Forward and Backward Kinematics on Manipulator

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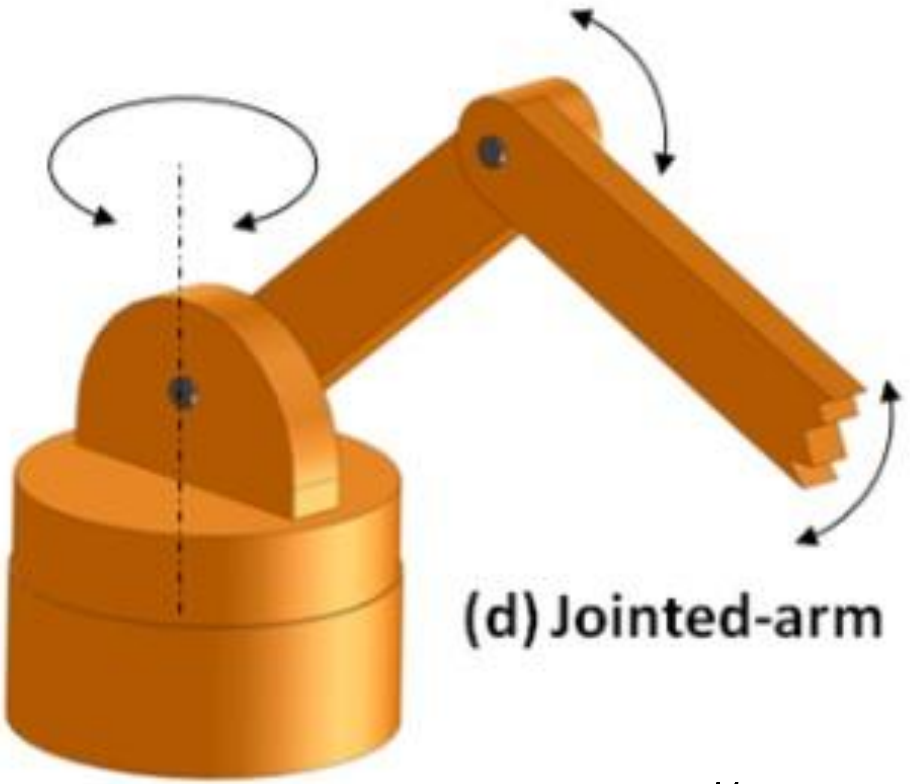
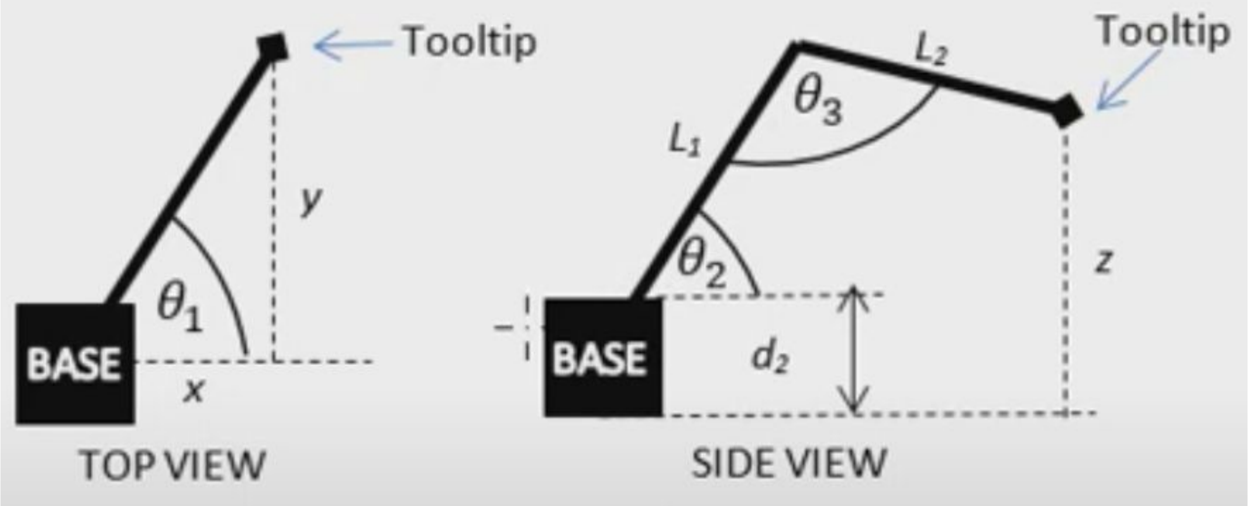
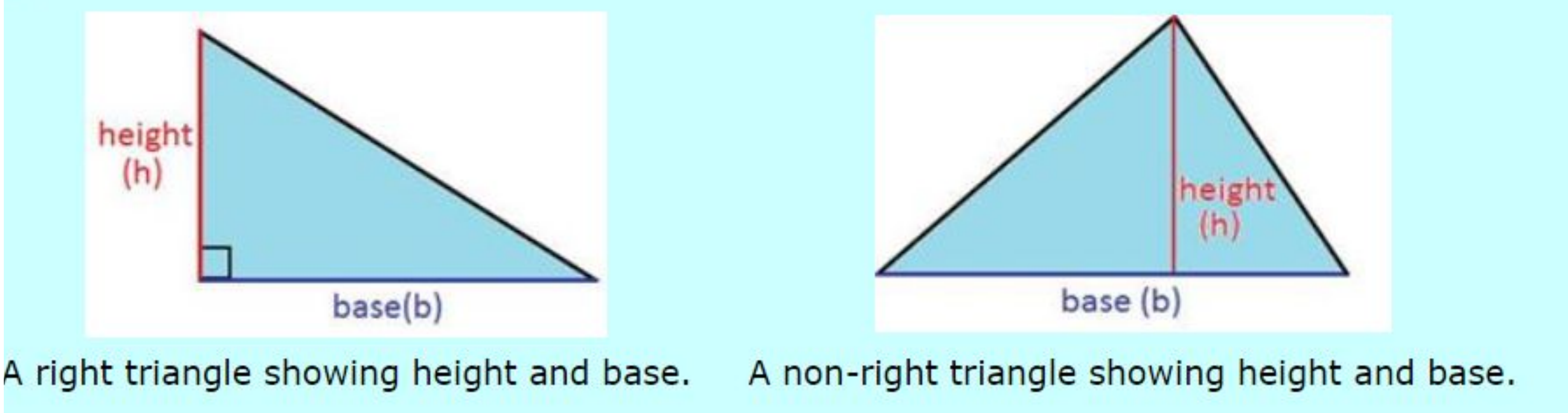
Forward Kinematics





https://www.youtube.com/watch?v=NRgNDlVtmz0

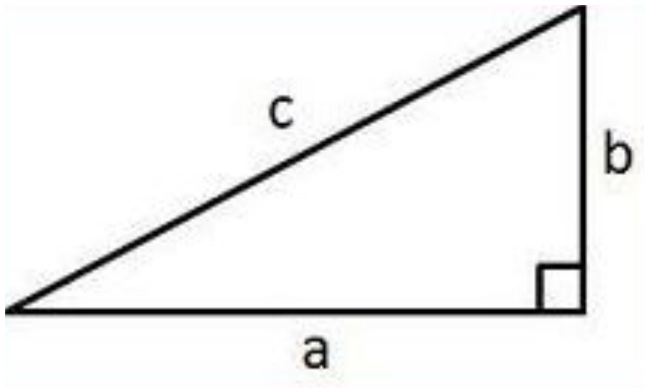
Area of a triangle

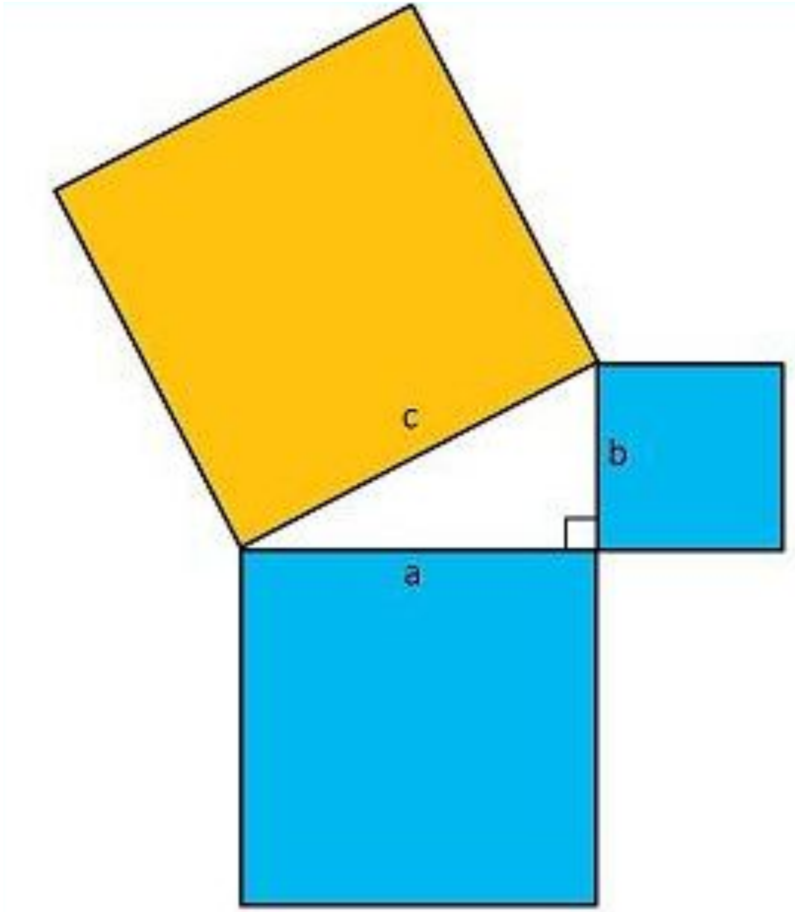
- 

A right triangle showing height and base. A non-right triangle showing height and base.

- $area\ of\ a\ triangle = \frac{1}{2}\ X\ base\ X\ height$

Pythagoras' Theorem for right triangle

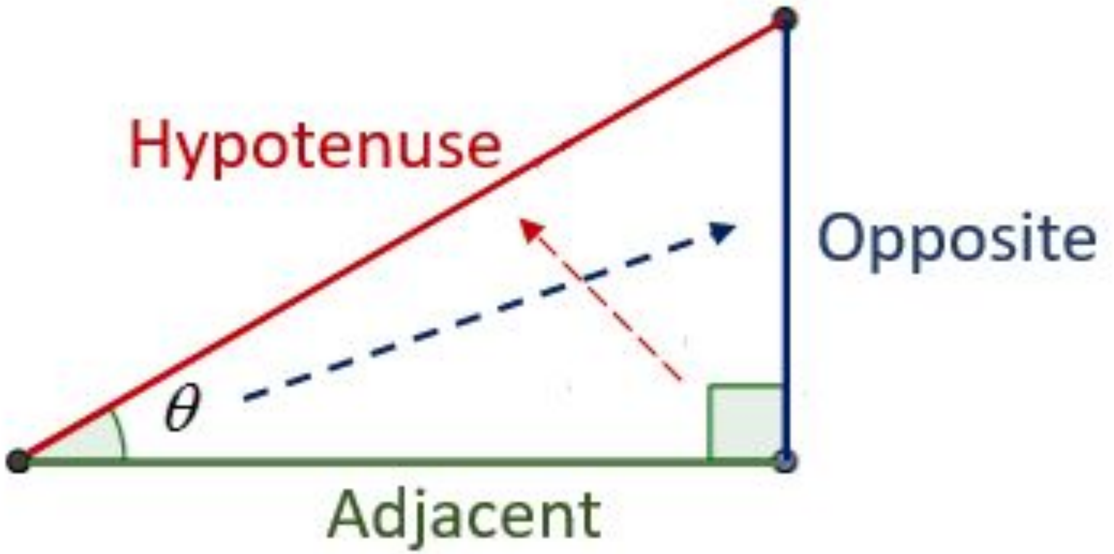
- 



- $c^2 = a^2 + b^2$

Basic Trigonometric Functions

SOHCAHTOA



SOH $\quad \text{Sin}\,\theta = \dfrac{\text{Opposite}}{\text{Hypotenuse}}$

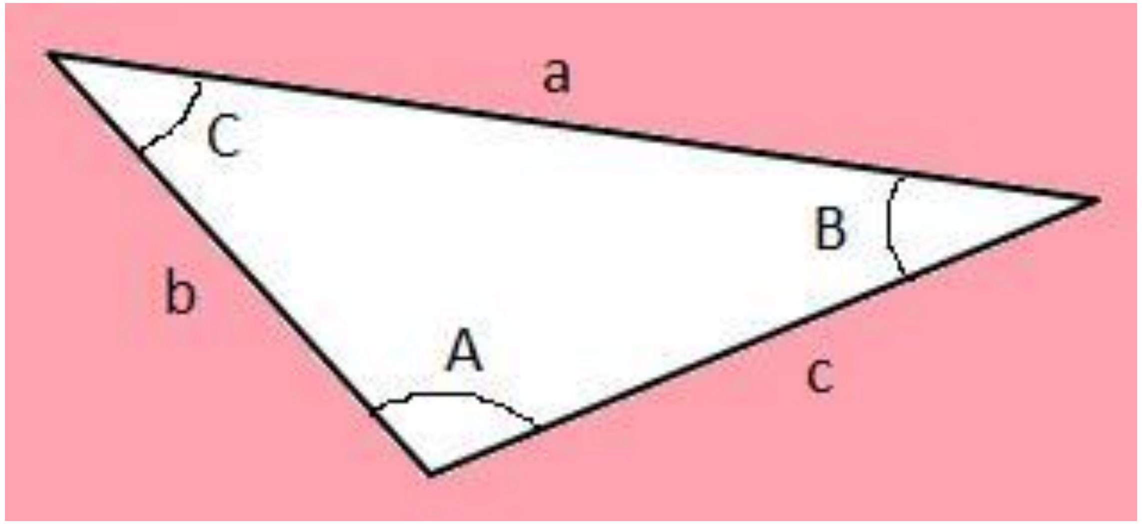
CAH $\quad \text{Cos}\,\theta = \dfrac{\text{Adjacent}}{\text{Hypotenuse}}$

TOA $\quad \text{Tan}\,\theta = \dfrac{\text{Opposite}}{\text{Adjacent}}$

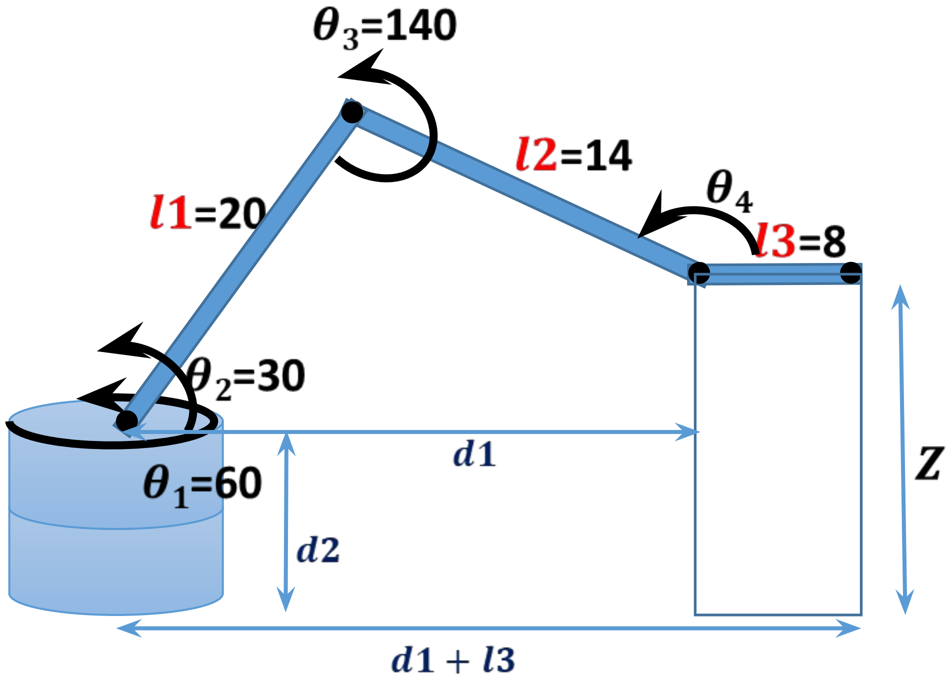
The Sine and Cosine Rules

- *Sine Rule:*
- $\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$

- *Cosine Rule:*
- $a^2 = b^2 + c^2 - 2bcCosA$
- *or*
- $CosA = \frac{b^2+c^2-a^2}{2bc}$



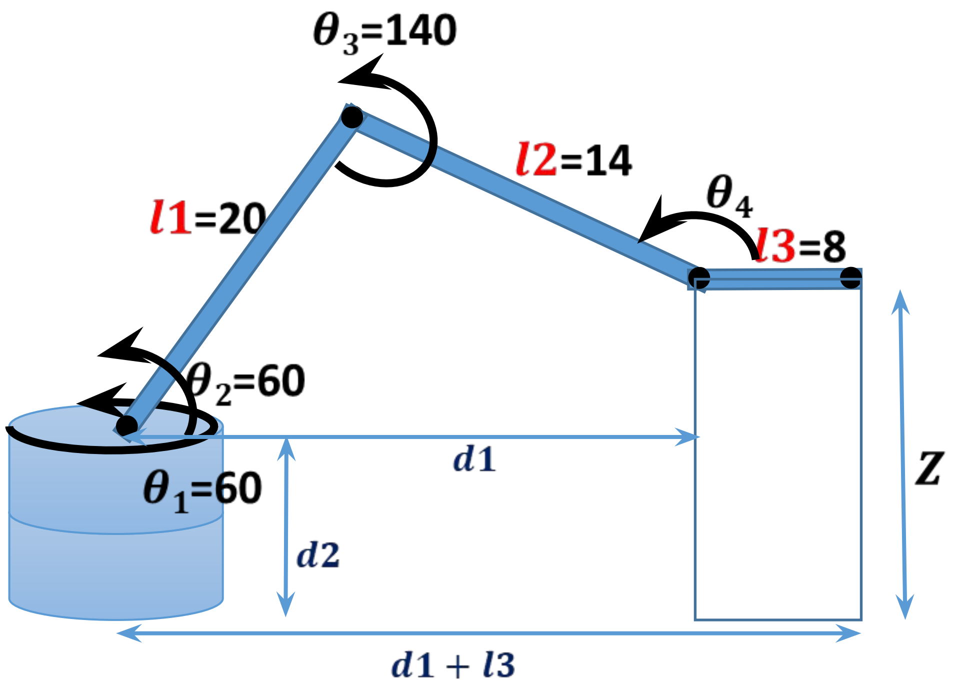
4 DOF Arm Calculation



- *l1=Length of first arm*
- *l2=Length of 2nd Arm*
- *l3=Length of 3rd Arm (end effector)*
- *d2=height of base*

- $\theta_1 = Angle\ of\ base\ rotation$
- $\theta_2 = Angle\ of\ first\ arm\ from\ horizon$
- $\theta_3 = Angle\ between\ 1st\ and\ 2nd\ arm$
- $\theta_4 = Angle\ between\ 2nd\ arm\ and\ end\ effector$

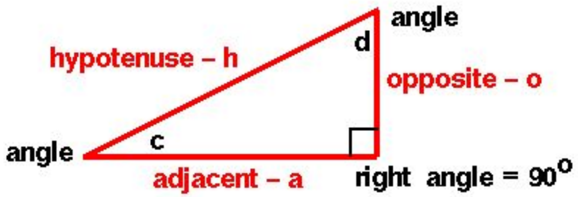
Example with a Value



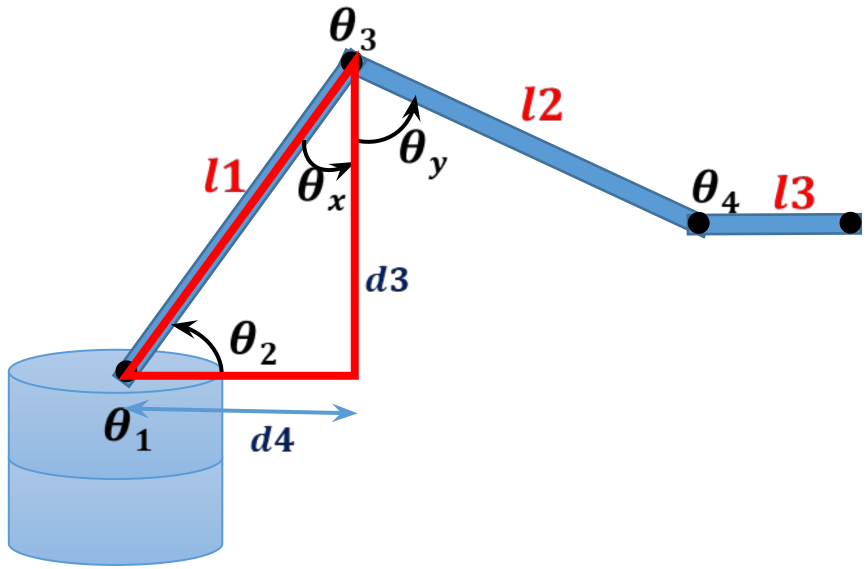
- *l1=20"*
- *l2=14"*
- *l3=8"*
- *d2=18"*

- $\theta_1 = 50$
- $\theta_2 = 60$
- $\theta_3 = 95$

Forward Kinematics Calculation

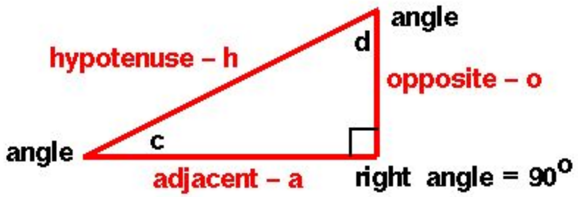


$$cos\theta = \frac{A}{H}; sin\theta = \frac{O}{H}; tan\theta = \frac{O}{A}$$

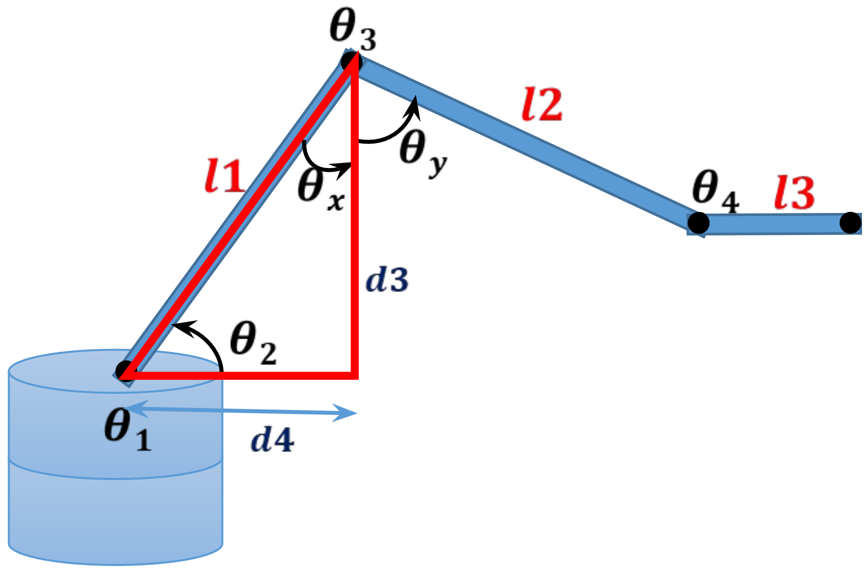


- $If\ we\ draw\ a\ right\ angle\ triangle\ with\ first\ arm,$
- $\theta_x\ is\ the\ angle\ with\ opposit - d3$
- $\theta_y\ is\ the\ angle\ with\ Adjacent - d4$
- $So, \theta_3 = \theta_x + \theta_y$
- $Here\ l1\ is\ hypotenus$
- $So, sin\theta = \frac{O}{H};\ sin\theta_2 = \frac{d3}{l1}$
- $d3 = sin\theta_2 * l1$
- $cos\theta = \frac{A}{H};\ cos\theta_2 = \frac{d4}{l1}$
- $d4 = cos\theta_2 * l1$

Example continuation…



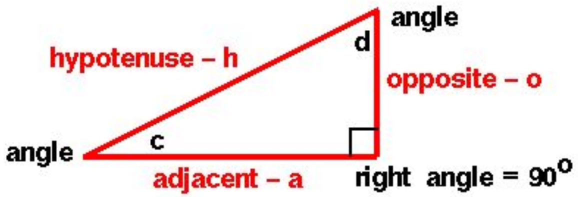
$$cos\theta = \frac{A}{H}; sin\theta = \frac{O}{H}; tan\theta = \frac{O}{A}$$



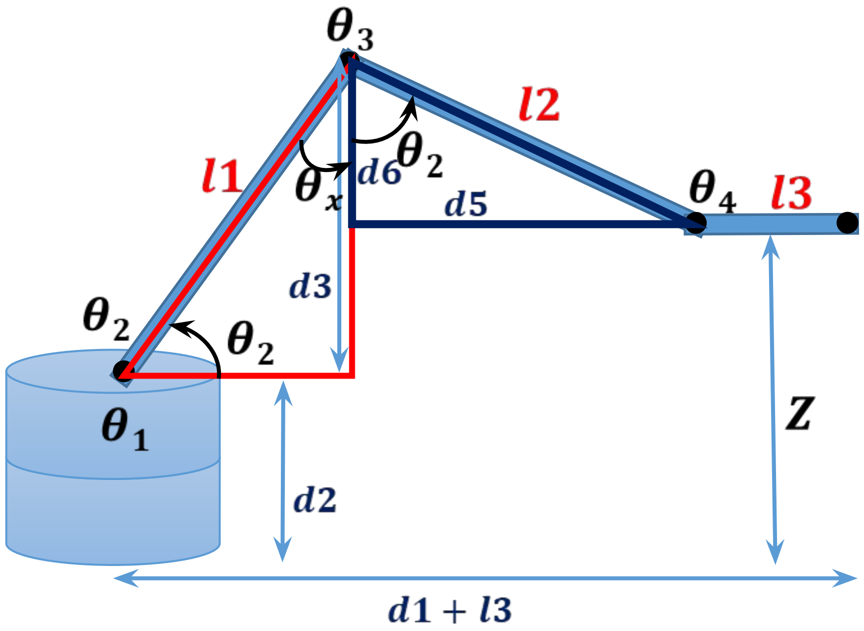
- $\theta_x = 180 - 90 - \theta_2$
- $\theta_x = 180 - 90 - 60 = 30$
- $\theta_3 = \theta_x + \theta_y$
- $\theta_y = (\theta_3 - \theta_x)$
- $\theta_y = 95 - 30 = 65$
- $d3 = sin\theta_2 * l1$
- $d3 = sin60 * l1$
- $d3 = 0.866 * 20 = 17.32$
- $d4 = cos\theta_2 * l1$
- $d4 = cos60 * l1$
- $d4 = .5 * 20 = 10$

l1=20”
l2=14”
l3=8”
d2=18”
$\theta_1 = 50$
$\theta_2 = 60$
$\theta_3 = 95$

Forward Kinematics Calculation

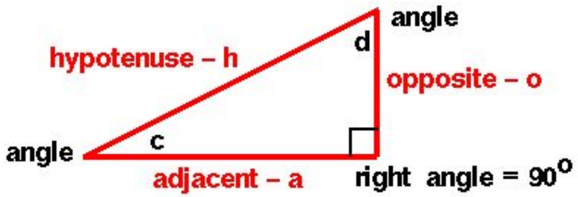


$cos\theta = \frac{A}{H}; sin\theta = \frac{O}{H}; tan\theta = \frac{O}{A}$

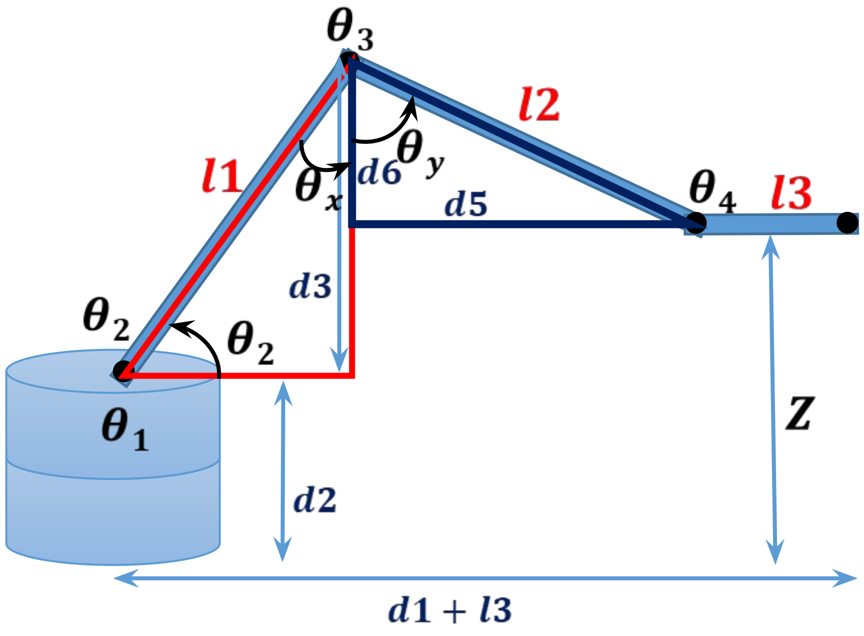


- $If\ we\ draw\ a\ right\ angle\ triangle\ with\ 2nd\ arm\ l2$
- $\theta_y\ is\ the\ angle\ with\ opposit - d6$
- $l2\ is\ hypotenus$
- $So,\ cos\theta = \frac{A}{H};\ cos\theta_y = \frac{d6}{l2};$
- $d6 = cos\theta_y * l2;$
- $sin\theta = \frac{O}{H};\ sin\theta_y = \frac{d5}{l2};$
- $d5 = sin\theta_y * l2$

Example continuation…



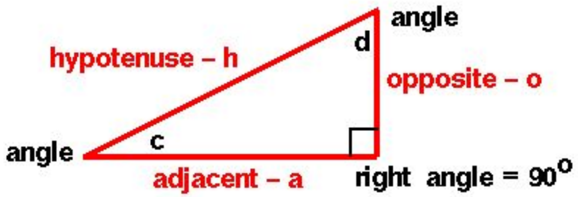
$$\cos\theta = \frac{A}{H}; \sin\theta = \frac{O}{H}; \tan\theta = \frac{O}{A}$$



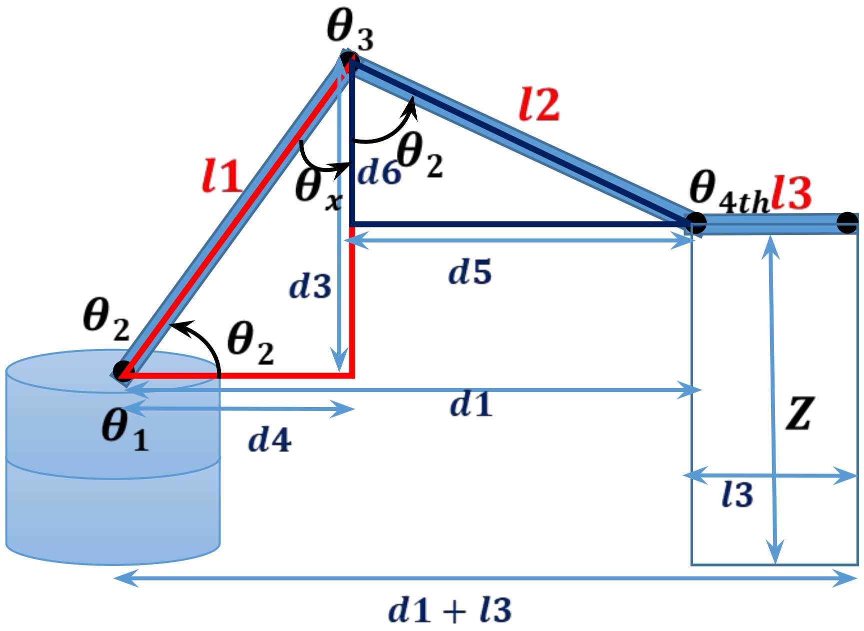
- $d6 = \cos\theta_y * l2;$
- $d6 = \cos 65 * l2;$
- $d6 = 0.4226 * 14 = 5.91$
- $d5 = \sin\theta_y * l2$
- $d5 = \sin 65 * l2$
- $d5 = .9 * 14$
- $d5 = 12.69$

l1=20”
l2=14”
l3=8”
d2=18”
$\theta_1 = 50$
$\theta_2 = 60$
$\theta_3 = 95$
$\theta_x = 30$
$\theta_y = 65$
$d3 = 17.32$
$d4 = 10$

Forward Kinematics Calculation



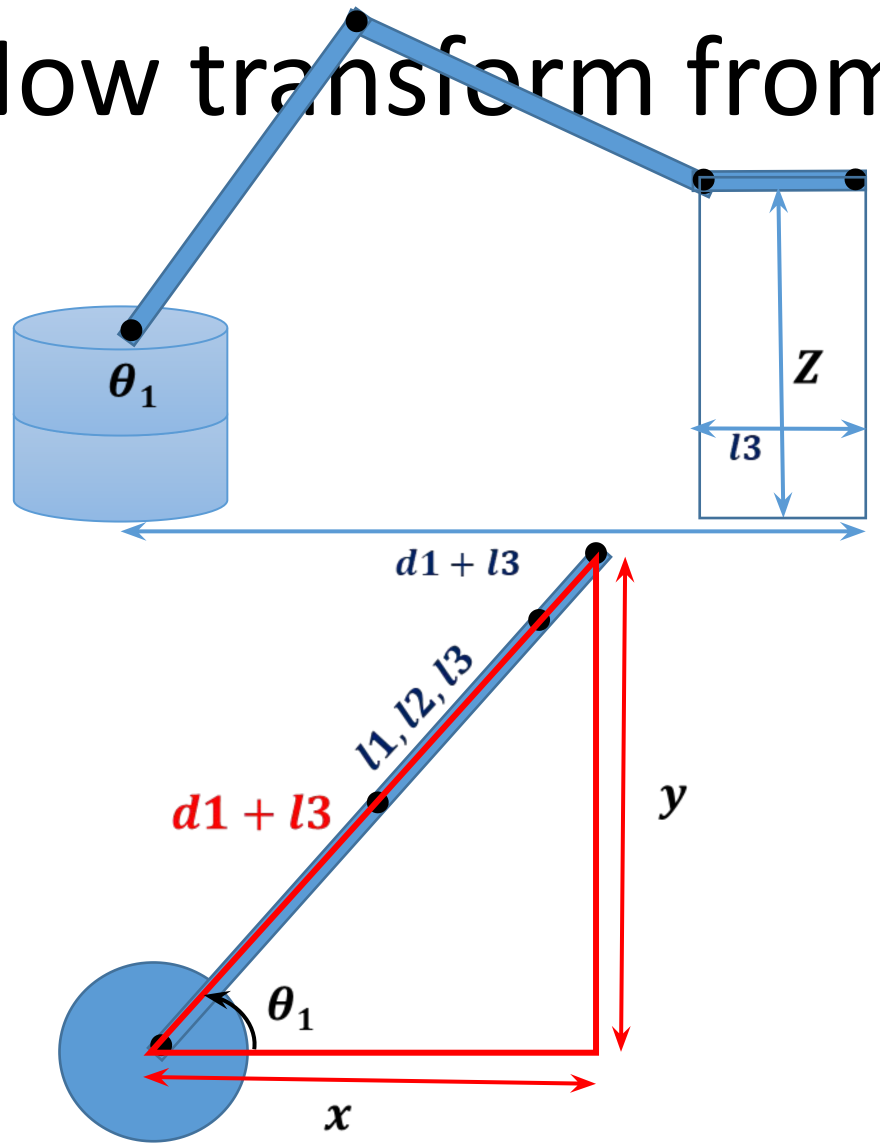
$$\cos\theta = \frac{A}{H}; \sin\theta = \frac{O}{H}; \tan\theta = \frac{O}{A}$$

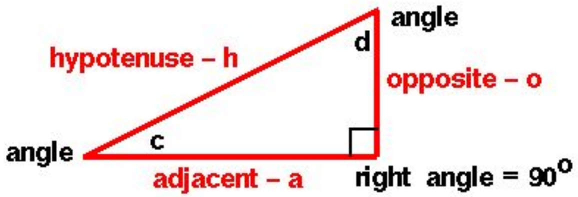


- $Z = d2 + (d3 - d6)$
- $d1 = d4 + d5$

- *So, d1+l3 is the current length of arm from base*
- *Z is the height of arm endpoint*

Now transform from top view

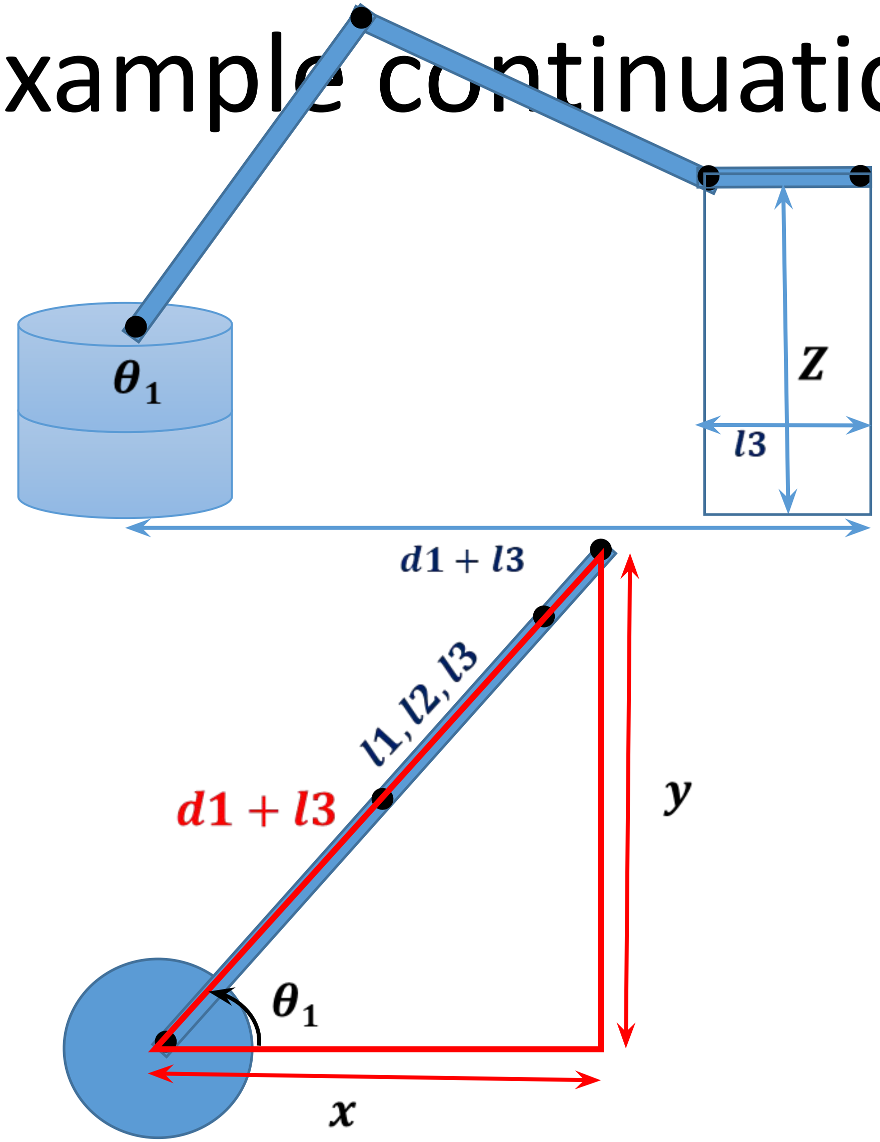


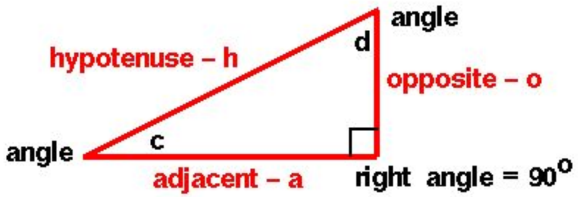


$cos\theta = \frac{A}{H}; sin\theta = \frac{O}{H}; tan\theta = \frac{O}{A}$

- $cos\theta_1 = \frac{x}{d1+l3};$
- $x = cos\theta_1 * (d1 + l3)$
- $sin\theta_1 = \frac{y}{d1};$
- $y = sin\theta_1 * (d1 + l3)$
- $Position\ is:\ (x, y, z)$

Example continuation…



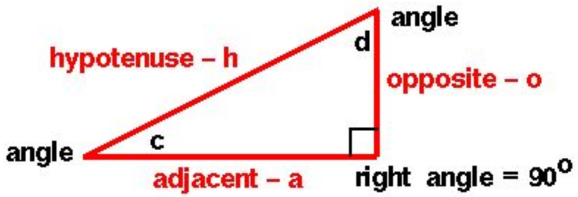


$cos\theta = \frac{A}{H}; sin\theta = \frac{O}{H}; tan\theta = \frac{O}{A}$

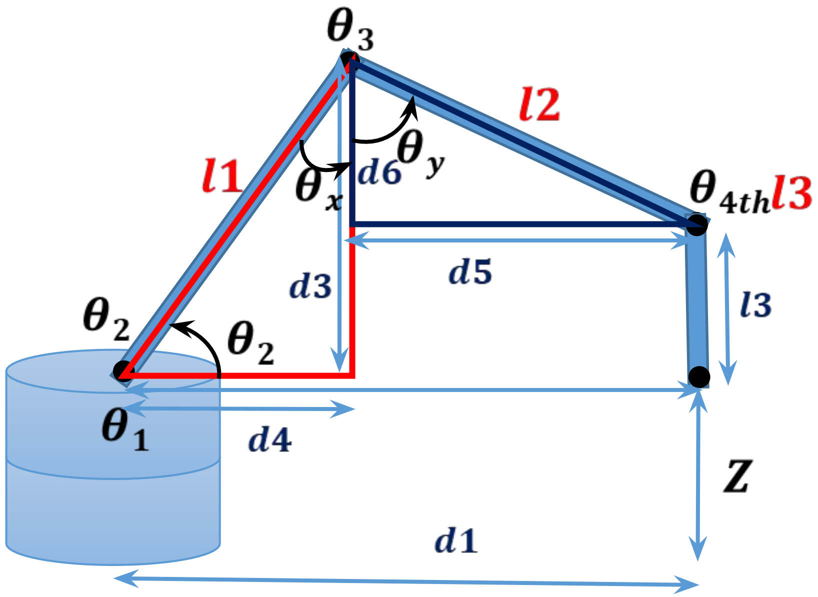
- $Z = d2 + (d3 - d6)$
- $Z = 18 + 17.32 - 5.916 = 29.4$
- $d1 = d4 + d5 = 10 + 12.69 = 22.69$
- $x = cos\theta_1 * (d1 + l3)$
- $x = cos50 * (22.69 + 8) = 19.73$
- $y = sin50 * (22.69 + 8) = 23.51$
- $Position\ is$:
- $(x, y, z) = (19.73, 23.51, 29.4)$

$l1=20''$
$l2=14''$
$l3=8''$
$d2=18''$
$\theta_1 = 50$
$\theta_2 = 60$
$\theta_3 = 95$
$\theta_x = 30$
$\theta_y = 65$
$d3 = 17.32$
$d4 = 10$
$d6 = 5.91$
$d5 = 12.69$

Forward Kinematics Calculation



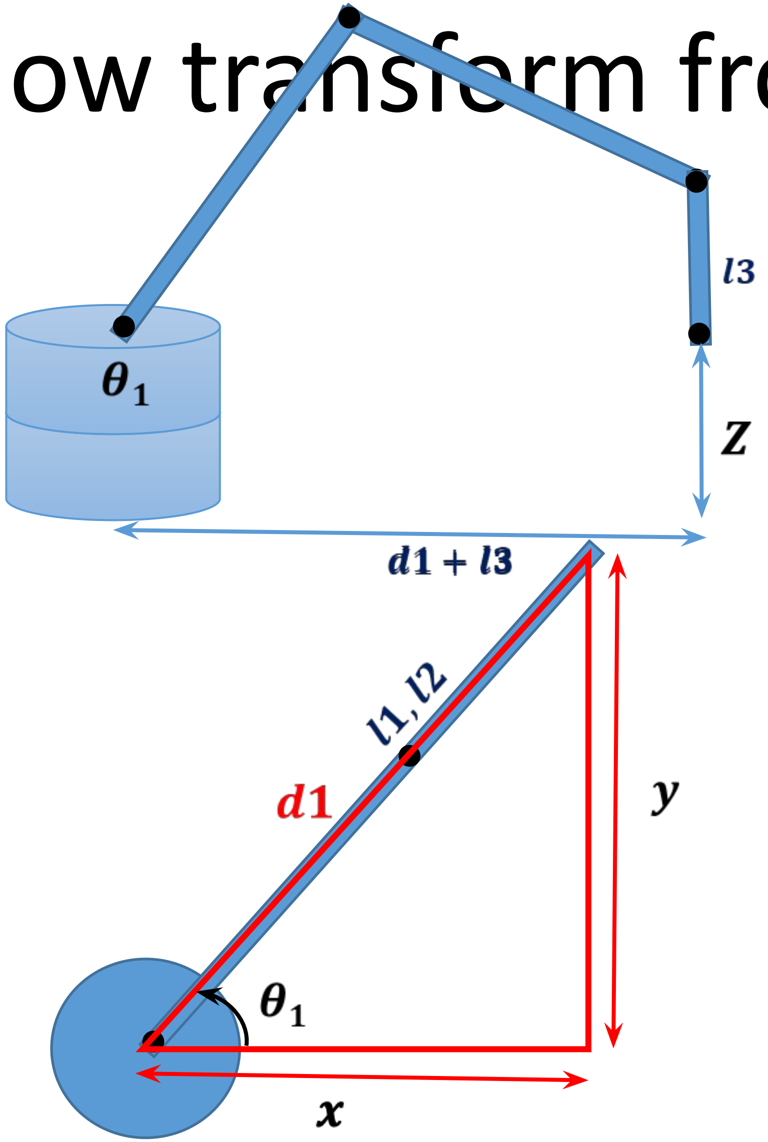
$$cos\theta = \frac{A}{H}; sin\theta = \frac{O}{H}; tan\theta = \frac{O}{A}$$

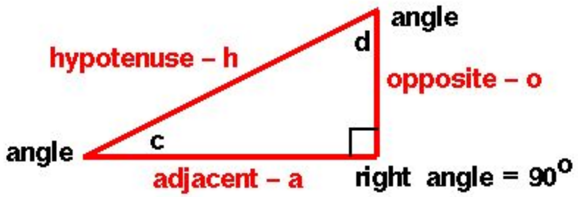


- $Z = d2 + (d3 - d6) - l3$
- $d1 = d4 + d5$

- *So, d1 is the current length of arm from base*
- *Z is the height of arm endpoint*

Now transform from top view

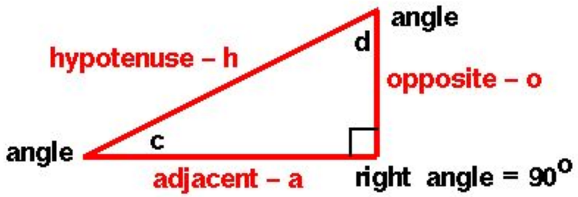




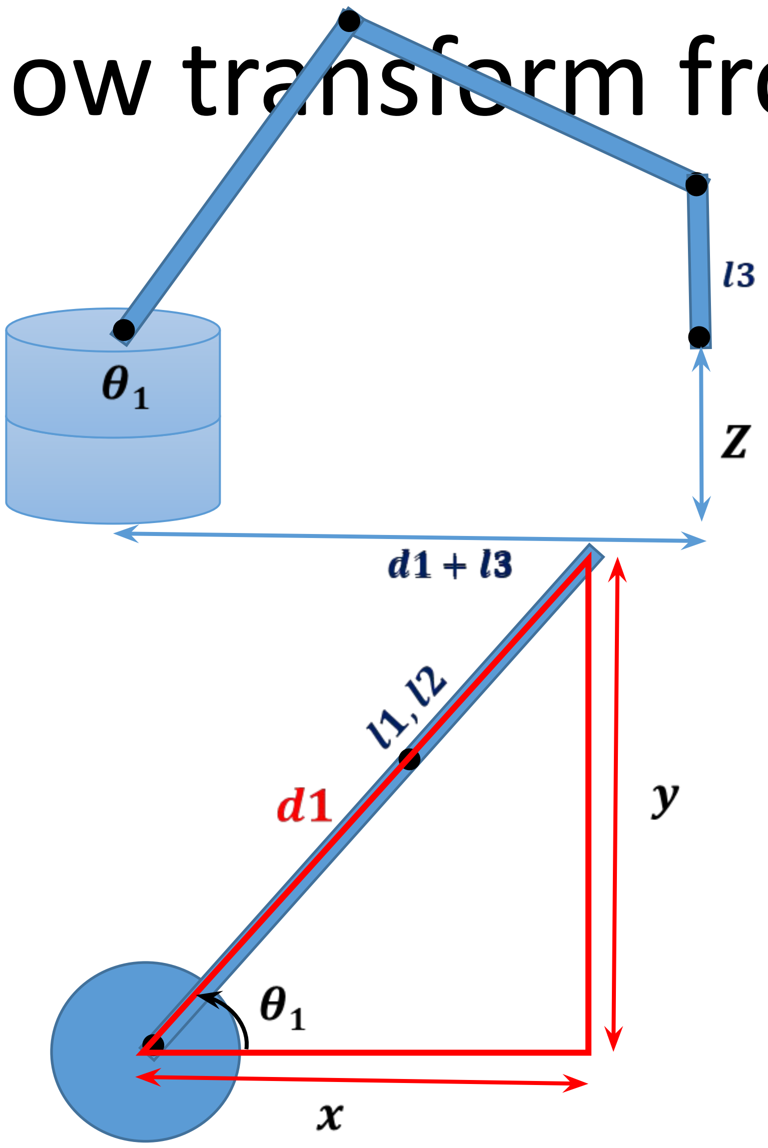
$$cos\theta = \frac{A}{H}; sin\theta = \frac{O}{H}; tan\theta = \frac{O}{A}$$

- $Z = d2 + (d3 - d6) - l3$
- $cos\theta_1 = \frac{x}{d1};$
- $x = cos\theta_1 * d1$
- $sin\theta_1 = \frac{y}{d1};$
- $y = sin\theta_1 * d1$
- $Position\ is:\ (x, y, z)$

Now transform from top view



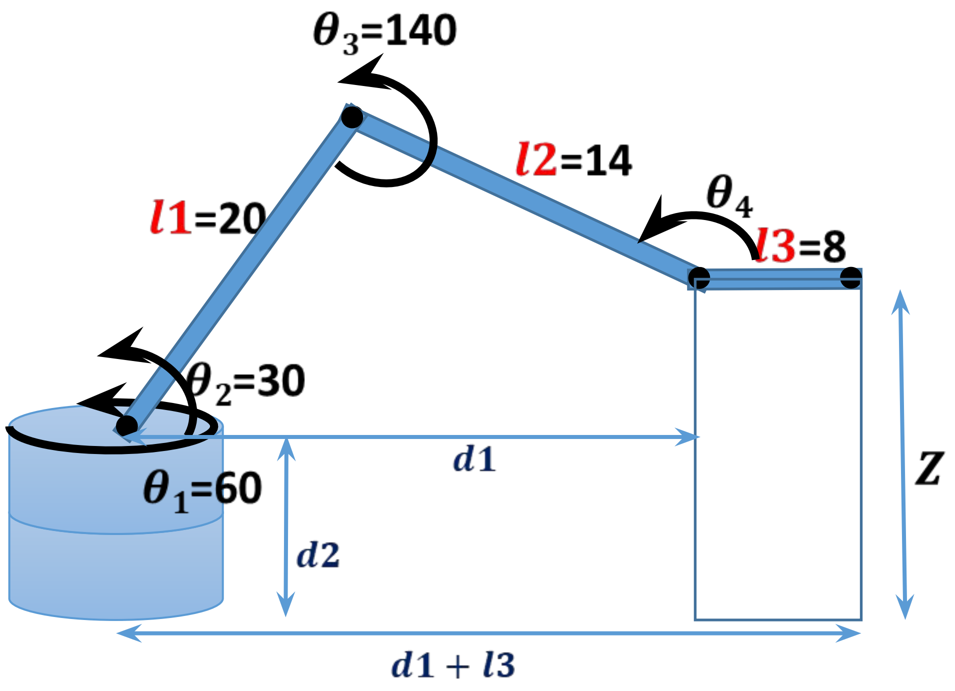
$$cos\theta = \frac{A}{H}; sin\theta = \frac{O}{H}; tan\theta = \frac{O}{A}$$



- $Z = d2 + (d3 - d6) - l3$
- $Z = 18 + 17.32 - 5.916 - 8$=21.4
- $d1 = d4 + d5 = 10 + 12.69 = 22.69$
- $x = cos\theta_1 * d1$
- $x = cos50 * 22.69 = 14.58$
- $y = sin\theta_1 * d1$
- $y = sin50 * 22.69 =$ *17.38*
- *Position is*:
- $(x, y, z) = (14.58, 17.38, 21.4)$

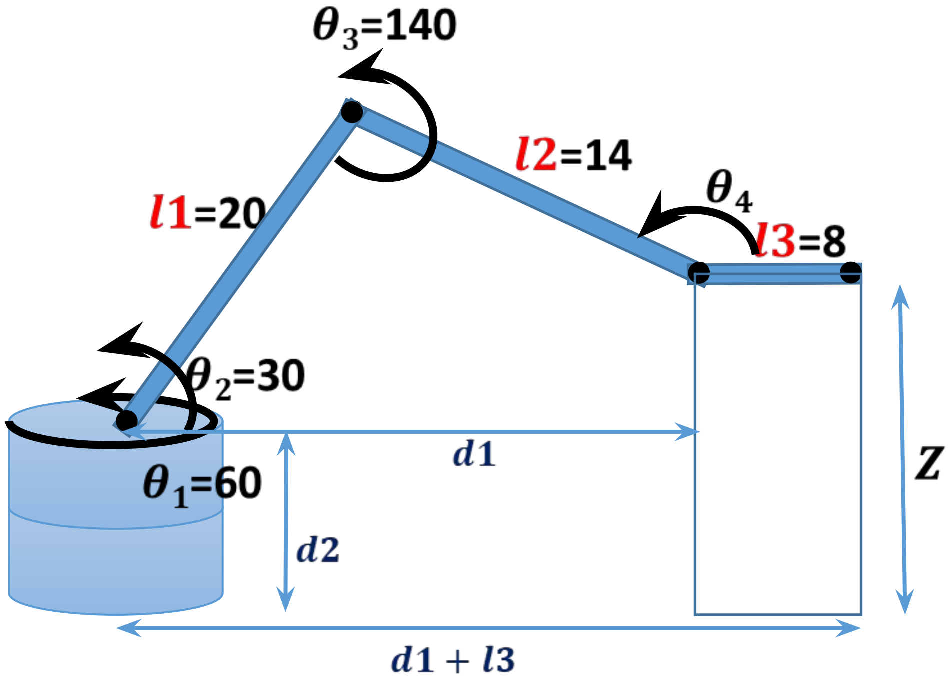
l1=20”
l2=14”
l3=8”
d2=18”
$\theta_1 = 50$
$\theta_2 = 60$
$\theta_3 = 95$
$\theta_x = 30$
$\theta_y = 65$
$d3 = 17.32$
$d4 = 10$
$d6 = 5.91$
$d5 = 12.69$

Reverse Kinematics



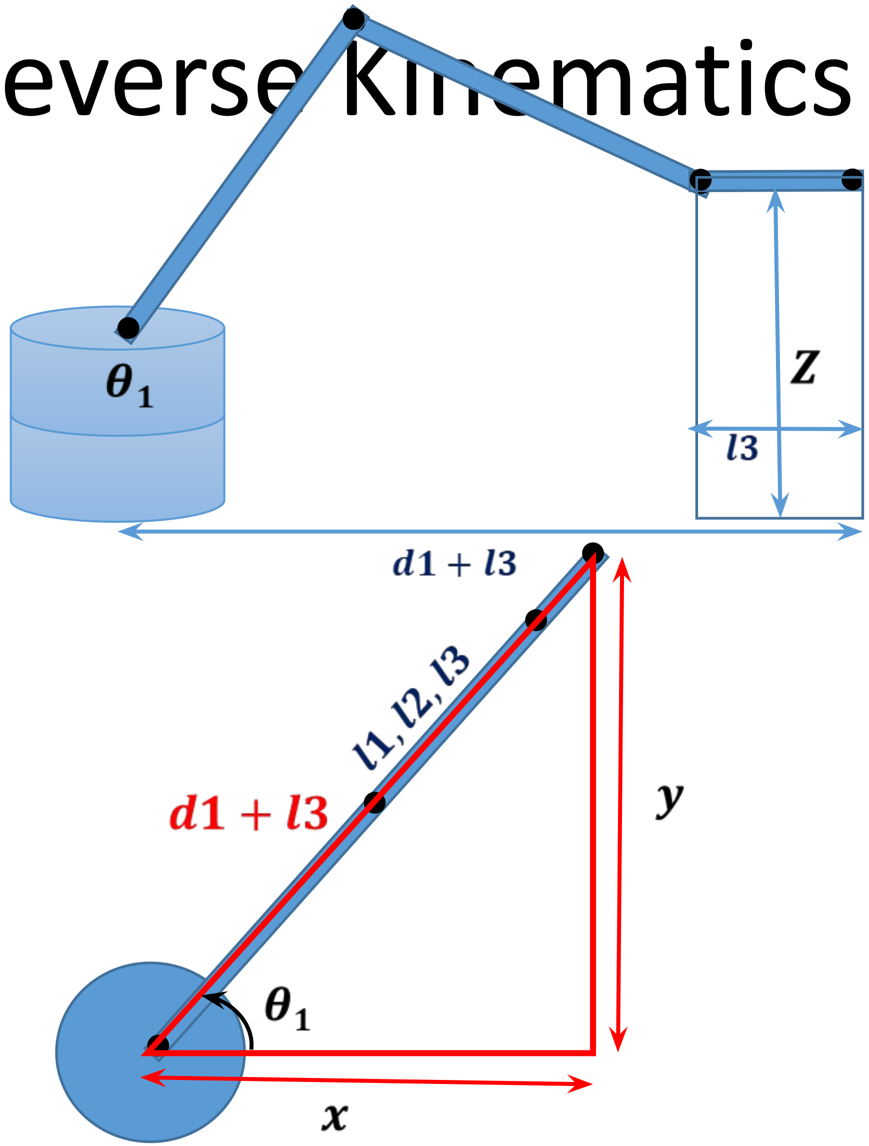
- $x = Position\ on\ front$
- $y = Position\ on\ left\ or\ right$
- $z = Position\ on\ height$
- $l1 = Length\ og\ first\ arm$
- $l2 = Length\ of\ second\ arm$
- $l3 = Length\ of\ third\ arm$
- $d2 = Height\ of\ base\ from\ ground$

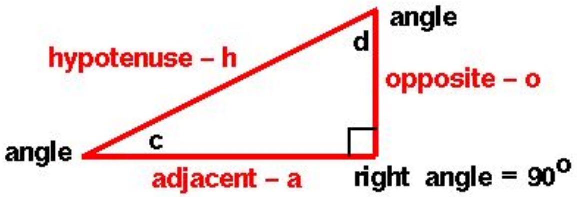
Reverse Kinematics Example value



- $x = 20$
- $y = 25$
- $z = 30$
- $l1 = 20$
- $l2 = 14$
- $l3 = 8$
- $d2 = 18$

Reverse Kinematics

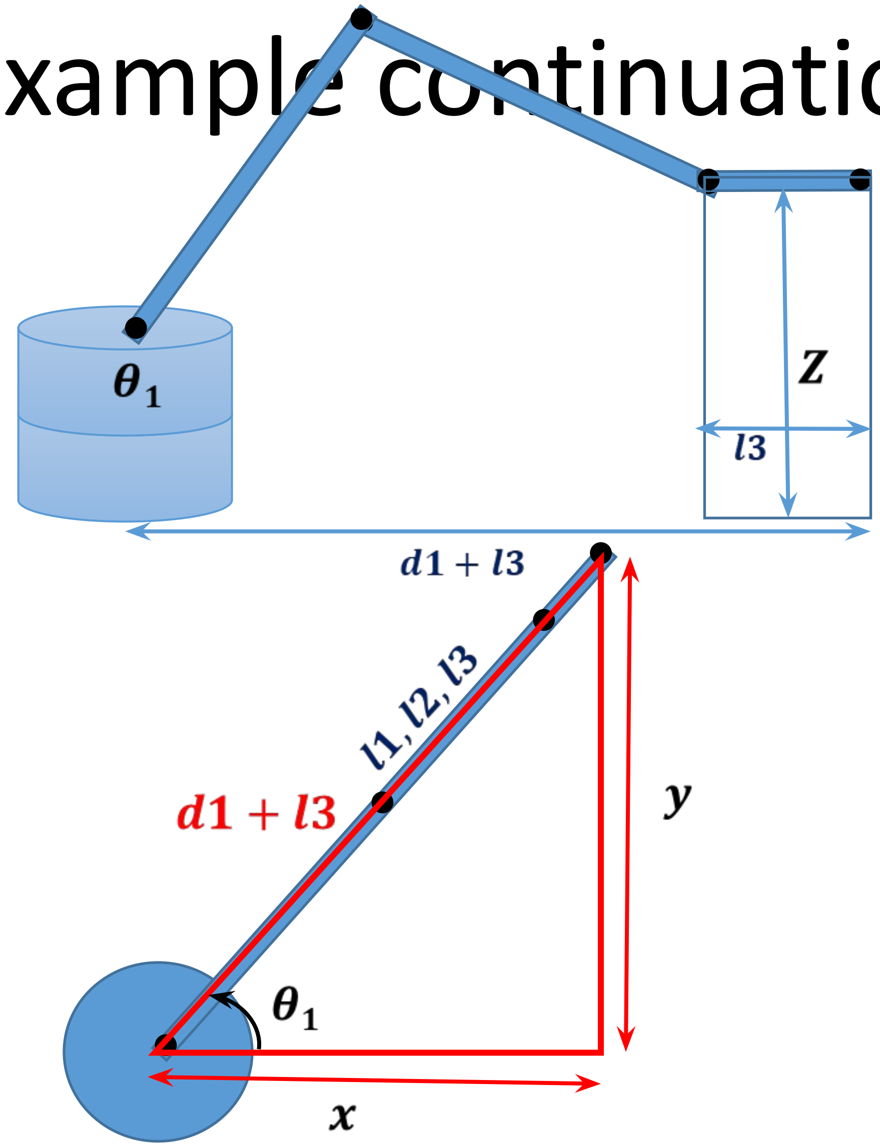


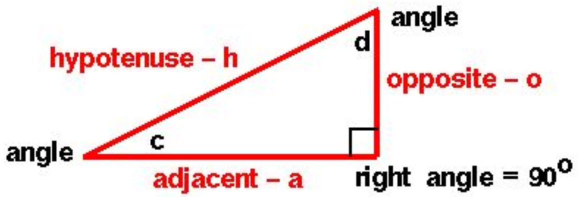


$$cos\theta = \frac{A}{H}; sin\theta = \frac{O}{H}; tan\theta = \frac{O}{A}$$

- $(d1 + l3)^2 = x^2 + y^2$
- $d1 = \sqrt{x^2 + y^2} - l3$
- $Cos\theta_1 = \frac{x}{d1+l3}$
- $\theta_1 = Cos^{-1}(\frac{x}{d1+l3})$

Example continuation…



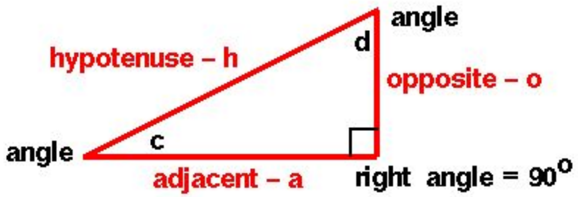


$$cos\theta = \frac{A}{H}; sin\theta = \frac{O}{H}; tan\theta = \frac{O}{A}$$

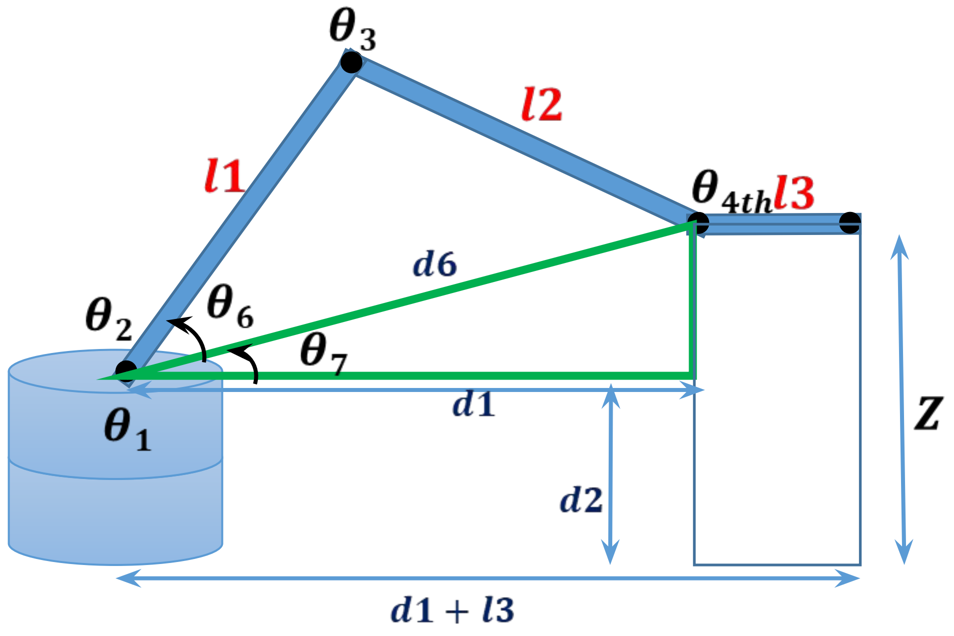
- $d1 = \sqrt{x^2 + y^2} - l3$
- $d1 = \sqrt{19.73^2 + 23.51^2} - 8$
- $d1 = 22.692$
- $\theta_1 = Cos^{-1}\left(\frac{x}{d1+l3}\right)$
- $\theta_1 = Cos^{-1}\left(\frac{19.73}{22.69+8}\right)$
- $\theta_1 = 50$

$x = 19.73,$
$y = 23.51,$
$z = 29.4$
$l1 = 20$
$l2 = 14$
$l3 = 8$
$d2 = 18$

Reverse Kinematics

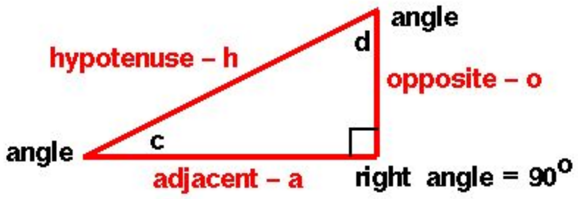


$$cos\theta = \frac{A}{H}; sin\theta = \frac{O}{H}; tan\theta = \frac{O}{A}$$

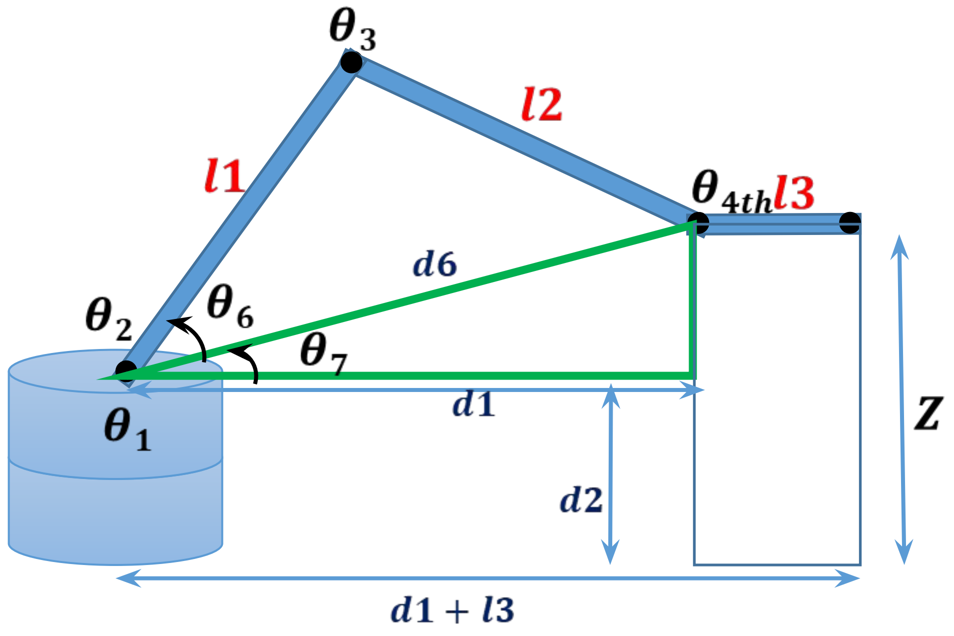


- $d6^2 = d1^2 + (z - d2)^2$
- $d6 = \sqrt{d1^2 + (z - d2)^2}$
- $Cos\theta_7 = \frac{d1}{d6};$
- $\theta_7 = Cos^{-1}(\frac{d1}{d6})$

Example continuation…



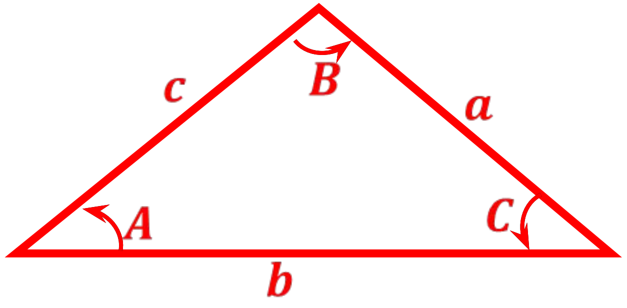
$$\cos\theta = \frac{A}{H}; \sin\theta = \frac{O}{H}; \tan\theta = \frac{O}{A}$$

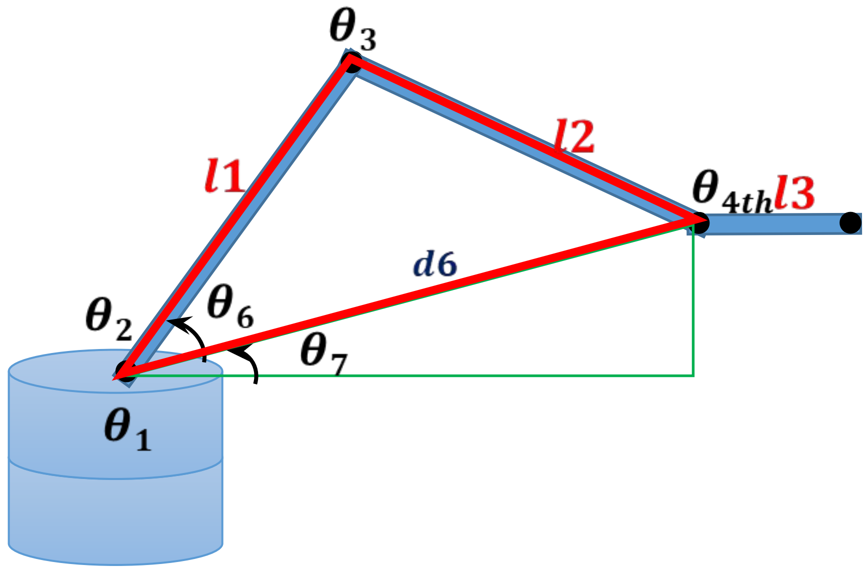


- $d6 = \sqrt{d1^2 + (z - d2)^2}$
- $d6 = \sqrt{22.69^2 + (29.4 - 18)^2}$
- $d6 = 25.393$
- $\theta_7 = Cos^{-1}(\frac{d1}{d6})$
- $\theta_7 = Cos^{-1}(\frac{22.692}{25.393})$
- $\theta_7 = 26.67$

$x = 19.73,$
$y = 23.51,$
$z = 29.4$
$l1 = 20$
$l2 = 14$
$l3 = 8$
$d2 = 18$
$d1 = 22.692$
$\theta_1 = 50$

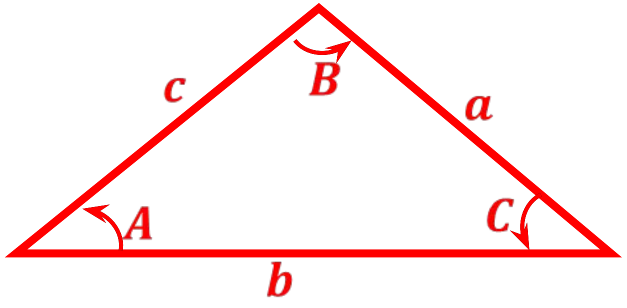
Reverse Kinematics



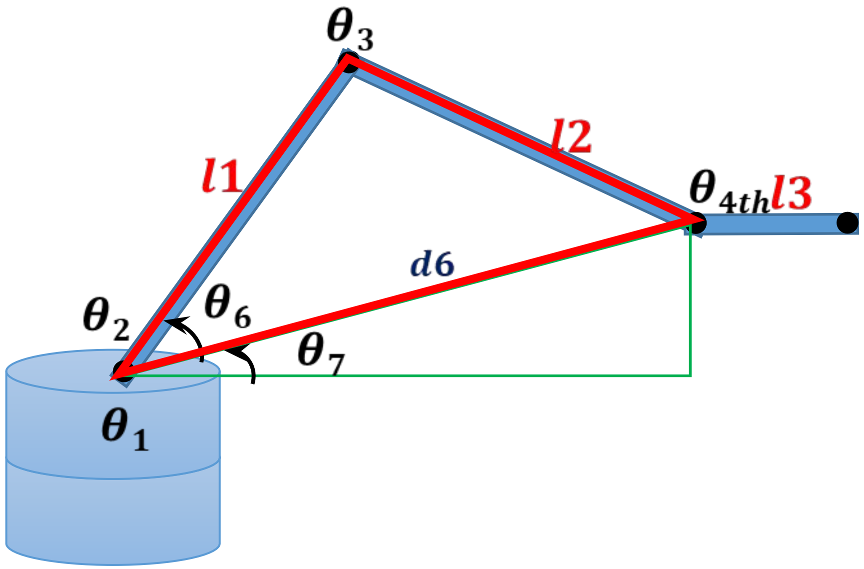


- $b^2 = a^2 + c^2 - 2acCosB$
- $Cos\theta_3 = \left(\frac{l1^2 + l2^2 - d6^2}{2*l1*l2}\right);$
- $\theta_3 = Cos^{-1}\left(\frac{l1^2 + l2^2 - d6^2}{2*l1*l2}\right)$
- $Cos\theta_6 = \left(\frac{l1^2 + d6^2 - l2^2}{2*l1*d6}\right);$
- $\theta_6 = Cos^{-1}\left(\frac{l1^2 + d6^2 - l2^2}{2*l1*d6}\right)$
- $\theta_4 = \theta_2 + \theta_3$

Example continuation…

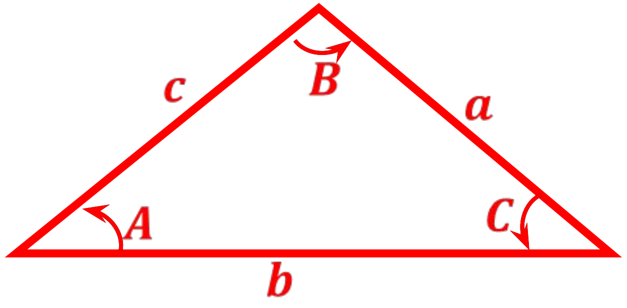


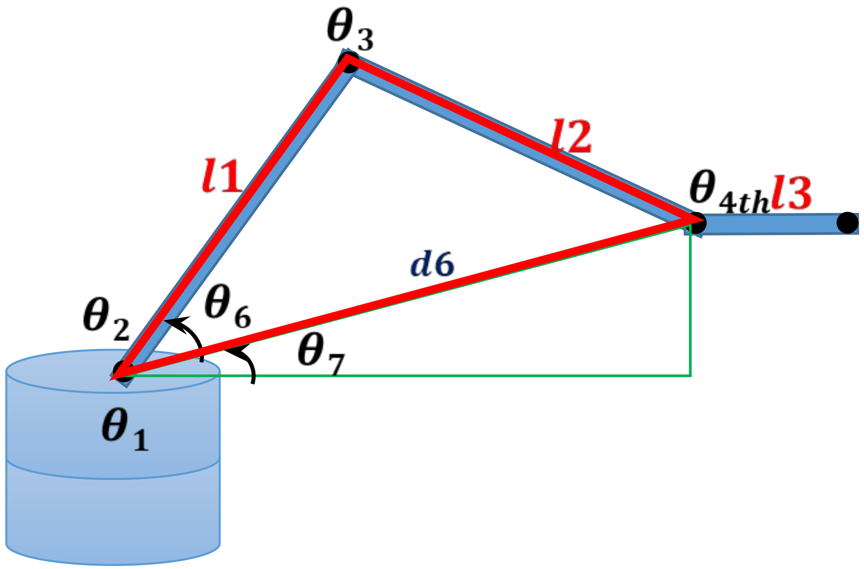
- $\theta_3 = Cos^{-1}\left(\frac{l1^2+l2^2-d6^2}{2*l1*l2}\right)$
- $\theta_3 = Cos^{-1}\left(\frac{20^2+14^2-25.39^2}{2*20*14}\right)$
- $\theta_3 = 95$



$$x = 19.73,$$
$$y = 23.51,$$
$$z = 29.4$$
$$l1 = 20$$
$$l2 = 14$$
$$l3 = 8$$
$$d2 = 18$$
$$d1 = 22.692$$
$$\theta_1 = 50$$
$$d6 = 25.393$$
$$\theta_7 = 26.67$$

Example continuation…

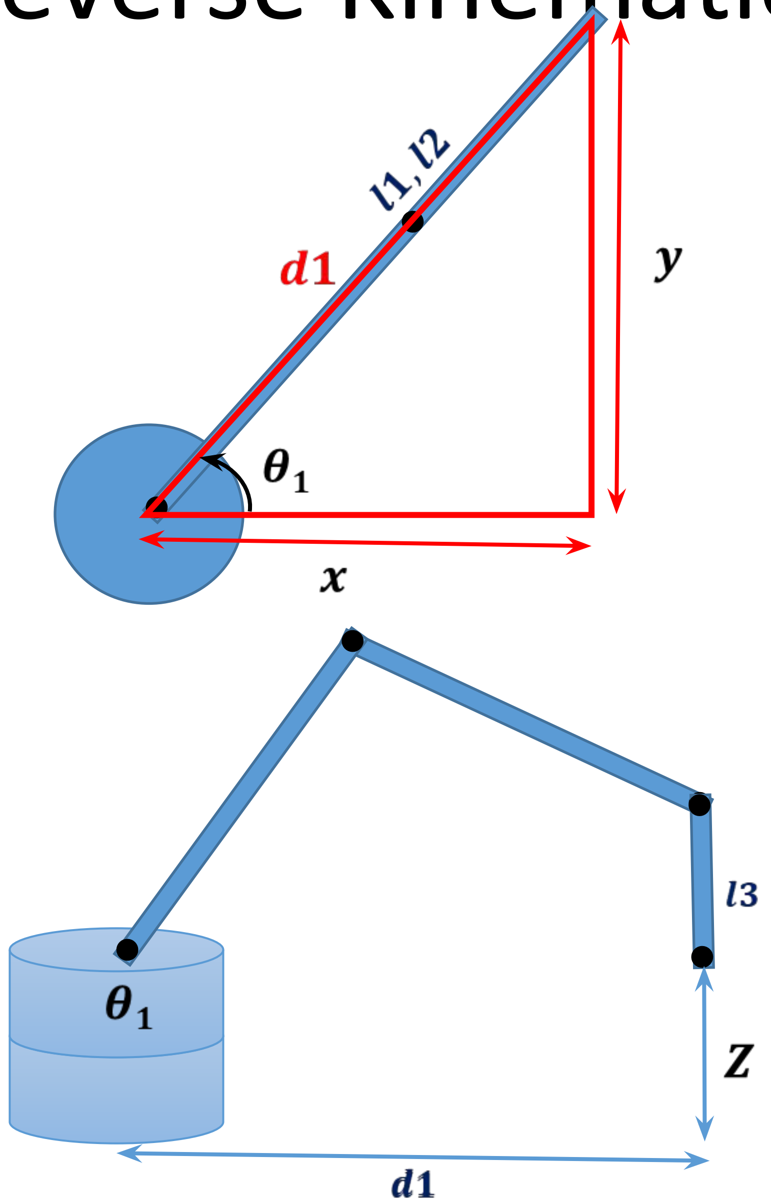




- $\theta_6 = Cos^{-1}\left(\frac{l1^2+d6^2-l2^2}{2*l1*d6}\right)$
- $\theta_6 = Cos^{-1}\left(\frac{20^2+25.39^2+14^2}{2*20*25.39}\right)$
- $\theta_6 = 33.31$
- $\theta_2 = 33.31 + 26.67$
- $\theta_2 = 59.98 \approx 60$
- $(\theta_1, \theta_2, \theta_3, \theta_4)$
- $(50, 60, 95)$

$x = 19.73,$
$y = 23.51,$
$z = 29.4$
$l1 = 20$
$l2 = 14$
$l3 = 8$
$d2 = 18$
$d1 = 22.692$
$\theta_1 = 50$
$d6 = 25.393$
$\theta_7 = 26.67$

Reverse Kinematics



- $d1 = \sqrt{x^2 + y^2}$
- $\theta_1 = Cos^{-1}\left(\frac{x}{d1}\right)$
- $d6 = \sqrt{d1^2 + (z - d2)^2}$
- $\theta_7 = Cos^{-1}\left(\frac{d1}{d6}\right)$
- $\theta_3 = Cos^{-1}\left(\frac{l1^2 + l2^2 - d6^2}{2 * l1 * l2}\right)$
- $\theta_6 = Cos^{-1}\left(\frac{l1^2 + d6^2 - l2^2}{2 * l1 * d6}\right)$
- $\theta_2 = \theta_6 + \theta_7$
- $\theta_4 = \theta_2 + \theta_3 + 90$
- $(\theta_1, \theta_2, \theta_3, \theta_4)$