

LINE DRAWING

1

Line-drawing Algorithm

(x_0, y_0)  Vector, ($\vec{v} = \vec{P}_1 - \vec{P}_0$)
start direction
is defined

$$\begin{cases} dx = x_1 - x_0 \\ dy = y_1 - y_0 \end{cases} \quad \begin{array}{l} \text{carries direction} \\ \text{and magnitude.} \end{array}$$

$$\begin{aligned} dx &\Rightarrow i |dx| & \rightarrow v \text{ & } -v \\ dy &\Rightarrow j |dy| & \text{defines the} \\ && \text{direction.} \end{aligned}$$

code {D,D,A}

line drawing algo's

$y = y_0$
 for ($x = x_0$ to $x \leq x_1$)
 draw Pixel (x, y)
 $y += m;$

We know,

$$m = \frac{y_1 - y_0}{x_1 - x_0}$$

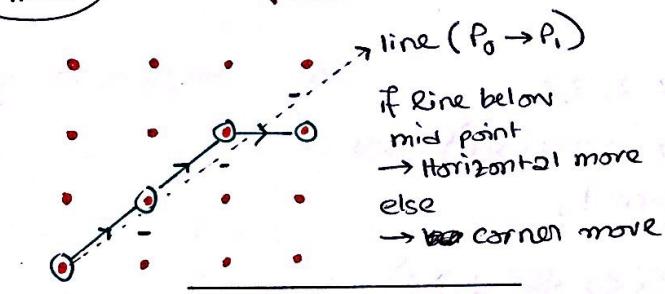
Q When step size α becomes 1

$$\phi_m = y_1 - y_0$$

Hence increment of $y = m$

IDEA

(Bresenham's Algorithm)



if line below
mid point
→ Horizontal move
else
→ ~~top~~ corner move

(Zones)

- Zone 0**: $(\theta \geq 0, < 45^\circ)$
- Zone 1**: $(\theta > 45^\circ, < 90^\circ)$
- Zone 2**: $(\theta > 90^\circ, < 135^\circ)$
- Zone 3**: $(\theta > 135^\circ, < 180^\circ)$
- Zone 4**: $(\theta > 180^\circ, < 225^\circ)$
- Zone 5**: $(\theta > 225^\circ, < 270^\circ)$
- Zone 6**: $(\theta > 270^\circ, < 315^\circ)$
- Zone 7**: $(\theta > 315^\circ, < 0^\circ)$

Conditions:

- Zone 0:** $|dx| > |dy|$, $dx \text{ +ve}$, $dy \text{ -ve}$
- Zone 1:** $|dy| \geq |dx|$
- Zone 2:** $|dy| \geq |dx|$, $|dx| > |dy|$
- Zone 3:** $|dx| > |dy|$
- Zone 4:** $|dx| > |dy|$
- Zone 5:** $|dx| > |dy|$
- Zone 6:** $|dx| > |dy|$, $dx \text{ +ve}$, $dy \text{ -ve}$
- Zone 7:** $|dx| > |dy|$

- Ø absolute values check
 - then determine the sets of zones
- Ø then check
 - dy, and dx's solid ± values to determine which zone.

Code : (zone)

```
int find_slope (int x0, int y0, int x1, int y1) {
    int dx = x1 - x0, dy = y1 - y0;
    if (abs(dx) > abs(dy)) // zone {0, 3, 4, 7}
        if (dx > 0 && dy > 0) // 0
            return 0;
        }
        else if (dx < 0 && dy > 0) // 3
            return 3;
        else if (dx < 0, && dy < 0) // 4
            return 4;
        else // 7
            return 7;
    }
    else // zone 1, 2, 5, 6
        if (dx > 0 && dy > 0) // 1
            return 1;
        }
        else if (dx < 0 && dy > 0) // 2
            return 2;
        else if (dx < 0 && dy < 0) // 5
            return 5;
        }
        else // 6
            return 6;
}
```

drawing the line:

(2)

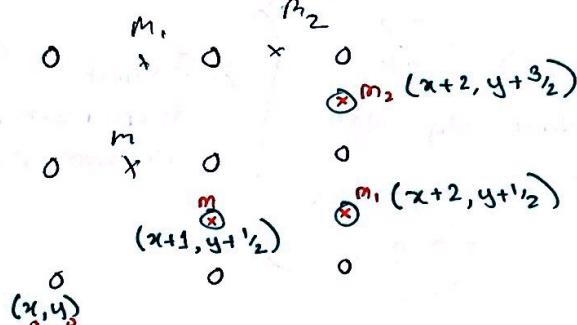
Eqn of line:

$$y = mx + c; \quad m = \frac{dy}{dx}$$

Multiply with dx (both side):

$$\begin{array}{c} dy \ x - dx \ y + dx \ c = 0 \\ \downarrow \quad \downarrow \quad \downarrow \\ A \ x + B \ y + C = 0 \end{array} \quad \left| \begin{array}{l} A = dy \\ B = -dx \\ C = dx \end{array} \right.$$

Zone 0



(deviation at M_1) $\rightarrow \Delta E$

$$\text{at } M_1 \rightarrow A(x+2) + B(y+1/2) + C = d_1$$

$$\text{at } M \rightarrow -A(x+1) + B(y+1/2) + C = d$$

$$\frac{A + 0 + 0}{A + 0 + 0} = d_1 - d \rightarrow \Delta E \quad \left\{ \begin{array}{l} \text{Horizontal movement} \\ \text{or} \\ \text{we need the deviation to form the whole line.} \end{array} \right.$$

$$\therefore \Delta E = A = dy$$

(deviation at M_2)

$$\text{at } M_2 \rightarrow A(x+2) + B(y+3/2) + C = d_2$$

$$\text{at } M_1 \rightarrow -A(x+1) + B(y+1/2) + C = d$$

$$\frac{A + B}{A + B} = d_2 - d \rightarrow \Delta NE \quad \left\{ \begin{array}{l} \text{NE movement} \\ \text{or} \\ \text{we need the deviation to form the whole line.} \end{array} \right.$$

$$\therefore \Delta NE = A + B = dy - dx$$

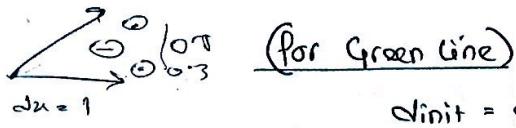
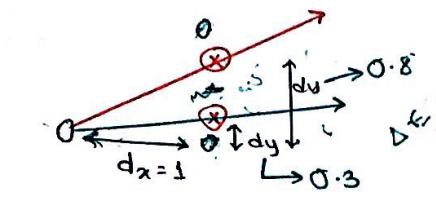
Dinit

$$A(x_0+1) + B(y_0+1/2) + C = d_{init}$$

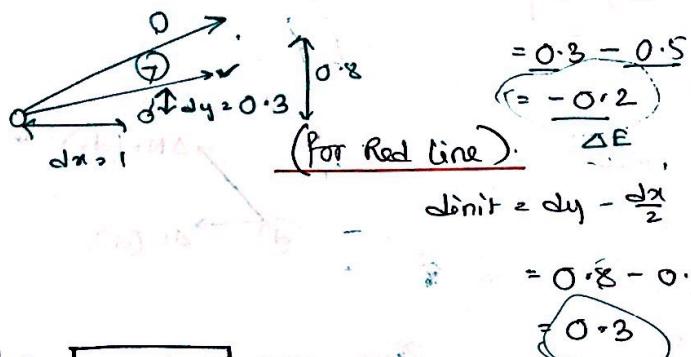
$$\therefore d_{init} = A + B/2$$

Since,
 $Ax_0 + By_0 + C = 0$
 \rightarrow line or upper part.

$$d_{init} = dy - dx/2$$



$$d_{init} = dy - \frac{dx}{2}$$



$$d_{init} = dy - \frac{dx}{2}$$

$$= 0.3 - 0.5 \\ (= -0.2)$$

$$= 0.8 - 0.5 \\ (= 0.3)$$

So,

if d_{init} is +ve
then next pixel is upper pixel.

if d_{init} is -ve
then next pixel is lower pixel.

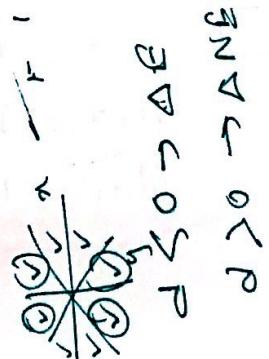
Code [drawline]

```

void drawline (int x0, int y0, int x1, int y1) {
    int dx = x1 - x0, dy = y1 - y0;
    int d = 2 * dy - dx; // mult by 2 to remove fraction.
    int dE = 2 * dy; // dNE = 2(dy - dx); // also mult by 2
    int y = y0, x = x0;
    drawPixel (x, y); // loop or bare first pixel draw hole.

    while (x < x1) {
        if (d > 0) { // Horizontal movement
            x++;
            d += dE;
        } else { // dNE movement
            x++;
            y++;
            d += dE;
        }
        drawPixel (x, y);
    }
}

```



Question 1(30, 50) to (40, 54) {
so 10x10 pixel size.void drawline - O { int x_0, y_0, x_1, y_1 }

$dx = 10;$

$dy = 4;$

$$d = -2; // 2dy - dx \\ = 8 - 10 \\ = -2$$

$\Delta E = +8; // 2dy = 8$

$\Delta NE = -12; // 2(dy - dx) = -12$

<u>x</u>	<u>y</u>	<u>d</u>	<u>$\Delta E/\Delta NE$</u>
30	50	-2	ΔE
31	50	6	ΔNE
32	51	-6	ΔE
33	51	2	ΔNE
34	52	-10	ΔE
35	52	-2	ΔE
36	52	6	ΔNE
37	53	-6	ΔE
38	53	2	ΔNE
39	54	-10	ΔE
40	54	→ end	

{ no redundant calculation.

Question No 2 $(x_0, y_0) = (100, 150)$ to $(x_1, y_1) = (0, 0)$

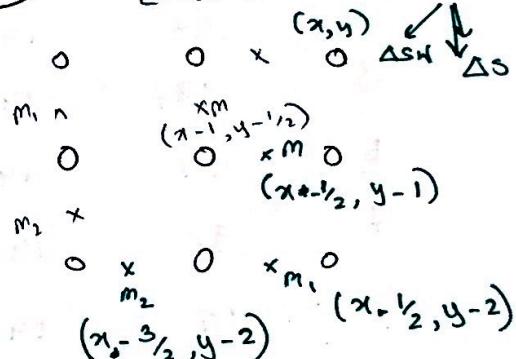
$dy = -150 \quad \{ \text{Since } \text{abs}(dx) < \text{abs}(dy) \}$

$dx = -100 \quad \} \text{ and}$

$(dx & dy) < 0$

So zone 5;

Zone 5

 Δs movement

$m_1 \quad A(x - 1/2) + B(y - 2) + C = d_1$

$m_2 \quad A(x - 1/2) + B(y - 1) + C = d$

$0 - A + 0 = d_1 - d \rightarrow \Delta s$

$\Delta s = -A = +dx$

 Δsw movement

$m_2 \quad A(x - 3/2) + B(y - 2) + C = d_2$

$m_1 \quad A(x - 1/2) + B(y - 1) + C = d$

$-A - B + 0 = d_2 - d_1 \rightarrow \Delta sw$

$\Delta sw = (B - A) + dx - \frac{dy}{2} \rightarrow \Delta sw$

$$d_{init} = A(x_0 - \frac{1}{2}) + B(y_0 - 1) + C$$

$$= -B - \frac{A}{2}$$

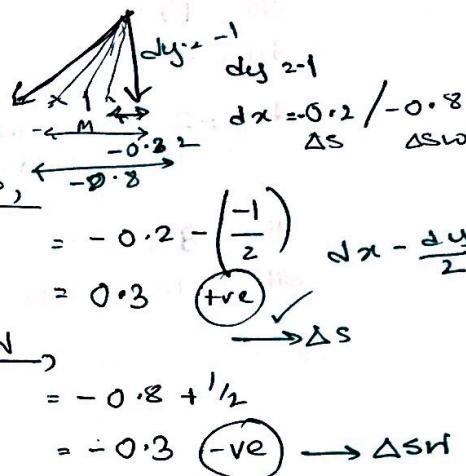
$$= dx - \frac{dy}{2}$$

for such question

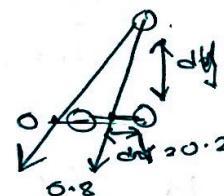
$$\sqrt{\Delta s} = dx = -100$$

$$\sqrt{\Delta SW} = dx - dy = -100 - (-150) \\ = 50$$

$$\sqrt{d_{init}} = -100 - \left(\frac{-150}{2}\right) \\ = -25$$



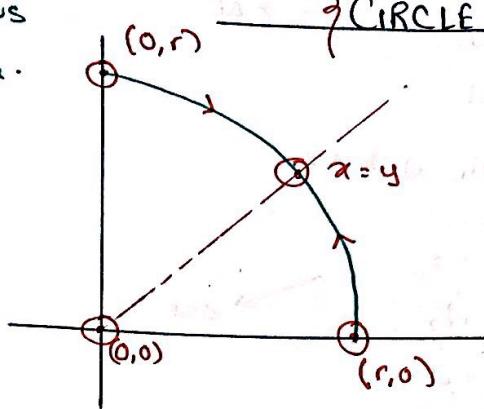
for 12 pixels



i. $d > 0 \rightarrow \Delta s$ $y--$
 $d < 0 \rightarrow \Delta SW$ $x--, y--$

Pixel	x	y	d	$\frac{\Delta s / \Delta SW}{\Delta SW}$
1	100	150	-25	ΔSW
2	99	149	25	ΔSW
3	99	148	-75	ΔSW
4	98	147	-25	ΔSW
5	97	146	25	ΔS
6	97	145	-75	ΔSW
7	96	144	-25	ΔSW
8	95	143	25	ΔS
9	95	142	-75	ΔSW
10	94	141	-25	ΔSW
11	93	140	25	ΔS
12	93	139	-75	ΔSW

r = radius
of circle.



CIRCLE DRAWING



2 operations

① If starting from $(0, r)$



ΔE movements.

ΔSE

② If starting from $(r, 0)$

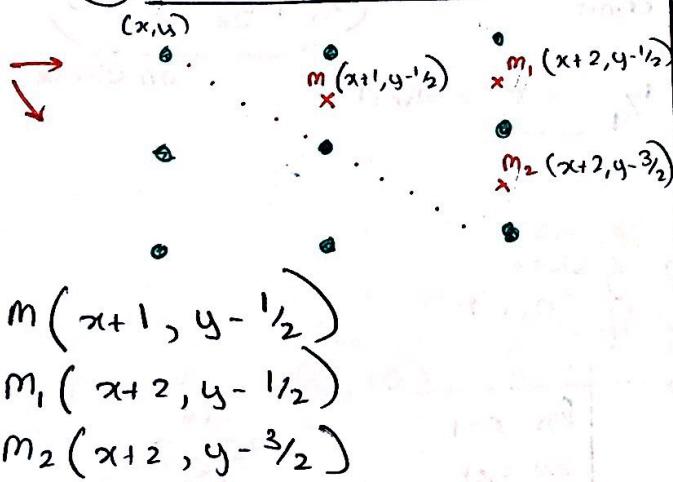
ΔNW

ΔN movements.

(parametric equation of a circle)

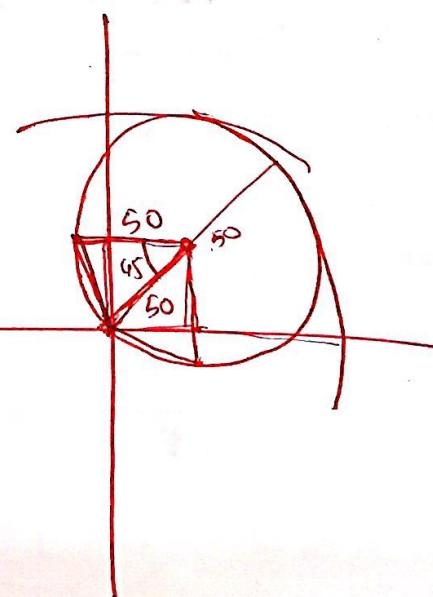
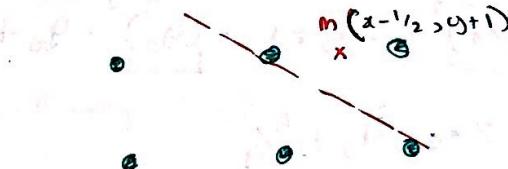
$$P(x, y) = x^2 + y^2 - r^2 = 0 \quad \text{--- } \textcircled{i}$$

① for $(0, r) \rightarrow \text{start}$



② for $(r, 0) \rightarrow \text{start}$

$$M_2(x-3/2, y+2) \quad M_1(x-1/2, y+2)$$



(For Operation 1) $\rightarrow (0, r)$

$$\begin{aligned} f(m_1) &= (x+2)^2 + (y - \frac{1}{2})^2 - r^2 = d_1 \\ f(m_2) &= (x+1)^2 + (y - \frac{1}{2})^2 - r^2 = d \end{aligned} \quad \rightarrow \Delta E$$
$$\Rightarrow 2x + 3 = d_1 - d = \Delta E$$

$\therefore \Delta E = 2x + 3$

$$\begin{aligned} f(m_2) &= (x+2)^2 + (y - \frac{3}{2})^2 - r^2 = d_2 \\ f(m_3) &= (x+1)^2 + (y - \frac{1}{2})^2 - r^2 = d \end{aligned} \quad \rightarrow \Delta SE$$
$$(2x + 3) + (-2y + 2) = d_2 - d = \Delta SE$$

$\Delta SE = 2x + 2y + 5$

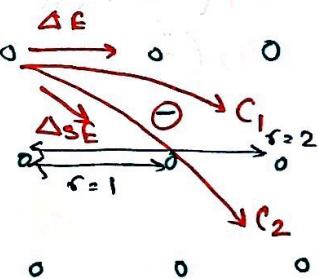
$$d_{init} = (x_0 + 1)^2 + (y_0 - \frac{1}{2})^2 - r^2 = d_{init}$$

$$\begin{aligned} &= (x_0)^2 + 2x_0 + 1 + (y_0)^2 - y_0 + \frac{1}{4} - r^2 = d_{init} \\ &= 2x_0 - y_0 + \frac{5}{4} \end{aligned}$$

$d_{init} = \frac{5}{4} - r$ Since $x_0 = 0$ $y_0 = r$ { start $(0, r)$

Since, $d_{init} = \frac{5}{4} - r$
 $= 1.25 - r$

+ve	So, ΔE goes below mid	for $r = 1$ we get $d_{init} + ve$
-ve	ΔE goes above mid	for $r = 2$ or greater $d_{init} - ve$



Code: {Zone 1 (start)}

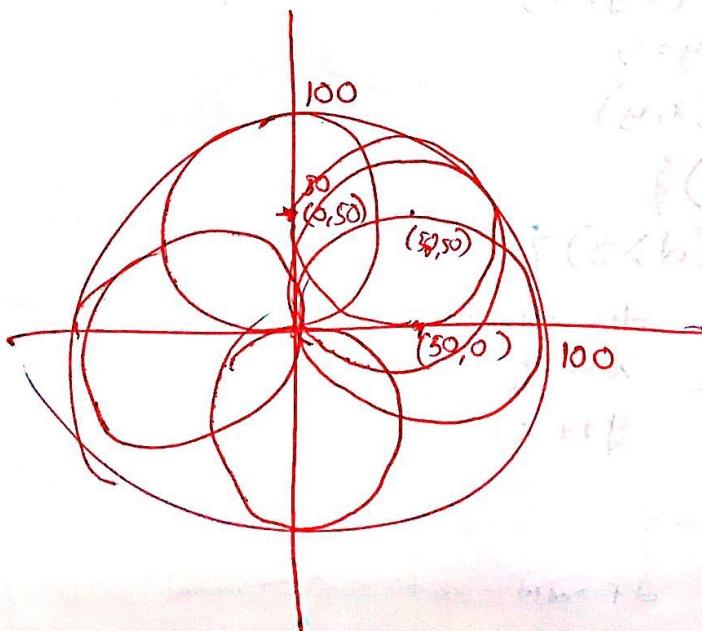
(2)

```

Void drawCircle_1 (int r) {
    int d = 5 - (4 * r); // mult 4
    int x = 0, y = r; // start (0,r)
    int dE = 4 * (2x + 3); // mult 4
    int dSE = 4 * (2x - 2y + 5); // mult 4
    draw8way(x,y);
    while (x < y) {
        if (d < 0) { // ΔE
            d += dE;
            x++;
        } else { // ΔSE
            d += dSE;
            x++;
            y--;
        }
        draw8way(x,y);
    }
}

```

Example : $(0,10)$ $\{r = 10\}$



$\frac{\Delta E}{\Delta SE}$
$\frac{\Delta E}{\Delta E}$
$\frac{\Delta E}{\Delta E}$
ΔE

→ 7 87

FOR CIRCLE Zone 0 (x_0, y_0)

NW $\nearrow \uparrow N$ ①

$$f(m_1) = (x - x_0)^2 + (y + 2)^2 - r^2 = d_1$$

$$f(m) = (x - x_0)^2 + (y + 1)^2 - r^2 = d$$

$$0 + (2y + 3) + 0 = d_1 - d \rightarrow \Delta N$$

$$\Delta N = 2y + 3$$

$$f(m_2) = (x - x_0)^2 + (y + 2)^2 - r^2 = d_2$$

$$f(m) = (x - x_0)^2 + (y + 1)^2 - r^2 = d$$

$$(-2x + 2) + (2y + 3) + 0 = d_2 - d \rightarrow \Delta NW$$

$$\Delta NW = 2y - 2x + 5$$

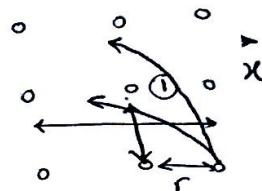
$$d_{init} = (x_0 - x_0)^2 + (y_0 + 1)^2 - r^2 = d_2 - d$$

$$d_{init} = 5/4 - r$$

$$so \quad d > 0 \quad \{ \Delta NW \}$$

$$so \quad d < 0 \quad \{ \Delta N \}$$

ΔNW
when $r \leq 1$



Code

```

void drawCircle_0 (int r) {
    int d = 5 - (4 * r);
    int dNW = 4 * (2y - 2x + 5);
    int dN = 4 * (2y + 3);
    int x = r, y = 0;
    draw8way(x, y);
    while (x > y) {
        if (d > 0) {
            d += dNW;
            x--;
            y++;
        } else {
            d += dN;
            y++;
        }
        draw8way(x, y);
    }
}

```

Question

(20, 0)

$$\Delta N = 2y + 3$$

$$\Delta NW = 2y + 2x + 5$$

r=20

$$d_{init} = -18.75$$

③

①

Pixel

x

y

d

$$\frac{\Delta N / \Delta NW}{\Delta N}$$

1

20

0

$$-18.75$$

ΔN

2

20

1

$$-15.75$$

ΔN

3

20

2

$$-10.75$$

ΔN

4

20

3

$$-3.75$$

ΔNW

5

20

4

$$5.25$$

ΔN

6

19

5

$$-21.75$$

ΔN

7

19

6

$$-8.75$$

ΔNW

8

19

7

$$6.25$$

ΔN

9

18

8

$$6.25$$

ΔNW

10

18

9

$$-6.75$$

ΔN

11

17

10

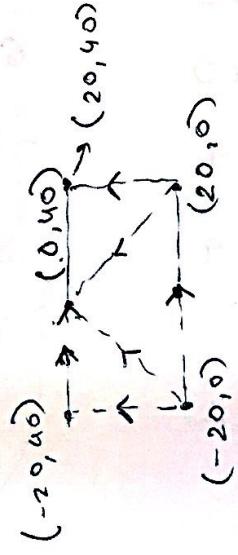
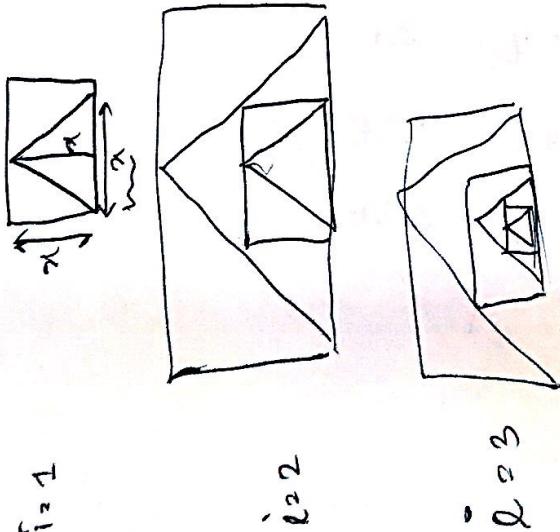
$$16.25$$

ΔNW

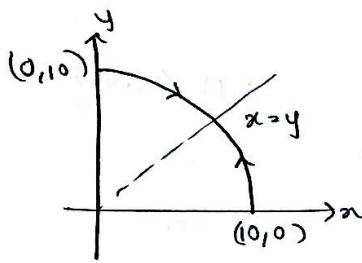
12

17

11



start $\rightarrow (0, 10)$



(x_0, y_0)

$x M_1(x+2, y-1/2)$

$M(x+1, y-1/2)$

$x M_2(x+2, y-3/2)$

$$f(x, y) = x^2 + y^2 - r^2 = 0$$

$$\begin{aligned} f(M_1) &= (x+2)^2 + (y-1/2)^2 - r^2 = d_1 \\ f(M) &= (x+1)^2 + (y-1/2)^2 - r^2 = d \end{aligned}$$

$$\frac{(2x+3)}{(2x+3) + 0} = d_1 - d = \Delta F$$

$$\textcircled{W} \Delta F = (2x+3)$$

$$f(M_2) = (x+2)^2 + (y-3/2)^2 - r^2 = d_2$$

$$\begin{aligned} f(M) &= (x+1)^2 + (y-1/2)^2 - r^2 = d \\ (2x+3) + (-2y+2) &= d_2 - d = \Delta S E \end{aligned}$$

$$\textcircled{W} \Delta S E = 2x - 2y + 5$$

$$\begin{aligned} \text{dinit} &= (x_0+1)^2 + (y_0-1/2)^2 - r^2 \\ &= 5/4 - r^2 \end{aligned}$$

$$\text{For } r \leq 1 \rightarrow \Delta F$$

$$\text{dinit} = +ve$$

$$\text{dinit} = -ve \rightarrow \Delta F$$

$$\begin{array}{c} x \\ \hline 0 \end{array}$$

$$\begin{array}{c} y \\ \hline 10 \end{array}$$

$$\begin{array}{c} d \\ -8.75 -35 \end{array}$$

$$\frac{\Delta F / \Delta S E}{\Delta F}$$

$$\begin{array}{c} -5.75 23 \\ \checkmark \end{array}$$

$$\begin{array}{c} -0.75 -3 \\ \checkmark \end{array}$$

$$\begin{array}{c} 10 \\ \checkmark 6.25 25 \end{array}$$

$$\begin{array}{c} 10 \\ -2.75 -4 \\ \checkmark \end{array}$$

$$\begin{array}{c} 9 \\ 10 \\ 8 \\ 7 \end{array}$$

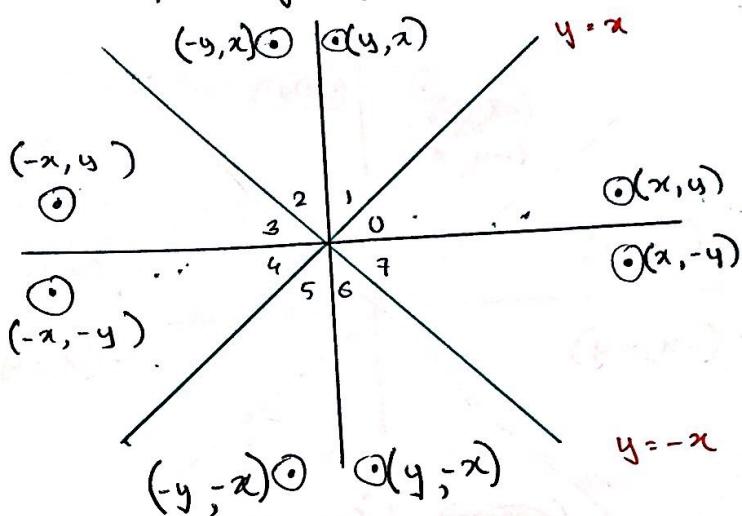
$$\begin{array}{c} 8.25 -33 \\ \checkmark 5.25 -20 \\ \checkmark \end{array}$$

$$\begin{array}{c} 8.25 -33 \\ \checkmark 5.25 -20 \\ \checkmark \end{array}$$

$$\begin{array}{c} 8.25 -33 \\ \checkmark 5.25 -20 \\ \checkmark \end{array}$$

end \rightarrow

38-way Symmetry?



Defn:

The line drawing code for each individual zone is not optimized, since, we need to write a lot of redundant lines of code.

Hence, we use 8-way symmetry.

If a ^{point} line is selected at zone 0, as (x, y) then we can derive its corresponding coordinates in the other zones by using reflection in the lines $y=x$ & $y=-x$.

From the above figure it can be determined that any point in another zone can be traced back to zone 0 (x, y) by going through the corresponding rearrangement pattern, by going through the corresponding rearrangement of the x & y coordinates in the other zones.

So, if we can derive the mid point line drawing algorithm for zone 0. We can use it draw a line

in any other zone, just by rearrangement of the coordinates in any other zone as shown in figure above.

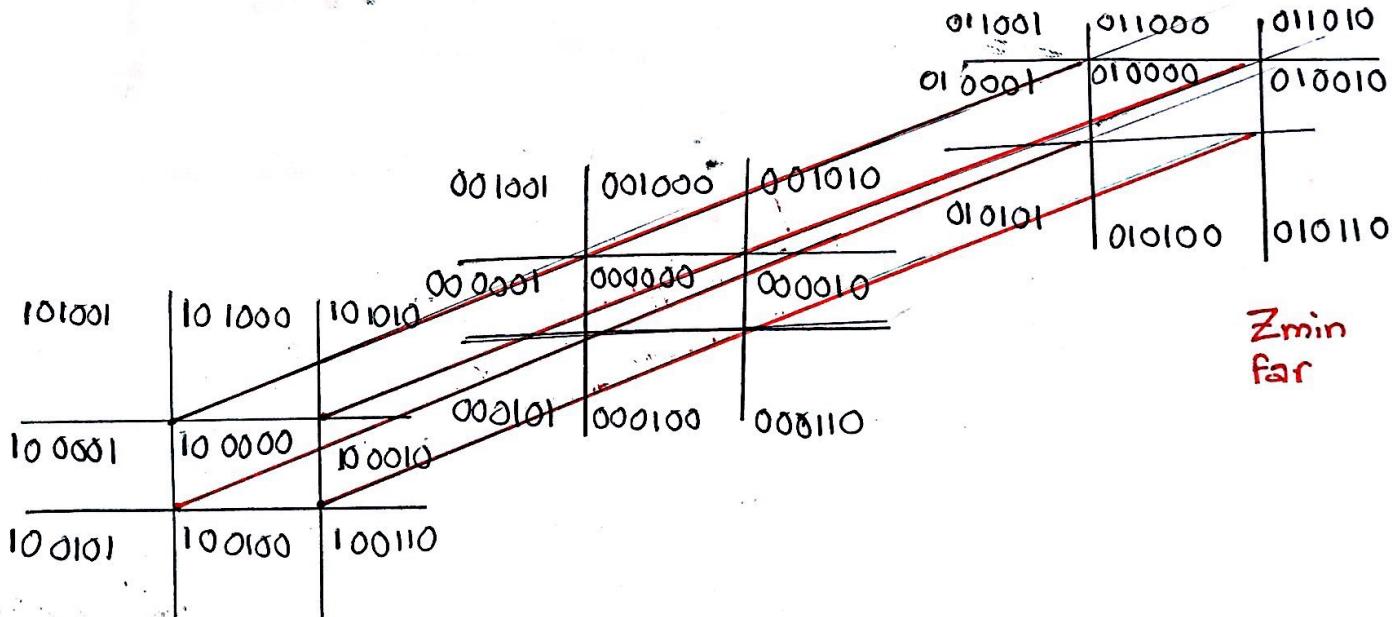
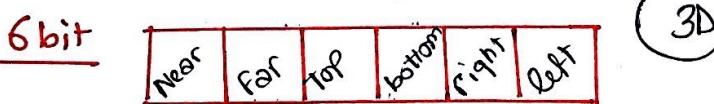
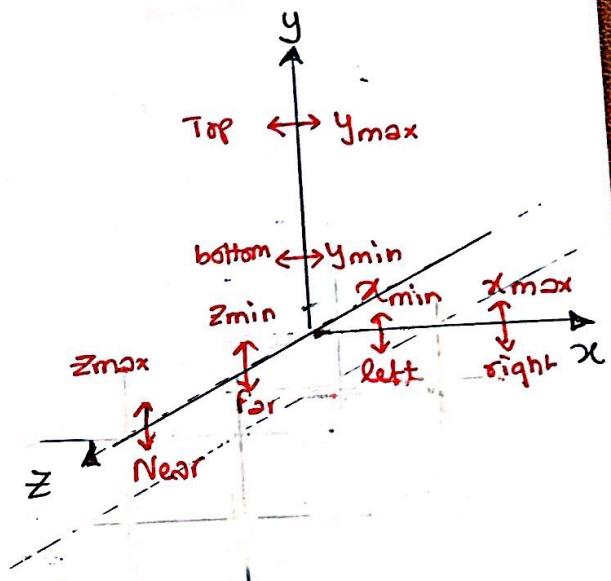
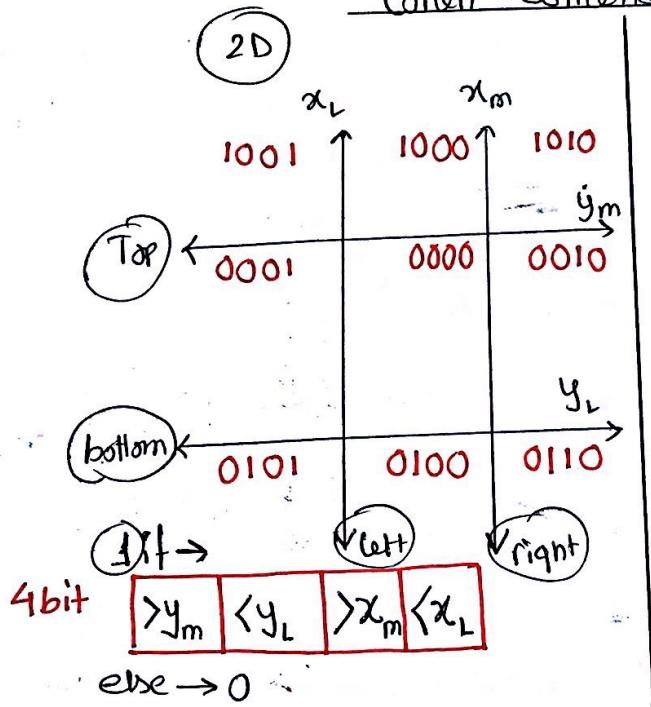
So we transform the points to zone 0. find the next pixel and by rearrangement, draw the pixel in its actual zone.

zone division

Explanation

How
MPG
works

Cohen-Sutherland LCA



\curvearrowleft z_max
Near

3D Cohen Sutherland Code

```
int ymax = 100, ymin = -100, xmax = 120, xmin = -120, zmax = 100, zmin = -100  
int Near = 32, far = 16, top = 8, bottom = 4, right = 2, left = 1  
int makeCode (int x, int y, int z)  
{  
    int Code = 0; // 1010001000  
    if (z > zmax) { // means adds 32 → so 32nd bit → 1  
        Code += Near; → 32  
    }  
    else if (z < zmin) {  
        Code += far; → 16  
    }  
    if (y > ymax) {  
        Code += top; → 8  
    }  
    else if (y < ymin) {  
        Code += bottom; → 4  
    }  
    if (x > xmax) {  
        Code += right; → 2  
    }  
    else if (x < xmin) {  
        Code += left; → 1  
    }  
    return Code;  
}
```

(2)

void Cohen - Sutherland (int $x_0, y_0, z_0, x_1, y_1, z_1$)

int Code0, Code1;

int x, y, z ;

Code0 = MakeCode (x_0, y_0, z_0) // start point

Code1 = MakeCode (x_1, y_1, z_1) // end point

while (1) {

if (Code0 | Code1 == 0) { // complete accept

drawline ($x_0, y_0, z_0, x_1, y_1, z_1$);

break;

}

else if (Code0 & Code1) { // complete reject

break;

}

else { // Partial Accept OR Reject

if (Code0) {

Code = Code0;

}

else {

Code = Code1;

}

if (Code & Near) { // $z > z_{max}$

$z = z_{max}$;

$x = x_0 + \frac{z - z_0}{z_1 - z_0} (x_1 - x_0)$;

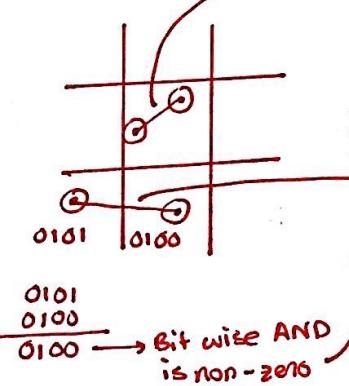
$y = y_0 + \frac{z - z_0}{z_1 - z_0} (y_1 - y_0)$;

else if (Code & Far) { // $z < z_{min}$

$z = z_{min}$

$x = x_0 + \frac{z - z_0}{z_1 - z_0} (x_1 - x_0)$;

$y = y_0 + \frac{z - z_0}{z_1 - z_0} (y_1 - y_0)$;



$$x = x_0 + t(x_1 - x_0)$$

$$\rightarrow t = \frac{x - x_0}{x_1 - x_0}$$

$$\therefore t = \frac{z - z_0}{z_1 - z_0}$$

$$\therefore t = \frac{y - y_0}{y_1 - y_0}$$

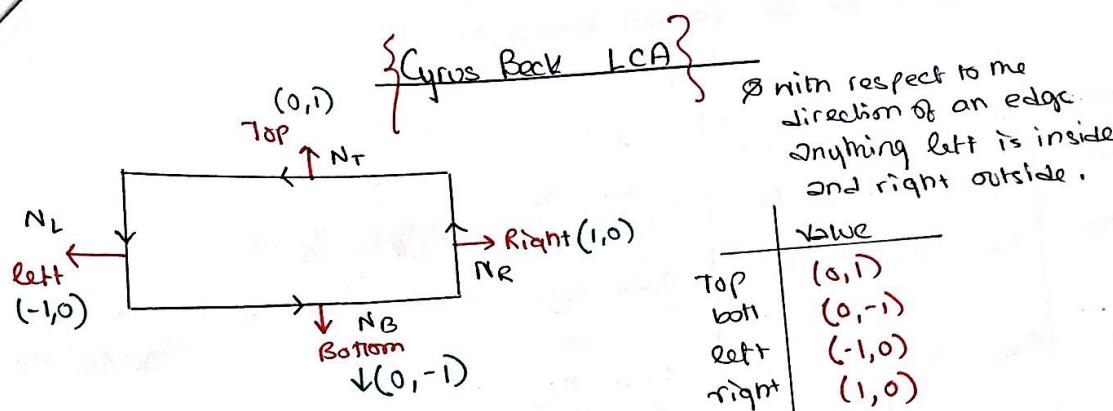
```

        if (Code & Top) { // y > ymax
            --- = some
        }
        else if (Code & Bottom) { y < ymin
            --- = some
        }
        if (Code & right) { x > xmax
            --- = some
        }
        else if (Code & left) { x < xmin
            --- = some
        }
        if (Code = Code 0) {
            x0 = x, y0 = y, z0 = z;
            Code 0 = Make Code (x0, y0, z0);
        }
        else {
            x1 = x, y1 = y, z1 = z;
            Code 1 = Make Code (x1, y1, z1);
        }
    } // Partial else
} // while
} // end

```

Summary

- will clip and make Code 0 & Code 1 again if outside, to fit it inside frame
- once inside from both points
 Point 0 → start → Code 0 → 000000
 Point 1 → end → Code 1 → 000000
 becomes fully accepted at first if cond and breaks.



Value of $\ell \approx (0 - 1)$

1 Parametric eqn of a line in view plane:

$$P_{(t)} = P_0 + \ell (P_i - P_0)$$

at $\ell=0, P_t = P_0$ & at $\ell=1, P_t = P_i$

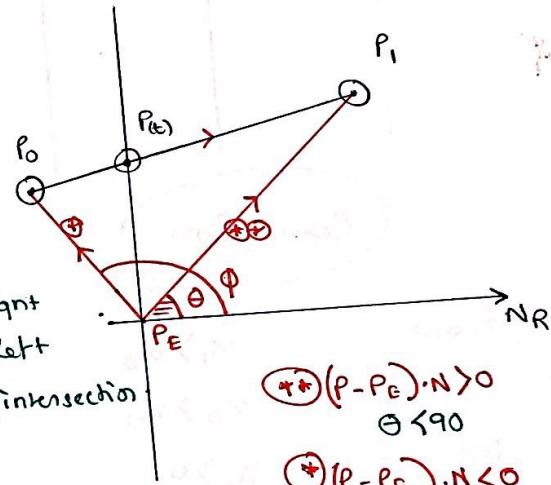
2 dot product considering "cos"

$$(P - P_E) \cdot N =$$

<0 , -ve, point on right

>0 , +ve, point on left

0, point on edge, intersection



So, if we consider, DERIVATION

$$P_{(t)} = P_0 + \ell (P_i - P_0) \quad \dots \text{I}$$

$$\text{and, for } P = P_{(t)} \rightarrow (P - P_E) \cdot N = 0 \quad \dots \text{II}$$

Substituting, $P = P_{(t)}$

$$(P_{(t)} - P_E) \cdot N = 0 \xrightarrow{\text{substitute eqn I}}$$

$$\Rightarrow (P_0 + \ell (P_i - P_0) - P_E) \cdot N = 0$$

$$\Rightarrow \{(P_0 - P_E) \cdot N\} + \{\ell (P_i - P_0) \cdot N\} = 0$$

$$\Rightarrow \ell = \frac{-(P_0 - P_E) \cdot N}{(P_i - P_0) \cdot N} = \frac{(P_E - P_0)}{(P_i - P_0)}$$

Conditions

i) $\theta > 90$

④ List of δ for all possible boundaries

Boundary	$P_E - P_0$	$P_i - P_0$	δ
Top	$y_{max} - y_0$	$y_i - y_0$	$\frac{y_{max} - y_0}{y_i - y_0} \cdot N$
Bottom	$y_{min} - y_0$	$y_i - y_0$	$\frac{y_{min} - y_0}{y_i - y_0} \cdot N = (\cancel{x_E} - x_0) \cdot Nx + (y_E - y_0) \cdot Ny$
Left	$x_{min} - x_0$	$x_i - x_0$	$\frac{x_{min} - x_0}{x_i - x_0} \cdot N$
Right	$x_{max} - x_0$	$x_i - x_0$	$\frac{x_{max} - x_0}{x_i - x_0} \cdot N = (x_i - x_0) \cdot Nx + (1 - c)(y_i - y_0) \cdot Ny$

Leaving Cnd

Entering Cnd

Right	$x_i > x_0$
Left	$x_0 > x_i$
Top	$y_i > y_0$
Bottom	$y_0 > y_i$

Right

$x_0 > x_i$

Left

$x_i > x_0$

Top

$y_i < y_0$

Bottom

$y_i > y_0$

(2)

$y_{max}, y_{min}, x_{max}, x_{min}$, (Algorithm)

Void Cyrus_beck (int x_0, y_0, x_1, y_1) {

float $t_{max} = 0.0, t_{min} = 0.0, t = 0;$

$$t = (y_{max} - y_0) / (y_1 - y_0);$$

if ($y_1 > y_0$) // leaving

if ($t < t_{min}$) {

$$t_{min} = t;$$

}

else // entering

if ($t > t_{max}$) {

$$t_{max} = t;$$

}

}

$$t = (y_{min} - y_0) / (y_1 - y_0)$$

if ($y_1 > y_0$) // entering

if ($t_{max} < t$) {

$$t_{max} = t;$$

}

}

else // leaving

if ($t_{min} > t$) {

$$t_{min} = t;$$

}

}

$$t = (x_{max} - x_0) / (x_1 - x_0)$$

if ($x_1 > x_0$) // leaving

if ($t_{min} > t$) {

$$t_{min} = t;$$

}

}

else // entering

if ($t > t_{max}$) {

$$t_{max} = t$$

}

}

```

 $t = (x_{min} - x_0) / (x_1 - x_0);$ 
if ( $x_1 > x_0$ ) {
    if ( $t > t_{max}$ ) {
         $t_{max} = t;$ 
    }
}
else {
    if ( $t < t_{min}$ ) {
         $t_{min} = t;$ 
    }
}

if ( $t_{min} > t_{max}$ ) {
    // unacceptable cond
    new P0 = Point( $t_{max}$ )
    new P1 = Point( $t_{min}$ )
    drawline(new P0.x, new P0.y, new P1.x, new P1.y)
}
POI = new P0, new P1
POI (R Point x, y Point t) {
    P0.x = t * P0.x + t * (P1.x - P0.x)
    P0.y = t * P0.y + t * (P1.y - P0.y)
    return x, y // POI
}

```

(continued on next page)

$$y_{\max} = 50, y_{\min} = -50, x_{\max} = 60, x_{\min} = -60$$

$$(x_0, y_0) = (100, 60)$$

$$(x_1, y_1) = (110, -70)$$

$$t_{\text{top}} = \frac{50 - (60)}{-70 - (60)} = 0.0769 \rightarrow y_1 < y_0 \rightarrow \text{entering}$$

$$t_{\text{bottom}} = \frac{-50 - (60)}{-70 - (60)} = 0.846 \rightarrow y_1 < y_0 \rightarrow \text{leaving}$$

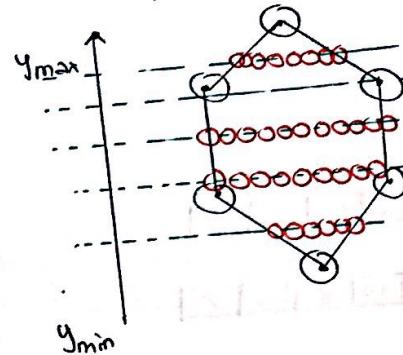
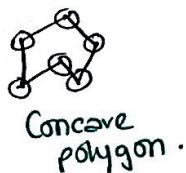
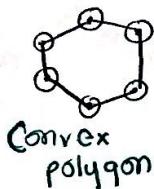
$$t_{\text{left}} = \frac{-60 - (-100)}{110 + 100} = 0.190 \rightarrow x_1 > x_0 \rightarrow \checkmark t_{\max}$$

$$t_{\text{right}} = \frac{60 - (-100)}{210} = 0.762 \rightarrow x_1 > x_0 \rightarrow \checkmark \text{leave } t_{\min}$$

\checkmark
 $t_{\text{left}} \rightarrow t_{\text{right}}$

$t_{\max} \rightarrow t_{\min}$
 line drawn.

POLYGON FILL SLIP



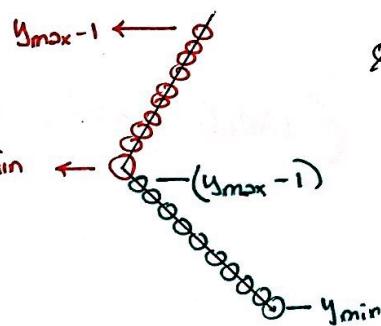
Filling starts

from y_{\min} to y_{\max} (iteration)

↓

fills from $(x_{\min} \text{ to } x_{\max})$

- ∅ Can't use midpoint algo, because it will have a lot of intersections
- ∅ and can create an ambiguous situation.



one draw from y_{\min} to $y_{\max-1}$, so no double overlaps

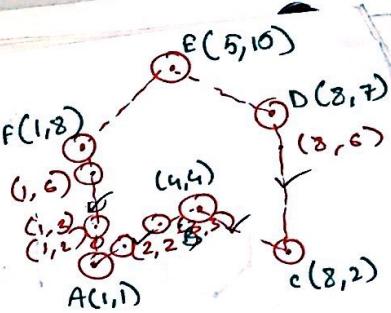
- (Edge Table)
- ∅ only considers the corner points
 - ∅ boundary set

∅ we don't consider horizontal lines. they are omitted.

- (Active Edge Table)
- ∅ has all the rows of x , for all the values of y .



Edge Table



$y=8$		$\rightarrow [10 \ 1 \ 2 \ 7]$		
$y=7$		$\rightarrow [10 \ 8 \ -1 \ \rightarrow]$		
$y=2$		$\rightarrow [4 \ 8 \ -2 \ \rightarrow]$	$\rightarrow [7 \ 8 \ 0 \ \rightarrow]$	
$y=01$		$\rightarrow [8 \ 1 \ 0 \ \rightarrow]$	$\rightarrow [4 \ 1 \ 1 \ \rightarrow]$	
\nearrow	y_{\max}	\downarrow current value of x	$\downarrow m$	\downarrow blank \rightarrow more is present \rightarrow ends.

$$y_{\max} = 10$$

$$y_{\min} = 1$$

$$\text{So } q_1 \rightarrow q_3$$

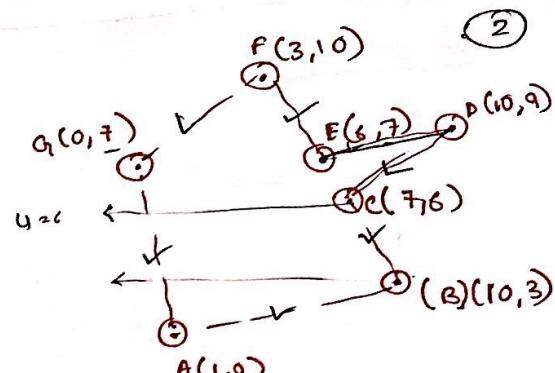
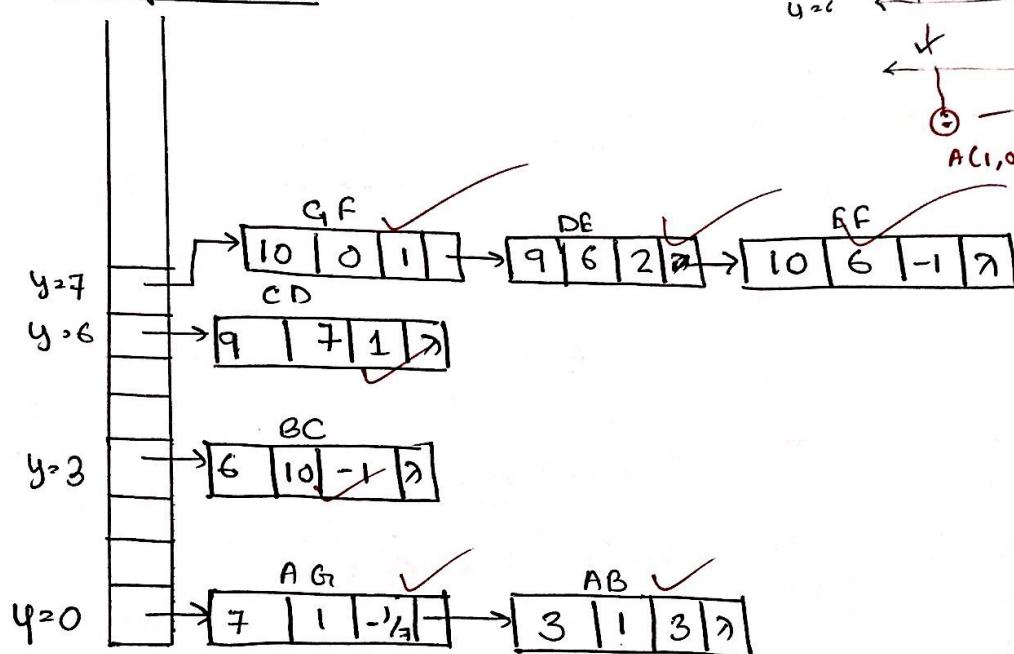
\rightarrow
rm)

Active Edge table

$y=10$		$\rightarrow [10 \ 3 \ 2 \ \rightarrow]$	$[10 \ 6 \ -1 \ \rightarrow]$	
$y=9$		$\rightarrow [10 \ 1 \ 2 \ \rightarrow]$	$[10 \ 7 \ -1 \ \rightarrow]$	
$\checkmark y=8$		$\rightarrow [10 \ 8 \ -1 \ \rightarrow]$	$[8 \ 1 \ 0 \ \rightarrow]$	
$\checkmark y=7$		$\rightarrow [7 \ 8 \ 0 \ \rightarrow]$	$[8 \ 1 \ 0 \ \rightarrow]$	
$y=6$		$\rightarrow [7 \ 8 \ 0 \ \rightarrow]$	$[8 \ 1 \ 0 \ \rightarrow]$	
$y=5$		$\rightarrow [7 \ 8 \ 0 \ \rightarrow]$	$[8 \ 1 \ 0 \ \rightarrow]$	
$y=4$		$\rightarrow [7 \ 8 \ 0 \ \rightarrow]$	$[8 \ 1 \ 0 \ \rightarrow]$	
$y=3$		$\rightarrow [4 \ 6 \ -2 \ \rightarrow]$	$[7 \ 8 \ 0 \ \rightarrow]$	$\rightarrow [8 \ 1 \ 0 \ \rightarrow]$
$\checkmark y=2$		$\rightarrow [4 \ 8 \ -2 \ \rightarrow]$	$[7 \ 8 \ 0 \ \rightarrow]$	$\rightarrow [8 \ 1 \ 0 \ \rightarrow]$
$\checkmark y=1$		$\rightarrow [8 \ 1 \ 0 \ \rightarrow]$	$[4 \ 1 \ 1 \ \rightarrow]$	
\nearrow	y_{\max}			

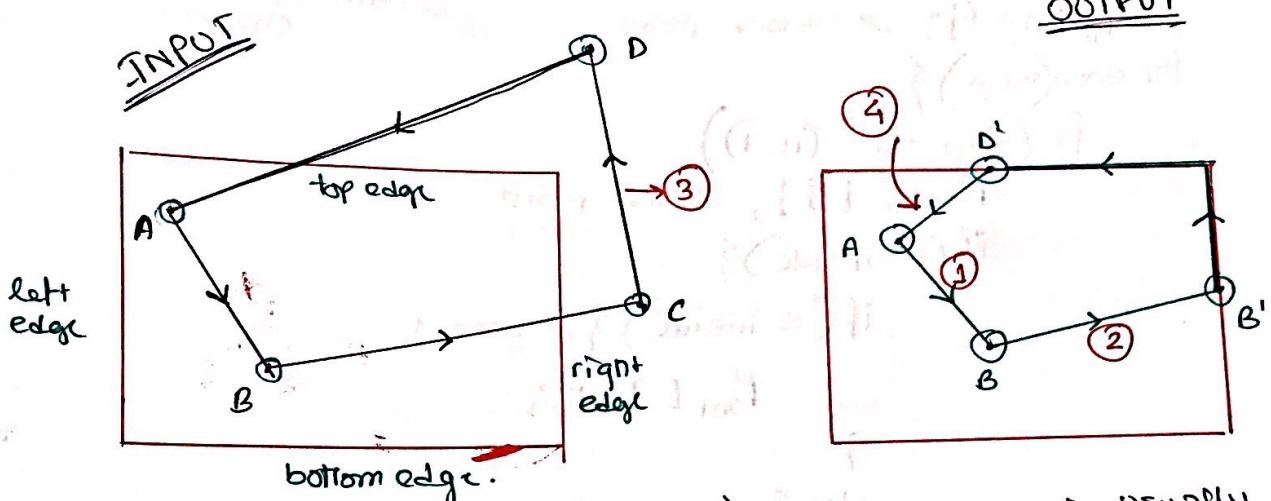
Question 2

Edge table



(Active Edge Table)

Polygon Clipping



Sutherland-Hodgeman PCA (4 rules)

- ① Both the start and end point are inside the frame. (in to in movement) $\rightarrow (A, B)$

Output : Destination vertex
 $= B$

- ② One point inside another outside the frame (movement in to out) $\rightarrow (B, C)$

Output : intersection point
 $= B'$

- ③ Both points outside the frame, so we ignore that (in case of CD)

Output : nothing.

- ④ One point outside, the other inside the frame.

(movement DA \rightarrow out to in)

Output : intersection point & destination point.
 $= D' A$

Usually done in 3 parts :

- ① left clip
- ② right clip
- ③ bottom clip
- ④ Top clip.

(Algorithm)

```
void PolygonClip ( // Input polygon  
    S = P_in[n-1]; // start point  
    for each(edge) {  
        for (j=0 to (n-1))  
            P = P_in[j]; // end point  
            if (P inside) {  
                if (S inside) // case 1  
                    P_out[] = P;  
                }  
                else { // case 4  
                    i = intersection (S,P);  
                    P_out[] = l; // small left  
                    P_out[] = P;  
                }  
            }  
        else { // P outside.  
            if (S inside) // case 2  
                i = intersection (S,P);  
                P_out[] = l;  
            }  
            else { // case 3  
                // do nothing.  
            }  
        S = P;  
    } // end for loop  
} // end of each edge.
```

$N \rightarrow$ no. of points
 $O \rightarrow (n-1)$ choice.