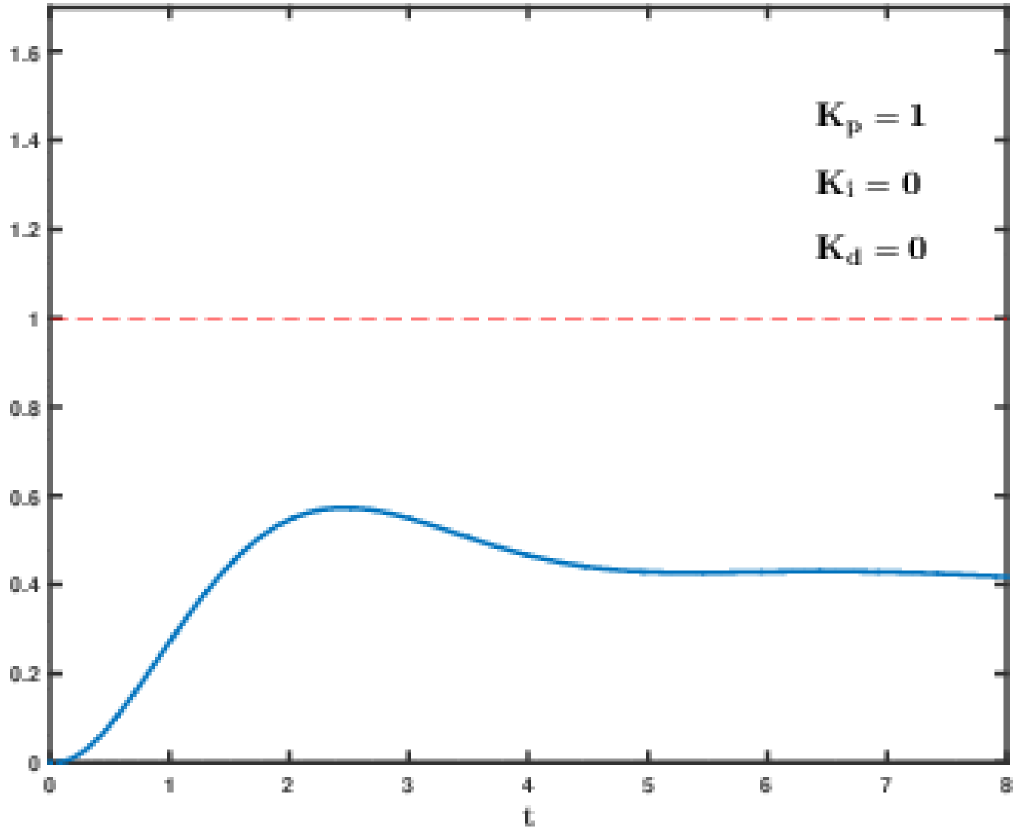
$$\text{Proportional control}: \quad u(t) = K_p e(t) \qquad \frac{U(s)}{E(s)} = K_p$$

$$\text{Integral control}: \quad u(t) = K_i \int_0^t e(t)dt \qquad \frac{U(s)}{E(s)} = \frac{K_i}{s}$$

$$\text{Differential control}: \quad u(t) = K_d \frac{d}{dt} e(t) \qquad \frac{U(s)}{E(s)} = K_d s$$

Effect of Controller Functions

- Proportional Action
  - Simplest Controller Function
- Integral Action
  - Eliminates steady-state error
  - Can cause oscillations
- Derivative Action (“rate control”)
  - Effective in transient periods
  - Provides faster response (higher sensitivity)
  - Never used alone



PID Tuning

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How to get the PID parameter values ?

- If we know the transfer function, analytical methods can be used (e.g., root-locus method) to meet the transient and steady-state specs.
- When the system dynamics are not precisely known, we must resort to experimental approaches.

Ziegler-Nichols Rules for Tuning PID Controller

Using only Proportional control, turn up the gain until the system oscillates without dying down, i.e., is marginally stable. Assume that K and P are the resulting gain and oscillation period, respectively.

Then Use

for P control	for PI control	for PID control
$K_p = 0.5\ K$	$K_p = 0.45\ K$	$K_p = 0.6\ K$
	$K_i = 1.2\ /\ P$	$K_i = 2.0\ /\ P$
		$K_d = P\ /\ 8.0$

Ziegler-Nichols Tuning for second or higher order systems

Conclusion

- PID control---most widely used control strategy today
- Over 90% of control loops employ PID control, often the derivative gain set to zero (PI control)
- The three terms are intuitive---a non - specialist can grasp the essentials of the PID controller’s action. It does not require the operator to be familiar with advanced math to use PID controllers
- Engineers prefer PID controls over untested solutions