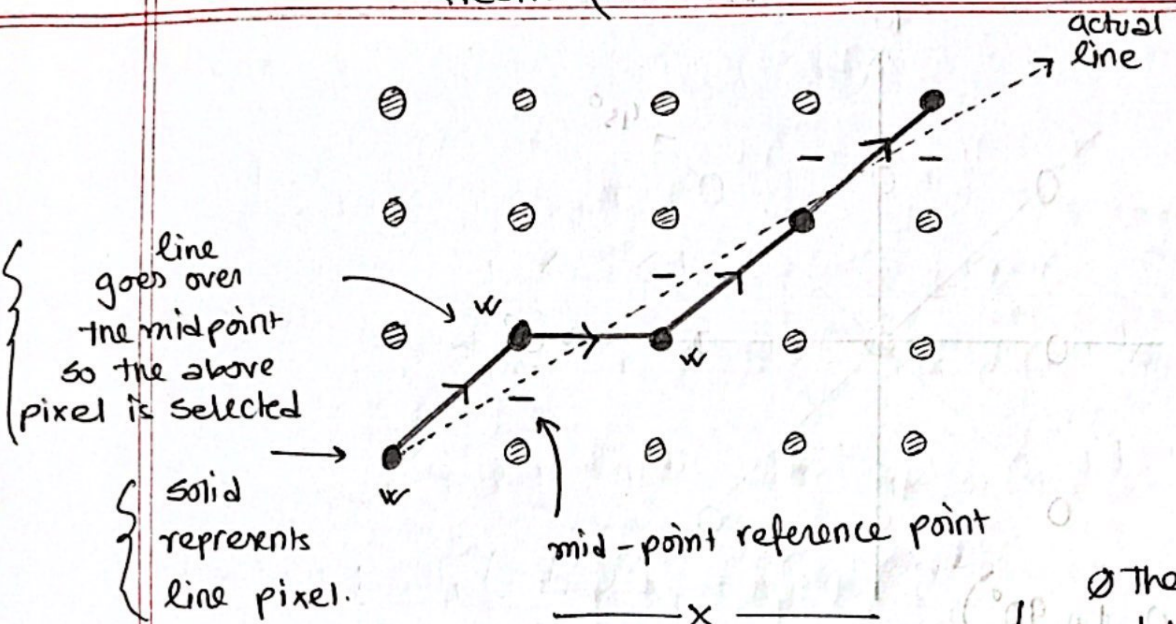
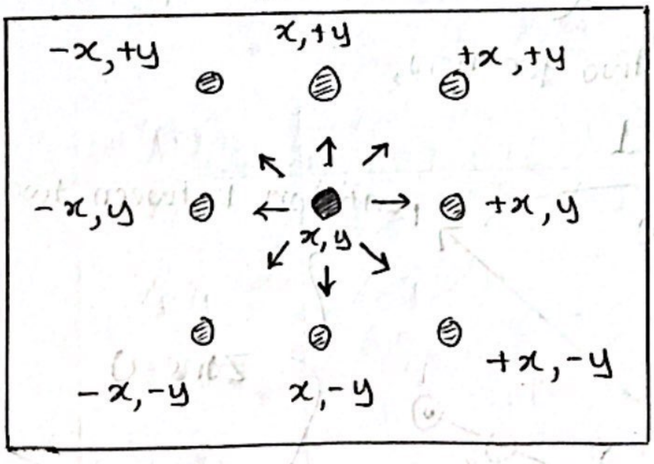
Week 2 (Mid Point line Derivation & Algorithm)

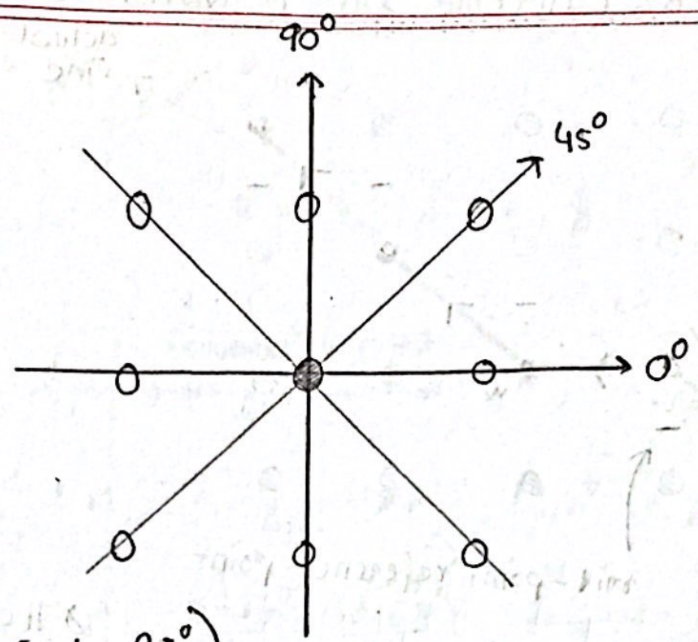


Ø A pixel has basically 8 neighbour pixels, the next pixel in line is defined by the Algorithm.



Ø The algorithm determines, basically if the line is selecting the pixel below or above the mid point.

→ Group (0 to 90°)

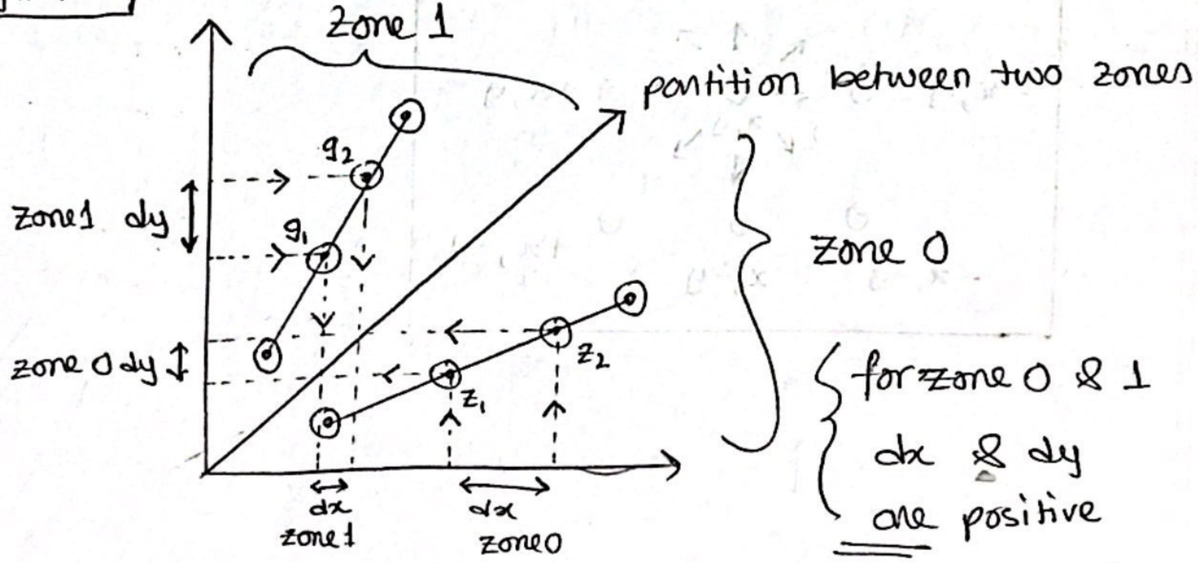


important:

$\boxed{\text{We need to determine the zone of the line first}}$

lets consider only these two for now,

- $0 \leq \theta < 45$ → is a group
- $45 \leq \theta < 90$ → is another group



for zone 0 & 1 dx & dy are positive

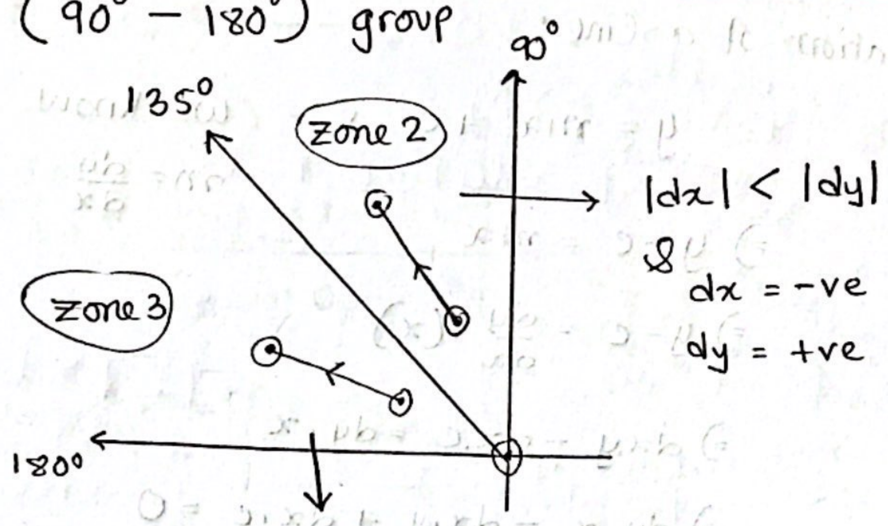
zone 0 → $|dx| > |dy|$

zone 1 → $|dy| > |dx|$

$dx = x_2 - x$

$dy = y_2 - y_1$

Now, $(90^\circ - 180^\circ)$ group



So we are done with zones 0, 1, 2, 3

only for 0,1,2,3

<u>Code:</u>

```
int find_zone (int x0, int y0, int x1, int y1){

        int dx = x1 - x0;
        int dy = y1 - y0;
        if (dx >= 0  && dy >= 0){   // zone 0 & 1
            if (dx >= dy){
                return 0;
            }
            else {
                return 1;
            }
        }
        else if (dx <= 0  && dy >= 0){
            if (abs (dx) >= dy){
                return 3;
            }
            else {
                return 2;
        }
}
```

Algorithm →

Equation of a line:

$$y = mx + c,$$

we know $m = \frac{dy}{dx}$

$$\Rightarrow y - c = mx$$

$$\Rightarrow y - c = \frac{dy}{dx} \cdot (x)$$

$$\Rightarrow dx \cdot y - dx \cdot c = dy \cdot x$$

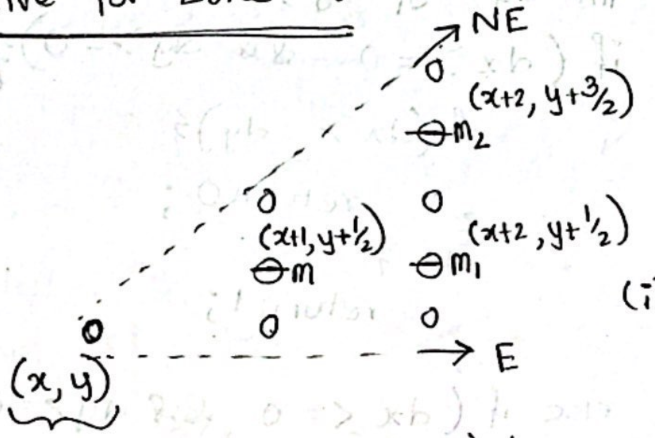
$$\Rightarrow \underbrace{dy}_{\text{reps} \to A} \cdot x \underbrace{- dx}_{B} \cdot y + \underbrace{dx \cdot c}_{C} = 0$$

So, $A = dy$
$B = -dx$

$$Ax + By + C = 0$$

lets deirve for zone 0 :

Step 1



an arbitrary starting point.

if a line is in zone 0, Then there cane be two types of movement

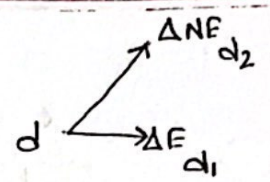
(i) NE → d_2

(ii) E → d_1

using eqn $Ax + By + c = 0$

Step 2

(deviation at M_1) $\longrightarrow \Delta E$



at $M_1 \longrightarrow A(x+2) + B(y+\frac{1}{2}) + c = d_1$

at $M \longrightarrow \underset{(-)}{} A(x+1) \underset{(-)}{+} B(y+\frac{1}{2}) \underset{(-)}{+} c = d$

$$A \quad +0 \quad +0 = d_1 - d = \underbrace{\Delta E}$$

the rate of change of pixel for a horizontal movement.

$\therefore \Delta E = A = dy$ //

(deviation at M_2) $\longrightarrow \Delta NE$

at $M_2 \longrightarrow A(x+2) + B(y+\frac{3}{2}) + c = d_2$

$M \longrightarrow \underset{(-)}{} A(x+1) \underset{(-)}{+} B(y+\frac{1}{2}) \underset{(-)}{+} c = d$

$$A + B + 0 = d_2 - d = \Delta NE \text{ \{NE movement}$$

$\therefore \Delta NE = A + B = dy - dx$ //

We've solved for ΔE, ΔNE, but we still need the initial deviation, which is represented using d_{init}.

$$A(x_0+1) + B(y_0 + \tfrac{1}{2}) + c = d_{init} \rightarrow \text{at } M$$

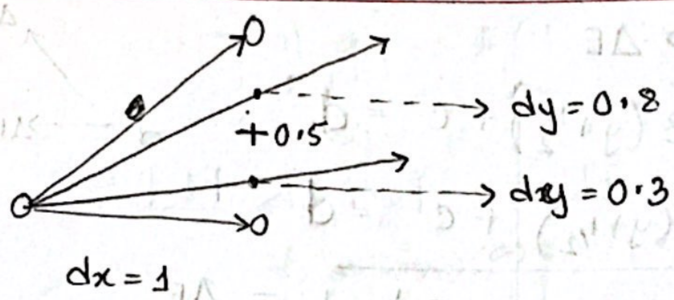
$\Rightarrow Ax_0 + A + By_0 + B/2 + c = d_{init}$

$\Rightarrow \underbrace{Ax_0 + By_0 + c}_{0} + A + B/2 = d_{init}$

since $Ax + By + c = 0$ & since x_0, y_0 belongs to the line.

$\therefore d_{init} = A + B/2 = dy - dx/2$

Step3



for dy = 0.8

$$d_{init} = dy - dx/2$$

$$= 0.8 - 1/2 = +0.3$$

for dy = 0.3

$$d_{init} = dy - \frac{dx}{2}$$

$$= 0.3 - 1/2 = -0.2$$

∴ if d is +ve, then the next pixel is upper pixel

&

if d is -ve, then the next pixel is lower pixel.

So, $\Delta E = dy$

$\Delta NE = dy - dx$

$d = dy - dx/2$ } → but we don't wan't fraction so will multiply by 2

$\times 2$

$d = 2dy - dx$

$\Delta NE = 2dy - 2dx$

$\Delta E = 2dy$

} since we changed d, we also need to change or multiply ΔE & ΔNE with 2

Example: (30,50) to (40,54)

(1) first calculate dx & dy,

$dx = 40-30 = 10$

$dy = 54-50 = 4$

So, dx & dy is +ve

& $|dx| > |dy|$ → Hence zone 0

———x———

(2) secondly, calculate the initial deviation → dinit

don't → $d = dy - \frac{dx}{2} = 4 - \frac{10}{2}$

(W)

$d = 2dy - dx$

$\Rightarrow d = 2(4) - 10$

$\Rightarrow d = 8 - 10 = -2$

→ if d +ve → ΔNE

if d -ve → ΔE

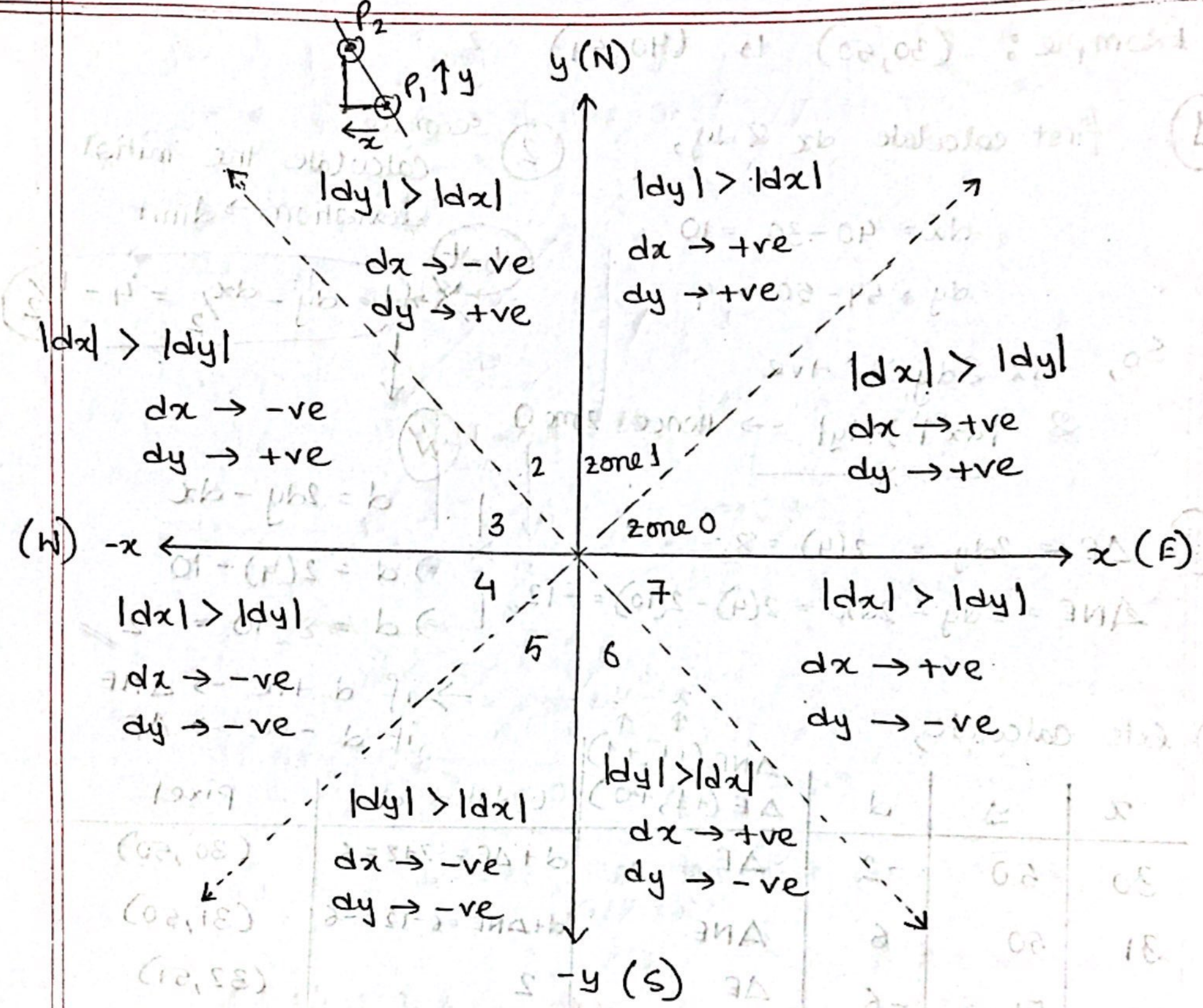
(3) $\Delta E = 2dy = 2(4) = 8$

$\Delta NE = 2dy - 2dx = 2(4) - 2(10) = -12$

(4) lets calculate,

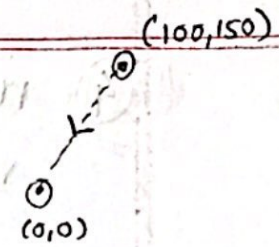
x	y	d	ΔNE (+1,+1) / ΔE (+1,+0) (x ↑, y ↑)	updated d	pixel
30	50	-2	ΔE	d + ΔE = -2+8 = 6.	(30,50)
31	50	6	ΔNE	d+ΔNE = 6-12 = -6	(31,50)
32	51	-6	ΔE	2	(32,51)
33	51	2	ΔNE	-10	(33,51)
34	52	-10	ΔE	-2	(34,52)
35	52	-2	ΔE	6	(35,52)
36	52	6	ΔNE	-6	(36,52)
37	53	-6	ΔE	2	(37,53)
38	53	2	ΔNE	-10	(38,53)
39	54	-10	ΔE	-2	(39,54)
40	54	-2	→ done here.		

(Zone Layout)



P2
P1 ↑y
←x
y (N)
|dy| > |dx|
dx → −ve
dy → +ve
|dy| > |dx|
dx → +ve
dy → +ve
|dx| > |dy|
dx → −ve
dy → +ve
|dx| > |dy|
dx → +ve
dy → +ve
2
zone 1
3
zone 0
(W) -x
x (E)
4
7
|dx| > |dy|
dx → −ve
dy → −ve
|dx| > |dy|
dx → +ve
dy → −ve
5
6
|dy| > |dx|
dx → +ve
dy → −ve
|dy| > |dx|
dx → −ve
dy → −ve
−y (S)

let's try to compute for another zone.

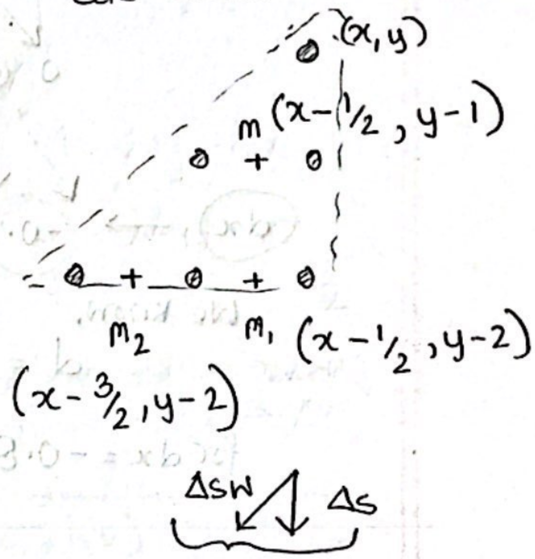


Example 2 $P_1(100,150)$ to $P_2(0,0)$

(1) $dy = 0 - 150 = -150$
$dx = 0 - 100 = -100$ } both -ve,

$|dy| > |dx|$ } So, zone 5

(2) lets draw zone 5



two types of movement possible.

(3) ΔS movement,

$M_1 \rightarrow A(x-1/2) + B(y-2) + C = d_1$

$M \rightarrow A(x-1/2) + B(y-1) + C = d$

$0 - B + 0 = d_1 - d = \Delta S$

$\therefore \Delta S \rightarrow -B$

$\Delta S = +dx$

ΔSW, movement,

$M_2 \rightarrow A(x-3/2) + B(y-2) + C = d_2$

$M \rightarrow A(x-1/2) + B(y-1) + C = d$

$-A - B + 0 = d_2 - d = \Delta SW$

$\therefore \Delta SW = -A - B = -dy + dx$

$= dx - dy$

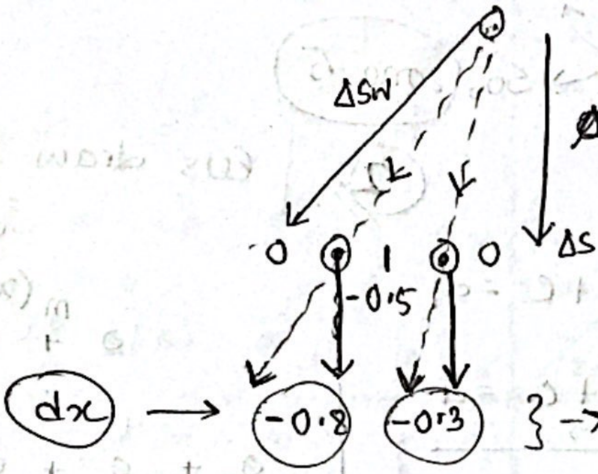
$\Rightarrow d_{init} = A(x_0 - 1/2) + B(y_0 - 1) + C$

$d_{init} = Ax_0 - \frac{A}{2} + By_0 - B + C$ → Since $Ax_0 + By_0 + C = 0$

$= -A/2 - B = dx - dy/2$

Scanned with CamScanner

(4) We need to figure out what happens for a +ve/-ve value of d, → if it is a ΔS/ΔSW movement.



} → arbitrary values greater than and lower than -0.5,

We know,

$$d = dx - \frac{dy}{2}$$

for $dx = -0.8$,

$$d = -0.8 - \left(-\frac{1}{2}\right)$$

$$= -0.8 + 0.5$$

$= -0.3$ → So if d is negative it is a ΔSW movement

for $dx = -0.3$

$$d = -0.3 - \left(-\frac{1}{2}\right)$$

$$= -0.3 + 0.5$$

$= 0.2$ → So if d is positive it is a ΔS movement.

$(100, 150) \longrightarrow (0,0)$

(5) What we have,

$d = dx - dy/2$ } → can't have fraction need to multiply by 2

$d = 2dx - dy$

$\Delta S = 2dx$ } → has also been multiplied by 2

$\Delta SW = 2dx - 2dy$ } → " " " " " "

$d_{init} = 2dx - dy = 2(-100) - (-150)$
$= -50$ → so initially ΔSW

$\Delta S = -200$
$\Delta SW = -200 + 300 = 100$

Lets only draw 12 pixels, → a little messed up sorry !!

No.	x	y	ΔS / ΔSW	d	(pixel)
1	100	150	ΔSW	-50	(100, 150)
2	99	149	ΔS	50	(99, 149)
3	99	148	ΔSW	-150	(99, 148)
4	98	147	ΔSW	-50	(98, 147)
5	97	146	ΔS	50	(97, 146)
6	97	145	ΔSW	-150	(97, 145)
7	~~97~~6	144	ΔSW	-50	(96, 144)
8	95	143	ΔS	50	(95, 143)
9	95	142	ΔSW	-150	(95, 142)
10	94	141	ΔSW	-50	(94, 141)
11	93	140	ΔS	50	(93, 140)
12	93	139	N/A	N/A	(93, 139) → done//

(Try at home for the other zones)