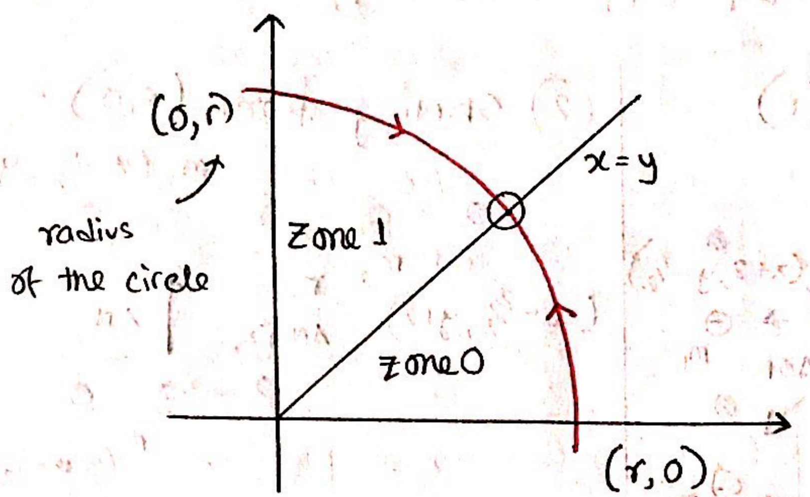
Week 4 (Mid Point Circle)



- In the mid point circle algorithm we can follow two approaches

(1) Starting from (0,r)

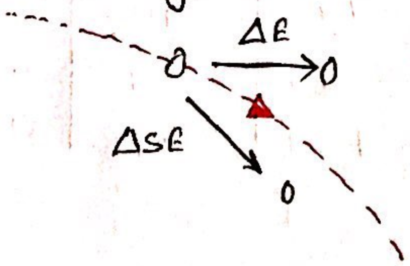
(2) Starting from (r,0)

(1) Starting from (0,r)

We basically draw the sogment of the circle in zone 1 by starting from (0,r) and the calculate each pixel along the zone 1 and the map them onto the other zones using 8-way ~~Symetm~~ Symmetry.

For zone 1

There can be basically two types of movement

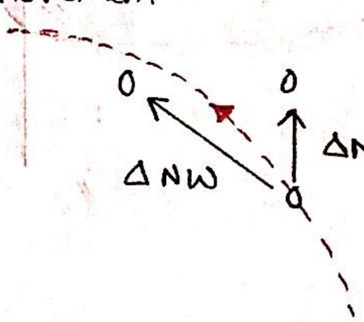


(2) Starting from (r,0)

We do the exact same thing as zone 1 but rather than starting from (0,r) we start from (r,0). And then map the points of zone 0 onto the other zones using 8-way Symmetry.

For zone 0

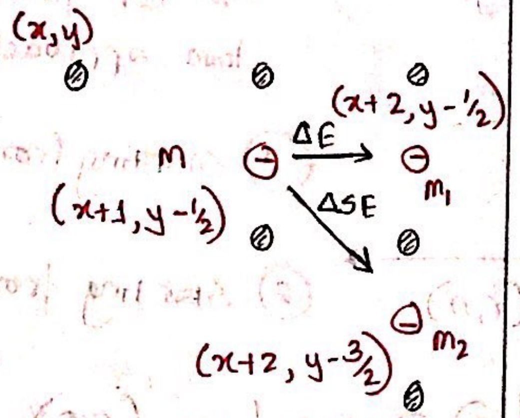
There can be two types of movement



The general equation of a circle:

$$f(x,y) = x^2 + y^2 - r^2 = 0 \quad \text{----} (1)$$

(1) Starting from $(0,r)$

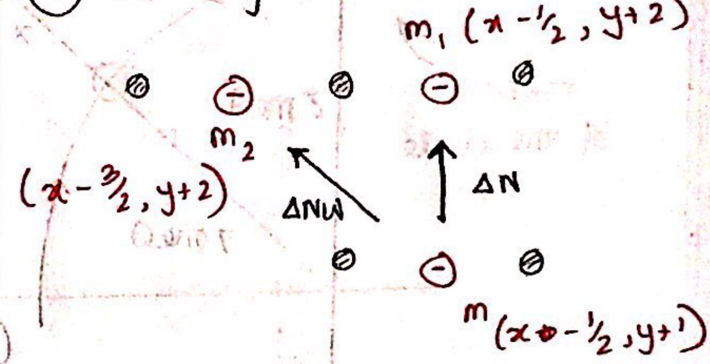


$m = (x+1, y-\frac{1}{2})$

$m_1 = (x+2, y-\frac{1}{2})$

$m_2 = (x+2, y-\frac{3}{2})$

(2) Starting from $(r,0)$



$m = (x-\frac{1}{2}, y+1)$

$m_1 = (x-\frac{1}{2}, y+2)$

$m_2 = (x-\frac{3}{2}, y+2)$

(x,y)

Derivation for $(0,r)$

For ΔE movement

$$f(m_1) = (x+2)^2 + (y-\tfrac{1}{2})^2 - r^2 = d_1$$

$$f(m) = (x+1)^2 + (y-\tfrac{1}{2})^2 - r^2 = d$$

$(-) \quad (-) \quad (+) \quad (-)$

$$\Rightarrow \quad 2x+3 + 0 + 0 = d_1 - d = \Delta E$$

$$\therefore \quad \Delta E = 2x+3$$

For ΔSE movement,

$$f(m_2) = (x+2)^2 + (y-3/2)^2 - r^2 = d_2$$

$$f(m) = \underset{(-)}{(x+1)^2} + \underset{(-)}{(y-1/2)^2} - \underset{(+)}{r^2} = \underset{(-)}{d}$$

$$(2x+3) + (-2y+2) + 0 = d_2 - d = \Delta SE$$

$$\therefore \Delta SE = 2x - 2y + 5$$

For d_{init},

$$f(m) = d_{init} = (x_0+1)^2 + (y_0 - 1/2)^2 - r^2$$

$$= (x_0)^2 + 2x_0 + 1 + (y_0)^2 - y_0 + 1/4 - (r^2)$$

There three acumulate to give 0.

general eqn of circle:

$f(x,y) = x^2+y^2-r^2=0$

$$= 2x_0 - y_0 + 5/4$$

$$d/d_{init} = 5/4 - r$$

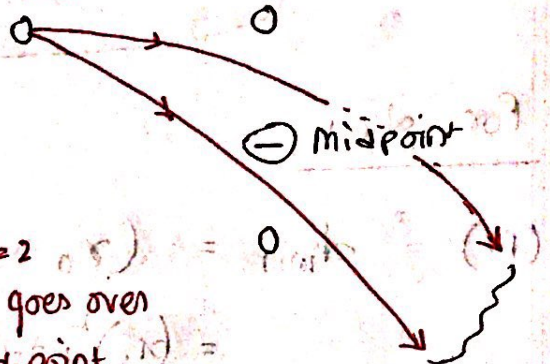
→ Since $x_0 = 0$ & $y_0 = r$ } This is because the starting point is $(0,r)$

How to decide if ΔE / ΔSE, ?

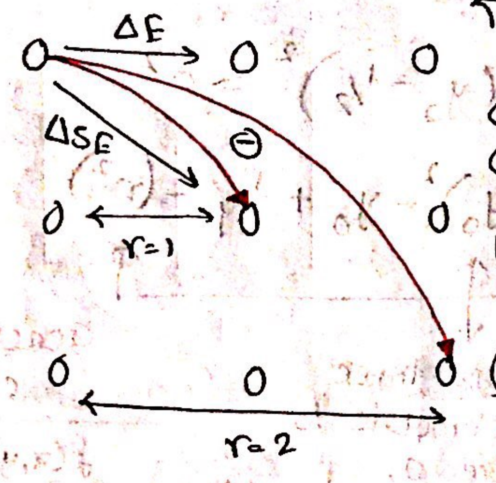
For that we need the value of d;

$$d = 5/4 - r$$

when draw a circle →



So,



For r=2 curve goes over the mid point

For r=1 curve goes below mid point.

These are the two possible outcomes, either the curve goes over the mid or below it.

(*) Remember r=1, r=2 are arbitrary values we use to understand the algorithm.

ΔE (when r=2)	ΔSE (when r=1)
$d = 5/4 - r$	$d = 5/4 - r$
$d = 5/4 - 2 = -0.75 =$ -ve	$d = 5/4 - 1 = 0.25 =$ +ve
So if d is -ve, The movement is ΔE	if d is +ve, The movement is ΔSE

$d = 5/4 - r$

multiply by 4

$d = 5 - 4r$

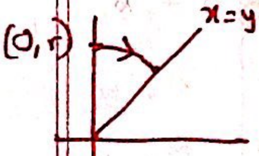
→ done to remove fraction.

Algorithm:

```
void drawCircle_zone1 (int r) {
    int d = 5 -(4*r);
  OR
    int d = 1-r;
    int x = 0;
    int y = r;
    draw8way (x,y);
    while (x <y) {
        if (d <0) {   // ΔE since -ve
            d = d+ 2x +3;
            x = x+1;
        }
        else {   // ΔSE since +ve
            d = d+ 2x - 2y +5;
            x = x+1;
            y = y-1;
        }
        draw8way (x,y);
    }
}
```

← OR since the video method says to use $d = 1 - r$ since 1 and 1.25 are insignificantly close, we'll go with that.

draws the pixel on all 8 zones.



works until x becomes equal to y.

For ΔSE move x increases by 1, y decreases by 1

draws the point on all 8 zones.

Example 1 Start $(0, 10)$, radius $= 10$

$d = 1 - r$, $\Delta E = 2x + 3$, $\Delta SE = 2x - 2y + 5$

x	y	d	$\Delta E / \Delta SE$	d update.
0	10	-9	ΔE	$= -9 + (2\times 0 + 3) = -6$
1	10	-6	ΔE	$= -6 + (2\times 1 + 3) = -1$
2	10	-1	ΔE	$= -1 + (2\times 2 + 3) = 6$
3	10	6	ΔSE	$= 6 + (2\times 3 - 2\times 10 + 5) = -3$
4	9	-3	ΔE	$= -3 + (2\times 4 + 3) = 8$
5	9	8	ΔSE	$= 8 + (2\times 5 - 2\times 9 + 5) = 5$
6	8	5	ΔSE	$= 5 + (2\times 6 - 2\times 8 + 5) = 6$
7	7	6	~	~

Answer

Example 2 : calculate the points for zone 1 of a circle who's radius is 6 and centered at (10, 10)

we'll basically carry out the normal calculations but since it is centered at (10, 10) we have to add +10 to each x pixel and +10 to each y pixel.

First carry out the normal calculation.

x	y	d	ΔE/ΔSE	d update	Actual pixel.
0	6	-5	ΔE	$= -5 + (0\times2+3) = -2$	$(0+10, 6+10) = (10,16)$
1	6	-2	ΔE	$= -2 + (1\times2+3) = 1$	(11, 16)
2	6	1	ΔSE	$= 1 + (2\times2 - 2\times6 + 5) = -2$	(12, 16)
3	5	-2	ΔE	$= -2 + (2\times3+3) = 7$	(13, 15)
4	5	7	ΔSE	$= 7 + (2\times4 - 2\times5 + 5) = 10$	(14, 15)
5	4	10		→ won't be accepted as $(x \not< y)$	

finally adjust for the centering.

(*) Try at home for (0, 20) centered around (5, 6)