Instructions for preparing the solution script:

- Write your name, ID#, and Section number clearly in the very front page.
- Write all answers sequentially.
- Start answering a question (not the pat of the question) from the top of a new page.
- Write legibly and in orderly fashion maintaining all mathematical norms and rules. Prepare a single solution file and rename it "ID#_FirstName_TheorySection#.pdf".
- Start working right away. There is no late submission form. If you miss the deadline, you need to use the make-up assignment to cover up the marks.

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1. Consider a set of data points (0, 1), (0.5, 1.6487) and $(1, 2.7183)$. Now answer the following:
   (a) (6 marks) Compute the Lagrange bases: $l_0(x)$, $l_1(x)$ and $l_2(x)$.
   (b) (3 marks) Write the interpolating polynomial and find the value of the interpolating polynomial at $x = 0.2$.
   (c) (3 marks) If the data are given by the function $f(x) = e^x$, what will be the maximum error of the Lagrange polynomial function at 0.2? (use Weierstrass Approximation)
   (d) (2 marks) Why is the Lagrange method better than the Vandermonde matrix method to find an interpolating polynomial?
2. Consider the data points (0, 4), (1, 16) and $(-1, 8)$. Using these data and Newton's divided/difference method, answer the following:
   (a) (6 marks) Compute the coefficients a_k for $k = 0, 1, 2$. Also write the interpolating polynomial.
   (b) (2 marks) Using the polynomial you found in the previous part, find $p_n(0.5)$ and $p_n(-0.9)$.
3. (a) (6 marks) Compute the upper bound of the error using Cauchy's Theorem for $f(x) = \sin^2(x/2)$ with nodes $\{-\pi/3, 0, \pi/3\}$ within the interval $[-1.2, 1.2]$.
   (b) (2 marks) Why are Chebyshev nodes an optimal choice in Interpolation?
4. Consider the following data set:

x	$f(x)$	$f'(x)$
0.1	-0.62050	3.58502
0.2	-0.28340	3.14033

   Answer the following based on the above data:
   (a) (8 marks) Compute the Hermite bases: $h_0(x)$, $h_1(x)$, $\hat{h}_0(x)$ and $\hat{h}_1(x)$.
   (b) (2 marks) Write the Hermite polynomial and find the value at $x = 0.15$.

☺☺☺ **Yes!!!!! Let the PHUN begin!!!** ☺☺☺